Derivation and Verification of RC Member Yield Deformation Including Bar Slippage and Shear Deformation Effects

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Abstract

In Japan, the capacity spectrum approach is widely used to predict the response of a structure subjected to a given earthquake motion. One major aspect of this is to predict a reduction factor, F_h, which accounts for the influence of hysteretic damping. In order to have a more reliable prediction of F_h, there is a need to estimate the yield displacement accurately. However, it is common practice to only consider flexural deformations when estimating yield displacement, and bar slippage and shear deformation effects are often ignored. Therefore, there is a need to consider all three effects for a more accurate prediction of yield response.

In this paper, the yield deformation of reinforced concrete (RC) members is separated into three independent deformation types; bar slippage, shear and flexural deformations. These are calculated separately considering basic member and material properties such as flexural stiffness, dimensions, and material strength. The sum of the displacements resulting from the individual deformation types is regarded as the yield deformation. To verify this method, a database of past results of experiments of tests performed in Japan on RC beams and columns was developed, and the experimental yield deformation from database was compared against those predicted by considering the three yield deformation components. In addition, comparisons were also made with other yield displacement estimation methods by Sugano (1973) and Priestley (1996). Parametric analyses of specific key parameters, such as bar anchorage length and the length of the hinge region, were performed.

Comparisons with the database show that the proposed method provided a reasonable prediction of the actual yield displacement. Moreover, the method mentioned in this paper can estimate the yield deformation better than Sugano formula and Priestley's method according to comparisons with calculation results. According to parametric analyses, increasing the anchorage length results in an increase in bar slippage deformation, and subsequently the yield deformation also increases. Similar observations were made regarding increasing the length of the hinge region. Based on these results, the method adopted in this study can be used to reliably predict the yield deformation of RC beams and columns.

Keywords: Reinforced Concrete Members, Yield Deformation, Bar Slippage, Shear Effect, Parametric Analysis

1. Introduction

To evaluate the seismic performance of reinforced concrete (RC) structures, the relationship between deformation and load, particular the yield deformation and yield load, needs to be properly defined. Moreover, according to Building Standards Act in Japan, "the calculation of response and limit strength" is one of the methods which can be used for seismic design. In this method, the response of structure should be smaller than the limit strength. However, under a major earthquake, a building may dissipate energy when deforming inelastically (i.e. hysteretic damping), which would reduce the demand on the structure. This effect can be approximated by multiplying demands by a response reduction coefficient, F_h, to describe its reduction. Based on formulas provided in the Japanese Building Code, F_h is related to the ductility factor, which is the building's displacement response divided by the yield displacement. Therefore, in order to reasonably estimate the ductility factor, it is important to accurately predict the structure's yield deformation.

Several studies have proposed methods to calculate yield deformations. Sugano [1] proposed a regression formula in 1973 by applying regression analysis to a database of RC beams and column experimental results. However, this material could not be applied to beams and columns using high strength materials. Different from Sugano's method, Morita and Kaku [2] considered the deformation caused by bar slippage in beams, and suggested a method including this part, but it did not consider the deformation caused by shear. For shear deformation, Shen and Kabeyasawa [3], Nagasaki et al. [4] and Jiang and Kitayama [5] proposed methods to estimate shear deformation effects based on arch model or truss model, but these approaches are difficult to apply due to the large number of member properties variables required. Priestley et al. [6] separated the yield deformation into several parts, and according to test results, provided a simple formula to calculate it.

In this study, the yield deformation of RC members is separated into three independent deformation types; bar slippage, shear and bending cross sectional deformations. A database of beam and column experimental tests performed in Japan was developed and used to verify the accuracy of the proposed method. Additionally, Sugano formula [1] and Priestley's method [6] are used to compare with it to verify the accuracy. Finally, this paper shows some parametric analysis of this method.

2. Derivation of yield deformation

2.1 Model of yield deformation

During elastic response, the lateral displacement at the top of an RC component can be assumed as the sum of three components; flexure (δ_b), shear (δ_s), and bar slip (δ_x) as Fig 1 shown.

