



## FRAGILITY CURVES FOR ENERGY ABSORPTION CAPACITY OF STRUCTURES UNDER EARTHQUAKE LOADING

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### Abstract

The foreseen challenge of resilient and sustainable infrastructure is the adaption of structures as per various codes for the disaster and thereby its emerging risk and management. Generally, the drift based damage index has been followed to calculate the loss of lives and economies for the structures for the disaster such as earthquakes, blasts etc. The energy based damage index which is more appropriate than the drift based damage index for more accurate estimation of losses for those disaster. In our paper we show the energy based damage index is better for the considered buildings under various earthquake ground motions. Energy concept is the future methodology as it addresses performance of a structure at global as well as local levels under varying dynamic loading. Drift based fragility curves are popular for damage assessment under various failure criterion. For the same drift, energy based fragility assessment is considered to be more effective because higher values probability of exceedance. A well conceived two story steel building structures designed for various performance levels has been modelled and analyzed under various earthquake ground motions. The incremental dynamic analysis is used to estimate the drift as well as energy components. Output data recorded as drift and energy are evaluated for their mean and standard deviation. Using mean and standard deviation values, fragility curves were drawn. Finally, observations and conclusions were made that for the given values of drift, energy based fragility assessment are more effective for identification and quantification of performance of a structure under varying earthquake ground motions.

**Keywords:** Incremental dynamic analysis, Energy based damage index, fragility curves, Perform 3D.

### 1. Introduction

In the case of structures, damage indices have been developed to quantify numerically, the seismic damage sustained by individual elements, story or complete structures. There are many ways of classifying the damage indices that have been proposed, but one of the best fundamental class among damage indices are local indices, which measure the damage level of each individual members or at particular joints and global indices, which calls for the damage state of the whole structure.

Overall, damage in structures is interrelated to inelastic deformations. Hence, any damage index variable would denote certain deformation quantity. Such deformation quantities are rotations at the member ends, strains in the materials, lateral storey displacements, curvatures at the cross sections and the structures inter storey drifts. The quantities such as lateral storey displacements and the structures inter storey drifts are used as the global damage index variables, while the curvatures, strains and rotations are used as the depiction of local damage index variables. Another evocative damage index variable is the amount of energy dissipated or the amount of energy absorbed by the particular member or the structure during its inelastic deformation. There have been various damage indexes established in the recent past years and each and every one of them uses certain variables to evaluate the damage in structures. Each and every one of these damage indexes might give certain valuable information on the structural damage, in view of the fundamental assumptions and its application boundaries as presented by their developers.

A huge volume of research work has been carried over till now, in the realm of the seismic damage indices for the structures, and every potential method seem to have been considered, although not subjected to the same level of astuteness. All damage indices recommended till now have been developed so as to give  $D=0$  when there is no collapse and  $D=1$  at total collapse, but considerably less amount of work has been carried with respect to energy based damage indices especially damage stages at the intermediate level. For any damage model it is important to define intermediate damage stages to estimate the status of the structure for



any deformation during or after any seismic event. The present study includes the assessment of damage of a structure at any stage, during its non-linear displacement path.

Damage indices are robust tools that could be integrated in the future design code procedures, in view of the likely lead to better effective and reasonable solutions. The primary goal should be the damage level check, which is the new direction in design of earthquake resistant structures. Actually, design procedures based on the usage of damage level indices have been proposed already by Miguel and Oscar (2004) [1], who presented the methodology of designing concrete frame in view of controlling damage level indices in structural members and maintain between the limits of tolerance. The strengths of the structural members are determined to meet the primary design objective, which necessitates the elastic performance under moderate earthquakes and inelastic performance within the tolerable limits under strong earthquake motions. Maximum horizontal displacement and the plastic energy dissipated are considered as the design standards.

Quantitative evaluation of structural level damage during strong earthquakes has continuously been a demanding problem for the structural engineers. Different damage level indexes were introduced with the aim of computing the structural level damage in prototype as well as the model structures when subjected to strong seismic excitation. A detailed comprehensive review of available damage indices was presented by Williams and Sexsmith (1995) [2], Padilla et al. (2009) [3] and Ghobarah et al. (1999) [4]. These indexes make use of different parameters such as deformation, energy, modal parameters and low cycle fatigue behaviour. In present study some of the most commonly used damage indexes have been discussed based on the parameters used and the energy based damage index are studied. The fragility curve is developed based on incremental dynamic analytic curve to compare the damage level difference of drift criterion and emery criterion.

## 2. Objective

A gap of study in the present state of art of damage based assessment under varying dynamic loadings is the thrust area of the content of this paper. Existing damage indices have been listed and further energy based fragility assessment curves were developed. The concept of normalization using mean and standard deviations of the output (post processed data) values under incremental dynamic analysis under varying earthquake ground motions have been numerically modelled in terms of energy and drift based probability of exceedance. Final observations and conclusions were achieved that energy based fragility index are better tools than drift based fragility curves because error is least for identification and quantification of performance of a structure at local as well as global levels. Energy based fragility assessment curves may be economically used for equal distribution of damages along throughout of the structures.

