



MULTI-OBJECTIVE DESIGN OF SLIDE BRIDGE BEARINGS UNDER SEISMIC EXCITATIONS BY DETERMINISTIC-STOCHASTIC APPROACH

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Abstract

In the process of parameter selection for slide bearings for the seismic performance-based design of bridges, there exist intrinsic tradeoffs between minimization of the response displacement of the bearings and that of the pier in strong earthquake events. However, difficulty in determining the optimal parameter combinations of these devices arises, caused by nonlinearity, complexity and requirement for computational resources for nonlinear time-history analysis. This paper proposes a stochastic-deterministic approach to achieve the multi-objective optimal design for two types of unconventional slide bearings, namely the uplifting slide shoe (UPSS) and functionally discrete bearings (FDB). The UPSS bearing is a simple and cost-effective sliding bearing consisting of one horizontal and two inclined plane sliding surfaces. This device is proposed to deal with the thermal problem of multi-span continuous girder bridges, as well as to control the excessive horizontal response displacement of the girder. The FDB system is a combination of the pure friction bearing and the elastomeric bearing set in parallel. Unlike the conventional rubber bearings, this system allows required period elongation of the structural systems and energy dissipation performance. To obtain the performance indices in the proposed procedure, seismic load is modeled as a stationary random process, whose characteristics are determined for the standard design ground motions in the Japanese codes, and the nonlinear behavior of the slide bearings is modeled as equivalent-linear elements by using the stochastic linearization technique. The objectives for optimization are defined in terms of the variance of stationary structural responses. As the result, a set of optimal parameter candidates is obtained as the Pareto-front solutions in the multi-objective function space. As the next step, the search of the optimal parameters is conducted by performing nonlinear time-history analysis only for the Pareto-front solution parameter sets to save the computational requirement. It is demonstrated that the seismic performance of the bridge for the case of the design parameters obtained by the proposed procedure is almost equivalent to the one with the optimal parameters found by the conventional exhaustive search approach.

Keywords: slide bridge bearings; multi-objective optimization; seismic excitations; stochastic-deterministic approach

1. Introduction

The bridge bearing with seismic functionality is recognized as a key component to facilitate the seismic performance. Through using these devices, elongation of the natural period and added supplementary energy dissipation capability are achieved so that both the bearing and pier responses are minimized under strong earthquakes. In addition, a better serviceability that includes the control of ambient vibration and the thermal problem of the girders can be achieved in regular maintenance and management.

Recently, some unconventional bridge bearings have been proposed to provide more flexible and feasible options for the seismic performance enhancement of bridges. The uplifting slide shoe (UPSS) bearing [1, 2] is a new type of bearing consisting of multiple sliding surfaces, as shown in Fig. 1(a). When the superstructure supported by UPSS is sliding on different sliding surfaces, the horizontal sliding surface of UPSS is to accommodate the thermal effects and small vibration of continuous girders, while the inclined sliding surface is to provide extra restoring force to effectively mitigate the excessive horizontal displacement response of the girders under strong earthquake scenarios. The functionally discrete bearing (FDB) system is a combination of elastomeric bearings and sliding bearings to separately provide restoring force and energy



dissipation ability, as shown in Fig. 1(b). Unlike the conventional rubber bearings, this system allows required period elongation of the structural systems and the energy dissipation performance.

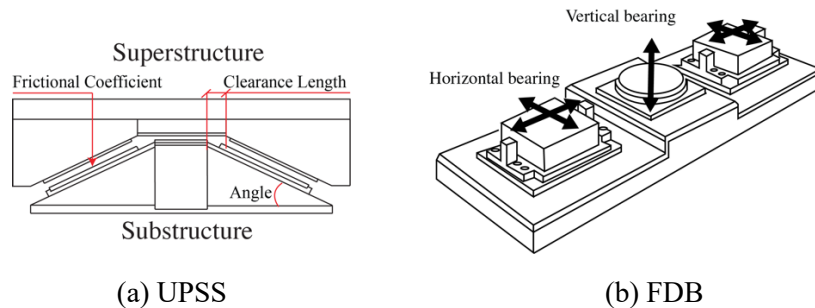


Fig. 1 Schematics of UPSS and FDB

In the process of parameter selection for slide bearings for the seismic performance-based design of bridges, there exist intrinsic tradeoffs between minimization of bearing displacement and that of pier response for strong earthquake events. However, difficulty in determining the optimal parameter selection of these devices arises. Conventional exhaustive search method requires numerous computational resources to determine the optimal design of the slide bearings, since all of the possible parameter combinations are of a large number, whereas only a small number of the parameter combinations satisfy the optimal design criteria. On the other hand, the objective function of the optimization problem is usually unknown due to the complexity of nonlinear time-history analysis. Therefore, a fast search algorithm to determine the optimal parameter selection is required.

