

A displacement-based seismic design procedure for buildings with linear and non-linear fluid viscous dampers considering damage control

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Abstract

Recent tendencies in earthquake engineering have as one of their main goals for the rehabilitation of existing and the design of new buildings to guarantee acceptable level of resilience are when they are subject seismic design demands during their life cycle. To fulfill this goal the control seismic performance of structures with fluid viscous dampers is an excellent alternative. Most of the seismic analysis procedures currently used to design structures use modal spectral analysis, which requires for its direct application that the damping matrix be proportional. However, the inclusion of these devices leads to a non-proportional damping matrix and an eigen-value analysis leading to complex modes and frequencies, and the requirement of two spectra to characterize the seismic demand, which is a procedure, not stipulated in the current design codes. For this reason, the effect of these devices is modeled as an equivalent proportional damping, which is not always the most appropriate option, since significant errors can occur in the structural performance obtained. Even though there is a considerable variety of passive energy dissipation devices, the fluid viscous dampers are one of the most used devices, since they only provides damping to the structure, for this reason the dynamic properties of the structure are not modified. The non-linear fluid viscous dampers are the most used in practical applications, since for relatively small velocities these devices produce larger forces than those produced by linear fluid viscous dampers, in addition to the axial forces in the columns are significantly reduced, which It can have a direct impact on the foundations of the structures. However, there is the difficulty of considering this type of devices in procedures whose formulation involves the approximate solution of the equation of motion of the structure directly using modal spectral analyses.

To overcome this problem, this paper presents an approximate seismic displacement-based design procedure for new and existing structures equipped with linear and no-linear viscous dampers, in which the participation of these devices is incorporated in the modal spectral analysis. To consider the non-proportional damping produced by these devices within a conventional modal spectral analysis their effect is approximated as the sum of a proportional damping matrix and a residual non-proportional damping matrix. The framework of the design procedure proposed is based on the concept that the performance of a multiple degree of freedom structure may be approximated from the performance of a reference bilinear single degree of freedom system, with properties generally those of the fundamental mode of the structure. To illustrate the application of this procedure proposed four regular plane frames: 8, 12, 17, and 20-storey were designed. The seismic demand used for design and validation was the record obtained at the SCT site during the 1985 Michoacan earthquake, the station is located on soft soil site. In order to validate the procedure proposed, the performances and damage distributions used as design targets were compared with the corresponding results from the nonlinear step-by-step analyses of the designed structures subjected to the same seismic demands. From the analysis and discussion of the results obtained, it is concluded that the procedure proposed allows the design of structures equipped with viscous dampers that satisfy a displacement-based design objective.

Keywords: displacement-based seismic design; damage control; linear fluid viscous dampers; non-proportional damping approximation; modal spectral analysis



1. Introduction

The main objective of seismic design is to guarantee adequate behaviour of a building by accomplishing given design performance levels, PL, when subjected to earthquake scenarios which may occur during its life cycle. Based on this objective, during the recent past the design methods based on forces have widely been used; however, these methods cannot guarantee acceptable performance under seismic demands associated with those used in the seismic design conditions. For this reason, performance-based design methods have been developed, particularly those based on displacements as these parameters are the most relevant to performance, e.g., [1], [2] and [3]. Nevertheless, there are many situations where it is not possible to guarantee the PLs considered due to architectural restrictions and/or code related issues. For this reason, it is necessary to consider other strategies to satisfy the PLs, such as the use of passive energy dissipation devices e.g., viscous dampers, which dissipate part of the input earthquake energy and as a consequence reduce the displacements of the structure and the corresponding earthquake induced damage. Moreover, in cases when it is required to add a large amount of supplementary damping to satisfy the PLs (i.e., $\xi_{Dampers} > 15\%$), and this amount is neither practical nor reasonable [4], the alternative is to accept damage in the structure, by using performance-based design procedures adapted to include passive energy dissipation devices.

