*17th World Conference on Earthquake Engineering, 17WCEE*
*Sendai, Japan - September 13th to 18th 2020*

# PHASE-BASED SEISMIC DESIGN METHOD FOR ANTI-CATASTROPHE CONCEPTS

R. Honda(1)*, Y. Liu(2)

(1) *Professor, Graduate School of Frontier Sciences, University of Tokyo, rhonda@k.u-tokyo.ac.jp*
(2) *Student, Institut National des Sciences Appliquées de Lyon*

## Abstract

Earthquakes may cause severe damages to society. Currently, risk-based analysis methodologies are predominantly used in the field of seismic design. They evaluate the probability of occurrence and the consequences of disastrous events. According to the importance of the risk of events evaluated by these two metrics, strategies adopted for the design of infrastructures are prescribed in the design code. However, given that these types of disaster types can cause extreme and critical damages, the definition of risks in the scheme of *risk-based design* is not appropriate. Furthermore, owing to the critical uncertainty associated with the referred phenomena, including earthquakes and structural behaviors, formulating decisions simply based on the estimation of the probability of damage occurrence may not be optimal from the viewpoint of safety. Specifically, to manage extreme types of damages, the deterministic seismic design scheme should be considered.

This study present a design prototype to implement seismic design based on the anti-catastrophic concept. This should also be based on the scheme of *risk-informed design*. This prototype needs to deal with extreme situations—that exceed the states considered in the conventional design—in which structures suffer severe damages. This phase can be referred to as the “next phase” (collapse phase). This process is associated with the risk of being ad hoc and inconsistent. Accordingly, it is necessary to set in place a clear, theoretically based procedure to formulate clear and consistent decisions. It is also essential to allow the usage of deterministic approaches to avoid probabilistic optimizations given that the probability characteristics of extremely rare events are not precisely known. A probabilistic approach may lead to the underestimation of the influence of rare but influential events. To embody severely damaged situations in a design procedure, the concept of “phase” is introduced in the presented design scheme. The design procedure consists of five stages: (1) identification of the damage modes, (2) modeling and parameterization of the target structure, (3) identification of uncertain factors and their uncertainties subject to the design conditions, (4) determination of phases based on numerical simulations, and (5) determination of parameter values for the design.

To illustrate the applicability of the proposed design scheme, it was applied on a simple antiseismic bridge pier structure. Monte Carlo simulations were conducted based on the assumption of a simplified model according to the use of physics engine software to deal with discrete element models. We also discussed the robustness of the parameter values based on numerical simulations at different conditions. The presented method yielded good performance. It was also indicated that the phase-based design method may not be effective when large uncertainty in ground motions must be assumed. If we can reduce the uncertainty in the design conditions, we would improve the reliability of the design.

*Keywords: anti-catastrophic concept; damage mode; phase; seismic design; high-intensity earthquakes*

## 1. Introduction

Earthquakes cause severe damages. Currently, risk-based analysis methodologies are predominantly used in the field of seismic design. It evaluates the probability of occurrence and consequences of a disastrous event. According to the importance of the risk of events evaluated with these two metrics, infrastructure design strategies are prescribed in the design code. However, these types of disasters can cause inevitable and extensive damages. Therefore, after a disastrous event strikes, the definition of risk in the scheme of the risk-based design is no longer appropriate. Furthermore, owing to the critical uncertainty of these phenomena, including earthquakes and structural behaviors, formulations of decisions simply based on the estimation of the probability of damage occurrence may not be the best option. Specifically, deterministic seismic design schemes should be considered to manage extreme damage scenarios.

Numerous concepts have been proposed for the design for extreme events. Let us review the transient of the concepts. It is essential to understand that these concepts do not claim that they can increase the strength of the structure against external forces. Instead, they identify the possible factors that may lead to serious damages and they take appropriate action to mitigate them.

### (1) Risk-informed decision making

In 2008, the National Aeronautics and Space Administration (NASA)1) introduced the risk-informed decision making scheme in the risk management framework of its programs and projects. This scheme focused on the types of direction settings of key decisions to complement continuous risk-management schemes. It is unique in that it considers the likelihood(s) of the scenario(s) and the consequences when these would occur. It differs from the traditional risk-based decision making given that risk-informed decision making (RIDM) is a fundamentally deliberative process that uses a diverse set of performance measures in conjunction with other considerations to inform decision-making personnel.

RIDM exploits the concept of quantification of margins and uncertainties (QMU)2). It identifies relevant parameters, quantifies the margins of these parameters relative to its failure point, and estimates the uncertainties associated with the parameters and the failure point. Takewaki et al.3,4) emphasized the importance of consideration of extreme events from the viewpoint of a mathematically rigorous approach.