The optimal design of structures can be achieved by different approaches according to previous studies. Feng et al. [3] derived the optimal parameters for the tall building with mega substructure configuration under white-noise excitations by minimizing the mean square of the responses of interest. A series solution of the H_{∞} optimization problem and a closed-form algebraic solution of the H_2 optimization problem for the dynamic vibration absorber attached to linear systems were given by Asami et al. [4]. The optimal design of the unconventional tuned mass damper to individually minimize the response quantity of interest was investigated in Refs. [5, 6]. Specifically, Hoang and Fujino [5] investigated the optimal parameter design of rubber bearing for the seismic response mitigation of a long-span truss bridge in Japan. Under the consideration of nonlinear structural elements, the optimal parameter design of nonlinear base isolation systems was investigated by Reggio [7] and De Domenico [8] based on stochastic dynamic analysis. In addition, a performance-based optimization procedure for nonlinear structures subjected to random seismic excitations is proposed by Xu and Spencer [9] to deal with the optimal parameter design for competing performance objectives.

In this study, a deterministic-stochastic approach is proposed to achieve the multi-objective design of slide bearings under seismic excitations. To obtain the performance indices, the seismic load is modeled as a stationary random process and the nonlinear behavior of the slide bearings is modeled as equivalent-linear elements using the stochastic linearization technique. As the result, a set of optimal parameter candidates is obtained as the Pareto-front solutions in the multi-objective function space. As the next step, the search of the optimal parameters is conducted by performing nonlinear time-history analysis only for the set of the Pareto-front solution parameter sets to save the computational requirement. As a numerical example, the proposed method is applied to the optimal parameter selection of UPSS and FDB in a girder bridge. It is demonstrated that the seismic performance of the bridge for the case of the design parameters obtained by the proposed procedure is almost equivalent to the one with the optimal parameters found by the conventional exhaustive search approach.



2. Modeling

A straight girder bridge supported by single columns with UPSS or FDB is studied. The dynamic behavior of the bridge in the longitudinal direction is simplified as a lumped-mass model with two horizontal degrees of freedom (DOF), as shown in Fig. 2.

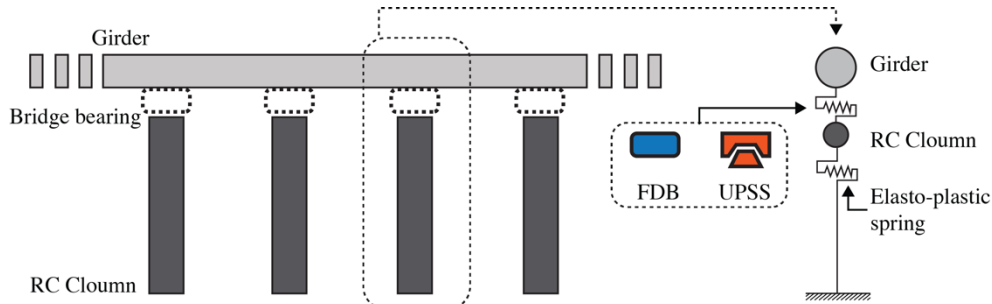


Fig. 2 Bridge model with the application of UPSS or FDB

2.1 Equation of motion

The governing equation of the system is given as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{g}(\mathbf{u}, \dot{\mathbf{u}}) = -\boldsymbol{\tau}\ddot{u}_g \quad (1)$$

where $\mathbf{M} = \text{diag}(m_1, m_2)$, m_1 and m_2 are the masses of the pier and girder, respectively; $\mathbf{u} = [u_1 \ u_2]^T$ is the vector of the nodal displacements and $\dot{\mathbf{u}} = [\dot{u}_1 \ \dot{u}_2]^T$ is the vector of the nodal velocities; $\mathbf{g}(\mathbf{u}, \dot{\mathbf{u}})$ is the reaction force vector, which can be linear or nonlinear; \ddot{u}_g is the ground acceleration; $\boldsymbol{\tau} = [1 \ 1]^T$ is the excitation vector to couple the input and the structural DOFs.

The mechanical behavior of UPSS and the superstructure can be simplified as a point mass sliding on three sliding surfaces, as shown in Fig. 3. Based on the dynamic equilibrium condition, the restoring force of UPSS is expressed as:

$$f_2 = \begin{cases} \mu m_2 g \text{sgn}(\dot{u}_b), & -L < u_b < L \\ N \sin \theta \text{sgn}(u_b) + \mu N \cos \theta \text{sgn}(\dot{u}_b), & u_b < -L \text{ or } u_b > L \end{cases} \quad (2)$$

where $N = m_2 g \cos \theta$ is the resistant force of the inclined sliding surface, and g is the gravitational acceleration; μ is the friction coefficient of sliding interface; θ is the inclined angle of the inclined sliding surfaces from the horizontal direction; L is the distance of the clearance length from the neutral position of the horizontal sliding surface to the beginning of the inclined sliding surface; u_b is the relative displacement of the bearing with respect to the pier; and $\text{sgn}(\cdot)$ is the signum function.