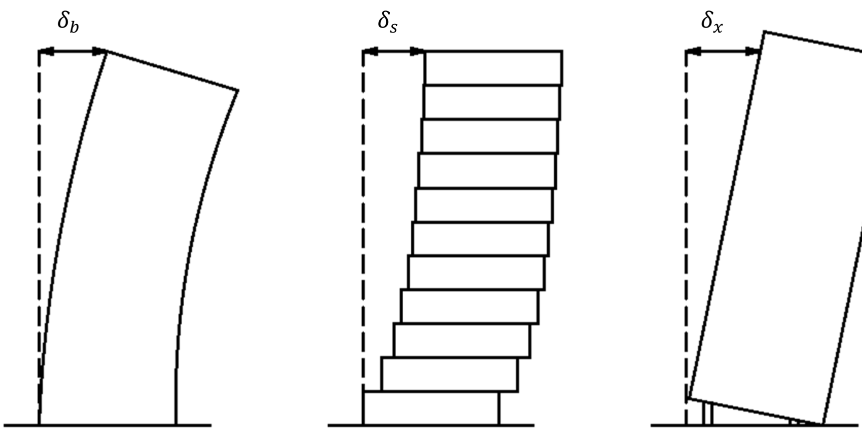


Fig. 1 –Three independent deformation types of RC component

If these lateral displacements are normalized by the member length, the drift rotation (hereby simply referred to as "deformation") can be defined directly by Eq. (1).

$$R_y = R_b + R_s + R_x \tag{1}$$

Where R_b is bending yield deformation, R_s is shearing deformation, and R_x is slip deformation. Estimation for each of the three parts are as follows.

2.2 Slip deformation

A simple geometric model, assuming the concrete member behaves as a rigid block and small angle theory, is proposed here to estimate the deformation caused by reinforcing bar slippage. Assuming the slip length is L_p, and the distance between the position of compression rebar and the tensile rebar is j,the slip deformation R_x can be expressed approximately by Eq. (2).

$$R_x = \frac{L_p}{j} \tag{2}$$

As the distribution of bond stress can be regarded as linear, the distribution of strain should also be linear. The slip length L_p in Eq. (2) can therefore be calculated by Eq. (3).

$$L_p = \frac{\varepsilon_y d_p}{2} \tag{3}$$

Where, ε_y is the strain of rebar and d_p is anchorage length. According to existing test results [7], the anchorage length is about 40 times of the diameter of the longitudinal reinforcement. This assumption will be adopted, though the influence of this will be examined in Section 5.1.

2.3 Shearing deformation

This study considers that concrete resists shear actions on its own, and according to elasticity theory, the relationship between the shear force, Q, and shearing deformation, R_s, can be expressed by Eq. (4).

$$Q = \beta_s \frac{GA}{\kappa L} \delta_s = \beta_s \frac{GA}{\kappa} R_s \tag{4}$$

Where, G is shear modulus, A is the gross cross-sectional area, κ is the shape-factor, and L is the shear span. Eq. (4) can be rearranged to obtain R_s as shown in Eq. (5).

$$R_s = \frac{\kappa Q}{\beta_s GA} \tag{5}$$

Assuming that κ is 1.5 for a rectangular cross section and the Poisson's ratio υ is $1/6$ (i.e. G=14$E/6$, where E is the Young's modulus). Because the concrete may be cracked in this process, the rigidity reduction rate β_s is assumed in $1/3$ in this paper. Eq. (5) can be transformed to Eq. (6).

$$R_s = \frac{10.5Q}{EA} \tag{6}$$

Where, Young's modulus of the concrete was calculated following the AIJ Standard for Structural calculation of RC Structures.

2.4 Bending yield deformation

The distribution of curvature under seismic loading when yielding was first reached was as assumed to be linear like Eq. (7) shown, where ϕ_y is the yield curvature at the critical cross-section and L is the shear span. Also, EI is assumed to be constant here.

$$\phi_{(x)} = -\frac{\phi_y x}{L} + \phi_y \tag{7}$$

Eq. (8) is given by integrating Eq. (7) twice and assuming slope and displacement is 0 at x = 0:

$$\delta_{(x)} = -\frac{\phi_y x^3}{6L} + \frac{\phi_y x^2}{2} \tag{8}$$

Denoting shear span as $L=aD$, where a is the ratio of span to section depth, D, the lateral displacement at the top of the column is expressed by Eq. (9).