### (2) Resilience-based design

Bruneau et al.5) presented the quantifiable concept of resilience. Resilience is defined as the ability of social units (e.g., organizations, communities) to mitigate hazards, contain the effects of disasters when they occur, carry out recovery activities in ways that minimize social disruption, and mitigate the effects of future earthquakes. The resilience-based design process aims to enhance the structural resilience by reducing the integration of the loss due to the deterioration of functionality. It employs the basic notions of the risk-based design. Practical applications have also been considered, while the American Society of Civil Engineers (ASCE) has published a special issue on this concept6–10).

After several recent earthquake events in Japan, such as the 2011 Tohoku earthquake, Takagi and Wada10) also emphasized the need for the change in the design philosophy to consider the structural performance after damages.

### (3) Anti-catastrophic concept

The anti-catastrophic concept has been intensively discussed, especially the 2011 Tohoku12) and 2017 Kumamoto earthquakes6) that constitute the motivations for this study. Perfect prevention of the damage is not possible. However, the proposed concept emphasizes that uncritical damages of infrastructures would not cause the loss of total serviceability. This could thus improve the resilience of society to these occurrences. Honda et al.11) and Akiyama et al.12) made efforts to formulate the design concept for extreme cases. Such discussion should be elaborated more.

## 2. Objectives

This study aims to present a prototype of a design scheme to implement a seismic design based on the anti-catastrophic concept. The proposed scheme needs to deal with extreme situations in which structures suffer severe damages, thus exceeding the state considered in the conventional design that can be referred to as the "next phase" (collapse phase). This may be an ad hoc and inconsistent process, and it is necessary to have clear procedures based on theoretical concepts to formulate clear and consistent decisions. It is also essential to allow the usage of deterministic approaches to avoid probabilistic optimizations given that probability characteristics of extremely rare events are not precisely known. Probabilistic approaches may lead to the underestimation of the influence of these rare but influential events. To embody severely damaged situations in a design procedure, the concept of "phase" is introduced in the presented design scheme. The design scheme can then be applied to a simple structure to illustrate the performance, feasibility, and reliability of the presented scheme.

## 3. Phase-based Seismic Design Scheme

### 3.1 Basic concepts

The anti-catastrophic design requires consideration of the structural behaviors when severe damages occur. It is assumed that the damage is provoked by the severities of extreme earthquakes. With this premise, the anti-catastrophic design must assume increased uncertainties and would require more complex simulations than those conducted for conventional design schemes.

The design should account for the fact that if unexpected situations occur and the structure is damaged, it will be less likely to suffer a catastrophe. However, given the major uncertainty related to external factors and conditions, such as the earthquake ground motions, and given the structural behavior after the damage, it is impossible to evaluate the probability of occurrence of the damage. Therefore, we do not consider the risk based on the probability, but we do use probability to estimate how *likely* the occurrence of the disastrous situation will be. This is a conditional probability that expresses how frequently the catastrophic situations will occur subject to the condition that the structure will be damaged. This is not deterministic owing to various uncertainty factors in structural and external parameters. Note that the *frequency* does not represent the true probability of occurrence because the probability of the condition that structure is damaged is not estimated but it is deterministically provided.

To evaluate the severity and consequences of the damage of one structure, we categorize the damage situations in different modes. The damage mode is defined as a pattern of the specific way the structure is damaged, and different modes should be based on different dominating mechanisms. It is also recognized that the dominant mechanism of any damage mode is different from the one before the damage. To reduce the occurrence probability of the damage modes that will cause catastrophic collapse of the structure, we should first understand the mechanisms of these damage modes and control the relevant key factors to attain our design objectives.

To implement this concept in the proposed design, we introduced the concept of "phase." The phase of the structural behavior is compounded with characteristic variation pertaining to the probabilities of occurrences of the damage modes. The phases were determined based on the conditional probabilities of the damage modes. Suppose that one damage mode is likely to occur in a specific situation, while another damage mode is likely to occur in another condition. Two conditions are defined as two different phases. It is assumed that a corresponding dominant damage mode exists for each phase. Changes of the phases imply the transition of a dominant mechanism pertaining to structural behavior.

The conditional probability is estimated based on the frequency of the damage obtained by the simulation results. The frequency of occurrence of the damage modes is dependent on the parameters of the structure. The designers must choose the values of the design parameters so that the conditional probability or the frequencies of occurrences of unfavorable damage modes do not exhibit precipitous increases. By restricting the design values within a safe-phase margin, the structure can be prevented from entering the "unsafe" phase in which the probabilities of occurrences of catastrophic damage modes are significantly higher than those of the "safe" domain.

### 3.2 Design procedures

A design workflow containing the critical steps of the proposed phase-based process is presented in Fig. 1.

