17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 13th to 18th 2020

TOWARDS IMPROVED MODELING OF DAMPING FOR SEISMIC ANALYSIS OF BUILDINGS

E. Miranda(1), C. Cruz(2),

(1) *Professor, John A. Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford University, Stanford, California, U.S.A., emiranda@stanford.edu*

(2) *Assistant Professor, Department of Civil Works, Universidad Técnica Federico Santa María, Santiago, Chile, cristian.cruzd@usm.cl*

Abstract

We have known for more than 50 years that damping ratios in buildings tend to decrease with increasing fundamental period of vibration. Over the years, this important observation has been independently confirmed by several different studies in various countries, yet it is typically ignored in building codes which commonly specify a constant damping ratio, often 5%, regardless of building height or fundamental period of the building. Similarly, based on data recorded in instrumented buildings, several studies have shown that damping ratios in higher modes are, in most cases, higher than those in the fundamental mode of vibration and tend to increase approximately linearly with increasing frequency, yet most seismic analyses use a Rayleigh damping which combines a stiffness- and mass-proportional damping model with the same damping ratio at two modal frequencies which are typically selected fairly arbitrarily. Practically no research has been conducted to study the reasons behind these empirical trends in damping ratios. The objective of this paper is to summarize the results of a five-year investigation aimed at improving our understanding on damping in buildings and on improving how damping is modeled for seismic analysis. Using relatively simple models, for both the superstructure and the foundation-soil system, it is shown that effective damping ratios, that is, damping ratios that when used together with fixed-base models are able to reproduce the dynamic characteristics and dynamic response of buildings with flexible base, decrease with increasing building height. This reduction with building height also corresponds to an approximately linear increase in effective damping ratio with increasing fundamental frequency of vibration. Similarly, the same relatively simple analytical models show that effective damping ratios of higher modes tend to be higher than those of the fundamental mode and that they tend to also increase approximately linearly with increasing frequency. It is shown that these trends observed in the analytical results match closely the trends in damping ratios inferred from strong motion records obtained in various earthquakes in more than 150 instrumented buildings in California by using system identification techniques, indicating that soil-structure interaction, and more specifically radiation damping, explains the trends in empirical data.

Keywords: Damping ratios, Soil-structure interaction, Radiation damping, System Identification, Modal damping

1. Introduction

The seismic response of buildings to earthquake ground motions depends on the level of damping in the soil-foundation-structure dynamic system. Relative to other dynamic characteristics of buildings, our present knowledge on damping mechanisms and their relative contribution to the overall energy dissipation in the system is more limited. Building codes all over the world usually use different values of damping ratios when analyzing the same building when subjected to wind loading and when subjected to seismic loading even though one would intuitively think that the level of damping at comparable levels of response would be the same or at least very similar regardless of the type of dynamic loading.

In most seismic analysis it is assumed that damping that is not explicitly modeled through hysteretic energy dissipation in structural elements is viscous, that is, that damping forces are proportional to relative velocities. Furthermore, in most cases a Rayleigh viscous damping is used with a linear combination of stiffness-proportional damping (damping ratios proportional to modal frequencies) and a mass-proportional damping (damping ratios inversely proportional to modal frequencies). The same level of damping ratio, typically 5%, is assumed to occur at two, rather arbitrarily-selected, modal frequencies (e.g. the fundamental frequency and fifth-mode frequency of vibration). Furthermore, the same damping ratio is assumed regardless of the height of the building, the fundamental period of the building, the slenderness ratio of the building, the lateral-resisting system or site conditions, even when several studies have indicated that these characteristics influence the level of damping in buildings [2-7].

The level of damping has an influence not only on peak values of many important response parameters such as peak interstory drifts, peak floor accelerations, peak story shears, peak overturning moments and peak base shear, but also on the duration of the response and therefore on the number of cycles that the structure and nonstructural components will undergo during earthquake ground motions. Unlike mass and stiffness which can be estimated at the element/component level and then assembled into mass and stiffness matrices at the structure/system level, material damping or damping in individual elements or in individual connections is not representative of the main energy dissipation mechanisms present in buildings and therefore damping cannot be computed or assembled from information at the element/component level to the structure/system level. Furthermore, damping cannot be measured directly and therefore it needs to be inferred from measured dynamic response of structures.

A few years after the development of the response spectra by Maurice Biot at the California Institute of Technology, Alford, Housner, and Martel were developing response spectra that could be used in the seismic analysis of buildings. In the technical report that preceded their well-known paper [1], they acknowledged that the damping ratio that can be expected in engineering structures was an unanswered question. Based on a series of large amplitude tests conducted by the Earthquake Engineering Research Institute on a four-story concrete building, they concluded: “From the meager evidence available, it appears possible that a minimum figure of the order of 5% of critical damping might be established as appropriate for conventional buildings” [1]. This early recommendation, despite being based on the tests results of a single building in the early 1950s provided the basis for the value of damping ratio of 5% that is recommended in many building codes.

