NEW ESTIMATION OF ULTIMATE SHEAR STRENGTH ON HEADED STUD CONSIDERING VARIABILITY

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Abstract

The formula of ultimate shear strength of a headed stud is shown in the design recommendations in US, Europe, and Japan respectively. However, the above design formula is not always corresponded to the ultimate shear strength obtained by the push-out tests. One of the reasons is that high-strength material such as concrete and headed studs, and the studs with large diameter being used recently are not considered in the design formula. Another reason is the variation of concrete characteristics affects the ultimate shear strength of headed studs. In previous studies, some calculation method of ultimate shear strength on headed studs had been considered, but these methods are not completely solved the issues.

Authors had been examined a calculation method of the ultimate shear strength of a headed stud to be able to use in case that there is more wide range of material characteristics by the database of push-out tests on headed studs. The calculation method was based on the multiple regression analysis between the ultimate shear strength and the five elements; i.e., cross sectional area of a headed stud ($_{sc}a$), Young's modulus of concrete (E_c), compression strength of concrete (F_c), the ratio of length to diameter of a headed stud (L/d), and tensile strength of a headed stud (F_u). The calculation method grasped the center of variation of the ultimate shear strength obtained by the push-out tests because the average values of the above five elements in the database were used to simplify the formula. However, the variation of ultimate shear strength was not estimated sufficiently in this method as similar as previous researches.

This study proposed the new evaluation of the ultimate shear stress of a headed stud. The proposed formula of the ultimate strength in this consideration had advantage to estimate the test results with wide range of material characteristics closer than the formulas in previous researches. This formula was modified by using the minimum values of the five elements in the database to be simplified. Furthermore, the estimation method was established according to statistical calculations and probability distributions. Based on the estimation by the trend whether the ultimate shear stress exceed than the formula of ultimate shear stress in design recommendations and our proposed formula, the structural designers can understand which formula is safer. Also, the proposed method was useful because structural designers can judge the number and construction detail of headed studs according to the probability the ultimate shear stress of a headed stud.

Keywords: Headed stud; Ultimate shear strength; Multiple regression analysis; Probability distributions

1. Introduction

Headed studs are well used as shear connectors between concrete and steel members. The formula of ultimate shear stress of a headed stud is shown in the design recommendations [1], [2] as followings;

$$_{d}q_u / {_{sc}a} = 0.5\sqrt{F_c \cdot E_c} \tag{1}$$

where, ${}_{sc}a$: Cross sectional area of a headed stud, E_c: Young's modulus of concrete, F_c: Designed compressive strength of concrete. The effective range of Eq. (1) is $\mathbf{500\ N/mm^2} \leq \sqrt{F_c \cdot E_c} \leq \mathbf{900}$ N/mm^2. If $\sqrt{F_c \cdot E_c} > \mathbf{900}$ N/mm^2, q_u keeps the value as $450 \cdot {}_{sc}a$.

However, the shear strength calculated by Eq. (1) is not corresponded to the ultimate shear strength obtained by the push-out tests which is calculated the product of the shear stress and cross sectional area of a headed stud as shown in Fig.1. This figure was that the comparison of these two shear strengths along 183 specimens based on the database of push-out tests of headed stud with flat slab [3]. The shear strengths obtained by the push-out tests have variation against the shear strength calculated by Eq. (1). One of the reasons of the difference between these two shear strengths is the variations of the elements that affect to the shear strength. Especially, the compressive strength of concrete and the Young's modulus of concrete affect the difference. Fig.2 shows the relationship of between the shear stress of a headed stud and the square root of the product of the compressive strength of concrete and the Young's modulus ($\sqrt{F_c \cdot E_c}$) in the 183 specimens [3]. The broken line in Fig.2 shows the value of Eq. (1). As shown Fig.2, if $\sqrt{F_c \cdot E_c}$ is less than 900 N/mm^2, the shear stresses have large variation in any $\sqrt{F_c \cdot E_c}$ values. In this range of $\sqrt{F_c \cdot E_c}$, the shear stresses obtained by the push-out tests tend to be overestimated. On the other hand, the shear stresses in the push-out tests using a headed stud of 25mm in the diameter are underestimated. In case that $\sqrt{F_c \cdot E_c}$ is larger than 900 N/mm^2, most of the shear stress in the push-out tests exceeds the shear stress in Eq. (1).