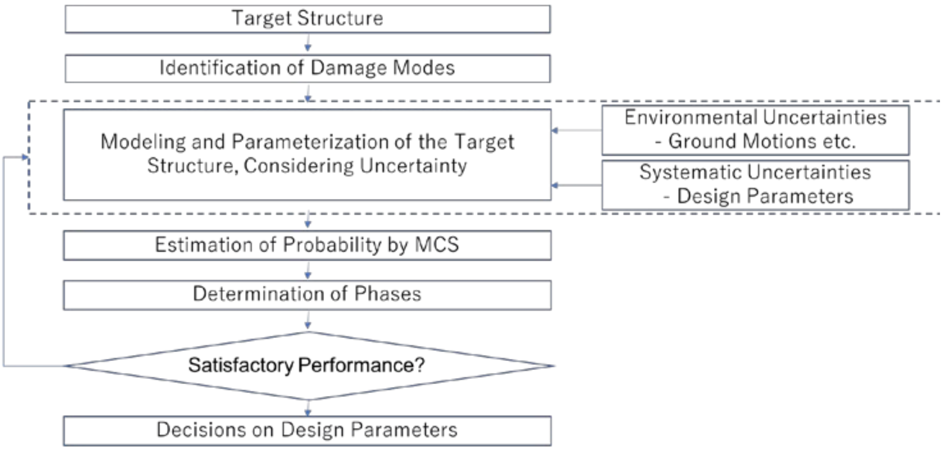

Fig. 1 General phase-based seismic design workflow

#### (1) Identification of damage modes

A damage mode is a specific pattern that a set of structural damage cases share. All of the possible destruction modes should be identified and organized in a development tree diagram.

For identified damage modes, their social influence should be discussed. For example, it depends on whether the structure is totally collapsed, or whether the structure can provide service for regional recovery after it is repaired. The occurrence frequencies of the most severe modes should be reduced by choosing the design values with rationale.

#### (2) Modeling and parameterization of the target structure

The target structure has to be modeled and parameterized to achieve effective numerical simulations. Parameters that govern the behavior of the structure during the earthquake need to be selected. They can change the occurrence probability of the damage modes of the structure in cases of severe earthquakes. These are referred to as design parameters. The design parameters should be chosen judiciously to achieve an effective design and to avoid the waste of computation resources.

### (3) Identification of uncertain factors subject to design conditions

To obtain occurrence frequencies of the damage modes in a large simulation set, we assign fluctuation to various parameters as the systematic and environmental uncertainties.

Environmental uncertainties include the uncertainty in the characteristics of the ground motions. It is recommended to use intense ground motions generated artificially with earthquake simulations with hypothetical fault parameters. It would be also possible to induce input ground motion fluctuations. Conversely, it would be also possible to use fixed deterministic ground motions, such as observed ground motion records.

Systematic uncertainties are applied to the design parameters and Monte Carlo simulations are conducted to estimate their conditional probabilities. Herein, we distinguish two categories of systematic uncertainties: the uncertainties owing to uncontrollable factors, such as the imperfections during the construction commonly referred to as aleatory uncertainties, and the uncertainties due to the lack of knowledge of the effects of the parameters commonly known as epistemic uncertainties. They are essentially different. In the numerical simulations, however, both uncertainties are employed as aleatory uncertainties but they are clearly distinguished.

### (4) Determine phases and formulate decisions on parameter values

As stated above, the phase of the accidental behavior of the target structure is the composition of all the damage modes. Each of these modes has a different level of probability. In different phases, different modes should be dominant. We define phases wherein different damage modes are likely to emerge. The likelihood is estimated with Monte Carlo simulations based on the conditional probabilities, that is, the occurrence frequencies.

For each parameter value, the probability of the damage could be plotted with the synthesis of the probability of each damage mode by considering the environmental and systematic uncertainties. In this curve, we identified empirically different parameter value intervals according to the variation of probability curves within these intervals. In one phase, the variation of probability damages should be consistent. The values of the design parameters should be constrained within an interval in which the phases of the damage modes that lead to the total collapse of the structure exhibit neither high values nor precipitous increases in probabilities.

## 4. Application of the Phase-based Seismic Design Scheme

To illustrate the applicability of the design scheme proposed in the previous section, it was applied on a simple antiseismic bridge pier structure proposed by Akiyama et al.[13, 14)]

### 4.1 Sliding reinforced concrete bridge pier

The deterministic phase-based design scheme formulated in the preceding section was applied to an innovative bridge pier structure proposed by Seto et al.[14)] The structural system of the research carried out by the research group of Akiyama et al. in Waseda University was utilized as the target structure in the process of the phase-based design scheme (Fig. 2).