Another early recommendation of damping ratios was done by Professor Jacobsen at Stanford University in a report to the Division of Architecture of the State of California, in 1959 [8]. The document aimed to provide damping values for different types of structures based on a literature review of 13 references. These recommended damping ratios varied from 4%for steel buildings with welded joints to 50% for buildings with nailed joints. They are much larger than any current standard. These damping values were not based on measurements of dynamic response but rather they correspond to equivalent damping ratios were obtained from static load tests of individual components and connections. That is, the recommended values are based on equating the ratio of dissipated work to the total work occurring during half or a full load cycle in the test, to the theoretical work ratio that a single-degree-of-freedom (SDOF) system with a linear viscous damper would have under the same load. Jacobsen himself admitted that the concept of equivalent damping ratios involves debatable assumptions, and wrote that "from a philosophical point of view it must be admitted that it is highly artificial to separate restoring forces and friction forces in vibrational systems where the hysteretic effect is a function of displacement only" [9].

Other early recommendations for damping ratios in structures were made in the early 1960s, when the first nuclear reactors were designed in the US. The design code for these reactors was the TID-7024, developed by the United States Atomic Energy Commission (AEC) [10]. Values ranged from 0.5% for welded steel structures to 7% for concrete structures, but it also included recommendation of 15 to 40% for masonry structures. Another well-known recommendation were made by Newmark and Hall [11, 12] but they did not properly documented the basis for those recommendations but they appear to be based on the recommendation by Jacobsen.

The first attempt to infer information on damping ratios in buildings subjected to earthquakes is the work of Tanaka et al. in 1969, who investigated the records of 17 buildings subjected to the Saitama earthquake of July 1, 1968 in Japan [13]. The fundamental period of the different buildings was obtained from peaks in the power spectral density function, and the damping ratios were estimated using the half-power bandwidth. They showed for the first time, that damping ratios tend to decrease with increasing fundamental period of vibration. In the U.S. Hart and Vasudevan analyzed the records of 12 buildings subjected to the 1971 San Fernando Earthquake and used a simplified method for estimating damping based on the half-power bandwidth method, and was one of the first investigations to systematically infer properties of buildings from their measured seismic response [14]. Their values ranges from 1.9% to 11.3%.

Goel and Chopra studied the records of 22 buildings subjected to the Northridge Earthquake of 1994, estimating the dynamic properties of the structures; including the modal periods and damping values [15]. More recently, Bernal et al. [4] inferred damping ratios of the fundamental mode of 72 buildings. More recently Cruz and Miranda [5-7, 16] inferred damping ratios on 154 instrumented buildings for more than 1,300 damping ratios and provided recommendations. They showed that the best individual predictor to estimate damping ratios in buildings was the building height and that adding information about the slenderness ratio, the fundamental period and lateral resisting system could provide modest improvements in the estimate. Data gathered and analyzed by Tanaka et al. [13] and Satake et al. [2] in Japan or by Goel and Chopra [15], Bernal [4] and Cruz and Miranda [5-7, 16] in the United States show that damping ratios decrease with increasing height and/or increasing fundamental period of vibration. However, none of these studies have investigations the reasons for with decreasing trend in damping ratios.

The objective of this investigation is to show that soil-structure interaction effects explain the reduction of first mode damping ratios with the building height, and also the linear increment with increasing frequency of damping in higher modes, observed in the empirical data. First, the method of Veletsos and Meek [17] for computing the effective periods and damping ratios of an SDOF system is extended to MDOF structures. Using the statistical relationships between the buildings' height, aspect ratios, and periods analytical models including soil-structure interaction are assembled and analyzed to examine the analytical variation of the effective damping ratio with the building height and with the effective modal frequency. Finally, the analytical results are compared to the empirical data showing that empirical data closely followed the analytical results.