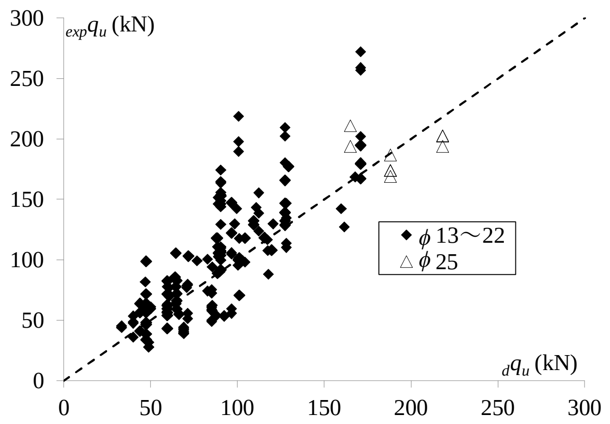


Fig. 1 – Comparison of ultimate shear strength between the test results and the designed values

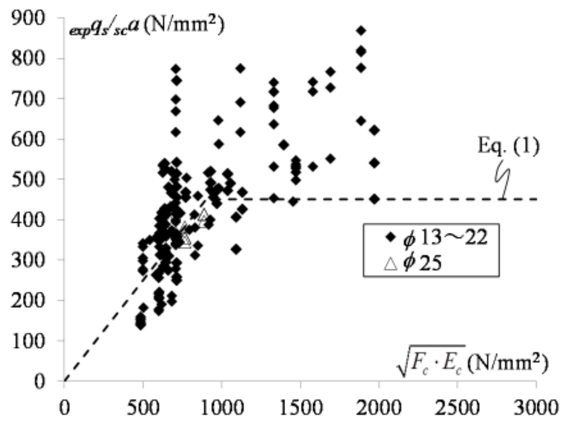


Fig. 2 – Distribution of ultimate shear stress obtained by the test results

The large variation of the real shear stress against the shear stress in Eq. (1) have already pointed out in previous studies [3-4]. In order to decrease the difference between the above two shear stress, some calculation method of the ultimate shear strength of a headed stud were proposed [3-5]. These proposed formulas are estimated whether they match the shear strength obtained by the push-out tests or not, not considering the distributions of elements. Thus, these proposed formulas were not completely reflected the variations of the elements. Since the strengths in composite structures using concrete generally have variations, the probability distribution is useful to estimate whether the strengths obtained in tests are reasonable or not based on the previous studies. For headed studs, the probability of ultimate shear strength of a headed stud is also helpful to consider how much strength, size, and position are needed in the design of structure with headed studs.

This study proposed the new estimation method of the ultimate shear strength of headed stud by statistical approach. At the beginning of study, new formula of the ultimate shear strength was proposed by previous database. To consider variation of the proposed formula, new database of the push-out tests of a headed stud was established. Based on the new database, the distributions of five elements (cross sectional area of a headed stud ($_{sc}a$), Young's modulus of concrete (E_c), compressive strength of concrete (F_c), the ratio of length to diameter of a headed stud (L/d), and the tensile strength of a headed stud (F_u) were calculated. Secondary, the expected value of the ultimate shear stress of a headed stud was estimated by the multiple regression analysis for the above five elements in the above database. The multiple regression analysis was also used to calculate the distribution and variance of the ultimate shear stress of a headed stud when it was a function of random multiple variables. Assuming that the logarithmic value of ultimate shear stress has the normal distribution, the probability of the ultimate shear stress of a headed stud was proposed by the calculation of the distribution of elements.

Using the proposed probability function, the probabilities that the ultimate shear stress of a headed stud was exceeded the shear stress in Eq. (1), or the proposed formula in this study were calculated. Considerations using these probabilities showed that the probability increases if L/d is larger, however, this increasing of probability is not linear accompanying with L/d. Compared to these two probabilities, the proposed formula in this study is effective to estimate more safely. The estimation using probability in this study is useful to understand whether the combination of elements in headed stud is safe or not. Also, the proposed method is the first step to estimate the shear stress of a headed stud reflecting variations of elements in composite structures.

2. Proporsal of formula for the shear strength of a headed stud by regression analysis

Authors had been shown a calculation method of the ultimate shear strength of a headed stud based on the multiple regression analysis [3]. The shear strength by this method, multiplying the modified coefficient to the value of Eq. (1), generally matched the ultimate shear strength obtained by the push-out tests because the average values of the five elements, i.e., cross sectional area of a headed stud ($_{sc}a$), Young's modulus of concrete (E_c), compressive strength of concrete (F_c), the ratio of length to diameter of a headed stud (L/d), and the tensile strength of headed stud (F_u). The method using the modified coefficient is easy to calculate the shear strength, however, simply using the results of multiple regression analysis is more effective to estimate the ultimate shear strength of a headed stud. The results of multiple regression analysis based on the database in [3] is shown in Eq. (2).

$$q_u = \mathrm{e}^{-6.93} \cdot {_{sc}a}^{1.05} \cdot E_c^{0.30} \cdot F_c^{0.32} \cdot (L/d)^{0.52} \cdot F_u^{0.12} \quad \text{(kN)} \tag{2}$$

Eq. (2) is hard to use for structural engineers because the calculation of index is complicated. Based on the consideration in [3], the product of the elements is simplified by changing some real numbers. Here, $_{sc}a$, E_c, F_c, and L/d are set as variables being valid to the first decimal place of the index. While, the parts of $_{sc}a$, E_c, F_c, and L/d having the index under the second decimal place are changed to real numbers by exponential calculation that the base is the minimum value of each element in database [3] and the exponent is each index under the second decimal place. F_u is also changed to real numbers by similar exponential calculation as described the above. Obtained formula is shown in Eq. (3).