$$\delta_b = \frac{a^2 D^2 \emptyset_y}{3} \tag{9}$$

Normalizing δ_b by span length results in Eq. (10).

$$R_b = \frac{aD\emptyset_y}{3} \tag{10}$$

Generally, $\emptyset_y$ can be approximated by Eq. (11)[6].

$$\emptyset_y = \frac{2\varepsilon_y}{D} \tag{11}$$

3. Database for verification

3.1 Overview of database

Using a database of past results from experimental tests, the accuracy of the proposed method was verified. As of December 2019, the database contained 243 beams and 905 columns from 210 papers and reports which had been published in the Journal of Structural Construction Engineering (Transactions of AIJ), conference proceedings of the Japan Concrete Institute, and many more. All tests included in the database were performed in Japan, most of which between 1980 to 2013. However, not all data entries were used for verification purposes. Instead, only the components which satisfied the following conditions were used; (1) members exhibiting flexural failure mode, (2) the data required for calculating yield deformation, such as sectional dimensions and material strength, were available, (3) the concrete compressive strength was less than 60 MPa and the yield strength of longitudinal reinforcement was less than 490 MPa, (4) the yield deformation was less than 1/50 rad, and (5) non-rectangular shaped members (i.e. T-shaped).

Based on these requirements, 79 beams and 360 columns were selected for verification purposes. The reason for majority of the database being filtered out was mainly due to lacking load-deformation results or using high strength materials. The distribution of some key parameters in this study are shown in Table 1.

Table 1 – The distribution of key parameters in this study

	Beams	Columns
Span to depth	1~5	0.69~9.8
Axial load ratio	-	-0.26~0.90
Tensile bar ratio	0.17%~2.68%	0.27%~4.28%
Concrete compressive strength	14.7N/mm^2~57.5N/mm^2	17.8N/mm^2~59.8N/mm^2
Yield strength of longitudinal reinforcement	114N/mm^2~454N/mm^2	297N/mm^2~485N/mm^2

3.2 Definition of experimental value

It should be noted that in some cases, the yield deformation was either not provided or the definition of yield deformation adopted was not consistent. To provide a standard definition for yield displacement, the method proposed by Kusunoki [8] was adopted. The origin method required α_1 for calculating the first corner, which is shown in Fig. 2, should be 1.05, but by analyzing, it may cause the second corner a quiet smaller. In order to describe the load-deformation curve well, this study changed the α_1 from 1.05 to 1.05~1.10. In this method, a trilinear curve was fitted to the load-deformation curve, and the deformation corresponding to the second corner was regarded as the yield deformation.

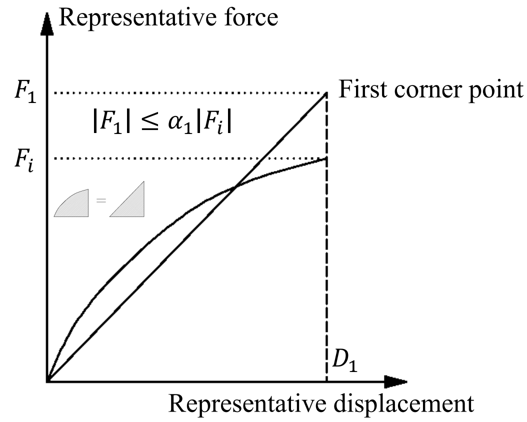
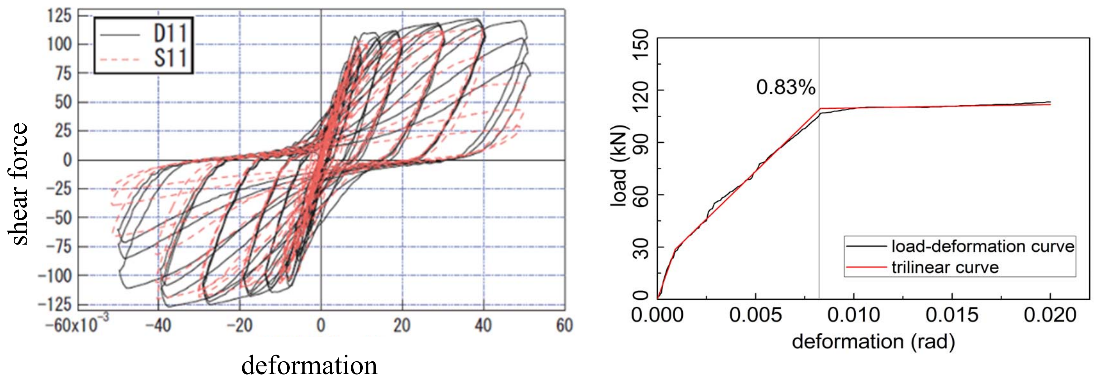