2. Dynamic properties of the soil-foundation-structure system

A series of idealized planar N-story soil-foundation-building models similar to that shown in Figure 1 were developed. The building's mass was lumped at floor levels, with each story having mass m_j. Each level is assumed to act as a rigid diaphragm, the rotational inertia of each story with respect to a horizontal axis is assumed to be negligible, as well as the axial deformations of the columns, so there is only one dynamic degree of freedom per floor. It was also assumed that the superstructure has classical damping, which allows

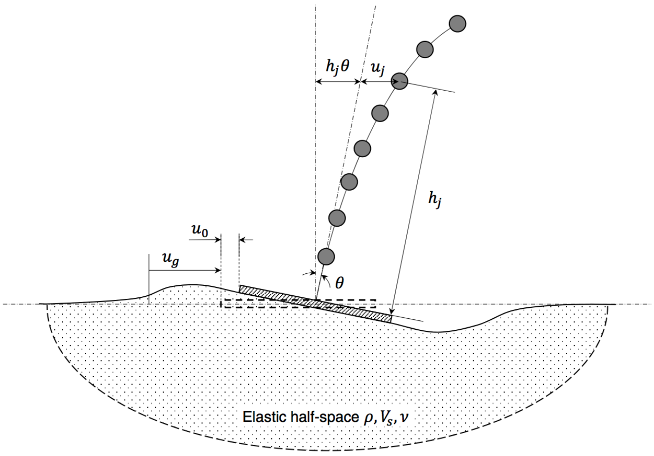

Fig. 1. Simplified building-foundation-soil system considered in this investigation.

decomposition into classical normal modes. The building has a rigid, circular foundation of radius *R0* and mass m_0, sitting on an elastic half space with no inherent damping. The transfer function relating absolute horizontal acceleration at any floor to the ground acceleration were computed in the frequency domain as a function of the transfer function of the superstructure and the dynamic stiffness of the circular rigid foundation on an elastic half-space using for the latter the results by Veletsos and Wei [18]. Figure 2 shows the simplified model of the soil-foundation system that was employed while figure 3 shows the frequency-dependent dynamic stiffness of the soil-foundation system which related horizontal forces with horizontal displacements and overturning moments at the base of the structure with rotations at the base. This dynamic stiffness is given by the following system of two equations with two unknowns:

$$\begin{Bmatrix} V \\ M \end{Bmatrix} = \begin{bmatrix} K_{VV}(a_0) & K_{VM}(a_0) \\ K_{MV}(a_0) & K_{MM}(a_0) \end{bmatrix} \begin{Bmatrix} H_{u_0} \\ H_\theta \end{Bmatrix} \tag{1}$$

where H_{u0} and H_θ are the frequency response functions associated with the swaying and rocking motions, respectively, and a_0 is the non-dimensional frequency defined as $a_0 = \omega R_0/Vs$.

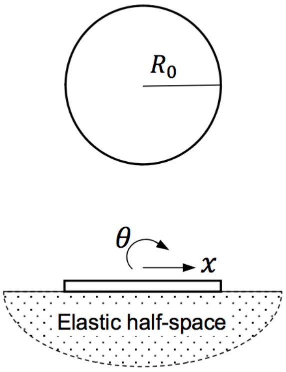

Fig. 2. Simplified model of the soil-foundation system.

The dynamic stiffness matrix of the soil-foundation interface is defined by the terms K_{VV}, K_{VM}, K_{MV}, and K_{MM}, which are given by:

$$\begin{aligned} K_{VV} &= K_x \cdot [k_{11}(a_0) + i a_0 c_{11}(a_0)] \\ K_{VM} &= K_x R_0 \cdot [k_{12}(a_0) + i a_0 c_{12}(a_0)] \\ &= K_{MV} \\ K_{MM} &= K_\theta \cdot [k_{22}(a_0) + i a_0 c_{22}(a_0)] \end{aligned} \tag{2}$$

where kij and cij are dimensionless functions of the soil's Poisson ratio ν and the non-dimensional frequency a_0. Exact values for these parameters can be found in Veletsos and Wei [18] and are shown in Figure 3 for $\nu=0.45$. As shown in this figure the horizontal stiffness (real part of the dynamic stiffness) relating horizontal forces and horizontal displacements is relatively frequency independent, but in the case of the rotational stiffness relating overturning moment at the foundational and rotations at the foundation (rocking) the stiffness shows significant reductions with increasing stiffness. Meanwhile the damping coefficients (imaginary part of the dynamic stiffness) is again fairly independent of frequency for the horizontal direction, that is the relationship between horizontal velocity and horizontal damping force, while the damping coefficient relating the change in rotational damping force and rotational velocity is again strong frequency dependent but in this case the rocking damping increases with increasing frequency.