### 2.1 Deformation based damage indexes

The indexes based on plastic ductility are one of the most prominent indexes in this category. Powell et al. (1988) [5] defined the damage index based plastic ductility in the structure as equation (Eq. 1.1). The capacities of the response variables are well-defined with respect to their corresponding maximum values during monotonically growing deformations. A portion of the ultimate capacity during deformation of the structure with monotonically growing horizontal deformations ( $u_{mon}$ ) had been used as the structure's deformation capacity under the strong earthquake motion. This naive concept and the comfort to use the damage index a popular one between engineers and researchers.

$$DI_{\mu} = \frac{u_{max} - u_y}{u_{mon} - u_y} = \frac{\mu - 1}{\mu_{mon} - 1} \quad (1.1)$$

where  $u_y$  and  $u_{max}$  are the yield and maximum deformations, respectively, and  $u_{mon}$  is the maximum capacity of deformation in the system during a monotonically growing horizontal deformation.  $\mu = u_{max}/u_y$  is the displacement ductility of the ground motion and  $\mu_{mon} = u_{mon}/u_y$  is the monotonic ductility capacity of the system. Anyhow, displacement ductility by itself does not disclose info of the inelastic deformations during repeated cycles of ground motion and the demand of energy dissipation. Roufaiel and Meyer (1987) [6], expressed global level damage of the system with respect to deflections at roof level as shown in Eq. (1.2).



$$D_{\text{global}} = \frac{\delta_m - \delta_y}{\delta_f - \delta_y} \quad (1.2)$$

where:  $\delta_y$  is the displacement at yield limit,  $\delta_f$  is the displacement at ultimate limit,  $\delta_m$  is the maximum displacement. Some authors recommended the height of the structure ( $H$ ) should be accounted and gave the expression  $\delta_f = 0.06H$ . The deformation based indices which are non cumulative, gives only a very approximate measure of the damage level sustained during a strong earthquake ground motions. But they remain the most widely used and recognized damage models, since they are easy to calculate and their physical meaning is clear. Banon et al. (1981) [7] used a normalized cumulative rotation to represent the damage as shown in Eq. 1.3. This was evaluated for flexure dominated members, while there was some broad correlation, the values of index shown considerable scatter at failure.

$$\text{NCR} = \frac{\sum |\theta_m - \theta_y|}{\theta_y} \quad (1.3)$$

where:  $\theta_m$  is maximum rotation in each cycle and  $\theta_y$  is yield rotation.

## 2.2 Energy based damage indexes

Energy absorption was first adopted as a measure of damage by Gosain et al. (1977) [8], the damage index was proposed as a simple cumulative energy ratio (Eq. 1.4).

$$D_e = \sum_i \frac{F_i \delta_i}{F_y \delta_y} \quad (1.4)$$

where:  $F_i$  is maximum failure force in a cycle,  $\delta_i$  is failure displacement in a cycle,  $F_y$  is yield force,  $\delta_y$  is yield displacement. Only hysteresis loops for which  $F_i/F_y \geq 0.75$  are included in the summation, the assumption being that, when the peak force has dropped below 75% of the yield value, the remaining capacity of the member is negligible, this assumption was not true in many cases.

Cosenza, et al. (2000) [9], expressed damage index for a global force-deformation relation, as ratio of irrecoverable hysteretic energy to the system capacity of hysteretic energy during monotonically growing deformation.

$$DI_H = \frac{E_H}{E_{Hmon}} \quad (1.5)$$

where:  $E_H$  is hysteretic energy,  $E_{Hmon}$  is the system capacity of hysteretic energy during monotonically growing deformation. This index fails to consider the distribution of plastic cycles, but takes the account of the amount of energy dissipated globally.

The most widely used global indices derived from local indices is, Park and Ang (1985) [10] damage function, which was introduced to find the member damage as the sum of maximum displacement ductility and cumulative energy dissipated (Eq. 1.6). Global damage is calculated as an average of the local damage indices, weighted by absorption of the energy locally. The Park and Ang damage index considers both, maximum plastic energy dissipated and the plastic deformation which is supported with a comprehensive correlation with the damage observed in structure. However, the determination of the variable  $\beta$  is hard as it is experimental.

$$D = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{(Q_y \delta_u)} \int dE \quad (1.6)$$

where:  $\delta_{max}$  is the maximum displacement corresponding to the maximum capacity of system,  $\delta_u$  is the ultimate displacement during monotonic increasing loading,  $\beta$  is variable which represents the cyclic effect of the loads,  $dE$  is incremental energy dissipated hysterically,  $Q_y$  is yielding force.