$$q_u = 2.75 \cdot {_{sc}a} \cdot E_c^{0.3} \cdot F_c^{0.3} \cdot (L/d)^{0.5} \quad \text{(kN)} \tag{3}$$

Fig. 3 shows the comparison between the ultimate shear strength calculated by Eq. (3) and the ultimate shear strength obtained by the push-out tests. The calculated strengths by Eq. (3) are closer to the ultimate shear strength in push-out tests than using the obtained strength by Eq. (1). One of the advantages using Eq. (3) is that the limitation of $\sqrt{F_c \cdot E_c}$ in Eq. (1) can be ignored. The correlation coefficient between the calculated strengths by Eq. (3) and the strengths obtained by the test results is 0.89 though the correlation coefficient between the calculated strengths obtained by Eq. (1) and strengths obtained by the test results is 0.81. Using Eq. (3) can improve the formula of ultimate shear strength as described above, however, the variation of the shear strengths obtained by the test results exists against the calculated strengths by Eq. (3). The estimation of ultimate shear strength according to the variations of the elements is needed because only proposal a formula of ultimate shear strength cannot be estimated its variation.

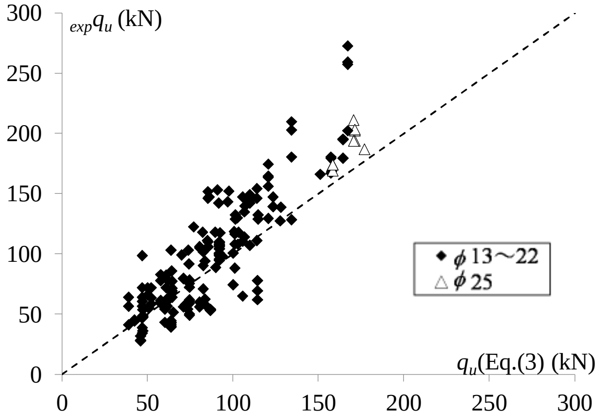


Fig. 3 – Comparison between the test results and the calculated values by Eq. (3)

3. Distribution of the elements affected shear stress of a headed stud

In order to consider the variation of ultimate shear strength of a headed stud, the new database is established. Based on the previous push-out tests in Japan [6-27], 129 specimens without metal deck are recorded in this database with the following information; the ultimate shear strength obtained push-out tests (${}_{exp}q_u$), cross sectional area of a headed stud (${}_{sc}a$), Young's modulus of concrete (E_c), compressive strength of concrete (F_c), the ratio of length to diameter of a headed stud (L/d), and the tensile strength of headed stud (F_u). These five elements i.e., ${}_{sc}a$, E_c, F_c, L/d, and F_u, were recorded because they are used to the designed formulas in specification or code in Japan, U.S., or Europe [1-2,28]. The reason of selecting these 129 specimens is that all of the five elements (${}_{sc}a$, E_c, F_c, L/d, and F_u) were obtained in the references. Especially, F_c is the results of concrete compressive test and F_u is the results of tensile test in stud material, not designed nor nominal values. All of the E_c in new database is unified as the value calculated by Eq. (4) [29] because the calculation formula of Young's modulus differs in each previous push-out test.

$$E_c = 33500 \cdot (\gamma/24)^2 \cdot (F_c/60)^{1/3} \tag{4}$$

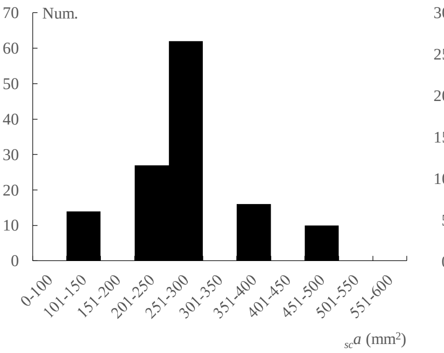
where, γ: Air-dried density (γ=23.0 kN/m^3 when $F_c \leq 36$ N/mm^2, γ=23.5 kN/m^3 when 36 N/mm$^2 < F_c \leq$ 48 N/mm^2, and γ=24.0 kN/m^3 when 48 N/mm$^2 < F_c$).

In the following considerations based on new database, the failure modes of specimens in the push-out tests are neglected. It is because that the multiple failure mode was mixed in some specimens and the failure modes were not recorded in 45% of the specimens in new database.