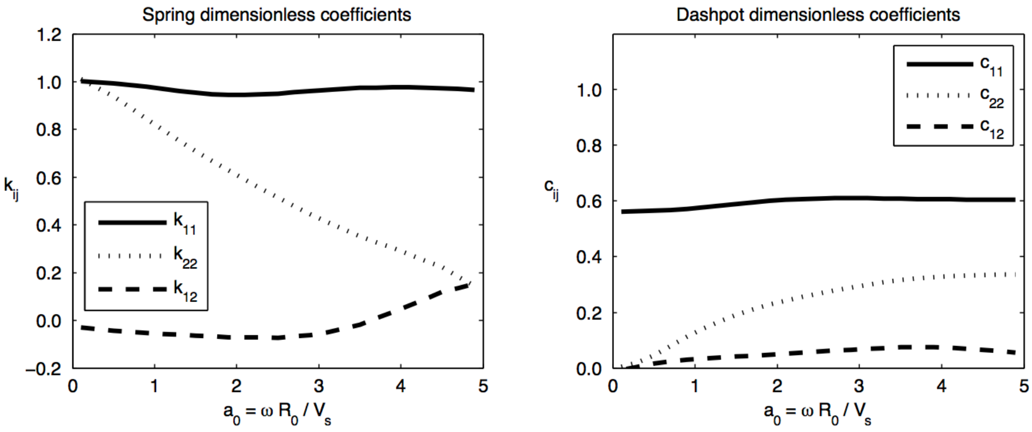

Fig 3. Dynamic stiffness for a rigid circular disk on an elastic half-space with $\nu = 0.45$.

3. Dynamic properties of the soil-foundation-structure system

A series of idealized planar N-story soil-foundation-building models similar to that shown in Figure 1 ranging from 1 to 80 stories were developed. Most buildings are designed using a linear elastic response modal spectrum analysis, in which the base structure is typically assumed as fixed. Therefore, it is of interest to evaluate what dynamic characteristics should a replacement fixed-base structure have in order to replicate as best as possible the transfer function of a building with a flexible base. An optimization routine was developed to find the equivalent fixed-base parameters that would best reproduce the transfer function of the building with a flexible base. Figure 4 shows the transfer function relating horizontal accelerations at the roof of the structure to the horizontal ground motion for a 25-story building The building has a height of 75 m, with a uniform story height of 3 m. The superstructure has a fundamental (fixed-base) period $T_1 = 3.0$ s. The

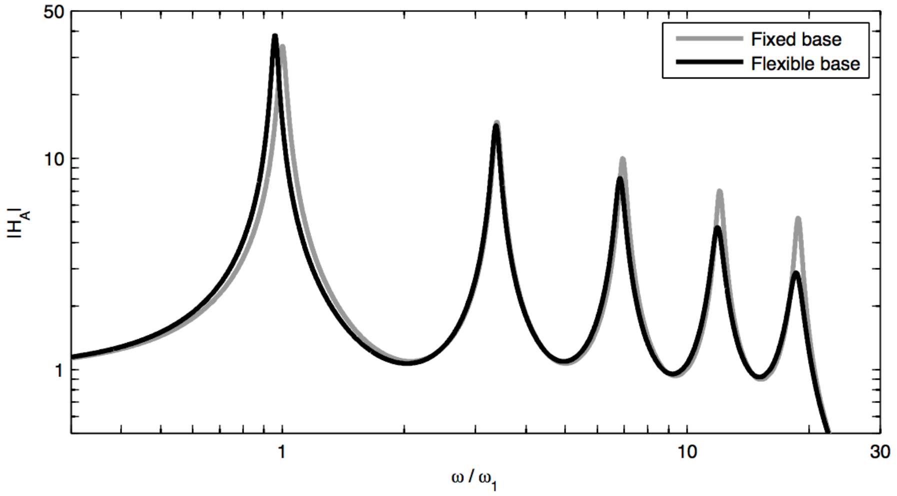

Figure 4. - Comparison of the absolute acceleration transfer function of the building at the roof considering fixed and flexible bases.

building has a dual lateral resistant system, consisting of moment frames and shear walls, i.e. $\alpha = 6$ [18]. The damping ratio of the superstructure was considered to be 2% for all modes. Each story was assumed to be circular with a radius R_0 = 15 m. The seismic weight is 10.5 kN/m2 (220 psf). The foundation has the same radius R_0, but only half the mass of a typical story. The building was assumed to be sitting on a soil with shear wave velocity Vs = 250 m/s (corresponding to NEHRP site class D), mass density ρ=2 kN ·s2/m4, and Poisson ratio ν=0.45. It was assumed that 5 modes contributed significantly to the structural response. Also shown in this figure is the transfer function for the same building with a fixed base. It can be seen that, as expected the modal frequencies of the fixed-base model are larger than those of the building on the elastic half space. However, the modal peak amplitudes in some cases are larger for the fixed-based model (e.g., for the fundamental mode) while in other cases they are smaller (higher modes). The optimization procedure consisted then on modifying the dynamic characteristics of the fixed-based model to match, as best as possible, the transfer function of the building on flexible base by minimizing the difference in transfer function of the replacement fixed-based structure and that of the flexible-base model. An example of the match after the optimization is shown in figure 5.