Bozorgnia and Bertero (2008) [11] established two damage level indices for the SDOF system in terms of displacement ductility and energy capacity on the structure. These damage level indices are given as



$$DI_1 = (1 - \alpha_1) \left( \frac{\mu - \mu_e}{\mu_{mon} - 1} \right) + \alpha_1 \left( \frac{E_H}{E_{Hmon}} \right) \quad (1.7)$$

$$DI_2 = (1 - \alpha_2) \left( \frac{\mu - \mu_e}{\mu_{mon} - 1} \right) + \alpha_2 \sqrt{\frac{E_H}{E_{Hmon}}} \quad (1.8)$$

where:  $\mu$  represents displacement ductility,  $\mu_{mon}$  is monotonic displacement ductility capacity,

$$\mu_e = \frac{u_{elastic}}{u_y} = \frac{\text{Maximum elastic portion of deformation}}{u_y} = \begin{cases} 1 & \text{for inelastic behaviour} \\ \mu & \text{for elastic behaviour} \end{cases}$$

$E_H$  is hysteretic energy,  $E_{Hmon}$  is the system capacity of hysteretic energy during monotonically growing deformation.

$0 \leq \alpha_1 \leq 1$ ;  $0 \leq \alpha_2 \leq 1$  are constants.

The fragility curve indicates the probability of exceedance of a given damage to the structure with respect to the engineering demand parameter which represents the earthquake ground motion. In our study peak ground acceleration is taken as engineering demand parameter (i.e. SPA, SPD). The lognormal cumulative distribution function is generally used as it simple and takes parametric form of the uncertainty quantity. The damage index for drift and the energy are presented in the form of fragility curve in our study.

### 3. Building Model and Ground Motions

To develop fragility curve for structure with energy criterion and drift criterion, a regular steel building of 2 stories (MODEL-I) has been considered under various earthquake loadings. The detailing and structural configuration of the structures resembled the typical buildings which are located in the seismic zone of India. The lateral loads and gravity loads are applied according to Indian code. The earthquakes chosen are El-Centro, Bhuj, Chamoli, India-Burma, Uttarkashi & Northridge ground motions (Fig 3 to Fig 8) with different scale factors ranging from 0.2 to 2. These earthquakes are scaled accordingly such that the frames won't collapse and/or cause excessive distress but yields. The building model comprises of two bays of 4.0 m bay width in X direction (i.e. longitudinal) and 5 m bay width in Y direction (i.e. transverse) designed and analyzed for various performance objectives. The dead load comprises non-structural and structural members self-weights. The live load on structure is taken as 1.5 kN/m<sup>2</sup> for roof and 3.0 kN/m<sup>2</sup> for floors. The input parameters as specified by BIS for the area are: Zone (V), response reduction factor (R) taken as 5.0 (SMRF), importance factor (I) as 1.0 and soil type (hard soil site, and spectral acceleration (Sa) for 5% damped spectra). Initially the above said models are examined by push over analysis (static analysis) for validation in SAP 2000, then the non-linear dynamic analysis was performed for these models using PERFORM 3D with the scaled ground earthquake motion records (i.e.) IDA.