Fig. 2–Definition of the first corner by Kusunoki's method

Fig 2 shows an example of fitting a trilinear model to the load-deformation curve. In this case, the experimental value of D11 in Fig. 3(1) is needed, where S11 is analysis result by FEM methods [9]. First, find the envelope curve of experiment result, and divide it into equal step dots, finally use the method above, the trilinear curve shown as Fig. 3(b) can be get. Here, the deformation at the second corner was 0.83%, which was adopted as the experimental value. This approach was applied to all 79 beams and 360 columns considered for comparison purposes.



(a) Load-deformation curve

(b) Trilinear model

Fig. 3–An example on transforming to a trilinear model

Moreover, this study also considers about the influence of P-Δ effect in columns, which could lead the yield load less than the reality. In order to correct it, an extra force calculated by axial force and constraints is added to the load of the second point, with the yield deformation is invariant, as Fig. 4 shown.

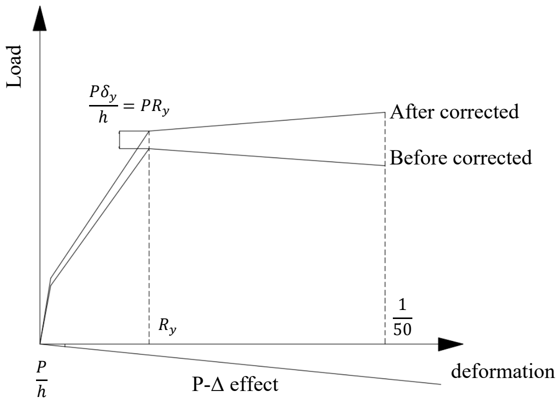
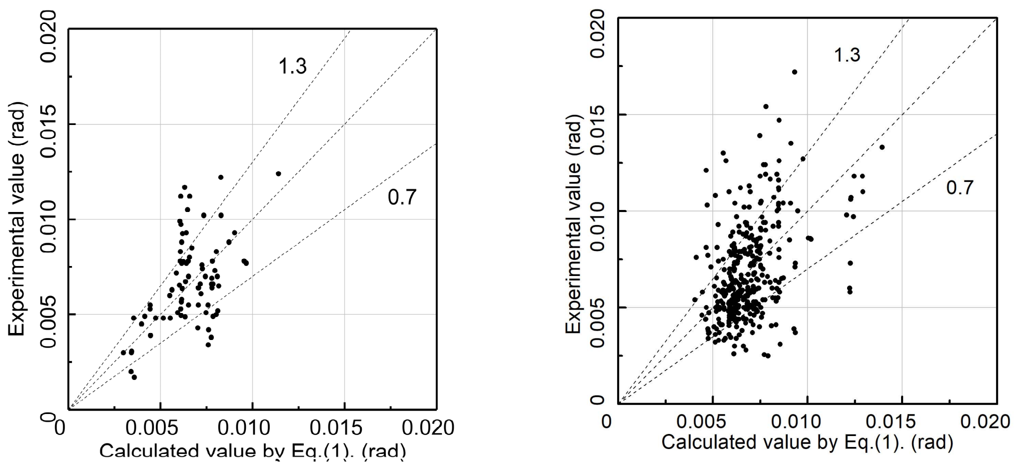


Fig. 4 – Correction of P-Δ effect

4. Verification results

4.1 Verification based on database

The comparation between calculated yield deformation and experimental value is shown in Fig. 5. The mean and the coefficient of variation (CV) of the ratio between the experiment and calculated yield deformations for beams were 1.03 and 0.30, respectively, while for columns the value was 1.00 and 0.33, respectively. The percent of data within the +/- 30% bounds were 70.9% and 68.0% for beams and columns, respectively. As the mean ratio was less than 1.05 (i.e. 5% error), it showed that this method provided good estimates of yield deformation for both beams and columns. The standard deviations and Fig. 5 show that this method is suitable for most situations.