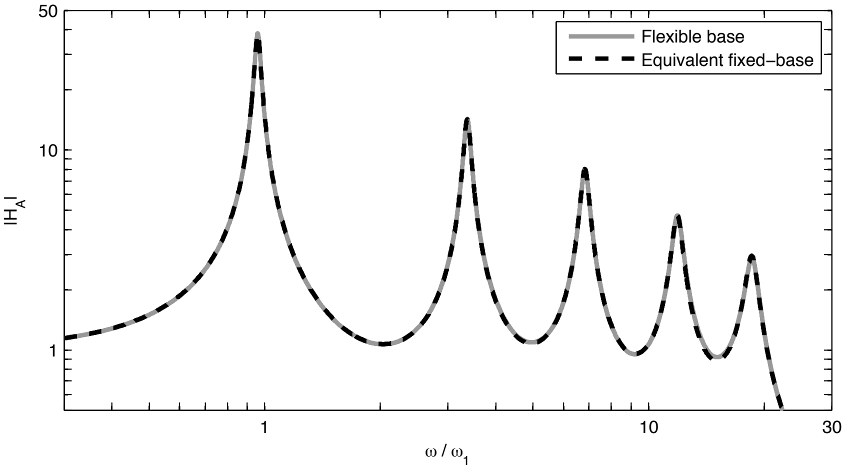

Figure 5. - Comparison of the absolute horizontal acceleration transfer function at the roof of the building with a flexible base (on elastic half-space) and the replacement building with a fixed base.

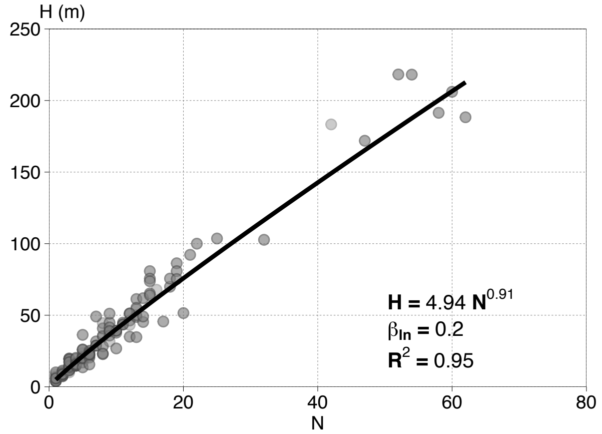

Figure 6. Relationship between the height and the number of stories in instrumented buildings in California.

4. Parametric analysis using 80 different building models

A parametric analysis was conducted to assess the influence of the different parameters that govern the effects of soil structure interaction on the effective damping of MDOF buildings. To this end, 80 different structures were modeled, and their effective properties were computed using the simplified analysis method for MDOF structures previously described in section 3. All the buildings were assumed to have a dual lateral resistant system consisting of shear walls and moment frames, therefore a value of 6 was used for the lateral stiffness ratio α. The damping ratio of the superstructure was set to 3% for all modes. The structures had different number of stories, with the first building having 1 story, the second one, 2, and so on until the 80th structure, which had 80 stories. In order to have buildings with realistic heights for a given number of stories, the height of each building was computed as a function of the number of stories using a regression performed on data obtained from instrumented buildings in California as shown in figure 6. In order to have relatively simple models all stories were assumed to have the same height. Once the building height H was computed, the aspect ratio, AR, of each building was calculated from the regression analyses done on data relating aspect ratio and building height in instrumented buildings in California shown in Figure 7. Consequently, the radius of the foundation was set to be R=H/AR. The seismic weight was assumed to be 10.5 kN/m^2 (220 psf)

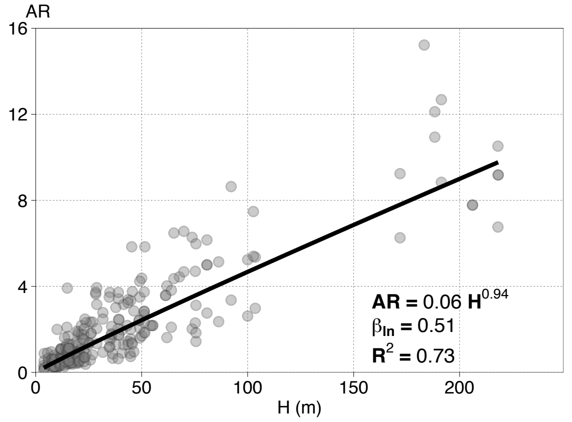

Figure 7. Relationship between the aspect ratio (AR) and the building height in instrumented buildings in California.