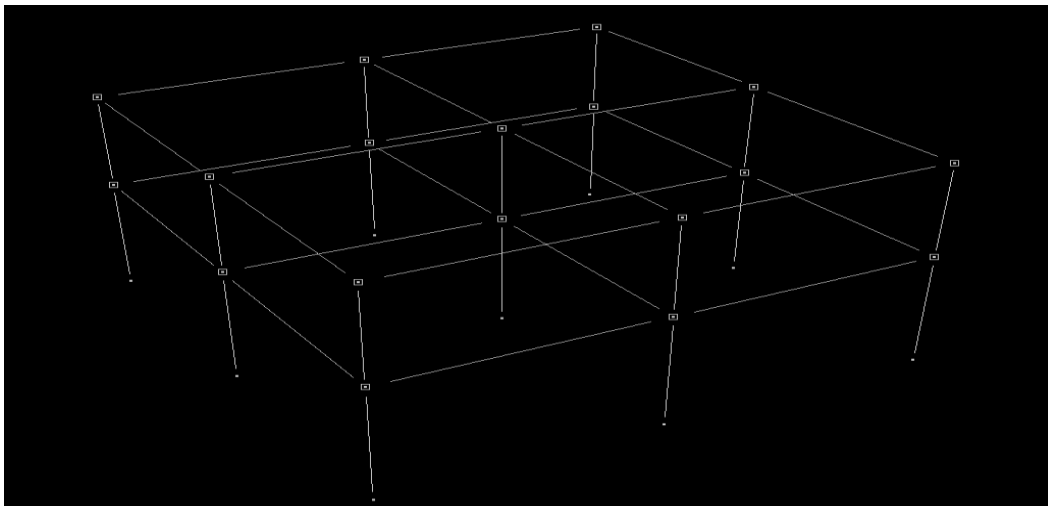


Figure 1 Two storey 2x2 bays steel building

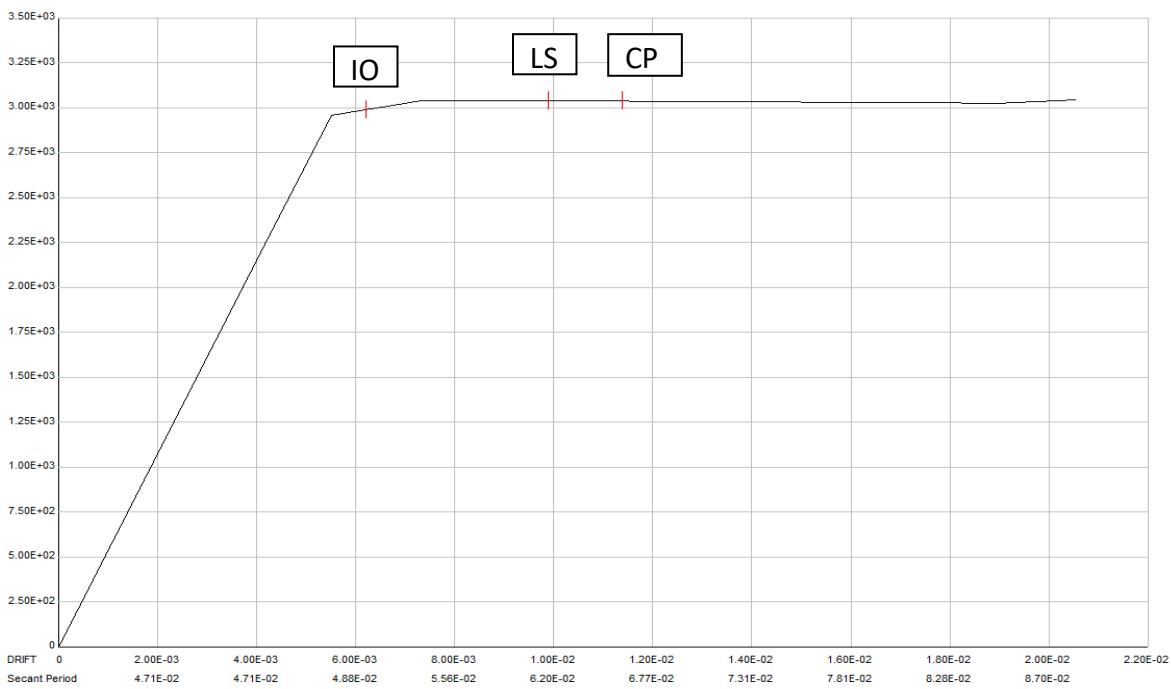


Figure 2 Pushover curve for referred building structure

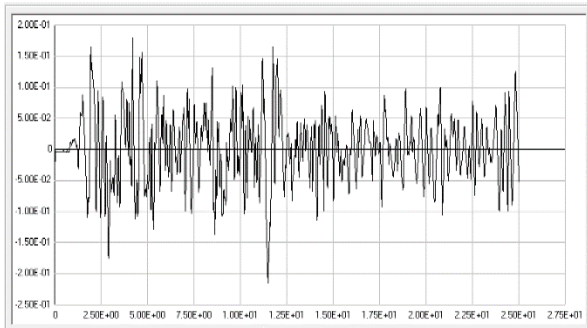


Figure 3 El Centro ground motion

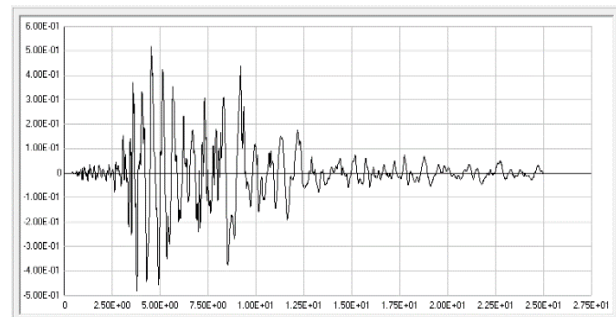


Figure 4 North ridge ground motion

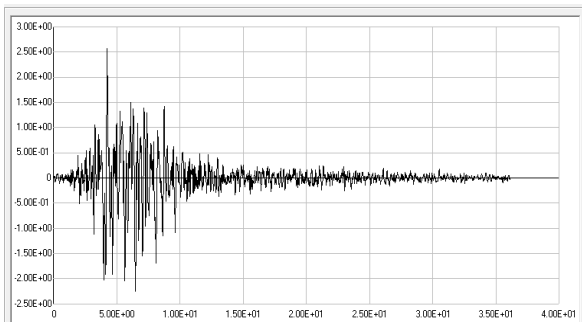


Figure 5 Uttarkashi ground motion

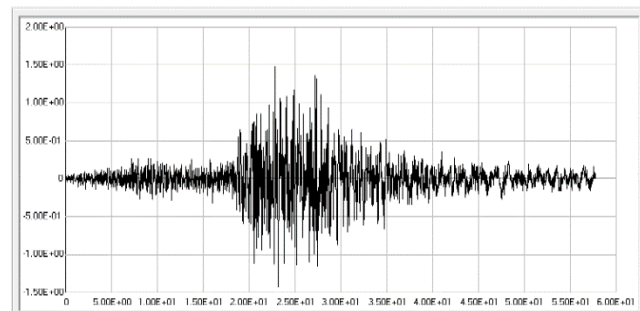


Figure 6 India Burma ground motion

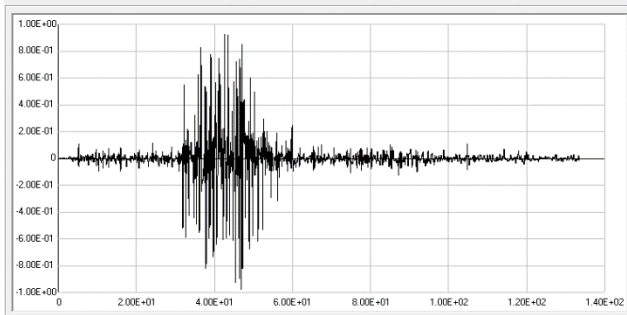


Figure 7 Bhuj ground motion

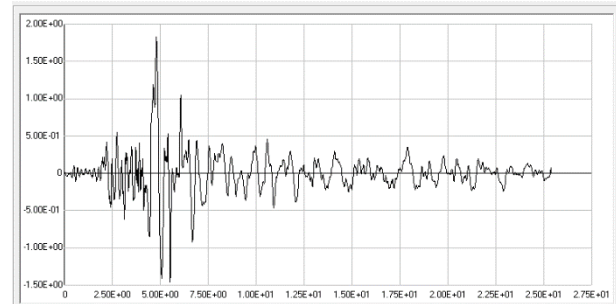


Figure 8 Chamoli ground motion

#### 4. Results and Discussions

The fragility curves generated from the analysis results are tabulated (Table 2 to Table 5) and used for further interpretation as target aim for the outcome of the present paper. The results of probability of damage and the fragility curves for the drift criterion and for various energies are plotted for different ground earthquake motions and are revealed in the following figures as Fig 9 to Fig 12. For considered structure the damage potential is more for the hysteretic energy, kinetic energy, strain energy and the modal damping energy than the drift criterion damage potential. Overall the results indicate that the energy criterion analysis gives better damage potential evaluation than the drift criterion for the structures subjected to earthquake ground motions.