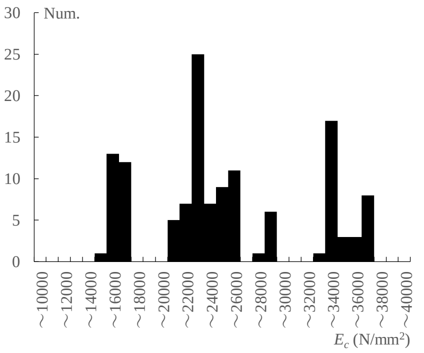
Table 1 shows the value ranges of the five elements in new database. The compressive strength of concrete has large range from lightweight concrete to high-strength in normal weight concrete. The diameters of a headed stud in new database is only five, i.e., 13 mm, 16 mm, 19 mm, 22 mm, and 25 mm based on the Japanese Industrial Standards (JIS B 1198 [30]). Both the concrete which is larger than 60 N/mm^2 in the compressive strength and the headed stud which is larger than 550 N/mm^2 in the tensile strength are included in new database. The histograms of the five elements are shown in Fig. 4. The diameter of a stud in the database is only five, therefore, the values of ${}_{sc}a$ are discrete. The values of other elements are continuous, however, the trend of some of the histograms such as the E_c's are not simple normal distributions.

Table 1 – Value range of the five elements in the database

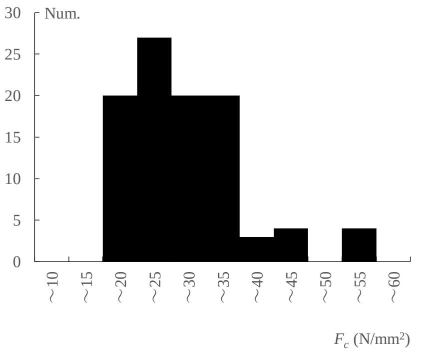
Diameter of a stud d (mm)	13, 16, 19, 22, 25				
${}_{sc}a$ (mm^2)	132.7～490.9				
Concrete kinds	Whole	(Detail)			
		Lightweight	Mortal	Normalweight	Normalweight (compressive strength is over 60 N/mm^2)
Number	129	33	15	61	20
F_c(N/mm^2)	15.5～99.5	15.5～34.5	42.8～99.5	18.2～54.4	60.3～79.7
E_c (×10^{4}N/mm^2)	1.48～3.68	1.48～2.34	2.75～3.64	2.07～3.24	3.35～3.68
L/d	3.1～9.4	3.9～7.9	3.1～9.4	3.6～7.7	3.6～6.3
F_u(N/mm^2)	349.1～826.7	349.1～533.5	402.0～687.4	420.0～559.0	470.7～826.7



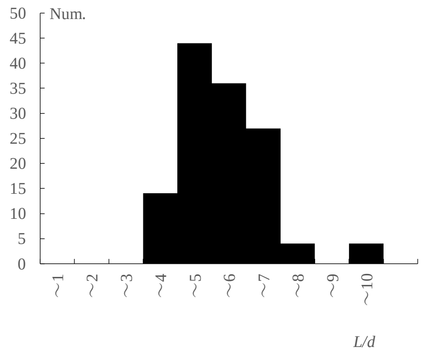
(a) Cross sectional area of a headed stud (${}_{sc}a$)



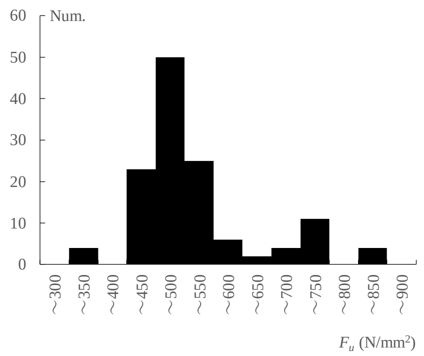
(b) Young's modulus of concrete (E_c)



(c) Compressive strength of concrete (F_c)



(d) The ratio of length to diameter of a headed stud (L/d)



(e) Tensile strength of headed stud (F_u)

Fig. 4 – Histogram of the five elements

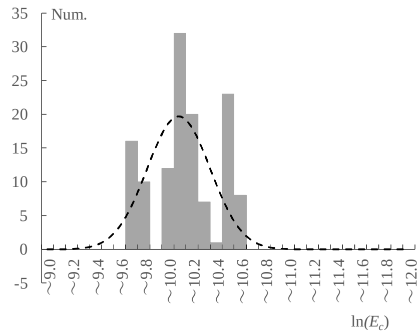
For the probability of ultimate shear strength of a headed stud, the distributions of the five elements are needed. Since the values of ${}_{sc}a$ is discrete as shown before, the distribution of ${}_{sc}a$ is treated as 'the ultimate shear stress' which divided the ultimate shear strength to cross sectional area (${}_{exp}q_u/{}_{sc}a$) in followings. Considering the multiple regression analysis which describes later (in Chapter 4), the four elements (E_c, F_c, L/d, and F_u) are supposed their logarithmic values can expressed some distribution. Fig. 5 shows the histograms of logarithmic values of the four elements (E_c, F_c, L/d, and F_u). The broken lines in Fig.4 are probability density function as assuming normal distribution. All of the histograms are close to the normal distribution. From these distributions, probability density functions of logarithmic value in the four elements (E_c, F_c, L/d, and F_u) are obtained by Eqs. (5)-(8) as followings;

$$f\{\ln(E_c)\} = \frac{1}{\sqrt{2\pi\cdot(0.26)^2}} \cdot exp\left\{-\frac{1}{2}\left(\frac{\ln(E_c)-10.1}{0.26}\right)^2\right\} \tag{5}$$