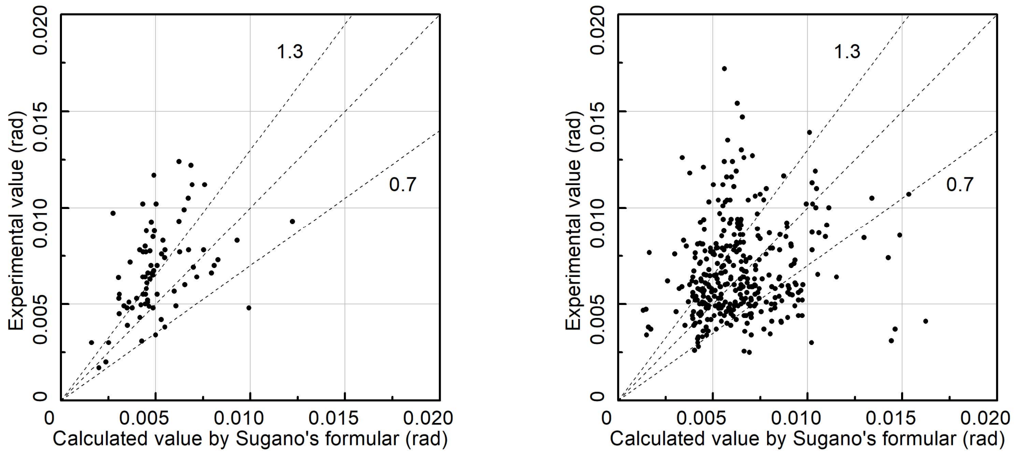
(a) beams (b) columns

Fig. 5 – Comparation between calculated value and database

4.2 Comparation with previous methods

Using the same filtered data described in section 3.1, comparisons were also made for predictions from Sugano [1] and Priestley et al. [6].

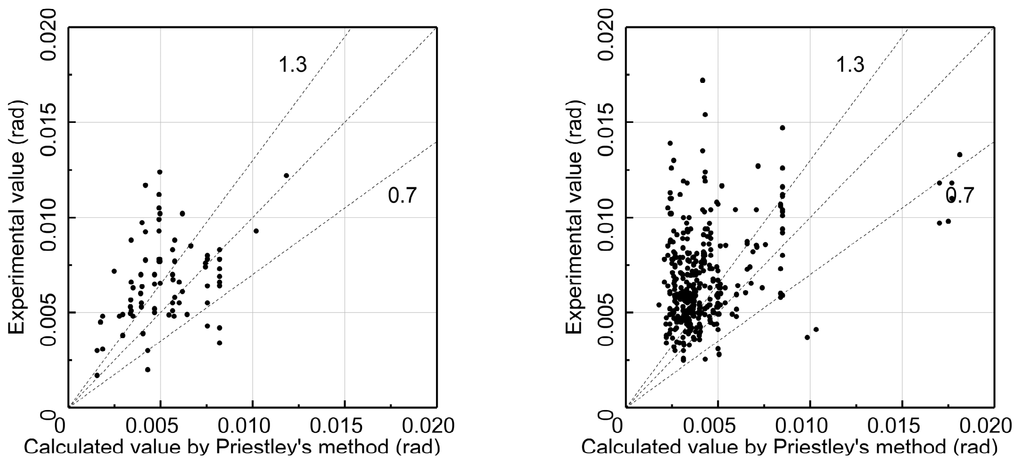
The comparison of experimental to calculated yield displacements using Sugano's formula is shown in Fig 5. It should be noted that the experiments included in these comparisons are identical to that considered in Fig. 5, even though Sugano's formula was only accurate in the following conditions; (i) the longitudinal reinforcement ratio is 0.4~2.8%, (ii) the aspect ratio is 2.0~5.0, and (iii) the axial load ratio is 0~0.55 [1]. The mean value and CV of the ratio between the experimental to calculated yield deformation was 1.37 and 0.34 in beams, respectively, and 1.15 and 0.50 in columns, respectively. The percent of the cases within the +/- 30% bounds was 39.2% and 50.8% for beams and columns, respectively.