Table 1

Standard deviation ( $\beta$ ) and Median ( $\theta$ ) values of drift and various energies for regular structure

Ground motions		Drift	Inelastic energy	Strain energy	Kinetic energy	Modal damping energy
Elcentro	$\theta$	0.5037	0.3194	0.4002	0.4194	0.4473
	$\beta$	0.3051	0.3726	0.3586	0.3439	0.3700
Bhuj	$\theta$	0.6384	0.4385	0.1943	0.5049	0.3711
	$\beta$	0.7057	0.6557	0.2979	0.4035	0.3537
Chamoli	$\theta$	0.6037	0.3184	0.2887	0.4559	0.4354
	$\beta$	0.6551	0.3519	0.3709	0.3235	0.3522
Ind-Burma	$\theta$	0.5937	0.2748	0.4428	0.4234	0.4340
	$\beta$	0.6958	0.3655	0.3675	0.3278	0.3758
Uttarkashi	$\theta$	0.4962	0.4081	0.2789	0.4017	0.4808
	$\beta$	0.4125	0.4226	0.3547	0.3288	0.3211
Northridge	$\theta$	0.4777	0.1111	0.4105	0.3989	0.3829
	$\beta$	0.3261	0.3333	0.3407	0.3401	0.3441



Table 2 Drift and Strain energy values under varying earthquake ground motions

PGA	Probability of damage - Strain energy						
	Drift	ElCentro	Northridge	Uttarkashi	IndiaBurma	Chamoli	Bhuj
0.1	0.00	0.00	0.00	0.00	0.00	0.00	0.01
0.2	0.05	0.03	0.02	0.17	0.02	0.16	0.54
0.3	0.14	0.21	0.18	0.58	0.14	0.54	0.93
0.4	0.25	0.50	0.47	0.85	0.39	0.81	0.99
0.5	0.36	0.73	0.72	0.95	0.63	0.93	1.00
0.6	0.46	0.87	0.87	0.98	0.80	0.98	1.00
0.7	0.55	0.94	0.94	1.00	0.89	0.99	1.00
0.8	0.63	0.97	0.97	1.00	0.95	1.00	1.00
0.9	0.69	0.99	0.99	1.00	0.97	1.00	1.00
1	0.74	0.99	1.00	1.00	0.99	1.00	1.00