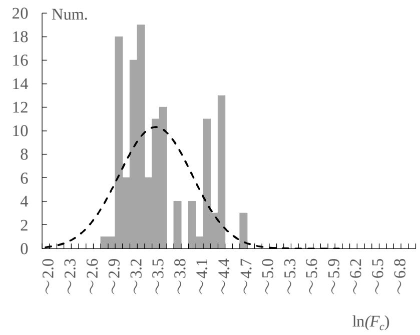
$$f\{\ln(F_c)\} = \frac{1}{\sqrt{2\pi(0.53)^2}} \cdot exp\left\{-\frac{1}{2}\left(\frac{\ln(F_c)-3.50}{0.53}\right)^2\right\} \tag{6}$$

$$f\{\ln(L/d)\} = \frac{1}{\sqrt{2\pi(0.18)^2}} \cdot exp\left\{-\frac{1}{2}\left(\frac{\ln(L/d)-6.23}{0.18}\right)^2\right\} \tag{7}$$

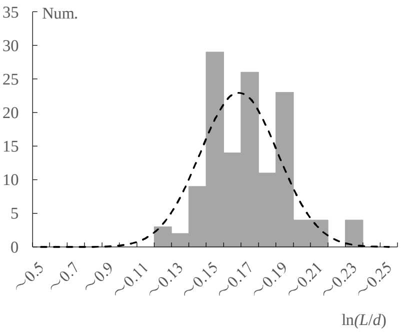
$$f\{\ln(F_u)\} = \frac{1}{\sqrt{2\pi(0.22)^2}} \cdot exp\left\{-\frac{1}{2}\left(\frac{\ln(E_c)-1.64}{0.22}\right)^2\right\} \tag{8}$$



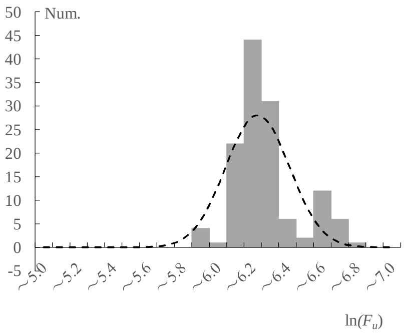
(a) Young's modulus of concrete (E_c)



(b) Compressive strength of concrete (F_c)



(c) The ratio of length to diameter of a headed stud (L/d)



(d) Tensile strength of headed stud (F_u)

Fig. 5 – Histograms and distributions of logarithmic value of the four elements

4. Proposal of the probability function of the ultimate shear stress of a headed stud

In previous studies [3], the proposed formulas of the ultimate shear strength of a headed stud were often shown based on the multiple regression analysis between the ultimate shear strength obtained push-out tests and the five elements ($_{sc}a$, E_c, F_c, L/d, and F_u). This study also conducted multiple regression analysis to understand the relationships between the ultimate shear stress and the four elements based on new database. Note that the random variable of the regression analysis is not the ultimate shear strength but the ultimate shear stress ($_{exp}q_u/_{sc}a$). Including the discrete distribution of $_{sc}a$ as shown in Chapter 3 in the ultimate shear stress is effective to express as the calculation using the distributions of the four elements. While, the predictor variables are the other four elements (E_c, F_c, L/d, and F_u).

If the logarithmic value of ultimate shear stress assumed to have a linear function, it is shown in Eq. (9) using the logarithmic values of the four elements based on the multiple regression analysis.

$$\ln\left({}_{exp}q_u/{}_{sc}a\right) = -19.85 + 2.52\ln(E_c) - 0.68\ln(F_c) + 0.24\ln(L/d) + 0.36\ln(F_u) \quad (9)$$

According that absolute values of T-test values in all of the four elements in the multiple regression analysis are over 2.0, the four elements are reasonable as the predictor variable. To simplify Eq. (9), the logarithmic values replace to other variables, i.e., Y, X, and the constant coefficients, i.e., a_i. the replaced function of the logarithmic value of ultimate shear stress is shown in Eq. (10).

$$Y = a_0 + a_1 \cdot X_1 + a_2 \cdot X_2 + a_3 \cdot X_3 + a_4 \cdot X_4 \quad (10)$$

If the logarithmic value of the four elements have any values as X_i=x_i, the logarithmic values of the ultimate shear stress (Y_i) can be written in Eq. (11);

$$Y_i = a_0 + a_1 \cdot x_1 + a_2 \cdot x_2 + a_3 \cdot x_3 + a_4 \cdot x_4 + E_i \quad (11)$$

where, E_i is a random variable representing prediction error of the ultimate shear stress. If E_i are assumed to have a normal distribution with zero mean and with constant variance despite of the values of X_i, the expected value of the logarithmic values of the ultimate shear stress $\mathrm{E}[\mathrm{Y}]_X$ is shown as Eq. (12).