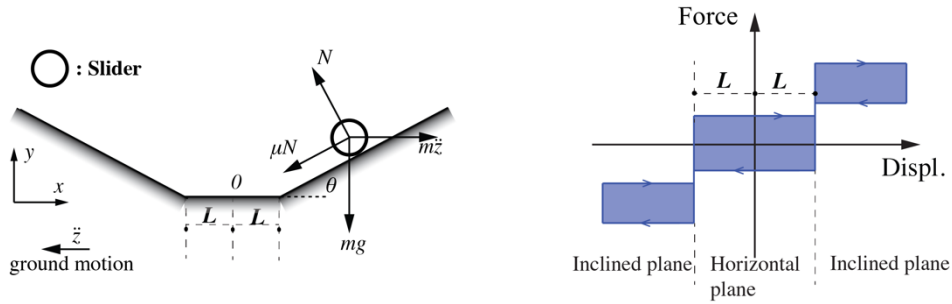


Fig. 3 Mechanism of UPSS

Since FDB is a combination of rubber bearings and slide bearings, the mechanical behavior of FDB is expressed as the sum of that of the two types of bearings, as shown in Fig. 4. The slide bearing is represented by the Coulomb friction model, and the rubber bearing is represented by an elastic linear spring. The restoring force of FDB is expressed as:

$$f_b = k_b u_b + \mu m_2 g \operatorname{sgn}(\dot{u}_b) \tag{3}$$

where $k_b = m_2(2\pi/T_b)^2$ is the stiffness of the rubber bearing, and T_b is the specified natural period when the superstructure is in the fixed bearing condition.

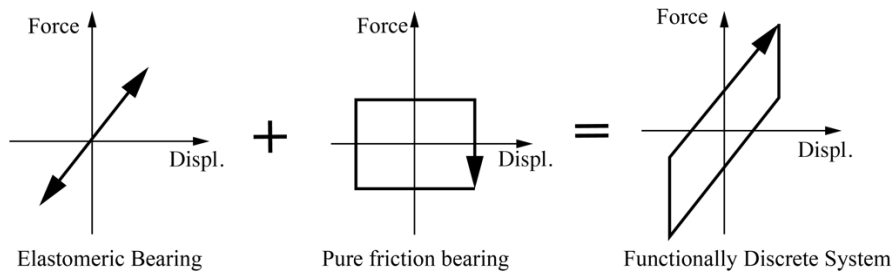


Fig. 4 Mechanism of FDB

The behavior of the RC pier supporting the bridge bearing is described by the Clough's degrading stiffness model in the nonlinear time-history analysis. The initial stiffness of the RC pier is specified so that the elastic period for the fixed bearing condition is 0.5 sec, and the yielding strength is 0.66g lateral force.

2.2 Stochastic representation of seismic excitations

The ground motion is represented by the modified Kanai-Tajimi model[10, 11]. In the time domain, this model can be expressed in a state-space form as Eq. (4).

$$\begin{aligned} \dot{\mathbf{x}}_f &= \mathbf{A}_f \mathbf{x}_f + \mathbf{B}_f w(t) \\ p(t) &= \mathbf{C}_f \mathbf{x}_f \end{aligned} \tag{4}$$

where,



$$\mathbf{x}_f = \begin{bmatrix} u_f \\ \dot{u}_f \\ u_g \\ \dot{u}_g \end{bmatrix}; \mathbf{A}_f = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_f^2 & -2\zeta_f\omega_f & \omega_g^2 & 2\zeta_g\omega_g \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_g^2 & -2\zeta_g\omega_g \end{bmatrix}; \mathbf{B}_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \mathbf{C}_f^T = \begin{bmatrix} -\omega_f^2 \\ -2\zeta_f\omega_f \\ \omega_g^2 \\ 2\zeta_g\omega_g \end{bmatrix}$$

in which $w(t)$ is the bedrock Gaussian white-noise process; ω_g and ζ_g are the fundamental circular frequency and damping ratio, respectively, of the surface soil layer in the Kanai-Tajimi model; ω_f and ζ_f are the parameters of the second filter suggested by Clough-Penzien.