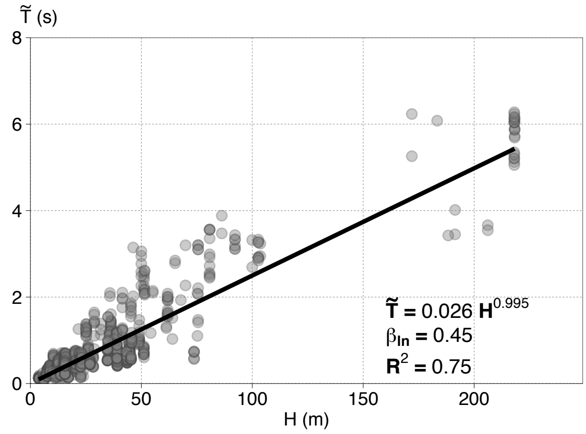

Figure 8. Relationship between the fundamental period and the building height in instrumented buildings in California.

for all floors, while the foundation was assumed to be massless. Finally, the fundamental period of the superstructure was computed using the results of a linear mixed-effects, LME, model shown on Figure 8 based on instrumented buildings in California. Note that the identified periods actually correspond to the effective period of vibration, which is larger than the period of the superstructure.

5. Results for the Effective Damping Ratio of the Fundamental Mode

Figure 9 shows the variation of the effective damping ratio of the fundamental mode, normalized by that of the superstructure, with the aspect ratio of the building. The figure shows the curves obtained after the analyses of the 80 structures of different heights, for the 5 different site classes. It can be seen that the effective damping ratio decreases with increasing aspect ratio, and that the rate of decrement is significantly affected by the shear wave velocity. Again, this plot shows that for practical purposes buildings located on firm soils, such as NEHRP site classes A or B (Vs = 1600 and 1000 m/s, respectively) are not affected by the

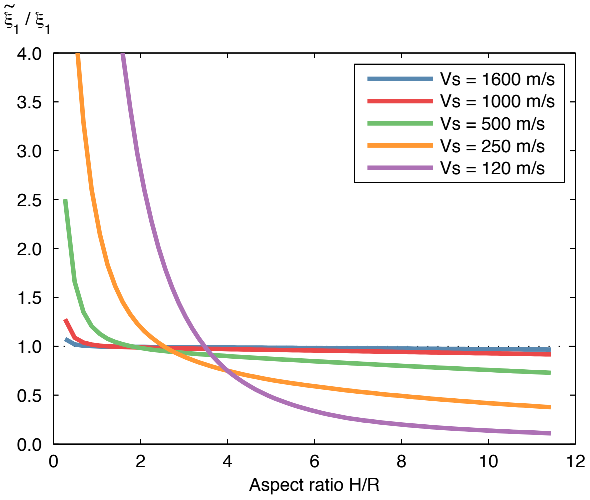

Figure 9. Variation of the effective damping ratio of the fundamental mode with changes in the aspect ratio of the buildings.

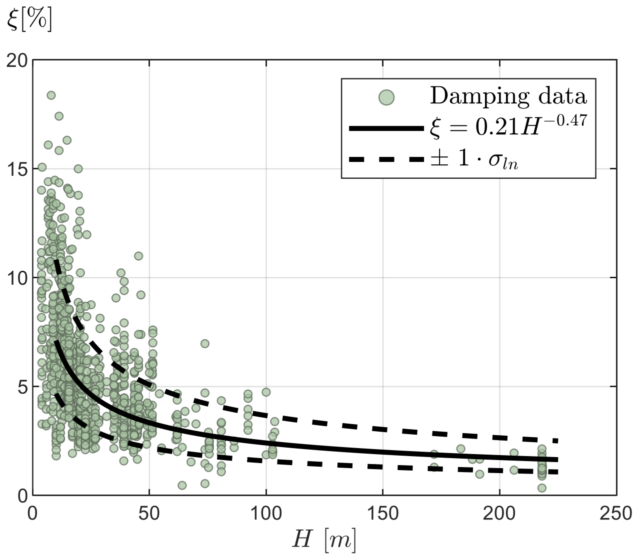

Figure 10. Empirical relationship between the inferred effective damping ratios in instrumented buildings in California and their building height. LME model of empirical data also shown [7].

effects of soil structure interaction. For buildings on softer soils, it can be seen that the effective damping ratio is higher than that of the superstructure for aspect ratios smaller than 2.0, 2.5, and 3.5 for shear wave velocities of 500, 250, and 120 m/s, respectively. The increment in the effective damping for squatty structures can reach values that double or even triple the damping ratio of the superstructure in some cases, highlighting the positive effects of soil-structure interaction in increasing the effective damping ratio. The opposite occurs for slender structures, where it is observed that buildings with large aspect ratios have an effective damping ratio lower than that of the superstructure. The reduction in damping can be significative for very slender buildings, for example, buildings built on soils with $V_S = 250$ m/s (site class D, the most common site class in California) and aspect ratios greater than 7 can have effective damping ratios lower than 50% that of the superstructure.