$$\mathrm{E}[Y]_X = a_0 + a_1 \cdot \mathrm{E}[X_1] + a_2 \cdot \mathrm{E}[X_2] + a_3 \cdot \mathrm{E}[X_3] + a_4 \cdot \mathrm{E}[X_4] = a_0 + \textstyle\sum_{i=1}^{4} a_i \cdot \mathrm{E}[X_i] \quad (12)$$

where, the constant values a_0 and a_i (i=1-4) are already known by the result of multiple regression analysis as shown in Eq. (9), that is;

$$a_0 = -19.85,\ a_1 = 2.52,\ a_2 = -0.68,\ a_3 = 0.24,\ a_4 = 0.37 \quad (13)$$

Assuming lognormal distribution, the probability of logarithmic value of the ultimate shear stress exceeds any logarithmic stress value; z, which is defined by a series of predictor variables. The probability function is shown in Eq. (14).

$$P\left[{}_{exp}q_u/{}_{sc}a > z | X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4\right] = 1 - \Phi\left(\frac{\ln(z) - E[Y]_X}{\sigma_Y}\right) \quad (14)$$

Here, σ_Y is the variance of the logarithmic value of the ultimate shear stress. This variance is shown in Eq. (15) by considering the correlations in the four elements (E_c, F_c, L/d, and F_u).

$$\sigma_Y^2 = \sum\sum a_i\, a_j \rho_{X_i X_j} \sigma_i \sigma_j \quad (15)$$

where, ρ_{ij} is collation between any two elements, σ_i is the variance of any element.

According to Chapter 2, the variances of the four elements are known by distributions in each element as followings;

$$\begin{cases} \sigma_{X_1} = \sigma_{\ln(E_c)} = 0.26 \\ \sigma_{X_2} = \sigma_{\ln(F_c)} = 0.53 \\ \sigma_{X_3} = \sigma_{\ln(L/d)} = 0.22 \\ \sigma_{X_4} = \sigma_{\ln(F_u)} = 0.18 \end{cases} \quad (16)$$

The variance of the logarithmic value of the ultimate shear stress is calculated by Eq. (13), (15), and (16).

$$\sigma_Y^2 = \sum\sum a_i\, a_j \rho_{X_i X_j} \sigma_i \sigma_j = 0.21$$

$$\therefore \sigma_Y = 0.45 \quad (17)$$

The probability of the ultimate shear stress exceeds the stress value which is defined by a series of predictor variables is calculated by Eq. (14) and (17).

$$P\left[{}_{exp}q_u/{}_{sc}a > z \mid X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4\right] = 1 - \Phi\left(\frac{\ln(z) - E[Y]_X}{0.45}\right) \tag{18}$$

5. Estimation of the ultimate shear stress of a headed stud by the proposed probability function

Previous studies have pointed out the variation of ultimate shear stress between the shear stress in Eq. (1) and the test results as described in Chapter 1. Estimating the variation of ultimate shear stress is important for more safer design of structures using headed studs, however, the estimation method have not been shown. A simple estimation of the variation is whether the ultimate shear stress obtained by the push-out tests tend to exceed the shear stress obtained by known formula or not. If a test result exceeds the shear stress in Eq. (1), the test result can be estimated as safe based on Eq. (1), in contract, a test result is below the shear stress in Eq. (1), the test result can be estimated as danger. Also, if the comparison of estimation changes from Eq. (1) to other formula such as Eq. (3), the calculation formula of the ultimate shear stress can be estimated. That is, the calculation formula with high possibility is considered safer.

In this chapter, the trend of the ultimate shear stress exceeding a shear stress is shown based on the probability distribution. The conditions of elements corresponding the trend is also described.

5.1 Estimation by the probability of the ultimate shear stress against the shear stress in design recommendations

In this section, the case that the comparison of estimation is the shear stress in Eq. (1) is considered. The variables to obtain the ultimate shear stress in Eq. (1) are E_c and F_c. The probability which the ultimate shear stress by test results exceeds the ultimate shear stress in Eq. (1) can be written as shown in Eq. (19).

$$P\left[\left({}_{exp}q_u/{}_{sc}a\right) > \left({}_{d}q_u/{}_{sc}a\right) \mid \sqrt{F_c \cdot E_c} = s\right] = 1 - \Phi\left(\frac{\ln\left({}_{d}q_u/{}_{sc}a\right) - E[{}_{exp}q_u/{}_{sc}a]_s}{0.45}\right) \tag{19}$$

where, s is any real value determined by E_c and F_c. In Eq. (19), L/d and F_u are assumed any constants.