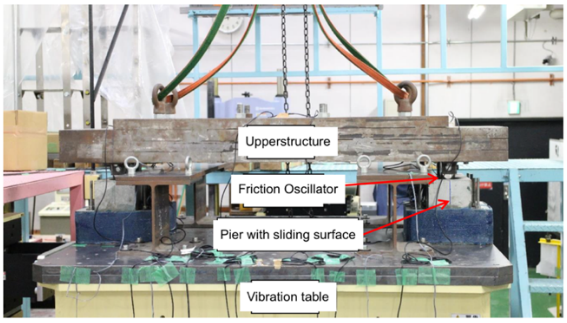

Fig. 2 Installation during the experiment along the transversal direction (Seto et al.[14])

Their research effort was based on the study of the various characteristics of a reinforced concrete (RC) bridge pier that included a friction oscillator in a seismic-free apparatus. In these types of piers, a structure (oscillator) extruded from the bottom of the bridge girder was posed in direct contact with the low-lying surface with specific forms engraved on the top of a column. The top structure was able to slide on the base-column system. It is proposed that this structure reinforces the bridge to avoid undesirable effects from the earthquake. Fig. 2 shows the installation of the structure.

In the experiments, several parameters, such as the length and slope angle of the interface, are optimized based on the experimental results. We use these optimized parameter values as the reference configuration of the target structure. Accordingly, the values of the parameters were varied in numerical simulations.

### 4.2 Application of phase-based design

In this study, the ground motion was amplified just before the upper structure fell from the pier system. We applied a phase-based design procedure to this structure because the structure did not enter the damage phase in the experiment.

We considered this structure as an example to demonstrate the application of the presented phase-based design method. It must be noticed that this case is a virtual design case of a simplified structure. Accuracy is limited, and the results are not applicable to the practical design of this structure.

The structure was simplified (Fig. 3). The figure shows the dimensions of the structure and the reference values of the parameters. We considered only the support structure. The girder was not considered. Unidirectional ground motion was applied on the model in a lateral direction. The structure involved four parameters: the angle of the slope *alpha*, the length of the bottom part of the lower pier *B*, the slope's horizontal length *Bl*, and the height of the oscillator on the top structure *Hu*.

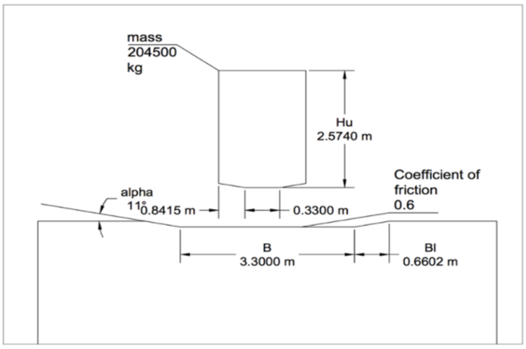

Fig. 3 Parameters and original values of the target structure

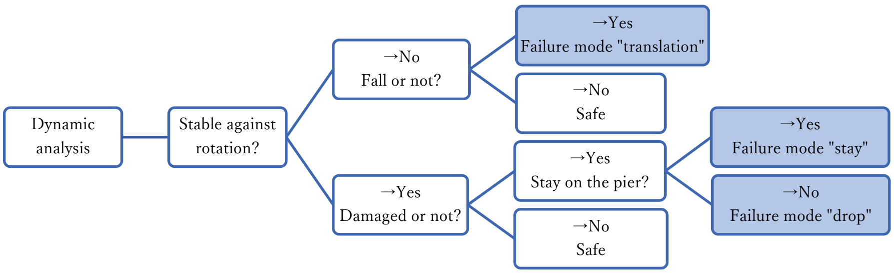

Fig. 4 Schematic of the development of the structural behavior and identification of damage modes

For this reduced structure, three damage modes, namely, “stay”, “translation”, and “drop”, are identified as shown in Fig. 4. Three modes are shown in Fig. 6. For the “stay” mode, the girder is considered to stay on the base structure, and with some repairs, the bridge could still partially offer the serviceability. For the other two modes, namely, “translation” and “drop,” the girder drops from the top of the pier and causes the total collapse of the bridge. Herein, the design objective was the suppression of the occurrences of the failure modes “translation” and “drop.”

### 4.3 Numerical simulations

We considered a severe structural collapse, as defined above, wherein the displacement of the structure was large, while the deformation of the media could be neglected. Accordingly, simulating the structure with rigid bodies is sufficient. Thus, the Open Dynamics Engine (ODE)$^{15,16)}$ was utilized for the numerical simulation of the dynamic behavior of the structure. ODE performs simulations of the articulated body structure. The articulations were realized by the joints of different types used to join one part to another in a specific way, such as slider fixed to the axis. In this research, the shapes of the structural members were realized by joining several bodies with fixed joints. The modeling of the structure is illustrated in Fig. 5.

In ODE, structures are modeled given knowledge of the environmental and systematic uncertainties. Correspondingly, the history of the trajectory of the oscillator and the base structure is output. The three collapse modes considered herein are shown in Fig. 6.