The peak ground acceleration (PGA) is related to the spectral intensity of the white-noise [8] as Eq. (5):

$$S_w = \frac{0.141\zeta_g\ddot{u}_{g0}^2}{\omega_g\sqrt{1+4\zeta_g^2}} \quad (5)$$

The parameters of the modified K-T model are determined by the PSD of the standard design ground motions in the Japanese codes through the least-square method.

2.3 Stochastic response of MDOF structures

The above equation of motion can be substituted into an equivalent linearized form, if $\mathbf{g}(\mathbf{u}, \dot{\mathbf{u}})$ satisfies some smoothness requirement and the input is a zero-mean stationary Gaussian process so as to minimize the mean square error between the linearized counterpart and the original nonlinear system [12], as shown in Eq. (6):

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{C} + \mathbf{C}_{eq})\dot{\mathbf{u}} + (\mathbf{K} + \mathbf{K}_{eq})\mathbf{u} = -\mathbf{r}\ddot{u}_g \quad (6)$$

where \mathbf{C} and \mathbf{K} are the damping matrix and stiffness matrix for linear elements, respectively; \mathbf{C}_{eq} and \mathbf{K}_{eq} are the equivalent linearized damping and stiffness matrix for nonlinear elements, respectively.

Then the governing equation can be rewritten into a state-space form as:

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathbf{A}_s\mathbf{x}_s + \mathbf{B}_s w(t) \\ \mathbf{y}_s &= \mathbf{C}_s\mathbf{x}_s + \mathbf{D}_s w(t) \end{aligned} \quad (7)$$

where $\mathbf{x}_s = [\mathbf{u}^T \quad \dot{\mathbf{u}}^T]^T$ is the state vector, \mathbf{y}_s is the response vector, and

$$\mathbf{A}_s = \begin{bmatrix} \mathbf{0}_{N \times N} & \mathbf{I}_{N \times N} \\ -\mathbf{M}^{-1}(\mathbf{K} + \mathbf{K}_{eq}) & -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{C}_{eq}) \end{bmatrix}, \quad \mathbf{B}_s = \begin{bmatrix} \mathbf{0}_{N \times 1} \\ \mathbf{M}^{-1}\boldsymbol{\tau} \end{bmatrix}$$

the matrices \mathbf{C}_s and \mathbf{D}_s are the response matrices, which are determined by the specified outputs.

Combined with the ground motion model, the state of the combined system can be expressed as Eq. (8):



$$\begin{aligned}\dot{\mathbf{x}}_a &= \mathbf{A}_a \mathbf{x}_a + \mathbf{B}_a w(t) \\ \mathbf{y}_s &= \mathbf{C}_a \mathbf{x}_a\end{aligned}\quad (8)$$

where $\mathbf{x}_a = [\mathbf{x}_s^T \quad \mathbf{x}_f^T]^T$ is the augmented state-space vector collecting displacement and velocities of both the system and the filter variables, and

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A}_s & \mathbf{B}_s \mathbf{C}_f \\ \mathbf{0}_{r \times 2N} & \mathbf{A}_f \end{bmatrix}, \quad \mathbf{B}_a = \begin{bmatrix} \mathbf{0}_{2N \times 1} \\ \mathbf{B}_f \end{bmatrix}, \quad \mathbf{C}_a = [\mathbf{C}_s \quad \mathbf{D}_s \mathbf{C}_f]$$

in which $r = 4$ is the number of the filter equations describing the modified K-T model.

The second order moment of the stochastic structural response is considered as the performance objective, which is defined as $\mathbf{\Gamma}_{x_a} = E[(\mathbf{x}_a - \boldsymbol{\mu}_a)(\mathbf{x}_a - \boldsymbol{\mu}_a)^T]$, where $\boldsymbol{\mu}_a$ is the expected value vector and $E[\cdot]$ denotes the expectation operator. In stationary cases, the covariance matrix is a constant matrix. With the assumption of a zero-mean Gaussian process as input and zero-mean initial conditions, the covariance of the structural response $\mathbf{\Gamma}_{x_a} = E[\mathbf{x}_a \mathbf{x}_a^T]$ is governed by the following Lyapunov equation: $\mathbf{A}_a \mathbf{\Gamma}_{x_a} + \mathbf{\Gamma}_{x_a} \mathbf{A}_a^T + 2\pi \mathbf{B}_a S_w \mathbf{B}_a^T = 0$. Note that the solution of the above equation is implicit, since the determination of linearized coefficients is dependent on the corresponding second-order moment responses which are unknown. Hence, an iterative procedure should be performed until the convergence is reached by initializing the procedure with a linear system.