The value of $\sqrt{F_c \cdot E_c}$ can express as a function of compressive strength of concrete (F_c) since Young's modulus (E_c) is calculated by F_c as shown in Eq. (4). The relationship between F_c and $\sqrt{F_c \cdot E_c}$ is drawn in Fig. 6 when F_c is set from 0 N/mm^2 to 60 N/mm^2. The broken line in Fig.6 is the trendline between F_c and $\sqrt{F_c \cdot E_c}$. The function of trendline can obtained as shown in Eq. (20).

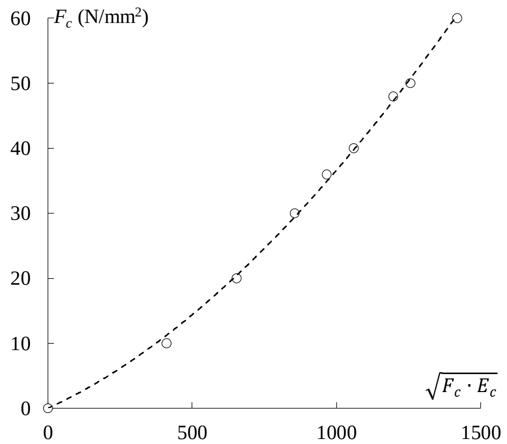


Fig. 6 –Relationship between F_c and $\sqrt{F_c \cdot E_c}$

$$F_c = -9.28 \times 10^{-9} \cdot \{\sqrt{F_c \cdot E_c}\}^3 + 3.41 \times 10^{-5} \cdot \{\sqrt{F_c \cdot E_c}\}^2 + 0.01 \cdot \sqrt{F_c \cdot E_c} \quad (20)$$

According to Eq. (20), F_c and E_c are calculated individually if $\sqrt{F_c \cdot E_c}$ have any value. That is, the expected value of the ultimate shear stress can be obtained in Eq. (12). It also means that the probability of ultimate shear stress can be considered if $\sqrt{F_c \cdot E_c}$ have any value.

First, the tensile strength of headed stud (F_u) is assumed constant, i.e., 400 N/mm^2 (this is the minimum of nominal tensile strength in general headed studs in Japan). Fig.7 shows the relationship between F_c and the probability of ultimate shear stress. In the range of F_c under 32.0 N/mm^2, the probabilities decrease as F_c is larger. While, in the range of F_c over 32.0 N/mm^2, the probabilities increase as F_c is larger. This switch of probability because the ultimate shear stress in Eq. (1) remains constant according that $\sqrt{F_c \cdot E_c}$ is larger than 900 N/mm^2. Changing the ratio of length to diameter of a headed stud (L/d) from 3.0 to 10.0, the probabilities grow as L/d is larger despite of the value of F_c.

Secondary, L/d is assumed constant, i.e., 5.0 (this value is close to mean of distribution in L/d). Fig.8 shows the relationship between F_c and the probability of ultimate shear stress under the assumption. The curve shapes of probabilities are similar to the shapes in Fig. 7. Changing F_u from 400 N/mm^2 to 550 N/mm^2, the probabilities grow as F_u is larger despite of the value of F_c.

Here, focusing on the probability if L/d is 5.0 and F_u is 400 N/mm^2. The probability is 58% if F_c is 17.7 N/mm^2 and the probability is 41% if F_c is 32.0 N/mm^2. Based on Eq. (1), the results of push-put tests with general headed stud tend to occur failure with about 50% probability in the range of F_c using in general floor slab or middle-low rise buildings. Also, compressive strength of concrete in cylinder tests is often larger than designed strength, the probability tends to be lower by using Eq. (1).

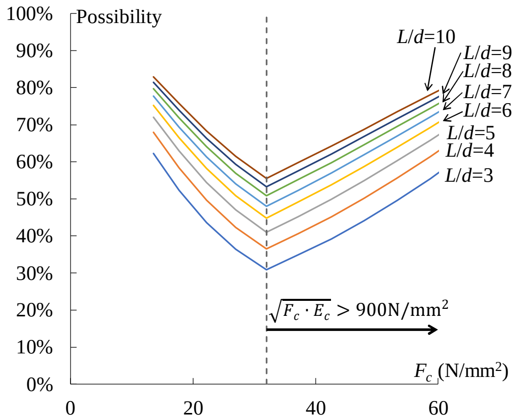


Fig. 7 – Probabilities of ultimate shear stress against Eq. (1) (case of changing L/d)

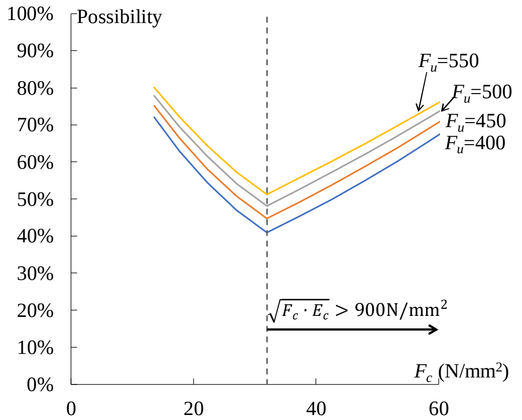


Fig. 8 – Probabilities of ultimate shear stress against Eq. (1) (case of changing F_u)