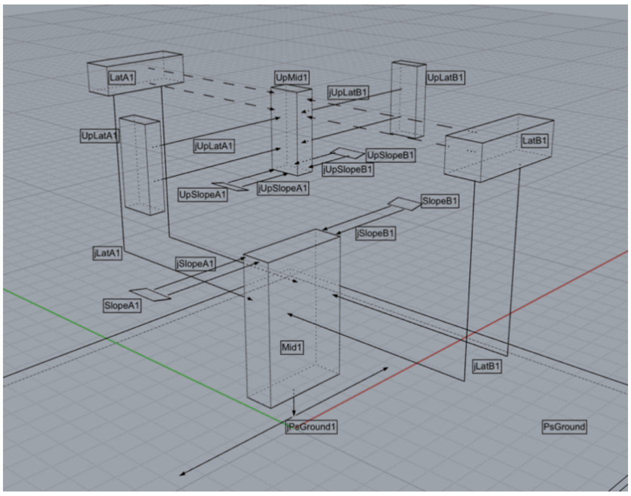

Fig. 5 Installation of all the bodies and their joints

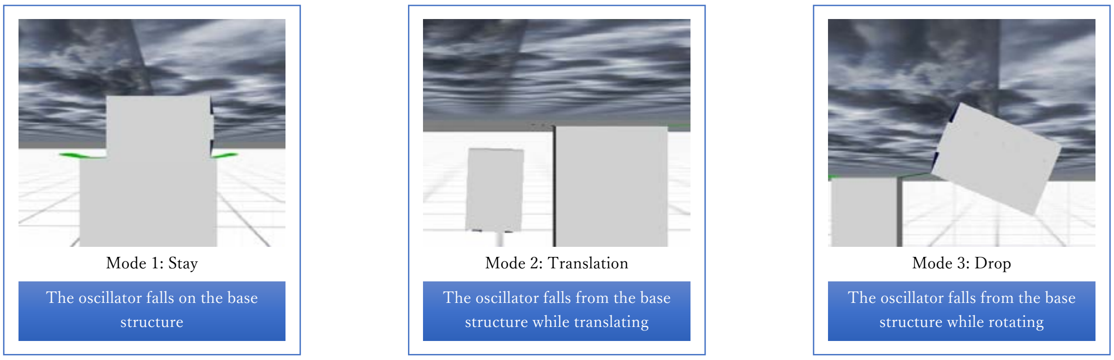

Fig. 6 Illustration of the three damage modes with simulation visualizations and their descriptions

As an input motion, we utilized a record obtained from the 2007 Noto–Hanto earthquake in Japan. The time history of acceleration in [cm/s$^2$] is shown in Fig. 7.

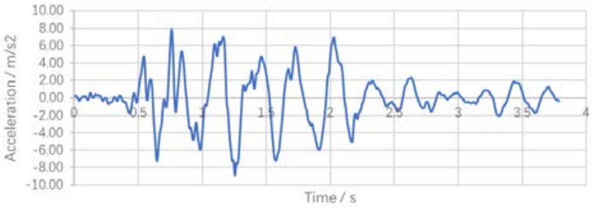

Fig. 7 Time history of the input ground motion

Table 1 Range of values of tested parameters

| Parameter | Reference value | Factors multiplied on the reference value |
|---|---|---|
| *B* | 3.3 m | 0.3, 0.4, 0.64, 0.88, 1, 1.12, 1.36, 1.6, 2, 3, 4, 5, 6, 7 |
| *Bl* | 0.66 m | 0.2, 0.3, 0.4, 0.5, 0.6, 0.64, 0.7, 0.8, 0.88, 0.9, 1.12, 1.36, 1.6, 2, 3, 4, 5 |
| *Hu* | 2.574 m | 0.2, 0.3, 0.4, 0.5, 0.6, 0.64, 0.7, 0.88, 1, 1.12, 1.36, 1.6, 2, 3, 4, 5 |
| *alpha* | 11° | 0.4, 0.64, 0.88, 1, 1.12, 1.36, 1.6, 2, 3, 4, 5, 6 |

### 4.4 Determination of design parameter values

In our design, we discuss the values of the parameters *B, Bl, Hu,* and *alpha*, as defined in Subsection 4.2. The values of these parameters are varied by factorizing the reference values, as shown in Table 1. For each parameter value, Monte Carlo simulations (MCSs) were conducted that allowed the variations of the other parameters. Based on the MCS results, the conditional probabilities of occurrence of the damage modes as a function of the parameter value was estimated by counting the frequency of each damage mode by integrating these situations. Regarding the damage, three damage modes were considered, as discussed in Subsection 4.2. The obtained conditional probability curves are shown in Figs. 8 to 11. Figures plot the conditional occurrence probabilities of each of the damage modes "drop," "stay," and "translation," as a function of the multiplication factors of tested parameters. Additionally, the fourth plot (denoted by the term "any") shows the accidental probability as a function of the multiplication factors when any of these three factors occur.