(a) beams (b) columns

Fig. 6 – Comparation between calculated value by Sugano [1] and database

The comparisons using Priestley's method considering the same set of experimental data used in Figs. 4 and 5 are shown in Fig. 7. Here, the mean value and CV was 1.41 and 0.40 in beams, respectively, and 1.81 and 0.48 in columns, respectively. The percentage of data entries with an error within 30% was 40.5% and 26.1% for beams and columns, respectively.



(a) beams (b) columns

Fig. 7 – Comparation between calculated value by Priestley et al. [6] and database

Compared with the results by Eq. (1), Sugano [1] and Priestley et al. [6] method underestimated the yield deformation both in beams and columns more significantly. Furthermore, the CV was also larger compared to using Eq. (1). On closer look, the calculated value by Priestley et al. [6] method did not appear to have any trends as the datapoints in Fig 6b were aligned almost vertically. Overall, the proposed approach of considering the three yield deformation components separately appeared to provide the most accurate prediction of all approaches considered.

5. Parametric analyses

In order to increase the precision of Eq. (1), parametric analyses were conducted. This study considered two parameters; the anchorage length, d_p, and the length of the hinge region.

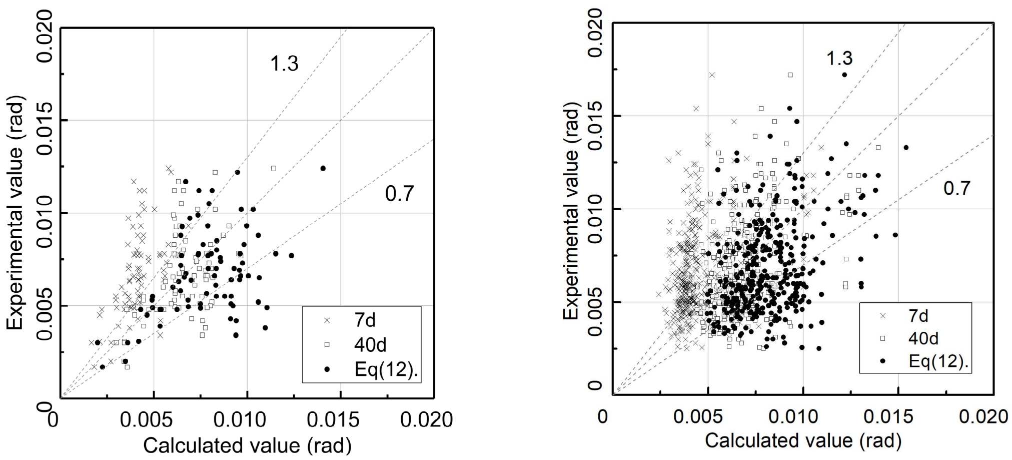
5.1 The anchorage length

In section 2.2, it was discussed that the anchorage length was often assumed to be 40 times the diameter of the longitudinal reinforcement. However, if one considered the average bond stress explicitly, the anchorage length can be calculated by Eq. (12).

$$d_p = \frac{f_y d_b}{4 f_b} \tag{12}$$

Where, f_y is the yield strength of the longitudinal reinforcement, d_b is the diameter of the longitudinal reinforcement, and f_b is average bond stress.

In addition to assuming $40d_b$ and Eq. (12), an anchorage length of $7d_b$ was also assumed. This was based on Neutron Diffractometer test results [10]. The comparison of these three assumptions are shown in Fig. 8 and Table 2. Different anchorage length did have a great influence on the deformation, and with an increase in the calculated anchorage length, the predicted yield deformation also increased. For an anchorage length of $7d_b$, the yield deformation was underpredicted for both beams and columns. Conversely, the anchorage length by Eq. (12) was around $50d_b$ to $60d_b$, which resulted in overestimation of yield deformation for both beams and columns. This result showed that choosing $40d_b$ provided the best estimation of yield deformation.



