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ANALYSIS OF NON-UNIFORM COUPLED SHEAR WALLS WITH VISCOELASTIC DAMPERS IN HIGH-RISE BUILDINGS

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Abstract

Non-uniform coupled shear walls, where the cross-section of the walls at upper levels are reduced, may become economical for high-rise buildings. Moreover, the viscoelastic coupling damper (VCD), which has been developed to enhance wind and seismic performance of high-rise buildings, can be used to replace reinforced concrete (RC) or steel coupling beams in coupled shear walls, to provide a significant amount of damping to the structure. In this study, the continuum method is used to derive the governing equation for free vibration analysis of coupled shear walls with viscoelastic dampers with an abrupt change in cross-section at some level, where the stiffness of the connecting elements and that of the slab directly above the dampers are also considered. The governing differential equation is solved numerically to evaluate the periods and (added) damping ratios for several modes of vibration. The accuracy of the solution method is verified using the finite element method (FEM), where two-dimensional shear wall systems with variable cross-sections are used as the model. In addition, 20-, 30-, and 40-story buildings with non-uniform coupled shear walls with viscoelastic dampers are used to investigate the effect of different coupled wall and damper parameters on the damping ratios, frequencies, and response of walls using analytical solution. It is found that with the increase in flexibility of the structure, the period of vibration and damping ratio both raise for the first mode of vibration. The stiffness of the connecting elements and that of the slab directly above the dampers have a significant influence on periods of vibration and damping ratios with respect to the height ratios. The proposed analytical formulation is useful for selecting the preliminary size of non-uniform coupled shear walls. Finally, response spectrum analysis (RSA) and time history analysis are also carried out using FEM for non-uniform coupled shear walls with viscoelastic dampers. The results from FEM are compared with the analytical results, which show that the differences are not so significant.

Keywords: continuum method; damping ratio; non-uniform coupled shear walls; viscoelastic dampers

1. Introduction

High-rise buildings (greater than 50 m) with reinforced concrete (RC) coupled shear walls have been popular in many countries. Researchers have been working on coupled shear walls for more than 50 years [1- 7]. Since buildings are getting taller and more slender, the amount of additional material required to achieve sufficient stiffness has become greater. As a result, adding additional damping rather than stiffness can be a more cost-effective solution in this scenario [8]. Additionally, significant damages in the coupling beams in high-rise buildings are observed in past earthquakes [9]. The viscoelastic coupling damper (VCD) has recently been developed to enhance wind and seismic performance of high-rise buildings, which provides both stiffness and damping to the system [9-12]. These dampers can be used to replace reinforced concrete (RC) or steel coupling beams in coupled shear walls, which can provide a significant amount of damping to the structure without occupying usable floor space or altering the structural layout of the building. On the other hand, improved structural behavior is provided by this high-performance system. Therefore, it reduces the cost of materials and construction of the structure. The VCD is made up of several layers of viscoelastic material and bonded to layers of steel plate. To build-up steel sections, these steel plates are anchored at alternating ends. VCDs transfer vertical forces and increase the lateral stiffness of the system [11].

The amount of damping ratio is an important parameter to evaluate the effectiveness of any damper. This damping ratio is commonly determined using the finite element method (FEM), which, however, lacks complex modal analysis features. In this scenario, the free vibration response history analysis (RHA) provides a better solution but involves a more time-consuming process. Additionally, introducing viscoelastic dampers makes the analysis more complicated since these are strain-, frequency-, and temperature-dependent. In a recent study, a free vibration analysis of tall coupled shear walls with viscoelastic dampers is conducted using the continuum method, and a sixth-order partial differential equation is derived to obtain the dynamic properties of tall uniform building structures [9].

As the seismic demand is not higher in upper stories, the use of non-uniform coupled shear walls, where the cross-section of the walls at upper levels is reduced, may become a cost-effective solution. The continuum method is used to analyze such coupled-wall structures with stepped variations in thickness [1]. Analytical studies based on the continuum method on coupled shear walls with an abrupt change in cross-sectional properties at some levels are also presented in a few studies [2, 3]. Previous analytical studies on coupled wall systems with [13, 14] and without [4-6] considering dampers assume the properties of shear walls to be uniform. Yet, none of these studies conducts free vibration analysis of non-uniform coupled shear walls with viscoelastic dampers.

In this study, non-uniform coupled shear walls with viscoelastic dampers are investigated analytically, considering the influence of the stiffness of the connecting elements and slab, where the analytical solution is able to provide an accurate estimation of its dynamic properties. A fourth-order differential governing equation of motion without considering the axial deformation of walls is derived and solved numerically using eight appropriate boundary conditions. The accuracy of the solution method is verified using two-dimensional finite element models. Moreover, this study also investigates damping ratios, frequencies, and responses of non-uniform coupled shear walls with different coupled wall and damper parameters.

