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PARAMETRIC STUDY ON ACCIDENTAL TORSION IN SEISMIC DESIGN OF BUILDINGS

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Abstract

Accidental torsional response of buildings during earthquake ground motion has many sources such as the base rotational motion and all the discrepancies that may exist between the plan-distributions of mass, stiffness and strength considered in the analysis model and those of the real building at the time of the earthquake. Because of the cumbersome and the ambiguity of accidental torsion provisions of most of worldwide seismic codes, a simplified method has already been developed to estimate its effect within the linear response. Considering the multitude of parameters controlling the torsional response and based on the research work aforementioned, a parametric study is carried out in this paper leading to a practical and conservative estimation of accidental torsion effects on low to medium-rise buildings. These effects are evaluated using the concept of accidental eccentricity with its most common value of 5% of the length of the building perpendicular to the direction of the considered ground motion. Time history analyses are used to investigate the maximum increase in edges displacements due to accidental eccentricity. Design envelopes are then proposed for both flexible and stiff edges of the building with respect to the ratio of its fundamental uncoupled torsional and lateral frequencies. Eventually, a conservative estimation of the effect of accidental torsion can easily be calculated in all lateral load resisting elements. A case study of a simulated multi-storey RC building has been conducted to evaluate the efficiency of the proposed procedure. Both linear and non-linear time history analyses were used to evaluate the greatest effect of accidental torsion from all possible combinations of accidental eccentricity through the height of the building. The results show a very good agreement in the linear range and some limitations for the non-linear response.

Keywords: accidental torsion, accidental eccentricity, building, earthquake, frequency ratio

1. Introduction

During an earthquake ground shaking, buildings undergo lateral as well as torsional vibrations simultaneously. There are many sources of the torsional oscillations. The obvious one is the static eccentricity (e_s) between the Centre of Mass (CM) and the Centre of Stiffness (CS) of a specific floor. The resulting rotational motion is named natural torsion.

Other source is the spatial non-uniformity of the ground motion, leading to base rotational motion in addition of the translational one. It concerns mostly long buildings where the probability of its base to capture different seismic waves is high. Torsional vibrations may also rise from the discrepancies that may exist between the plan-distributions of mass, stiffness and strength used in the analysis model and those of the real building at the time of the earthquake. Because it's difficult to accurately identify and to evaluate the aforementioned sources related to non-uniform ground motion and uncertainties in building properties, the term of accidental torsion has been used for a long time to designate their effect on the building response.

Because of accidental torsion, even symmetric-plan buildings can undergo torsional motion. This has been highlighted from real earthquake records of existing nominally symmetric-plan buildings [1]. Accidental torsion is imposed to the designer whereas the natural one can be controlled depending on the static eccentricity (e_s) of the building analysis model. This latter eccentricity divides the building into two sides separated by the position of the CS: The Flexible Side (FS) where the lateral and the rotational motions are complementary and the Stiff Side (SS) where they are opposite. Then the stiff and the flexible edges can be defined (Fig.1).

1.1 Accidental torsion provisions in seismic codes

Most of worldwide seismic codes consider the effect of accidental torsion on buildings by the use of an additional eccentricity, called accidental eccentricity (e_{acc}). Its value is a proportion (β) of the length of the building (b) proportional to the direction of the considered ground motion. This approach makes sense since sources of accidental torsion are likely to increase with the length of the building. The value of this proportion and how to incorporate it in the design are the fundamental questions. Torsional provisions of seismic codes give different answers. The two values of $5\%b$ and $10\%b$ of accidental eccentricity are the most common values in seismic codes, with the exception of the Japanese seismic code (Building Standard Law) where the concept of accidental torsion is not explicitly mentioned. For the dynamic analysis, the design eccentricity (e_d) calculated from the CS, is the addition of both static and accidental eccentricities (Fig.1).