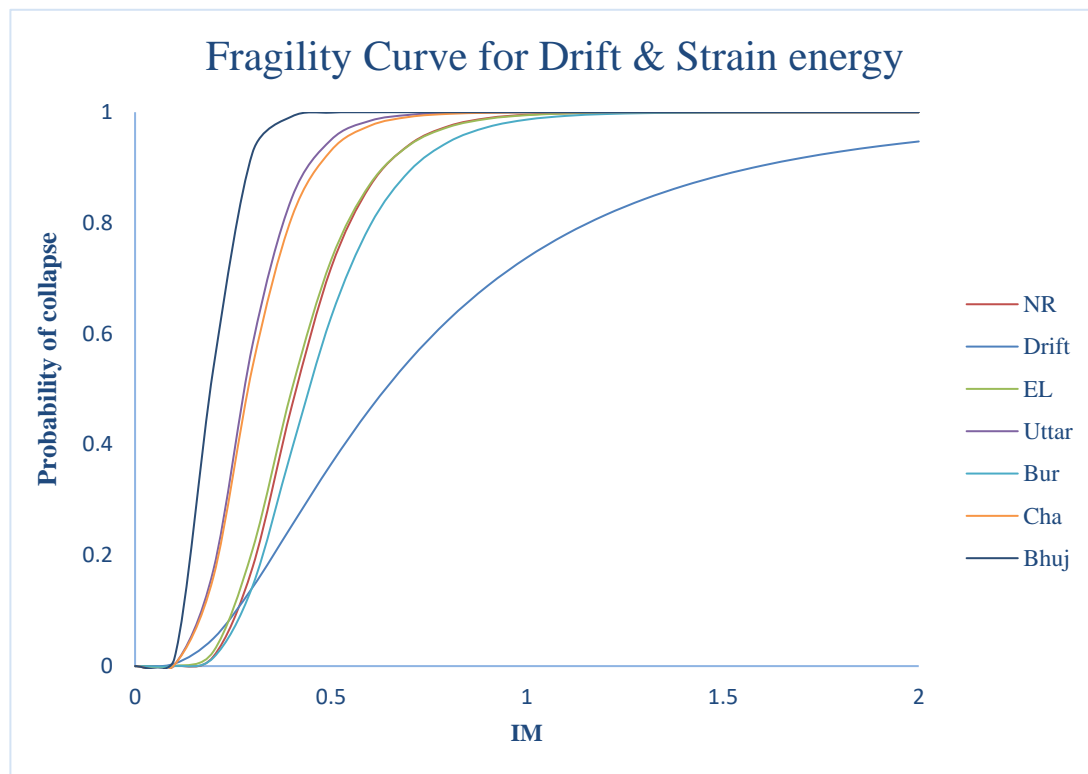


Fig 9 Fragility Curve for Drift &amp; Strain energy



Table 3 Drift and Kinetic energy values under varying earthquake ground motions

PGA	Probability of damage - Kinetic energy						
	Drift	ElCentro	Northridge	Uttarkashi	IndiaBurma	Chamoli	Bhuj
0.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.05	0.02	0.02	0.02	0.01	0.01	0.01
0.3	0.14	0.17	0.20	0.19	0.15	0.10	0.10
0.4	0.25	0.45	0.50	0.49	0.43	0.34	0.28
0.5	0.36	0.70	0.75	0.75	0.69	0.61	0.49
0.6	0.46	0.85	0.88	0.89	0.86	0.80	0.67
0.7	0.55	0.93	0.95	0.95	0.94	0.91	0.79
0.8	0.63	0.97	0.98	0.98	0.97	0.96	0.87
0.9	0.69	0.99	0.99	0.99	0.99	0.98	0.92
1	0.74	0.99	1.00	1.00	1.00	0.99	0.95