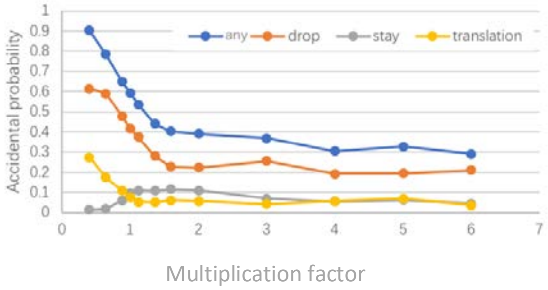

Fig. 8 Probability of damage of individual damage modes and all of these modes ("any") for parameter *alpha*

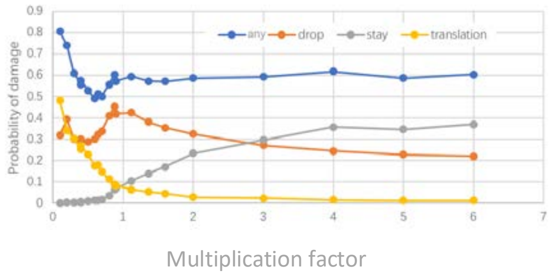

Fig. 10 Probability of damage of each individual damage mode and all of these modes ("any") for parameter *Bl*

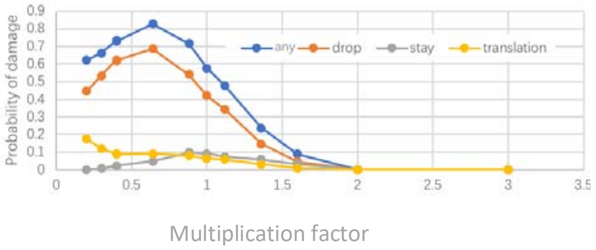

Fig. 9 Probability of damage of each individual damage mode and all of these modes ("any") for parameter *B*

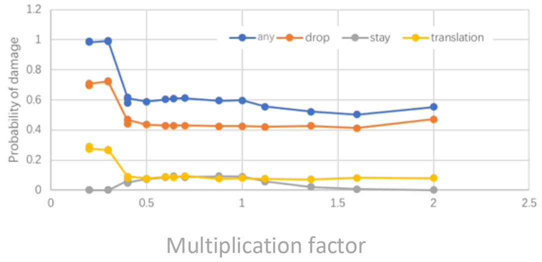

Fig. 11 Probability of damage of each individual damage mode and all of these modes ("any") for parameter *Hu*

In these plots, the probability of damage of each mode varies heterogeneously and yields different phases of structural behavior. Specifically, for Figs. 8, 9, and 11, we can clearly identify the phase because the probability of mode damage abruptly decreases beyond a certain threshold. This indicates that we can determine the design value of the corresponding parameter so that we can suppress the occurrence probability of the two unfavorable damage modes. For these three parameters, *alpha, B,* and *Hu*, the multiplication factor should be two or higher. Regarding the parameter *Bl* in Fig. 10, the plot referred to as "any" does not exhibit a clear decreasing trend, but the "drop" and "translation" decay curves do when the multiplication factor values exceed two and three, respectively. Accordingly, values in the vicinity of this region can be adopted.

### 4.5 Robustness of the determined design parameters

Let us confirm the robustness of the determined parameter values. For this purpose, we input another ground motion, introduced another systematic uncertainty source, and checked the performance of the determined parameter values. The temporal history of the ground motion used in this simulation was obtained from the 1995 Kobe earthquake. This ground motion has different characteristics in terms of frequency and duration (Fig. 12). The ground motion was applied to the structure whose parameter values were set to the values obtained in the previous subsection.

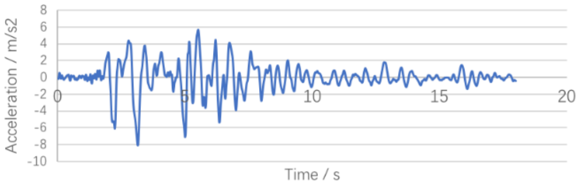

Fig. 12 Time history of another ground motion record from the 1995 Kobe earthquake

Given that the intensities of the two ground motions are not identical, the probability of the damage occurrence cannot be the same. However, it was observed that the probability of occurrence of the two damage modes "drop" and "translation" were considerably low, as shown in Fig. 13. These results indicated the robustness of the parameter values toward the uncertainty of the input ground motions.