5.2 Estimation by the probability of the ultimate shear stress against the shear stress by the proposed formula

As similar to 5.1, the case that the comparison of estimation is the shear stress in Eq. (3) is considered in this section. The probability which the ultimate shear stress by test results exceeds the ultimate shear stress in Eq. (3) can be written as shown in Eq. (21).

$$P\left[\left({}_{exp}q_u/{}_{sc}a\right) > \left({}_{prop}q_u/{}_{sc}a\right) \middle| \sqrt{F_c \cdot E_c} = s\right] = 1 - \Phi\left(\frac{\ln\left({}_{prop}q_u/{}_{sc}a\right) - E[{}_{exp}q_u/{}_{sc}a]_s}{0.45}\right) \quad (21)$$

where, s is any real value determined by E_c and F_c. In Eq. (21), L/d and F_u are assumed any constants.

E_c is calculated if $\sqrt{F_c \cdot E_c}$ have any value by Eq. (20). Fig.9 shows the relationship between F_c and the probability of ultimate shear stress in case that F_u is assumed constant (400 N/mm^2). The probabilities are not shown significant change because the ultimate shear stress in Eq. (3) can neglect the limitation of $\sqrt{F_c \cdot E_c}$. Changing L/d from 3.0 to 10.0, the probabilities decrease as L/d is larger despite of the value of F_c. This trend is opposite to case that probabilities obtained using Eq. (1).

If the ratio of length to diameter of a headed stud (L/d) is assumed as constant, 5.0, Fig.10 shows the relationship between F_c and the probability of ultimate shear stress. Similar trend that the probabilities grow as F_u is larger despite of the value of F_c is observed.

Here, focusing on the probability if L/d is 5.0 and F_u is 400 N/mm^2 as similar as 5.1, the probability is 67% if F_c is 17.7 N/mm^2 and the probability is 59% if F_c is 32.0 N/mm^2. The probability is about 10% larger than the case using Eq. (1). Also, the probability in the case using Eq. (3) is almost larger than 60% in whole range of F_c. This means that the proposed formula is effective to estimate more safely the shear stress of a headed stud.

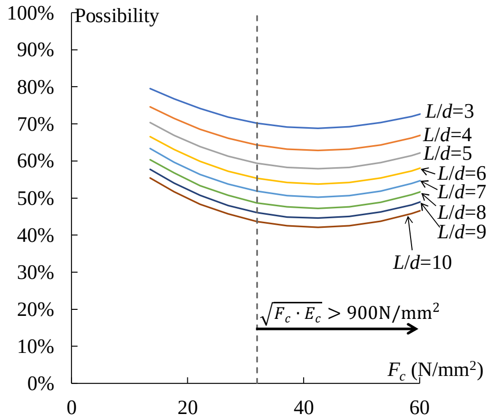


Fig. 9 – Probabilities of ultimate shear stress against Eq. (3) (case of changing L/d)

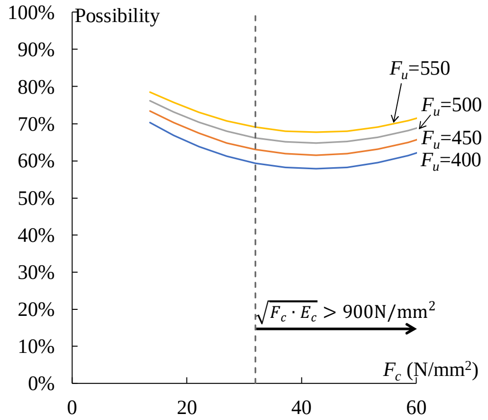


Fig. 10 – Probabilities of ultimate shear stress against Eq. (3) (case of changing F_u)

Note that the probability which the ultimate shear stress by test results exceeds a shear stress as shown above is based on Eqs. (4) and (20). Thus, the trend of probability is strongly controlled by the calculation of E_c by F_c and the effective range of F_c in the calculation. Especially, the calculation of Young's modulus varies depending on the standards or codes in each country and region. To obtain the probability with higher accuracy, these differences and effects for probability should also be estimated.

6. Conclusions

Headed studs in composite structure are affected by variation of material and construction detail, however, the formulas of the ultimate shear strength of a headed stud in previous studies and specifications have not considered these variations. This study proposed a new formula based on the previous database and multiple regression analysis. For estimation of the ultimate shear stress of a headed stud reflecting the distributions of five elements (${}_{sc}a$, E_c, F_c, L/d, and F_u), the probability of the ultimate shear stress of a headed stud is established based on new database. The obtained probability is useful to understand the trend compared to designed stress value or other specific values. Proposed method is strongly based on the database of push-out test of headed studs, thus, it is necessary to expand the data ranges and number of the database to improve accuracy of the estimation. In addition, the effect of calculation of E_c for the proposed method should be considered in order to establish general estimation method including not only Japanese database but also database in other countries and regions.

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