2.3 Stochastic linearization

If the structural responses are assumed to be stationary random processes, the equivalent linearized damping and stiffness can be determined through the stochastic linearization technique [12] as Eq. (9).

$$K_{eq}^{(i,j)} = E \left[\frac{\partial g_i}{\partial u_j} \right], \quad C_{eq}^{(i,j)} = E \left[\frac{\partial g_i}{\partial \dot{u}_j} \right] \quad (9)$$

Since the inclined surfaces of UPSS are responsible for controlling the excessive horizontal response under strong earthquakes, only the action of UPSS sliding on the inclined surfaces is considered here. The restoring force of UPSS in the horizontal direction can be simplified as Eq. (10).

$$f_b = N \sin \theta \operatorname{sgn}(u_b) + \mu N \cos \theta \operatorname{sgn}(\dot{u}_b) \quad (10)$$

where the same definition applies to the symbols as mentioned above.

Hence, the stochastic linearization coefficients for UPSS can be solved by Eq. (11).

$$K_{eq}^{(b,b)} = E \left[\frac{\partial f_b}{\partial u_b} \right] = N \sin \theta E \left[\frac{\partial \operatorname{sgn}(u_b)}{\partial u_b} \right] = N \sin \theta E [2\delta(u_b)] = N \sin \theta \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_{u_b}} \quad (11)$$



$$C_{eq}^{(b,b)} = E \left[\frac{\partial f_b}{\partial \dot{u}_b} \right] = \mu N \cos \theta E \left[\frac{\partial \text{sgn}(\dot{u}_b)}{\partial \dot{u}_b} \right] = \mu N \cos \theta \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_{\dot{u}_b}}$$

where $\delta(\cdot)$ denotes Dirac delta function.

Similarly, the stochastic linearization coefficients for FDB can be solved by Eq. (12).

$$K_{eq}^{(b,b)} = k_b, \quad C_{eq}^{(b,b)} = \mu m_2 g \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_{\dot{u}_b}} \quad (12)$$

2.4 Stochastic and deterministic analysis

The constraint conditions for parameter selection and the structural information for stochastic analysis are listed in the Table I. Since minimization of the pier response is considered to be one of the optimization objective, the stiffness and damping properties of the bridge pier are assumed to be constant in the stochastic analysis. On the other hand, the structural information for the NTHA is presented in Table II.

Table I. Design variables and structural parameters for stochastic analysis

| | | |
|------------------------------|-----------------------|---------------|
| Mass ratio m_2/m_1 | | 3.0 |
| Pier | Natural period T_1 | 0.5 (sec) |
| | Damping ratio ξ_1 | 0.02 |
| Elastomeric bearing | Natural period T_b | 1.0~2.0 (sec) |
| | Damping ratio ξ_b | 0.03 |
| Friction coefficient μ | | 0.05~0.15 |
| UPSS inclined angle θ | | 5 ~30 (°) |

Table II. Structural parameters for NTHA

| | |
|---|------------|
| Mass of girder m_2 | 900 (tons) |
| Mass of RC bridge pier m_1 | 300 (tons) |
| Initial natural period of RC pier $T_{0,1}$ | 0.5 (sec) |
| Yield strength of RC pier | 0.66g |
| Clearance of UPSS L | 0.03 (m) |

3. Multi-objective design

3.1 Pareto-front solutions

Since minimization of both the bearing and bridge pier responses is a multi-objective optimization problem, the Pareto-front solution is introduced in the form of the ε -Constraint method. This method is to minimize the pier response for a given bearing displacement demand as expressed by Eq. (13).

$$\begin{aligned} & \min J_I(\mathbf{x}) \\ \text{Subjected to: } & J_{II}(\mathbf{x}) < \varepsilon_m, \quad m = 1, 2, 3, \dots, M \\ & \mathbf{x}_{lb} < \mathbf{x} < \mathbf{x}_{ub} \end{aligned} \quad (13)$$



where the performance objectives J_I and J_{II} are the second-order moment of the displacement response of the bridge pier and that of the bearing, respectively; ε_m is the constraint condition in terms of the bearing response; \mathbf{x}_{lb} and \mathbf{x}_{ub} are the lower and upper bounds of the design variable vector, as shown in Table I.

3.2 Procedure for optimal parameter selection

The procedure to determine the optimal design is given as follows and as Fig. 5:

- I. Determine the parameters of the modified K-T model for a given standard ground motion accelerogram of the Japanese codes. The PSD of the ground motion accelerogram is firstly determined by the Welch's method, and then the parameters of the modified K-T model is determined by the least square method.
- II. Compute the second-order moment of the structural responses for all possible parameter combinations through stochastic dynamic analysis.
- III. Select a set of optimal parameter candidates through the Pareto solutions of the all stochastic responses. The constraint condition (ε_m) of the Pareto-front is ranging from the minimum bearing displacement to the maximum bearing displacement responses (σ_{ub}).
- IV. Search the optimal design for the given bearing displacement demand by performing nonlinear time-history analysis to only the set of the optimal parameter candidates obtained in the previous step.