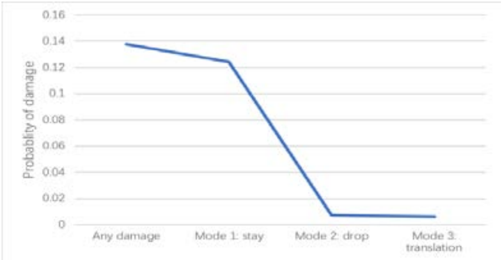

Fig.13 Probability of damage of individual and all the damage modes when subjected to the ground motion from the 1995 Kobe Earthquake

## 5. Effects of Variations of Input Ground Motions

To discuss the applicability of the presented scheme, we evaluated the performance of the method for different ground motions. We considered five strong ground motion records. In addition to the two ground motions mentioned above, three ground motions from the 2016 Kumamoto earthquake, the 2010 Chile earthquake, and the 2011 Tohoku earthquake. Time series are shown in Fig. 14. As it can be inferred from the temporal histories, their characteristics are significantly diverse.

The curves of occurrence probability against the two parameters alpha and b are plotted in Figs. 15 and 16, respectively. Although the curves exhibit abrupt occurrence probability decreases, the thresholds are not so clear as those in the cases in which the single ground motion was considered. Hence, the phases were not clearly defined. It would be naturally inevitable that when the diversity of the ground motion was large, the thresholds used to determine the phase would become obscure. Efforts should be expended to reduce the uncertainties of the design conditions.

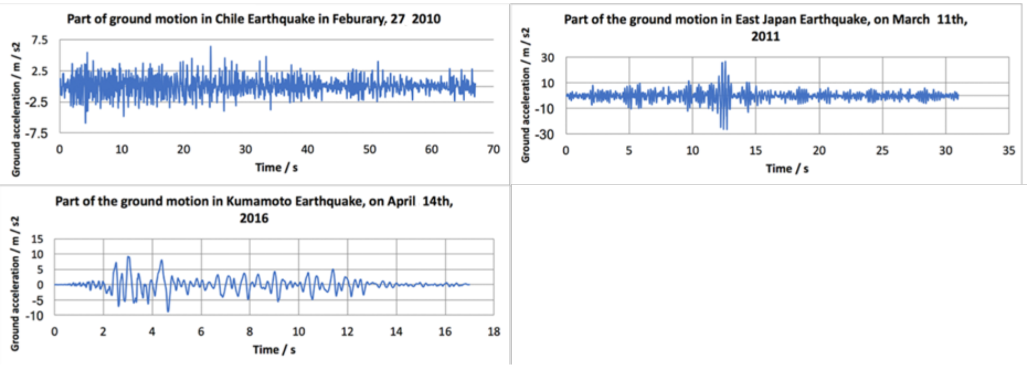

Fig.14 Time series of three additional ground motion records

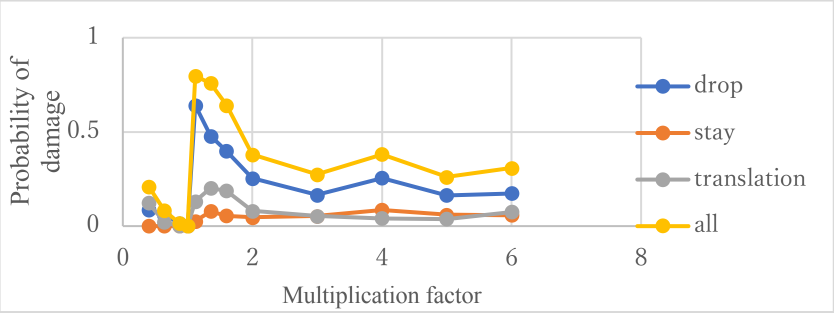

Fig. 15 Probabilities of damage of the individual and all damage modes for parameter *alpha*

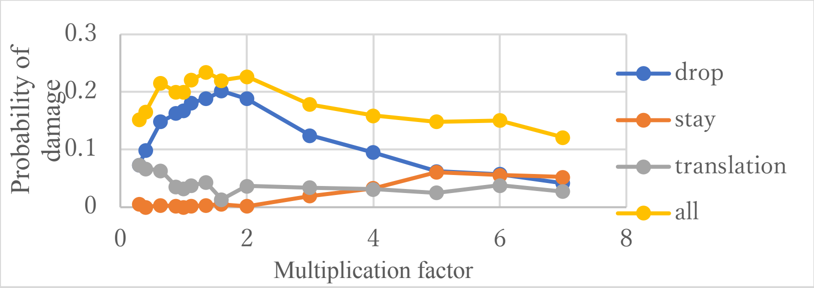

Fig. 16 Probabilities of damage of the individual and all damage modes for parameter *b*