(a) beams (b) columns

Fig. 8 – Comparation in different achorage length

Table 2 – The sample statistics under the analyze of different anchorage length

	Beam				Column			
Calculation method	Mean	Standard deviation	CV	Percent of the error within 30%	Mean	Standard deviation	CV	Percent of the error within 30%
Eq. (12)	0.88	0.28	0.32	62.0%	0.85	0.29	0.34	59.4%
$7d_b$	1.65	0.46	0.29	21.5%	1.59	0.55	0.35	31.1%
$40d_b$	1.03	0.31	0.30	70.9%	1.00	0.33	0.33	68.1%

5.2 The length of hinge region

In section 2.4, the bending yield deformation was calculated by assuming the distribution of curvature is regarded as linear. Denoting the length of hinge region as βD, the curvature can be expressed by Eq. (13).

$$\phi_{(x)} = \begin{cases} -\dfrac{\phi_y}{L-\beta D}(x-\beta D)+\phi_y & x \geq \beta D \\ \phi_y & x < \beta D \end{cases} \tag{13}$$

Using the same way mentioned in section 2.4, the displacement of the end can be expressed by Eq. (14).

$$\delta = \left(\frac{1}{3}a^2+\frac{1}{3}\beta a-\frac{1}{6}\beta^2\right)\phi_y D^2 \tag{15}$$

And the flexural yield deformation R_b is shown in Eq. (16).

$$R_b = \left(\frac{1}{3}+\frac{1}{3}\frac{\beta}{a}-\frac{1}{6}\frac{\beta^2}{a^2}\right)\phi_y Da \tag{16}$$

Furthermore, some previous studies [10-11] showed that the length of the hinge region is related to shear span ratio l/d and full depth D, as shown in Eq. (17).

$$l_p = \alpha\left(\frac{l}{d}\right)\cdot D \tag{16}$$

A study by Yoshioka [11] estimated α to be 0.5, while Suzuki [12] assumed this to be 0.2. Here, α was treated as a variate. Fig. 9 shows the relationship between the error, defined as $|R_{yexp}/R_{ycal}-1|$, where R_{yexp} is the experimental value of yield deformation and R_{ycal} is the calcualted value, and α. Based on Fig. 9, it can be seen that hinge region had an influence on the yield deformation. In particular, the prediction was reasonably accurate when α was around 0.08~0.10 for beams. However, even if the effect of plasticity was ignored, the error would still be 4% or less which is reasonable.

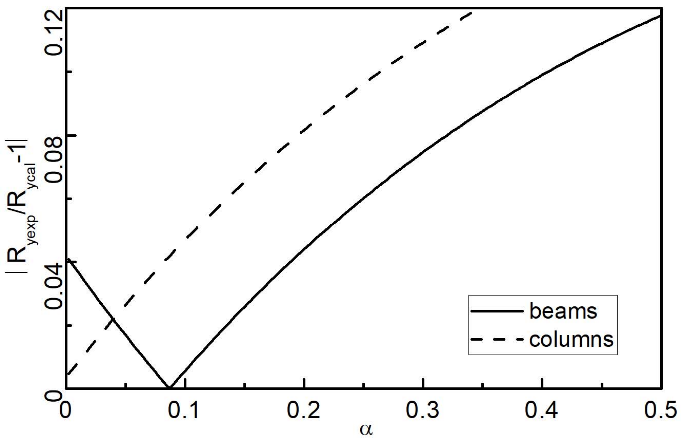


Fig. 9 – The relationship between the error of experimental value/calculated value and α

6. Conclusions

This paper proposed and demonstrated application of a method to predict the yield deformation of reinforced concrete beams and columns by explicitly considering the influence of bar slippage and shear deformation effects. A database of experimental results from tests done in Japan was developed and used to evaluate the accuracy of this method. The mean ratio of experimental to calculated yield deformation was 1.03 for beams and 1.00 for columns, highlighting the accuracy of this method.

Using the same database, a mean ratio of experimental to calculated yield deformation of 1.37 and 1.14 for beams and columns, respectively, was obtained following Sugano [1], and 1.41 and 1.81 for beams and columns, respectively, by Priestley et al. [6]. This showed that the proposed method had a better estimate of yield deformation than these two established approaches.

Parametric analyses were performed to evaluate the influence of anchorage length and length of hinge region on the prediction of yield deformation. It was shown that increasing the anchorage length and the length of the hinge region results in the predicted yield deformation also increasing. However, it was found that an assumed anchorage length of 40 times the bar diameter gave the best estimate, while consideration of plasticity had minor improvements on the predicted yield deformation.

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