Since the selected optimal parameter candidates are a subset of all possible parameter combinations, the computational resources are considerably reduced in comparison with the conventional exhaustive search approach.

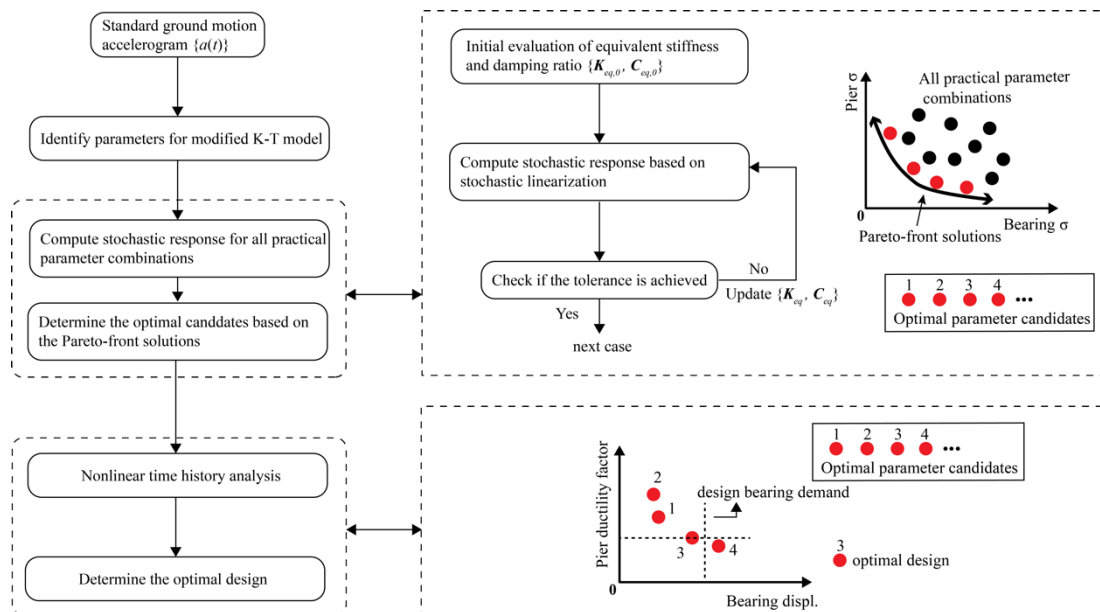


Fig. 5 Flowchart of the proposed procedure

3.3 Numerical example

An example to determine the optimal parameter selection for UPSS and FDB is presented. The corresponding structural parameters are given in section 2.

The standard ground motion II-I-2 based on the Japanese Highway Bridges Design Specification [13] is selected. The corresponding time-history and the PSD of this ground motion are shown in Fig. 6. As indicated



by this figure, the PSD of the modified K-T model determined by the least-square method has a good agreement with the PSD of the standard ground motion.

Then, the optimal parameter candidates of the bearings are determined based on the Pareto solutions of the stochastic dynamic response of all possible parameter combinations, as shown in Fig. 7.

Finally, the optimal design is searched by performing the nonlinear time-history analysis using the limited set of the Pareto solution parameters (optimal parameter candidates). For all possible bearing displacement demands, the maximum pier response ductility factors obtained by the proposed procedure are plotted in the Fig. 8 with red points. As indicated by this figure, the seismic performance of the bridge for the case of the assumed optimal design with red points are close to that of the conventional exhaustive search approach indicated with black points.