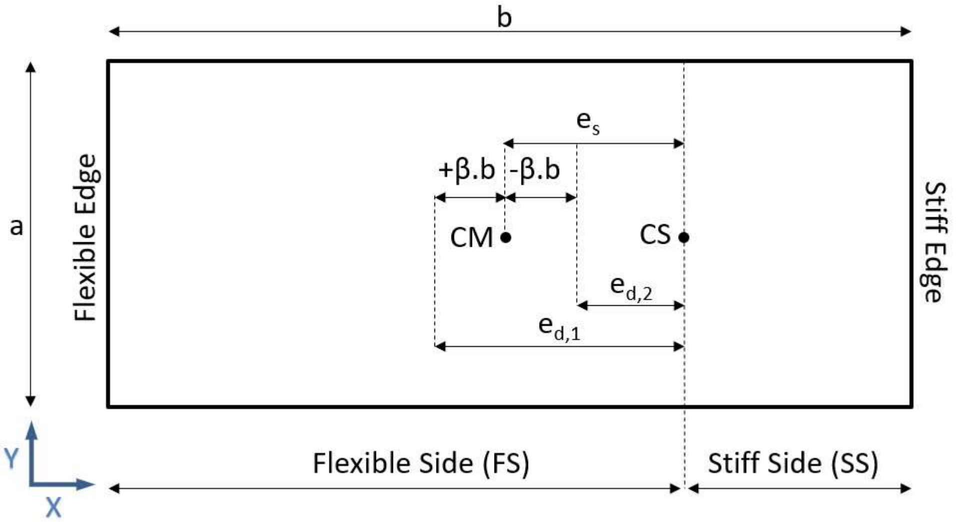

Fig. 1 – Floor plan view: Design eccentricities for a dynamic analysis

For the equivalent static analysis, the static eccentricity is sometimes multiplied by a dynamic amplification factor to consider at an early stage the real dynamic performance of the building. Other codes evaluate this dynamic effect differently as explained further in this chapter. Shown in Eq. (1) and Eq. (2) the combinations used to calculate the design eccentricities and Table 1 summarizes the values of coefficients used in these combinations for different seismic codes. Two design eccentricities need to be considered to get the greatest effect in both flexible and stiff sides of the building.

$$e_{d,1} = \alpha e_s + \beta b \tag{1}$$

$$e_{d,2} = \gamma e_s - \beta b \tag{2}$$

Table 1 – Design eccentricities for equivalent static seismic analysis

Seismic Code	**Country/Region**	**α**	**γ**	**β**
ASCE 7-16	USA	1.0	1.0	0.05
Eurocode 8	Europe	1.0	1.0	0.05
NBCC 2015	Canada	1.5	0.5	0.10
NZS 2004	New Zealand	1.5	0.5	0.10
RPS2000 V2011	Morocco	1.5	1.0	0.05

Moreover, the Eurocode 8 proposes an alternative way to consider the effect of accidental torsion on plan-symmetric buildings. The member response calculated by the equivalent static analysis can be amplified by the coefficient δ given in Eq. (3), where x is the distance in plan of the considered element from the CM and L_e is the distance between the farest resisting elements perpendicular to the direction of calculation. The ASCE 7-16 considers a factor A_x, varying from 1.0 to 3.0, to amplify the accidental torque resulting from accidental eccentricity in case of buildings with extreme torsional irregularity.

$$\delta = 1 + 0.6\frac{x}{L_e} \tag{3}$$

How to incorporate the accidental eccentricity in the analysis is another challenge for practitioner engineers. In a dynamic analysis, shifting the location of the CM of a specific floor in a calculation software is not always that easy. First, it requires the access to the mass matrix of the building. Reason of which, most of seismic codes propose the incorporation of an additional torque due to accidental eccentricity. Then how to vary accidental eccentricity through the height of the building, is still an unanswered question. Considering only the source of base rotational motion, shifting all CMs to the same direction seems reasonable as required in the Eurocode 8. Unfortunately, it's not the only source of accidental torsion and three possible positions of the CM for each floor and each direction of calculation need to be considered to get the greatest effect of accidental torsion as required by ASCE 7-16. The latter approach is more realistic but very cumbersome to adopt by practitioner engineers.