The most appropriate procedure to analyse structures with viscous dampers is the dynamic step-by stepanalysis; nevertheless, current design practices and most building codes recommend for the seismic evaluation and design of structures the use of modal spectral analysis with seismic demands given by design spectra. Due this situation, in the majority of cases in which viscous dampers are used as passive energy dissipation devices, the associated damping is non-proportional leading to non-classical eigen-values and vectors, making the conventional modal spectral analysis used in the practice of the seismic design of structures, not feasible to apply.

To overcome this limitation [5] propose an approximate simplified procedure, in which the effect of the viscous dampers is considered as supplemental modal viscous damping. In this procedure, the approximate amount of damping, associated to the viscous dampers, is easily determined in terms of the dynamic properties of the structure such as the fundamental mode shape and period. Using this approximation, [6], propose a simplified sequential search algorithm to calculate the distribution of dampers and damping coefficients according to interstorey velocities. Along the same lines, [7] propose a damper distribution based on storey shear strain energy. The assumption of non-proportional damping as proportional is not always appropriate because it can produce significant errors in the response of structures [8]. Therefore, diverse studies focusing on the development of simplified procedures to use the modal spectral analysis in structures with viscous dampers have been carried out.

In this paper present, a simplified displacement-based seismic evaluation and design procedure that considers explicitly a combination of damage control and added viscous damping to comply with target design displacements. To illustrate the application of this method and to evaluate the performance of structures designed with the method proposed, four structures are designed. The additional damping required by the structure under design conditions with the accepted damage distribution is calculated using the approximation proposed. To distribute the nominal damping coefficients of the devices that produce the design amount of damping, assuming that they are located at the central spans of each floor, a procedure using as relative weights of these damping coefficients the drifts in the structure. As design seismic demand, the EW component of the SCT record of the 1985 Michoacan earthquake in Mexico is considered. The design obtained, is evaluated by comparing the interstorey drifts and overall structural performance. The accuracy of the results is assessed by comparing the performances extracted from the results of the step-by-step non-linear dynamic analysis of the structure designed with the target design performance. Finally, some conclusions about the design method and the results obtained are presented, stressing the most relevant advantages of the displacement-based design of structures with viscous dampers and damage control.



2. Fundamentals of the procedure proposed

2.1 Fundamentals of the procedure proposed

The procedure proposed is based on the assumption that the performance of a non-linear multi-degree of freedom (MDOF) structure may be approximated from the performance of a reference simplified non-linear single-degree of freedom (SDOF) system, normally associated to the fundamental mode of the structure [9]. The principle of this evaluation/design method is that the non-linear capacity curve of a MDOF structure may be approximated by a bilinear curve, using the equivalence of deformation energies corresponding to the real capacity curve and its bilinear approximation, and that, in accordance with basic principles of structural dynamics, the bilinear capacity curve of the reference SDOF system, also referred to as the behaviour curve of the reference system, may be directly extracted from this capacity curve. The behaviour curve of the reference system is obtained from the results of two conventional modal spectral analyses, one for the elastic phase of behaviour, i.e. structure without damage, and other for the inelastic phase, i.e. structure with the assumed distribution of damage. The slope of the first branch of the behaviour curve represents the elastic stiffness of the reference SDOF system whereas the slope of the second branch, the stiffness corresponding to the inelastic range. The slope of this second branch is defined by a previously assumed damage distribution associated to the proposed maximum displacement of the target PL. To satisfy the target PL of the structure with viscous dampers, the spectrum is modified to consider the added damping due at the effect of these energy dissipation devices.