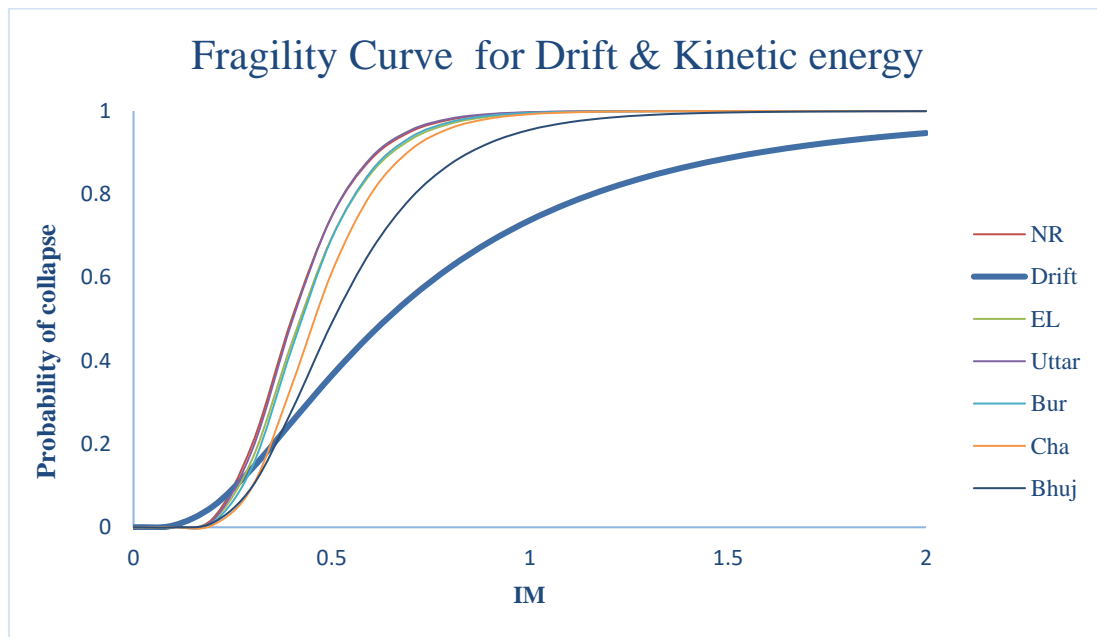


Fig 10 Fragility Curve for Drift &amp; Kinetic energy





Table 4 Drift and Inelastic energy values under varying earthquake ground motions

PGA	Probability of damage - Inelastic energy						
	Drift	ElCentro	Northridge	Uttarkashi	IndiaBurma	Chamoli	Bhuj
0.1	0.00	0.00	0.38	0.00	0.00	0.00	0.01
0.2	0.05	0.10	0.96	0.05	0.19	0.09	0.12
0.3	0.14	0.43	1.00	0.23	0.59	0.43	0.28
0.4	0.25	0.73	1.00	0.48	0.85	0.74	0.44
0.5	0.36	0.89	1.00	0.68	0.95	0.90	0.58
0.6	0.46	0.95	1.00	0.82	0.98	0.96	0.68
0.7	0.55	0.98	1.00	0.90	0.99	0.99	0.76
0.8	0.63	0.99	1.00	0.94	1.00	1.00	0.82
0.9	0.69	1.00	1.00	0.97	1.00	1.00	0.86
1	0.74	1.00	1.00	0.98	1.00	1.00	0.90