1.2 Previous research works on accidental torsion effects

More researchers studied about natural torsion rather than accidental one. And most of those who studied about accidental torsion used the approach of accidental eccentricity (e_{acc}) to evaluate its effect on the building response. It's mainly due to the lack of data related to the building base rotational motion and the probabilistic aspect of the uncertainty in mass, stiffness and strength plan-distributions in a building. A serial research work done by De la Llera and Chopra [2, 3, 4] is one of the rare studies that treated deeply and separately the effect of different sources of accidental torsion on the increase of building response. Considering the latter increase as the "true" value, it has been shown that the one calculated using $5\%b$ of accidental eccentricity corresponds to an exceedance probability of about 30% of the "true" mean value [5]. Because of the probabilistic nature of the seismic design itself, this level of exceedance probability has been adopted by the same authors to propose a simplified method to estimate the linear effect of accidental torsion on a one-story building [5]. This method can also be used for a special class of multistory buildings, as they can be associated to an equivalent one-story building having the same frequency ratio Ω (the ratio of the fundamental uncoupled rotational to lateral frequencies), the same uncoupled lateral period and the same static eccentricity [6]. An extension and an evaluation of the proposed method have been carried out from recorded linear torsional response of 12 instrumented plan-symmetric buildings subjected to different earthquakes [7]. Good agreements were observed but not in all cases. The main advantage of this method is to avoid performing many extra seismic analyses to evaluate the effect of accidental torsion, as implicitly required by most of seismic codes. It evaluates the effect of accidental torsion using a design envelope with respect to the frequency ratio Ω and the ratio of b over the mass radius of gyration r of the floor.

The parametric study carried out in this paper results to design envelopes considering the effect of three more parameters. In the research work aforementioned, spectral analyses were conducted. The frequency content of the real earthquake ground motion cannot be captured and may alter the results, particularly for torsionally flexible buildings [8]. Time history analyses are adopted in this paper. Second, only one design envelope calculated for symmetric-plan buildings was proposed previously. It was highlighted that symmetric buildings are more sensitive to accidental eccentricity [6], but some slightly asymmetric-plan buildings show higher increase than symmetric ones having the same frequency ratio [8]. Thus, other small values of static eccentricity are added in this paper resulting in design envelopes for both flexible and stiff edges. Eventually, different values of the fundamental uncoupled lateral period were considered to include their sensitivity to the frequency content of artificial ground motions adopted in this study.

2. Dynamic torsional response: Equation of motion

The model structure used in the parametric study is a one-story Reinforced Concrete building. The floor plan view is shown in Fig.1 excluding the lateral load resisting elements for a clear view. The building is kept symmetric in the X-direction. The static eccentricity e_s in the X-direction is variable in the parametric study resulting in both symmetric and asymmetric buildings in the Y-direction. The floor is considered rigid with a total mass m. Considering a translational input ground motion $u_{\ddot{g}y}$ only in the Y-direction, the undamped motion of the CM $(u_y\ ,u_\theta)$ is given by the following equation [4]:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{pmatrix} u_{\ddot{y}} \\ r u_{\ddot{\theta}} \end{pmatrix} + \begin{bmatrix} k_y & \frac{e_d}{r}k_y \\ \frac{e_d}{r}k_y & \frac{k_{\theta,CM}}{r^2} \end{bmatrix}\begin{pmatrix} u_y \\ r u_\theta \end{pmatrix} = -m.\begin{pmatrix} u_{\ddot{g}y} \\ 0 \end{pmatrix} \quad (4)$$

Where r is the mass radius of gyration, k_y the lateral stiffness in the Y-direction, $k_{\theta,CM}$ the rotational stiffness around the CM. Considering the rotational stiffness around the CS ($k_{\theta,CS}$), the uncoupled lateral and rotational circular frequencies are respectively given in Eq. (5) and Eq. (6). The frequency ratio Ω is then calculated by Eq. (7). The undamped equation of motion of the CM can be rewritten as shown in Eq. (8).

$$\omega_y = \sqrt{\frac{k_y}{m}} \quad (5)$$

$$\omega_\theta = \sqrt{\frac{k_{\theta,CS}}{mr^2}} \quad (6)$$

$$\Omega = \frac{\omega_\theta}{\omega_y} \quad (7)$$

$$\begin{pmatrix} u_{\ddot{y}} \\ r u_{\ddot{\theta}} \end{pmatrix} + \omega_y^2\begin{bmatrix} 1 & \frac{e_d}{r} \\ \frac{e_d}{r} & \Omega^2 + \left(\frac{e_d}{r}\right)^2 \end{bmatrix}\begin{pmatrix} u_y \\ r u_\theta \end{pmatrix} = -\begin{pmatrix} u_{\ddot{g}y} \\ 0 \end{pmatrix} \quad (8)$$