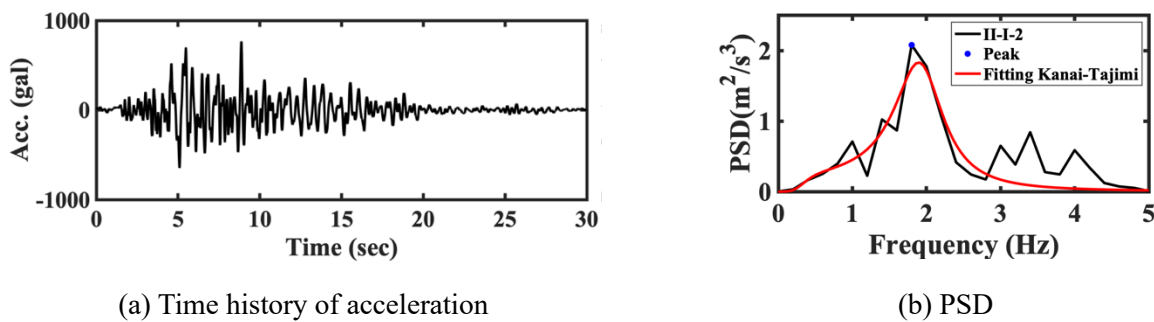


Fig. 6 Standard ground motion, case II-I-2

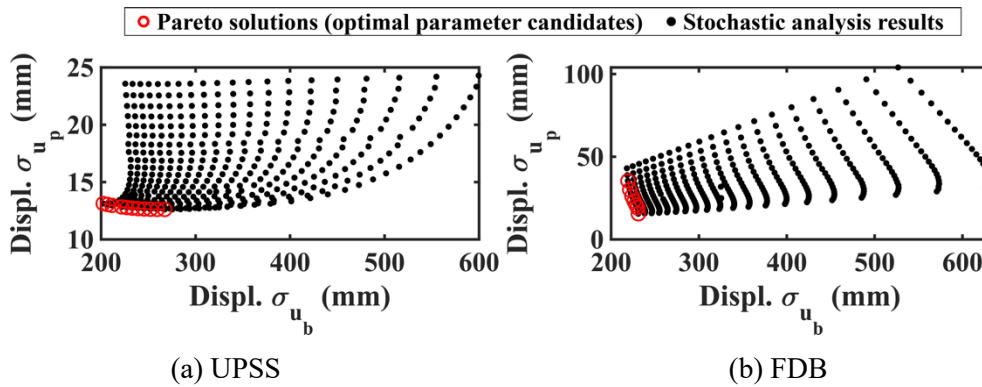


Fig. 7 Pareto solutions of stochastic analysis results, case II-I-2

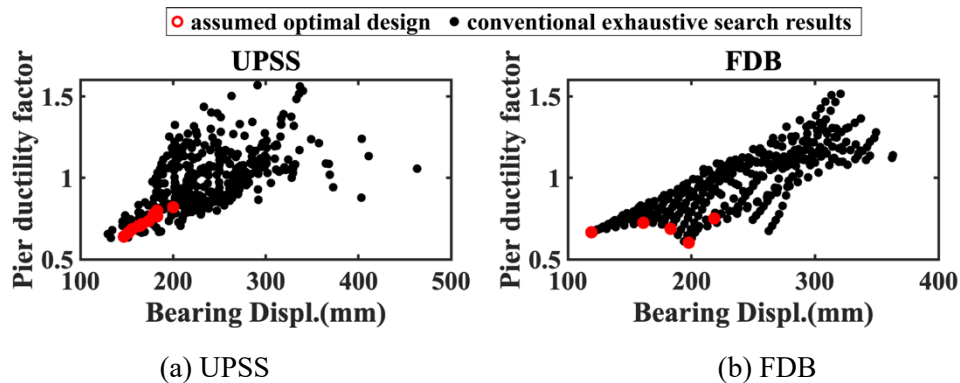


Fig. 8 Comparison with conventional exhaustive search, case II-I-2



More specifically, for a given bearing displacement demand (200mm), the optimal design for UPSS is determined as $\mu = 0.11$, $\theta = 5^\circ$ and that for FDB is $\mu = 0.15$, $T_b = 1.45$ sec. The corresponding bridge responses are shown in Fig. 9.