2.2 Added damping consideration

As mentioned above, most seismic analysis procedures for structures with viscous dampers consider the effect of these devices in the response of the structure by assuming a proportional damping matrix, which is representative of the actual non-proportional damping matrix. This assumption, however, is not strictly valid, as the response of the structure may show significant errors when compared against that obtained using a stepby-step dynamic analysis of the structure with non-proportional damping matrix. To minimize the errors involved in this approximation, this paper proposes a correction to the original procedure. This correction is described in the following paragraphs:

The equation that describes the dynamic equilibrium of a MDOF structure can be written as:

$$[M]{\ddot{u}(t)} + [C]{\dot{u}(t)} + [K]{u(t)} = {p(t)}$$
(1)

where

[M]= Mass matrix[C]= Damping matrix[K]= Stiffness matrix{p(t)}= Force vector

 $\{u(t)\}, \{\dot{u}(t)\}, \{\ddot{u}(t)\} =$ Displacement, velocity and acceleration vectors

For a structure with viscous dampers, the damping matrix can be represented as:

$$[C] = [C_0] + [C_D]$$
(2)

where

 $[C_0]$ = Inherent damping matrix (assumed as proportional)





Since the damping matrix associated to the viscous dampers is non-proportional, neither to the mass nor the stiffness matrices, it is possible to approximate it as the sum of a proportional damping matrix ($[C_{DP}]$) and a residual damping matrix($[C_{DR}]$), *i.e.*, :

$$[\mathsf{C}_{\mathsf{D}}] = [\mathsf{C}_{\mathsf{DP}}] + [\mathsf{C}_{\mathsf{DR}}] \tag{3}$$

The first order approximation of the proportional matrix, also referred as Rayleigh damping may be written as:

$$[C_{\rm DP}] = a_{\rm 0D}[M] + a_{\rm 1D}[K] \tag{4}$$

To uncouple the equilibrium equations given by Eq. 1 by the following change of coordinates is required: $\{u(t)\} = [\Phi]\{x(t)\}, \{\dot{u}(t)\} = [\Phi]\{\dot{x}(t)\}, \{\ddot{u}(t)\} = [\Phi]\{\dot{x}(t)\}, \text{ to give:}$

$$[M][\Phi]{\dot{x}(t)} + [C][\Phi]{\dot{x}(t)} + [K][\Phi]{x(t)} = {p(t)}$$
(5)

where

 $[\Phi]$ = Normalized modal matrix

Pre-multiplying each term in Eq. 5 by $\{\phi\}_i^T$ gives:

$$\{\phi\}_{i}^{T}[M]\{\phi\}_{i}\{\ddot{x}(t)\} + \{\phi\}_{i}^{T}[C]\{\phi\}_{i}\{\dot{x}(t)\} + \{\phi\}_{i}^{T}[K]\{\phi\}_{i}\{x(t)\} = \{\phi\}_{i}^{T}\{p(t)\}$$
(6)

Assuming that the inclusion of viscous dampers in the structure does not significantly modify the modal characteristics of the structure, a set of uncoupled dynamic equilibrium equations expressed in terms of modal coordinates $x_i(t)$ is obtained:

$$\ddot{x}_{i}(t) + 2 \left(\xi_{0i} + \xi_{DPi}\right) \omega_{i} \dot{x}_{i}(t) + \{\phi\}_{i}^{T} [C_{DR}] \{\phi\}_{i} \dot{x}(t) + \omega_{i}^{2} x_{i}(t) = \{\phi\}_{i}^{T} \{p(t)\}$$
(7)

where

 ω_i = Frequency for mode i

 ξ_{0i} = Inherent viscous damping ratio for mode i

 ξ_{DPi} = Proportional viscous damping ratio of devices for mode i

 Γ_i = Modal participation factor

Even though the term $\{\phi\}^T [C_{DR}] \{\phi\}$ is not a diagonal matrix, experimental evidence has shown that as the damping ratio of a structure is increased, the contribution of the higher modes of the structure to its total response may be ignored. Therefore, for practical applications in the approximate method proposed, only the contribution of the fundamental mode of the MDOF system is usually considered [7]. Based on this



consideration, the damping ratio associated to the residual damping matrix can be expressed as: $\xi_{DRi} = \frac{\{\varphi\}_i^T [C_{DR}] \{\varphi\}_i}{2 \omega_i}$, and Eq. 7 can be rewritten as:

$$\ddot{x}_{i}(t) + 2\left(\xi_{0i} + \xi_{DPi} + \xi_{DRi}\right)\omega_{i}\dot{x}_{i}(t) + \omega_{i}^{2}x_{i}(t) = \{\varphi\}_{i}^{T}\left\{p(t)\right\}$$
(8)

where

 ξ_{DRi} = Residual viscous damping ratio of devices for mode i

The proportional viscous damping ratio associated to the added devices can be calculated using the energy based approximation proposed by [5]:

$$\xi_{\rm DPi} = \frac{T_{\rm i} \sum_{j=1}^{\rm nd} (C_{\rm j}) (\cos \theta_{\rm j}) (\phi_{\rm i} - \phi_{\rm i-1})^2}{4 \pi (\sum_{i=1}^{\rm n} m_{\rm i} \phi_{\rm i}^2)}$$
(9)

where

 T_i = Natural period of vibration of mode i

 θ_i = Angle of the viscous damper j

 ϕ_i = Horizontal modal displacements of mode i

 C_j = Damping coefficient of the damper at storey j

The residual viscous damping ratio of the devices for mode *i* can be defined as:

$$\xi_{\rm DRi} = \frac{\{\phi_i^{\rm T}\} [C_{\rm DR}] \{\phi_i\}}{2 \,\omega_i} = \frac{\{\phi_i^{\rm T}\} [C_{\rm D} - C_{\rm DP}] \{\phi_i\}}{2 \,\omega_i} \tag{10}$$

Thus, ignoring the response contribution of higher modes, the total damping ratio for the first mode is defined as:

$$\xi_{\text{Design1}} = (\xi_{01} + \xi_{\text{DP1}} + \xi_{\text{DR1}})$$
(11)

In the order to considering non-linear viscous dampers in the procedure proposed, it can be used an equivalence, *e.g.*, [10], Eq. 12.

$$C_{\alpha_{j}} = \frac{(2)^{(2+\alpha_{j})} \cdot (\pi)^{(2+\alpha_{j})} \cdot (\phi_{rj,1})^{(1+\alpha_{j})} \cdot (D_{roof})^{(1-\alpha_{j})} \cdot (C_{L_{k}}) \cdot (f_{j})^{(1-\alpha_{j})}}{\lambda_{k} + \left(\frac{1}{T_{1}}\right)^{(\alpha_{j})}}$$
(12)



3. Displacement-based design method

In accordance with the aforementioned concepts, the application of the displacement-based design method proposed to structures with viscous dampers intended to satisfy the life safety limit state, LSLS, can be summarized in the following steps:

- 1. Preliminary configuration and dimensioning of the elements of the structure according to engineering judgement and/or designer experience. The objective of this step is to define a realistic stiffness distribution of the structural elements throughout the height of the structure, so that design displacement shape is in accordance with those of real structures.
- 2. Modal analysis of the elastic bare structure designed in the previous step. From this analysis, the modal participation factor PF, the fundamental period of the structure, T_E , and the displacement shape ϕ of the fundamental mode are obtained. From this displacement shape, the spectral yield displacement of the reference SDOF, Sd_y, may be calculated using the following equation:

$$Sd_{y} = \frac{IDT_{y} H_{k}}{PF_{1}^{E} \left(\phi_{k,1}^{E} - \phi_{k-1,1}^{E}\right)}$$
(13)

where

 IDT_v = Yield interstorey drift

 H_k = Height of the critical storey k (where maximum drift occurs)

 PF_1^E = Modal participation factor of the fundamental mode (elastic structure)

 Φ_{k1}^{E} = Modal shape ordinate of the critical interstorey, k

The yield interstorey drift for a reinforced concrete structure may be calculated using the Eq. (14) [2]

$$\Delta_{\rm y} = \frac{0.5 \,\varepsilon_{\rm y} \,\mathrm{L}_{\rm 1}}{\mathrm{h}_{\rm v1}} \tag{14}$$

where

 $\varepsilon_{\rm v}$ = yield strain of the reinforcing steel

 L_1 = beam length

 h_{v1} = beam depth

3. Definition of a design damage distribution for the PL in accordance with the characteristics of the structure and the design demands, using a strong-column weak-beam strategy and considering the contribution of the viscous dampers added to the structure. Structural damage is introduced at the ends of the elements, where damage is accepted to occur under design conditions by adding hinges with zero or residual rotational stiffness, equal to a reduced bending stiffness of the damaged element sections.