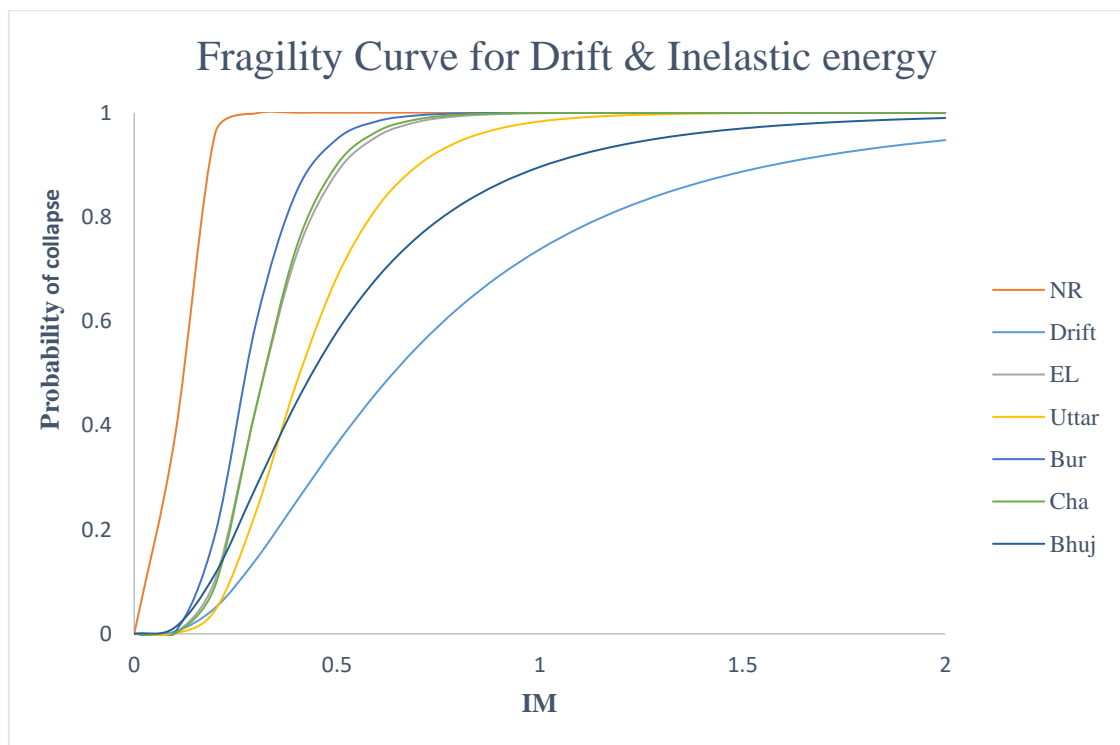


Fig 11 Fragility Curve for Drift &amp; Inelastic energy



Table 5 Drift and Damping energy values under varying earthquake ground motions

PGA	Probability of damage –Modal damping energy						
	Drift	ElCentro	Northridge	Uttarkashi	IndiaBurma	Chamoli	Bhuj
0.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	0.05	0.01	0.03	0.00	0.02	0.01	0.04
0.3	0.14	0.14	0.24	0.07	0.16	0.15	0.27
0.4	0.25	0.38	0.55	0.28	0.41	0.40	0.58
0.5	0.36	0.62	0.78	0.55	0.65	0.65	0.80
0.6	0.46	0.79	0.90	0.75	0.81	0.82	0.91
0.7	0.55	0.89	0.96	0.88	0.90	0.91	0.96
0.8	0.63	0.94	0.98	0.94	0.95	0.96	0.99
0.9	0.69	0.97	0.99	0.97	0.97	0.98	0.99
1	0.74	0.99	1.00	0.99	0.99	0.99	1.00

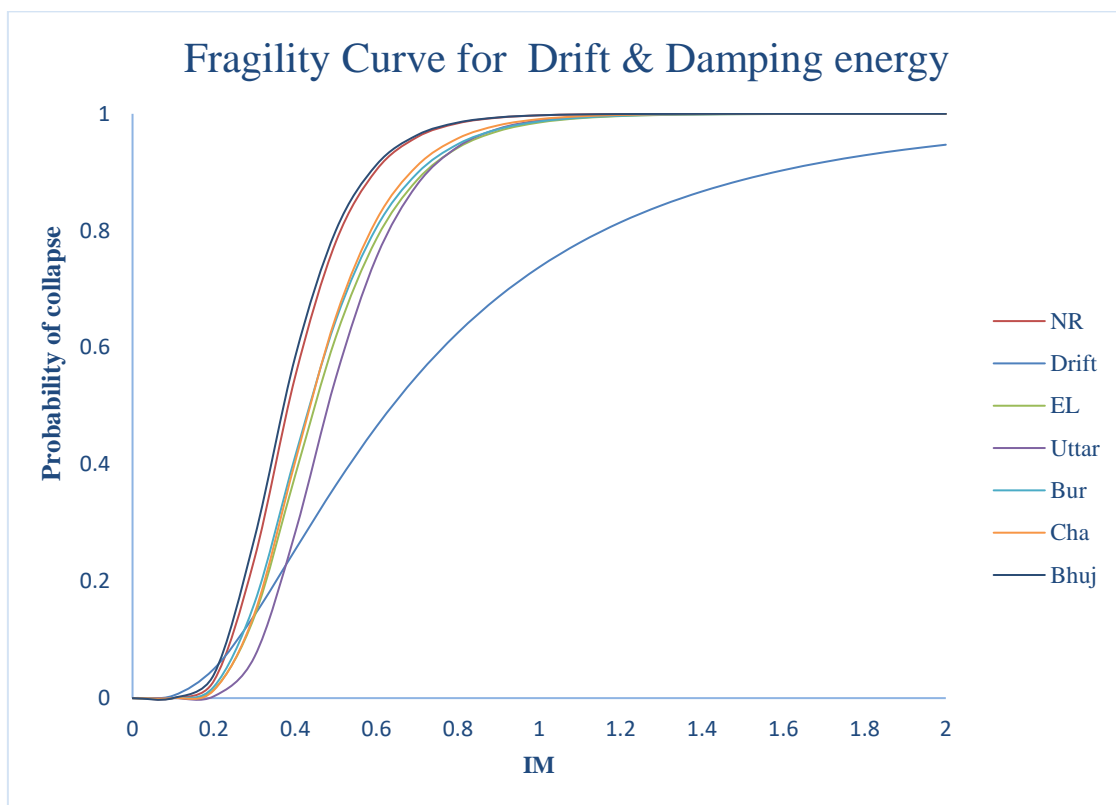


Fig 12 Fragility Curve for Drift &amp; damping energy



## 5. Conclusions

The following are salient conclusions of this paper

1. Damage measure based on energy concept using incremental dynamic analysis facilitate for speedy assessment of the structure's damage state and gives information about margin left for total collapse.
2. Proposed methods clearly give information about distribution of the damage among the structure for any global damage state and any intermediate damage state of the structure can be easily estimated.
3. The parabolic damage profile which coincides with realistic deflection profile of the structure. Conceptually it gives clear meaning of expended energy to cause damage, unlike other methods includes linear members energy in calculation of damage.
4. When the damage methods applied to numerical study, it revealed that increase of aspect ratio of structure decreases ductility capacity of the structure and leads to sudden collapse, and when aspect ratio decrease, damage shifted to upper floors.

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