According to Eq. (8), five parameters control the undamped response of the building: the lateral uncoupled period T_y, the mass radius of gyration r, the ratio of the design eccentricity e_d over r, the frequency ratio Ω and the earthquake ground motion $u_{\ddot{g}y}$. For a rectangular floor ($b \geq a$) as shown in Fig.1, the Aspect Ratio (AR) is defined by Eq. (9). If the mass is uniformly distributed, Eq. (10) gives the relation between the mass radius of gyration r and the AR. In the parametric study of this paper, the accidental eccentricity is fixed to 5%b.Thus, only the following parameters are considered: $u_{\ddot{g},y}$, e_s, T_y, Ω and the AR.

$$\mathrm{AR} = \frac{b}{a} \quad (9)$$

$$\frac{r}{b} = \sqrt{\frac{1+AR^{-2}}{12}} \quad (10)$$

3. Parametric study

3.1 Investigated parameter

The increase in edge displacement due to accidental eccentricity of $5\%b$ is the parameter investigated in this study. It's defined by the ratio of the edge displacement considering $\pm e_{acc}$ in the analysis, over the one without considering e_{acc}. $I_{f,dy}$ (respectively $I_{s,dy}$) is the maximum increase in displacement of the flexible edge (respectively stiff edge) obtained from time history analysis results. For each model building and translational ground motion in Y-direction, three different analyses have to be performed to get the maximum increase in edges displacement due to accidental eccentricity. One with the original position of the CM and two other analyses where the CM is shifted by $\pm 5\%b$ by modifying the mass matrix of the building.

3.2 Variant parameters

3.2.1 Input ground motion $\ddot{u_{g,y}}$

Three artificial ground motions are used to perform the time history analyses. They all have been generated from the same target spectrum but with different phases: El Centro phase, Kobe phase and a random phase using level 1 of Jennings envelope. The target spectrum corresponds to the design spectrum of the zone with the highest seismicity in Morocco, including the surface soil amplification. Shown in Fig.2 the target and the artificial elastic acceleration response spectra for the case of the artificial ground motion with a random phase. It should be noted that the target spectrum does not affect the investigated parameter in the linear range.

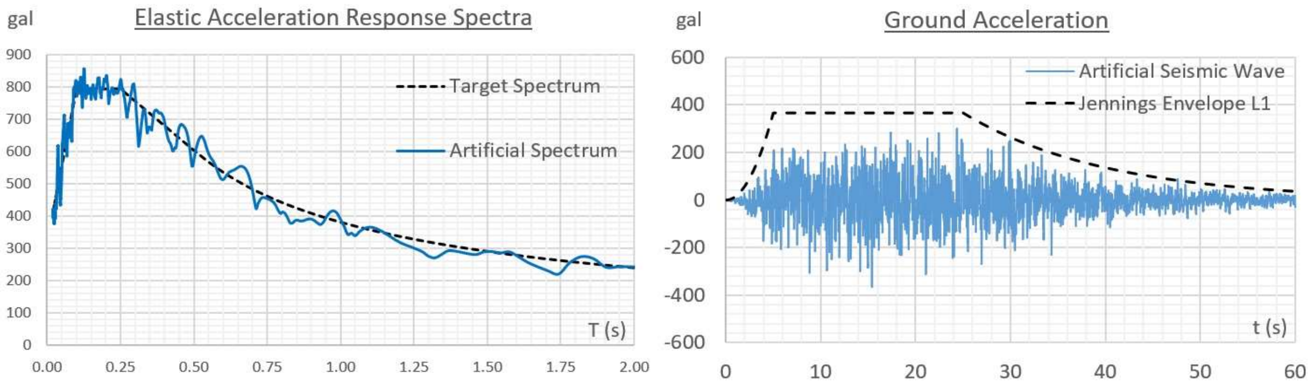

Fig. 2 – Target and artificial response spectra (left) of an artificial ground motion (right)

3.2.2 Static eccentricity e_s

Three different values of static eccentricity e_s are considered: $0.00b$, $0.05b$ and $0.09b$. For greater values, the investigated parameter tends to decrease as already mentioned [6]. Recommendations on how to estimate the effect of accidental torsion in case of other values of e_s are given in the conclusion.