4. Modal analysis of the damaged bare structure to obtain the fundamental modal shape, participation factor and period, T_D, and, from this period, the slope of the second branch of the idealized bilinear behaviour curve of the reference SDOF system. From this modal shape, the target spectral displacement of the reference SDOF, Sd_{PL}, is obtained using the following equation:

$$Sd_{PL} = \frac{IDT_{PL} H_{k}}{PF_{1}^{D} \left(\phi_{k,1}^{D} - \phi_{k-1,1}^{D} \right)}$$
(15)

where

 IDT_{PL} = Interstorey drift of PL

5. Calculation of the target yield and ultimate spectral displacements of the reference SDOF system, Sd_{PL} and Sd_y respectively, corresponding to the fundamental mode using the results of modal analysis. Its ductility, μ , and post-yielding to initial stiffness ratio, α , are obtained using Eqs. (16) and (17)

$$\mu = \frac{Sd_{PL}}{Sd_y} \tag{16}$$

$$\alpha = \left(\frac{T_{\rm E}}{T_{\rm D}}\right)^2 \tag{17}$$

6. Modification of the effective viscous damping ratio from the inelastic displacement spectrum for the given μ and α , until the spectral displacement associated to the fundamental period is equal to the target spectral displacement of the structure (step 4). See Fig. 1:



Fig. 1. Inelastic displacement spectrum

7. Calculation of the damping coefficients of the viscous dampers, using Eq. (9), where the proportional part (ξ_{DP1}) may be defined using the [5] proposal Eq. (18), and the residual part ξ_{DR1} as defined by Eq. 10. The damping coefficient for each interstorey is calculated in proportion to the relative modal displacement drifts normalized with the interstorey height of the building.



$$C_{t} = \frac{4 \pi \xi_{DPi} \left(\sum_{j=1}^{nd} \frac{\phi_{i} - \phi_{i-1}}{H_{j}} \right) \left(\sum_{i=1}^{n} m_{i} \phi_{i}^{2} \right)}{T \sum_{j=1}^{nd} (\cos \theta_{j}) \left(\frac{(\phi_{i} - \phi_{i-1})^{3}}{H_{j}} \right)}$$
(18)

$$C_{j} = \left(\frac{\left(\frac{\Phi_{i} - \Phi_{i-1}}{H_{j}}\right)}{\sum_{i=1}^{nd} \left(\frac{\Phi_{i} - \Phi_{i-1}}{H_{j}}\right)}\right) C_{t}$$
(19)

where

- H_i = Height of the storey j
 - 8. Determination of the yield strength, R_y/m , using the period of elastic model T_E in the inelastic strength spectrum, corresponding to the values of μ and α previously calculated, Fig. 2:
 - 9. Calculation of the ultimate strength, Ru/m, of the reference system using the following equation:



 $\frac{R_{u}}{m} = \frac{R_{y}}{m} [1 + \alpha (\mu - 1)]$ (20)

Fig. 2. Strength per unit mass spectrum for μ and $\alpha,$ associated to the PL

10. Determination of the design forces of the elements using the results of three different analyses: a gravity load analysis of the undamaged structure; a modal spectral analysis of the undamaged structure, using the elastic design spectrum scaled by the ratio of the strength per unit mass at the yield point of the behaviour curve and the elastic pseudo-acceleration for the initial period, λ_E , and a modal spectral analysis of the damaged structure, using the elastic spectrum scaled by the ratio of the difference of ultimate and yield strengths per unit mass and the pseudo-acceleration for the period of the damaged structure, λ_D . Each modal spectral analysis considers the total viscous damping ratio (ξ_{Design}). The



design forces are obtained by adding the forces due to gravity loads and the forces of the modal spectral analyses of the undamaged and damaged structure.