## 6. Conclusion

Seismic design based on the anti-catastrophic concept considers the situation wherein structures are exposed to extremely intense ground motions. For this type of design, the prevalent risk-based design process may not be inappropriate. We proposed a phase-based design scheme, wherein phases were defined according to the change of the dominant mechanism. This scheme intended to avoid requiring the accurate estimation of the probabilities of occurrence of severe damages, because they are impossible to estimate. Alternatively, the presented scheme defined phases that used the conditional probability (or frequency) of occurrence of modes that caused severe damages. To investigate the performance of the presented scheme, it was applied to a support system proposed by Akiyama et al. Monte Carlo simulations were conducted based on the assumption of a simplified model with the use of a physical engine software to deal with discrete element models. The simulation results indicated the validity of the presented scheme. We also discussed the robustness of the parameter values. The estimated design parameter values yielded desirable performance of the structure even against different ground motion. It was also recognized, however, that the phase-based design method may not be effective when large deviations of ground motions must be assumed. Accordingly, it becomes difficult to identify the phases. The phase-based design method may help us make rational decisions about the values of structural parameters even when we must accept considerable uncertainty. However, if we make efforts to reduce the uncertainty in the design conditions, we would improve the reliability of the design. Additional research is required to establish a practical method to exploit the presented framework of the design.

## Acknowledgements

This research was partially supported by JSPS Kakenhi 16H02357. The help on the usage of the physical Open Dynamics Engine offered by Dr. Toyooka A. at the Railway Technical Research Institution (RTRI) is also gratefully appreciated. It is noted that any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of RTRI.

## References

[1] NASA. (2010). Risk-Informed Decision Making Handbook. Office of Safety and Mission Assurance NASA Headquarters.

[2] Goodwin, B. T., & Juzaitis, R. (2006). National Certification Methodology for the Nuclear Weapons Stockpile. Lawrence Livermore National Laboratory, U.S. Department of Energy.

[3] Takewaki, I. (2015). Beyond Uncertainties in Earthquake Structural Engineering. Frontiers in Built Environment 1:1. doi: 10.3389/fbuil.2015.00001.

[4] Takewaki, I., Moustafa, A., & Fujita, K. (2013). Improving the Earthquake Resilience of Buildings. London: Springer.

[5] Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O'Rourke, T. D., Reinhorn, A. M., Shinozuka, M., Tierney, K., & von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. Earthquake Spectra, 19(4), pp. 733-752.

[6] Cimellaro, G. P., Dueñas-Osorio, L., & Reinhorn, A. M. (2016). Resilience-Based Analysis and Design of Structures and Infrastructure Systems. Journal of Structural Engineering, 142(8).

[7] Cimellaro, G. P. (2013). Resilience-based Design (RBD) Modelling of Civil Infrastructure to Assess Seismic Hazards. In Tesfamariam, S., & Goda, K. (Eds.), Handbook of Seismic Risk Analysis and Management of Civil Infrastructure Systems (pp. 268–303). Woodhead.

[8] Echevarria, A., Zaghi, A. E., Christenson, R., & Accorsi, M. (2016). CFFT Bridge Columns for Multihazard Resilience. Structural Engineering, 142(8), p. 16.

[9] Lucia, T., Serban, O., Lin, L., Wang, M., & Lin, N. (2016). Improving the Seismic Resilience of Existing Braced-Frame Office Buildings. Structural Engineering, 142(8), p. 14.

[10] Takagi, J., & Wada, A. (2019). Recent Earthquakes and the Need for a New Philosophy for Earthquake-Resistant Design. Soil Dynamics and Earthquake Engineering, 119, p. 499–507.

[11] Honda, R., Akiyama, M., Kataoka, S., Takahashi, Y., Nozu, A., & Murono, Y. (2016). Seismic Design Method to consider "Anti-Catastrophe" Concept - A Study for the Draft of Design Codes. Journal of JSCE, 72(4), p. I_459–I_472.

[12] Akiyama, M., Takahashi, Y., Hata, Y., & Honda, R. (2016). Lessons from the 2016 Kumamoto earthquake based on field investigations of damage to bridges. International Journal Earthquake and Impact Engineering, 1(3), p. 225–252.

[13] Seto, T., Hattori, R., Ishigaki, N., Suezaki, M. & Akiyama M. (2017). Bidirectional Shaking Table Test of Concrete Bridge Pier with Friction Pendulum Isolation System. Journal of Structural Engineering A, JSCE, 63A, p. 385–396 (in Japanese).

[14] Seto, T. (2018). Experiment of Oscillation in both Horizontal Directions of a RC Bridge Installed with Friction-Oscillator-type Earthquake-proof Structure whose Abrasion Resistance and Slip Performance are Improved. Graduate School of Wasada University, School of Creative Science and Engineering. Tokyo: Waseda University.

[15] Kosei, D. (2007). Robot Simulation - Robot Programming based on Open Dynamics Engine. Tokyo: Morikita Press SARL (in Japanese).

[16] Smith, R. (2007). Open Dynamics Engine: http://www.ode.org (Last visited on June 4th, 2018).