3.2.3 Uncoupled lateral period T_y

Four values of uncoupled lateral period are considered in the parametric study: 0.2s, 0.5s, 1.0s and 2.0s. It can be assumed that the proposed envelopes cover all the cases of low to medium-rise buildings.

3.2.4 Frequency ratio Ω

Some measures on existing buildings showed that the frequency ratio Ω is in the range of 0.8 to 1.5 [5]. More field investigations have to be done to confirm this statement. Given this lack of data, values of Ω varying from 0.7 to 1.6 were adopted in the parametric study with a step of 0.05.

3.2.5 Aspect ratio AR

All model buildings used in the analyses have the same Aspect Ratio of 3.5. Thus, the proposed envelopes concern buildings with the same Aspect Ratio. For a rectangular floor, the AR is related to the ratio of r over b as shown in Eq. (10). Based on the former method proposed by De la Llera and Chopra [5], the Correction Factor CF proposed in Eq. (11) is used in this study for other values of AR.

$$CF(AR) = \frac{1+3.5^{-2}}{1+AR^{-2}} \tag{11}$$

3.4 Seismic analyses

Combining all the parameters aforementioned, more than 2000 time-history analyses have been performed to obtain the final results. The Damping matrix is considered stiffness-proportional with 5% of equivalent viscous damping ratio for the first mode. The software STERA_3D is used for the analysis [9].

4. Design envelopes

The design envelopes have been decided for a conservative and practical design. The increase in edge displacement due to accidental eccentricity of 5%b, can be easily calculated with respect to Ω. Results are presented for buildings with aspect ratio of 3.5.

4.1 Flexible Edge

Fig.3 shows the design envelopes and all intermediate results for the three cases of static eccentricity. Fig.4 summarizes the design envelopes for the flexible edge.