11. Determination of the design of every structural element in accordance with the forces obtained from the analysis of the simplified models and using the applicable design rules. The design process must be carried out in such a way that the design criteria of the code do not alter significantly the expected performance.

4. Application examples

To illustrate the application of the design method developed in this paper, four structures, was designed. The nominal properties of the materials used in the design are for the concrete, a compressive strength $f'c=3.00x10^4$ kN/m², a modulus of elasticity Ec=27.00x10⁶ kN/m², and a weight density $\gamma=23.53$ kN/m³, and for the steel reinforcement a yield stress fy = 4.50x105 kN/m² and a modulus of elasticity Es = 2.00x108 kN/m². Based on the results of the preliminary design of the frame, the sections of the structural elements were defined (Fig. 3).

To validate the results, obtained from the displacement-design method proposed, the seismic design demand considered was the response spectra of the EW component of the 1985 the earthquake in Mexico recorded at the SCT site. To validate the seismic performance of the designed structure, the drifts, obtained from the non-linear step-by-step analysis of the structure, were compared with the maximum drift considered as design target. The non-linear step-by-step analysis was carried out with the Perform 3D V5 [11], program with the following considerations: 1) nearly elasto-plastic bilinear stable hysteretic behaviour for all beams and columns, 2) non-classical damping matrix due to the incorporation the viscous dampers, 3) nominal yield moments for beam and columns obtained from the design method proposed and 4) the drift considered as design target was 0.015, as specified by [12] to satisfy the LSLS.

5. Design results

Figure 4 shows the maximum interstorey drifts calculated using step by step analyses for four different cases: (a) bare structure (SA), (b) structure with viscous damping (AV), (c) structure design with the damping coefficients calculated by procedure proposed (PP) and (d) structure design with the damping coefficients calculated by procedure [5]. It may be observed that when the bare structure is subjected to the design seismic demand corresponding to the LSLS, some interstoreys lightly exceed the target drifts for this limit state (IDT LSLS). However, when the viscous dampers are added to the structure, the interstorey drift prescribed by the code is not exceeded at any interstoreys. In addition, the procedure proposed gives a better approximation when is compared with the results of the non-linear step-by-step analysis (see Fig. 4). Regarding the damage distributions, the results of the non-linear step-by-step analysis.

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Fig. 3 Application examples





Fig. 4 Interstorey drifts in the application examples.

6. Conclusions

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This paper presented the formulation and a practical application of a new displacement-based seismic evaluation/design method for structures equipped with viscous dampers and/or when these devices are used as a retrofit measure. To validate the design method proposed, the performance of the designed structure was obtained using non-linear step-by-step dynamic analysis considering as seismic demand the same demand used for its design. From the analysis of the results obtained the following conclusions may be extracted:

1. For the structures designed with the procedure proposed, the maximum interstorey drifts, produced by the design demand applying the correction, are closer to those obtained from the non-linear step-by-step analysis of the structure, considering a non-proportional damping matrix, than the ones computed through the procedure without the correction. Nevertheless, considering that the design interstorey drift is recommended to limit the damage of the LSLS, both results can be considered satisfactory. However, to guarantee this conclusion it is necessary to carry out additional evaluation/design examples, considering structures of different configurations and subjected to different seismic demands.



- 2. The distribution of damage, used as target within the design procedure proposed was reproduced in the results of non-linear step-by-step analysis of the structure with a non-proportional damping matrix. This result was due to the correction to the added damping coefficient in the procedure, something that guaranteed that the response of the structure was reduced, and the accomplishment of the design objective of controlling the distribution of damage in the structural elements.
- 3. The comparison of the effort involved in the application of this method using the computational tools available in most design offices, e.g., SAP 2000[13] and the quality of results obtained compared with those of other design methods, place it as an excellent design tool, since requires only two modal spectral analyses.

7. Conclusions

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