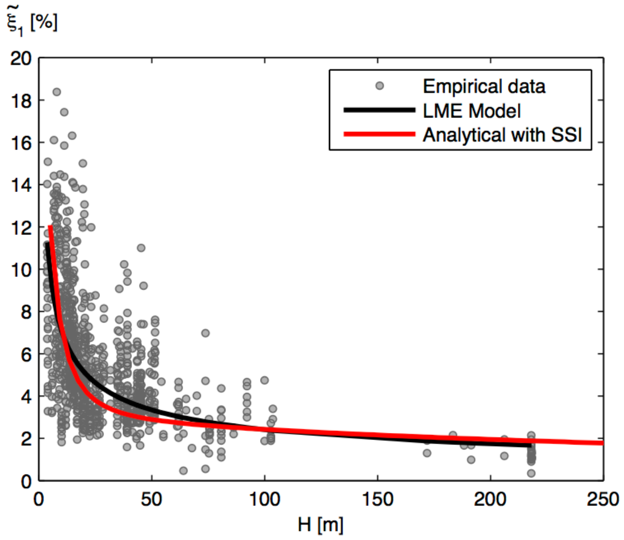

Figure 11. Comparison of empirical and analytical variation of the effective damping ratio of the fundamental mode with the building height.

As shown previously in Figure 7, the aspect ratio AR is strongly correlated with the building height H. Therefore, for a given value of Vs, the effective damping ratio will have a somewhat similar descending trend with increasing building height. Figure 11 compares the 1,335 damping ratios of the fundamental modes inferred from records obtained in instrumented buildings in California during various earthquakes [20-22] with those computed the analytical models shown in figure 1 obtained for the 80 different structures using a Vs value of 360 m/s, which defines the boundary of site classes C and D – where the vast majority of the buildings in the data set are located. The figure also includes the LME statistical model obtained from the empirical data. As expected, the effective damping decays with the building height following a nonlinear trend. More importantly, there is a remarkable similarity between the analytical results and the median damping ratios obtained by the statistical LME model, suggesting that soil-structure interaction is the most important factor controlling the observed reduction of damping ratios with increasing building height.

7. Summary and conclusions

The effect of soil-structure interaction on the overall damping ratio of multi-story buildings has been evaluated, and theoretical findings have been compared with empirical data inferred from the seismic response of buildings in California. A method was developed for obtaining the effective periods and effective damping ratios of replacement fixed-base MDOF structure capable of reproducing the response of the corresponding structures on an elastic half-space. The method uses modal superposition to decompose the structural response into a series of SDOF structures, whose effective properties are computed using the method developed by Veletsos and Meek for SDOF systems.

A parametric analysis was conducted to assess the influence of parameters governing the effective damping ratios of the fundamental mode. In these analyses, the number of stories in the structure was varied while the building's height, aspect ratio, and fundamental period were calculated to match relationships inferred from empirical data. It was shown that that for a given value of the shear wave velocity of the soil, the aspect ratio is the parameter that primarily controls the effects of the soil-structure interaction. On soft soils – like NEHRP site classes C, D and E – the effective damping ratio decreases with increasing values of the aspect ratio following a nonlinear trend. Squatty buildings with low aspect ratios will have an effective damping ratio of the fundamental mode higher than that of the superstructure, hence benefiting from soil-structure interaction effects. On the other hand, on slender structures with aspect ratios greater or equal to 3, the effective damping ratio will be lower than that of the superstructure, reaching reduction rates than can be higher than 50%. A similar descending nonlinear trend was obtained when examining the variation of the effective damping with the building height.

The analytical results were then compared with a database of 1335 damping ratios inferred from the seismic responses of 154 buildings in California. It was shown that the theoretical variation of the effective damping ratio with the building height, obtained for shear wave velocities representative of NEHRP site classes C and D – were the vast majority of the buildings in the data set are located, follows closely the median trend of the empirical data.

These results show that soil-structure interaction, that is, the combination of radiation damping along with the rotation and swaying of the base of the building, explains the observed empirical variations of the effective damping ratio with building height, and with the effective modal frequency. This suggests that the effects of soil-structure interaction is the most important contributor, and therefore what primarily controls, the overall damping ratio of buildings subjected to earthquakes.