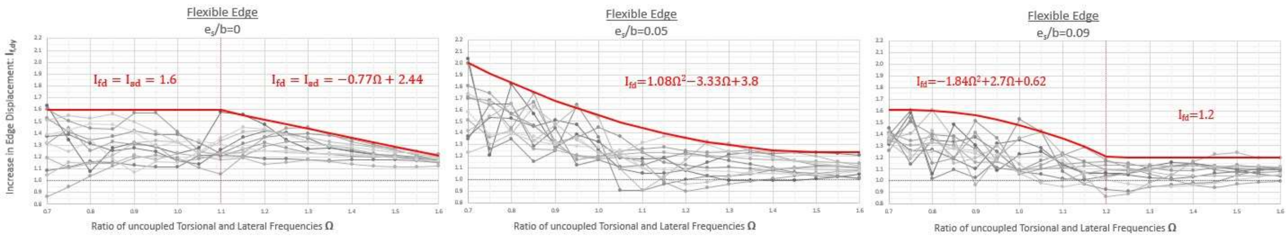

Fig. 3 – Design envelopes and intermediate results for the Flexible Edge

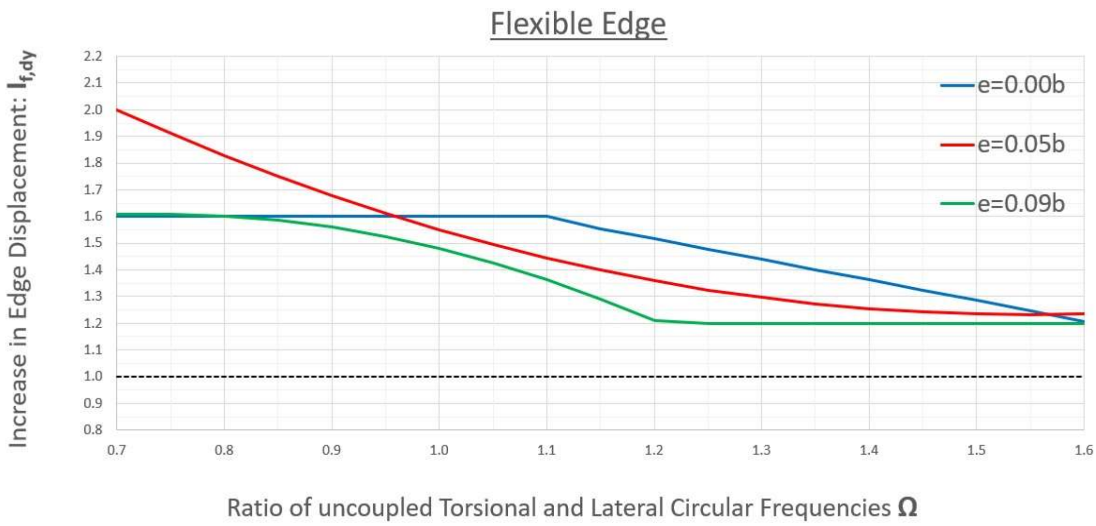

Fig. 4 – Summary of design envelopes for the Flexible Edge

4.2 Stiff Edge

Fig.5 shows the design envelopes and all intermediate results for the three cases of static eccentricity. Fig.6 summarizes the design envelopes for the stiff edge.

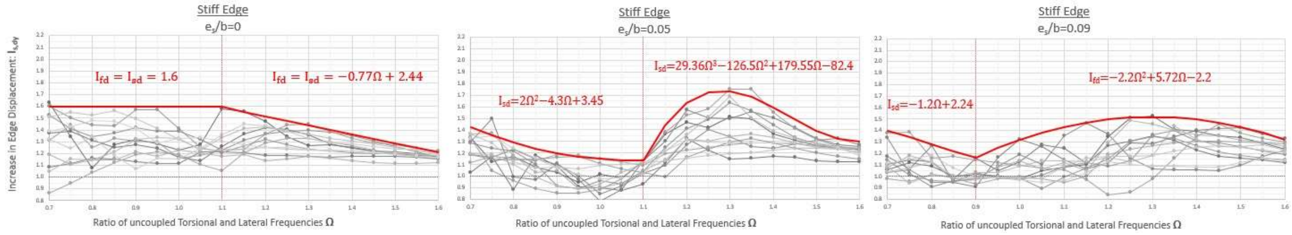

Fig. 5 – Design envelopes and intermediate results for the Stiff Edge

Fig. 6 – Summary of design envelopes for the Stiff Edge

5. Proposed procedure to estimate accidental torsion effects in a building

To evaluate the increase in displacement at any lateral load resisting element due to accidental torsion, the following steps are sufficient if considering a translational input ground motion in the Y-direction:

- ✓ Determine the associated single degree of freedom Ω_{eq} and $e_{s,eq}$ [6],
- ✓ Calculate the increase in edge displacement $I_{f,dy}$ and $I_{s,dy}$ using the design envelopes,
- ✓ Adjusting the value for an aspect ratio other than 3.5 using Eq. (11) and Eq. (12):

$$I^{*}_{f/s,dy} = 1 + \left(1 - I_{f/s,dy}\right) * CF(AR) \tag{12}$$

- ✓ Calculate the increase in displacement at any element in a distance x from the CS separation (Fig.7) using Eq. (13) for the flexible side or Eq. (14) for the stiff side. b_{FS} (respectively b_{SS}) is the length of the Flexible Side (respectively Stiff Side) perpendicular to the direction of calculation (Fig.7).

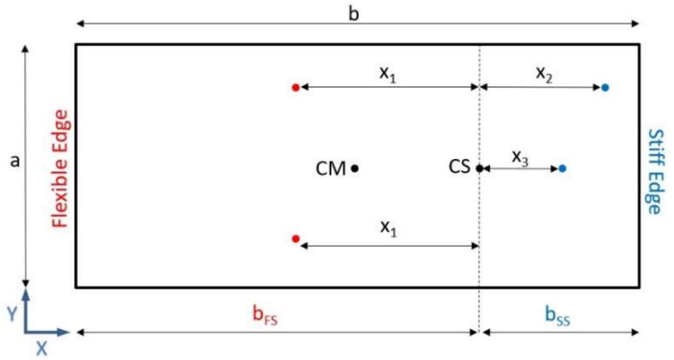

Fig. 7 – Distance of lateral force resisting elements from the CS separation

$$I_{FS}(x) = 1 + \frac{x}{b_{FS}} \times (I^*_{f,dy} - 1) \tag{13}$$

$$I_{SS}(x) = 1 + \frac{x}{b_{SS}} \times (I^*_{s,dy} - 1) \tag{14}$$

✓ In the elastic range, the increase in force is the increase in displacement of the same element.