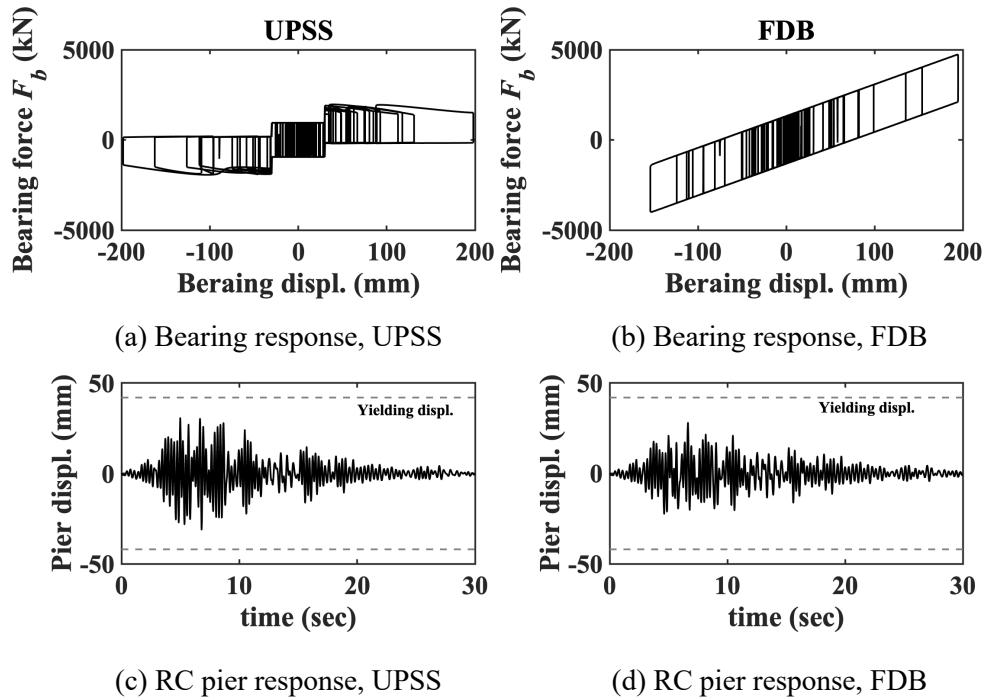


Fig. 9 Seismic response of the bridge for the optimal parameter selection, case II-I-2

4. Effectiveness

To validate the effectiveness of the proposed procedure compared with the conventional exhaustive search approach, the definition of the error is firstly given. In the ideal situation, the assumed Pareto solutions perfectly match with the exact Pareto solutions obtained by the conventional exhaustive search. However, the error appears, if the minimum pier response obtained from the proposed procedure is higher than the minimum value obtained from the conventional exhaustive search, for a given bearing displacement demand, as show in Fig. 10. Thus, the value of the error is defined as:

$$\text{error} = \frac{\Delta d}{D} \times 100\% \quad (14)$$

where Δd is the difference between the two minimum pier responses obtained from the two approaches, and D denotes the difference between the maximum and minimum pier responses obtained from the conventional exhaustive search for the given bearing displacement demand.

The results for UPSS and FDB under 18 standard ground motions covering three different soil conditions and two earthquake source types are shown in Fig. 11, where the horizontal axis is normalized by the maximum and minimum bearing displacements of each case. As indicated by this figure, the error of each ground motion case shows different precision depending on the characteristics of the ground motion. The mean error over most of the bearing displacement range is lower than 4% for UPSS and is around 11.5% for FDB, while the mean error tends to rise when the bearing displacement demand is in the lower level.

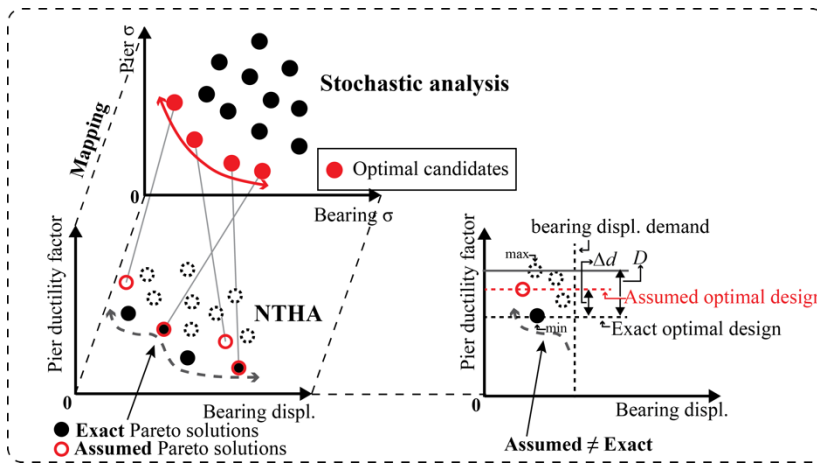


Fig. 10 Illustration of error

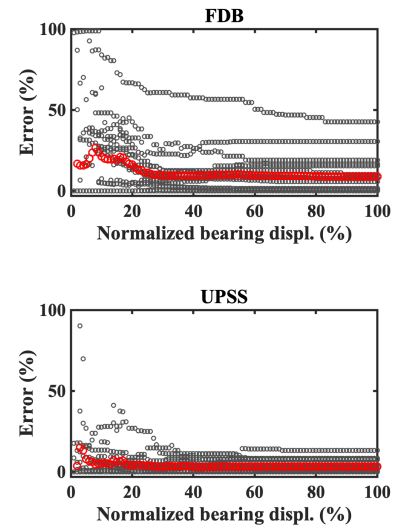


Fig. 11 Error of the proposed method (18 standard ground motions in Japanese codes)

5. Conclusions

For the seismic design of bridges with unconventional slide bridge bearings, a stochastic-deterministic approach to determine bearing parameters is proposed to achieve the multi-objective optimal design that minimizes the bearing displacement as well as the pier response to save the computational requirement. A numerical example is given for the optimal design of slide bearings of two types, namely UPSS and FDB, under strong ground motions. It is demonstrated that the seismic performance of the bridge for the case of the design parameters obtained by the proposed procedure is almost equivalent to the one with the optimal parameters found by the conventional exhaustive search approach.

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