8. Acknowledgements

The authors would like to acknowledge CONICYT – Becas Chile, the John A. Blume Earthquake Engineering Center at Stanford University, and the Shaw Family Fund for the financial aid provided to the second author to conduct this investigation in collaboration with the first author. Ground and structural motions employed in this study were obtained from the California Strong Motion Instrumentation Program of the California Geological Survey and from the United States Geological Survey. Efforts to install, operate, and maintain seismic instrumentation in buildings, as well to process and disseminate earthquake records by these organizations are gratefully acknowledged.

9. References

[1] Alford, J. L., and Housner, G. W. (1953). A dynamic test of a four-story reinforced concrete building. *Bulletin of the Seismological Society of America*, 43(4), 7–16.

[2] Satake N, Suda K, Arakawa T, Sasaki A, Tamura Y. Damping evaluation using full-scale data of buildings in japan. *Journal of Structural Engineering* 2003; **129**(4): 470–477.

[3] Fritz WP, Jones NP, Igusa T. Predictive models for the median and variability of building period and damping. *Journal of Structural Engineering* 2009; 135(5): 576–586.

[4] Bernal D, Dohler M, Kojidi SM, Kwan K, Liu Y. First mode damping ratios for buildings. *Earthquake Spectra* 2015; 31(1): 367–381.

[5] Cruz, C., and Miranda, E. (2017). Evaluation of damping ratios for the seismic analysis of tall buildings. *Journal of structural engineering*, 143(1), 04016144.

[6] Cruz, C., and Miranda, E. (2018). First mode damping ratios inferred from the seismic response of buildings. *In 11th US national conference on earthquake engineering*, EERI, Los Angeles, California.

[7] Cruz, C., and Miranda, E. (2020). Damping ratios of the first mode for the seismic analysis of buildings. Accepted for publication in *Journal of structural engineering*, ASCE.

[8] Jacobsen, L. S. (1959). Frictional effects in composite structures subjected to earthquake vibrations. *Tech. Rep. CSA No. 1976*, Department of Mechanical Engineering, Stanford University, Stanford, CA.

[9] Jacobsen, L. S. (1960). Damping in composite structures. *In Proc. 2nd World Conference on Earthquake Engineering*, (pp. 1029–1044). Tokyo and Kyoto, Japan.

[10] U.S Atomic Energy Commission (1973). Damping values for seismic design of nuclear power plants. *Regulatory Guide 1.61*, U.S Atomic Energy Commission, Washington, DC.

[11] Newmark, N. M. (1967). Design criteria for nuclear reactors subjected to earthquake hazards. *Tech. Rep.* , University of Illinois at Urbana-Champaign, Urbana, Illinois.

[12] Newmark, N. M. (1972). Earthquake Response Analysis of Reactor Structures. *Nuclear Engineering and Design*, 20(2), 303–322.

[13] Tanaka, T., Yoshizawa, S., and Osawa, Y. (1969). Period and damping of vibration in actual buildings during earthquakes. *Bulletin of the Earthquake Research Institute*, 47, 1073–1092.

[14] Hart, G., and Vasudevan, R. (1975). Earthquake design of buildings: damping. *Journal of the Structural Division*, 101(1), 11–30.

[15] Goel, R. K., and Chopra, A. K. (1997b). Vibration properties of buildings determined from recorded earthquake motions. *Tech. Rep. EERC-97-14*, Earthquake Engineering Research Center, University of California, Berkeley, Berkeley, CA.

[16] Cruz, C., and Miranda, E. (2017). Evaluation of damping ratios inferred from the seismic response of buildings, *Report No. 196*, John A. Blume Earthquake Engineering Center, Dept. of Civil and Envir. Engrg., Stanford University, Stanford, California, pp. 249.

[17] Veletsos, A. S., and Meek, J. (1974). Dynamic behaviour of building-foundation systems. *Earthquake Engineering and Structural Dynamics*, 3, 121–138.

[18] Veletsos, A. S., and Wei, Y. T. (1971). Lateral and rocking vibration of footings. *Journal of Soil Mechanics and Foundations Division*, 97, 1227–1248.

[19] Miranda, E., & Reyes, C. J. (2002). Approximate lateral drift demands in multistory buildings with nonuniform stiffness. *Journal of Structural Engineering*, 128(7), 840-849.

[20] Cruz, C., and Miranda, E. (2017). Evaluation of soil-structure interaction effects on the damping ratios of buildings subjected to earthquakes. *Soil Dynamics and Earthquake Engineering*, 100, 183-195.

[21] Cruz, C., and Miranda, E. (2020). Insights into the damping ratios in buildings. Under review in *Soil Dynamics and Earthquake Engineering*.

[22] Miranda, E. and Cruz, C. (2018). Evaluation of the role of soil-structure interaction on modal damping ratios in buildings. *In 11th US national conference on earthquake engineering*, EERI, Los Angeles, California.