The increase in displacement is considered to vary linearly from its value at the edge to 0 at the CS separation.

6. Case study

6.1 Simulated building

The simulated building is a symmetric three-story RC frame building designed with the Japanese standard. The frames are composed with the same structural elements (Fig.8). The aspect ratio is 2.5 ($30 \times 12\ m^2$) and the fundamental lateral period in the Y-direction is 0.32 seconds. The damping is stiffness proportional with 5% of equivalent viscous damping ratio for the first mode. Input ground motions are all derived from scaled NS components of El-Centro and Kobe earthquakes and are applied only in the Y-direction. Time history analyses were performed using the software STERA_3D [9]. To evaluate the procedure proposed in this paper, the effect of accidental torsion was first calculated using all possible positions of the CMs (Fig.8). The case 1 is the common analysis not considering the accidental eccentricity. Then the maximum increase in edge displacement at each story is selected and compared with the one easily evaluated by the proposed procedure.

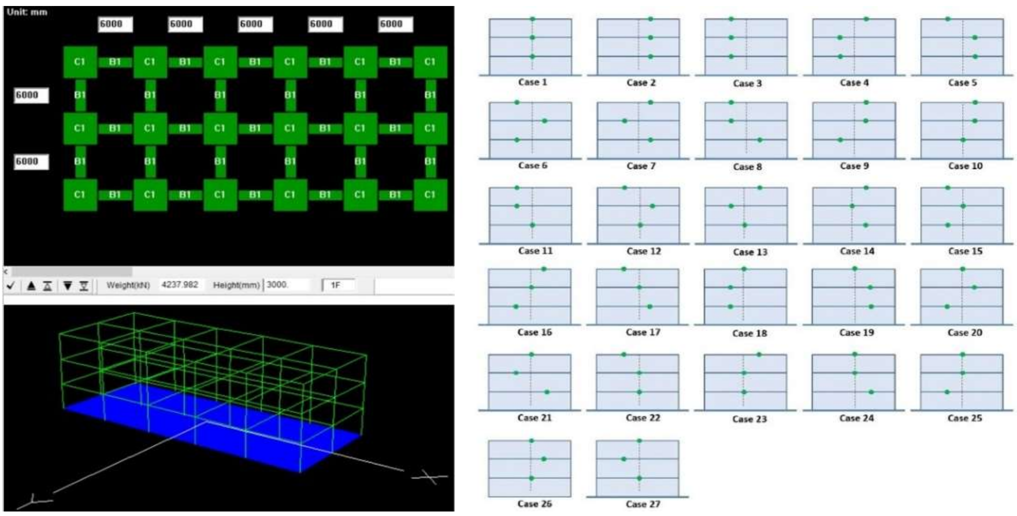

Fig. 8 – Simulated RC building and all combinations of positions of CMs with an $e_{acc} = \pm 5\% b$

The building is plan-symmetric ($e_s = 0.00$) and the frequency ratio in the elastic range is $\Omega_{elastic} = 1.22$. Using the design envelope of Fig.4 or Fig.6, the increase in displacement of both edges is evaluated to $I_{f/s,dy} = 1.50$. This value is adjusted to consider the aspect ratio of the building: $I^*_{f/s,dy} = 1.46$.

6.2 Linear range

In the linear range, the NS components of El-Centro and Kobe earthquakes are scaled to 0.15. Table 2 shows the maximum increase in edge displacement at each story and for both input ground motions in the Y direction. The combinations of cases 2 and 3 (Fig.8) produce the maximum effect for both input ground motions. It's the combination required by the Eurocode 8. The maximum value is 1.38 which is less than 1.46. The relative error of the estimated increase by the proposed procedure is in the range of [5.80%; 8.15%], which is acceptable for a conservative design.

Table 2 – Maximum increase in edges displacement in the linear range

Floor	Maximum Increase in edge displacement	
	0.15 El-Centro NS	0.15 Kobe
	Cases 2/3	Cases 2/3
3rd	1.38	1.37
2nd	1.38	1.35
1st	1.37	1.35

6.3 Non-linear range

The lateral load resisting elements are symmetric and identical. If we assume that the damage at each instant is symmetric as well, it can be deduced from Eq. (12) that $\Omega_{plastic} \geq \Omega_{elastic} = 1.22$. However, it's not easy to calculate the exact value of $\Omega_{plastic}$ and it's out of the scope of this paper. Considering the design envelope, the increase in edge displacement in the non-linear range should be less than $I^*_{f/s,dy} = 1.46$.

$$r^2\left(\Omega^2_{plastic} - \Omega^2_{elastic}\right) = \left(\sum k_{yi}.x_i^2\right).\left(\frac{1}{k_{x,plastic}} - \frac{1}{k_{x,elastic}}\right) \tag{12}$$

In the non-linear range, the NS component of El-Centro and Kobe earthquakes are used. The El-Centro input causes a moderate damage (ductility less than 5) whereas the Kobe input causes severe damage (ductility more than 5). For that reason, another input of NS component of Kobe scaled to 0.7 was considered to get a moderate damage. Thus, three input ground motions are applied separately in the Y direction. Table 3 shows the maximum increase in edge displacement at each story for the three input ground motions. The combinations of cases 2 and 3 (Fig.8) are still producing the maximum effect for both El-Centro and 0.7 Kobe inputs, both causing a moderate damage. The increases are effectively less than the values obtained in the linear range (Table 2). Thus, less than $I^*_{f/s,dy} = 1.46$. The relative error of the estimated increase by the proposed procedure in the case of moderate damage is in the range of [8.95%; 19.67%]. The estimation starts to be over-conservative. However, the NS component of Kobe earthquake produces an unexpected increase in edge displacement of the 1st floor of 1.53 which is greater than values obtained in the linear range. Moreover, the combinations producing the maximum increase are Cases 6, 7, 10 and 11 (Fig.8) which don't correspond to the common practice to shift all the CMs to the same side. The relative error of the estimated increase by the proposed procedure in the case of severe damage is in the range of [-4.57%; 19.67%]. It seems that when the building is heavily damaged, the proposed procedure is not any more reliable and can lead to incorrect results.

Table 3 – Maximum increase in edges displacement in the non-linear range

Floor	Maximum increase in edge displacement			
	El Centro NS	0.7 Kobe NS	Kobe NS	
	Cases 2/3	Cases 2/3	Cases 6/7	Cases 10/11
3rd	1.24	1.22	-	1.22
2nd	1.24	1.22	-	1.25
1st	1.26	1.34	1.53	-

7. Conclusions

The parametric study conducted in this paper results in a very simplified method to estimate the effect of accidental torsion on a building subjected to ground motion. Conclusions of this study are the following:

1. The proposed design envelopes include the effect of many parameters such as the frequency content of the ground motion and its interaction with the fundamental uncoupled lateral period of the building. They can be used to evaluate the effect of accidental torsion in seismic design of low to medium-rise buildings.
2. Plan-symmetric or slightly symmetric buildings are more sensitive to accidental eccentricity than plan-asymmetric ones. Three values of e_s are considered in this study: $0\%b$, $5\%b$ and $9\%b$. For intermediate values, the greater increase from the surrounding two values is to be considered for a conservative design. For values greater than $9\%b$, the minimum value from the three design envelopes can be considered.
3. The estimated increase in displacement due to accidental eccentricity has shown a very good agreement with actual results of the elastic response. The maximum relative error obtained in the case study is 8.15% which is acceptable for a conservative design.
4. In the non-linear range, the proposed method still gives conservative results in case of a low to moderate damage (ductility less than 5), but with a greater error of 19.67%. For heavy damage, the method shows some limitations and is not anymore reliable. Further studies need to be performed to estimate the variation of the frequency ratio Ω in the non-linear range for a more accurate estimation of accidental torsion effects.

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