2. Analytical Study

2.1 Devising the problem

Two shear walls coupled using viscoelastic dampers, having an abrupt change of cross-section at some height, H_1 subjected to uniformly distributed loading of intensity, p per unit height is shown in Fig. 1. For generality, the structure is subdivided into two zones considering cross-sections being constant along their

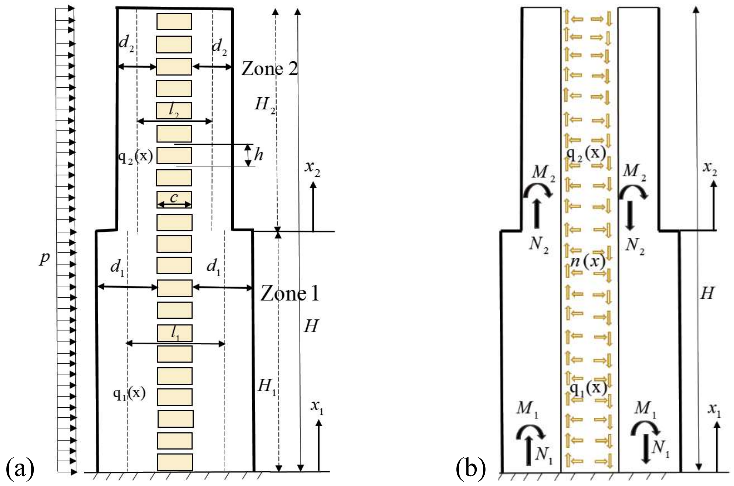

Fig. 1 – Schematic of the coupled wall system under lateral load [3]: (a) equivalent continuum model; (b) internal forces after cutting along the midspan of the connecting medium.

respective height. Rigid diaphragm assumption is used at each floor so that two walls deflect equally at each floor level with a point of contraflexure at the mid-span of the viscoelastic dampers. The story height, h, and the properties of the walls and dampers are constant along the height. The axial deformations of the walls are assumed as small, hence neglected in this study. The inherent damping is not considered during analysis since the added damping usually causes large damping ratios. The behavior of the shear walls is assumed to be linear elastic, and plane sections remain plane both before and after the bending.

For the continuum method, if it is assumed that the vertical medium is cut along the midspan of the connecting medium, the only forces acting in the midspan are the shear flow of intensity, $q(x)$ per unit height and axial forces of intensity, $n(x)$ per unit height, as shown in Fig. 1(b) [7]. The axial force, N acting in each wall at any level, x is expressed as

$$N(x) = \int_{x}^{H} q(\lambda) d\lambda. \tag{1}$$

In this study, the sign convention is considered as the positive relative displacement, which means that the left-hand side will move downwards relative to the right-hand side of the damper and vice-versa. As axial deformations are neglected in the free vibration analysis, the relative displacements due to axial deformation of the walls are not considered here. Therefore, the vertical compatibility occurs along the midspan of the connecting medium due to

(i) wall rotations caused by the bending of walls [7],

$$\delta_1 = l \frac{dy(x)}{dx}. \tag{2}$$

Here, $y(x)$ = lateral displacement and l = center-to-center distance of the walls.

(ii) shear deformations in the viscoelastic dampers, obtained using equivalent complex shear modulus, where the dampers are connected to the walls with steel beams embedded into them and slab stiffness where the slabs are placed directly above the dampers [9]. The equivalent complex shear modulus, G_e^* considering connecting element stiffness and slab stiffness is expressed as

$$G_e^* = G' e_c (1 + i\eta) + r_s G', \tag{3}$$

where, G' = storage modulus of the viscoelastic material, η = loss factor of the viscoelastic material, $r_s = k_s/k_d$, where k_s = slab stiffness above the damper, and k_d = stiffness of the viscoelastic damper, and e_c is referred as

$$e_c = \frac{r_k G'}{r_k G' + G'(1+i\eta)}, \tag{4}$$

where, $r_k = k_c/k_d$. and k_c = stiffness of the connecting element.

The relative displacement, δ_2 using the stress-strain relationship of the viscoelastic damper, $\gamma = -\tau/G_e^*$, at height x is

$$\delta_2 = -\frac{q(x)}{G_e^* n_e}, \tag{5}$$

where, γ = harmonic strain time history, τ = stress time history and $n_e = nA_{ve}/ht_{ve}$, where, n = number of viscoelastic layers, A_{ve} = area of each viscoelastic layer, h = story height, and t_{ve} = thickness of each viscoelastic layer.

At the point of contraflexure of the connecting medium, no relative vertical displacement exists under lateral loads. As a result, vertical compatibility at this midspan is

$$\delta_1 + \delta_2 = 0. \tag{6}$$

Substituting the values of δ_1 and δ_2 in Eq. (6)

$$l\frac{dy(x)}{dx} - \frac{q(x)}{G_e^* n_e} = 0. \tag{7}$$

The moment-curvature relationship [4] for the combined system is

$$EI\frac{d^2y(x)}{dx^2} = M_o(x) - l\int_x^H q(\lambda)d\lambda. \tag{8}$$

Here, E = modulus of elasticity of concrete, I = total moment of inertia of walls, and M_o = total moment due to external loading.

According to D'Alembert's principle, the dynamic equation of motion of a vibrating shear wall structure can be derived by considering the inertia forces along with the external forces. However, while considering free vibration, only inertia forces act and external forces vanish [4], which leads to

$$\frac{\partial^2 M_o(x,t)}{\partial x^2} = -m\frac{\partial^2 y(x,t)}{\partial t^2}. \tag{9}$$

Here, m is the mass per unit height of the shear walls.

Embracing the similar strategy described in [4] and [9], the governing equation becomes

$$\frac{d^4Y(x)}{dx^4} - \frac{G_e^* n_e l^2}{EI}\frac{d^2Y(x)}{dx^2} - \frac{m\omega^2}{EI}Y(x) = 0. \tag{10}$$

Here, ω is the angular frequency of vibration.

Non-dimensionalizing by $\bar{Y} = \frac{Y}{H}$ and $\bar{X} = \frac{x}{H}$, and using non-dimensional parameters yield the required governing equation of motion as follows

$$\frac{d^4\bar{Y}(\bar{X})}{d\bar{X}^4} - \alpha^4 \frac{d^2\bar{Y}(\bar{X})}{d\bar{X}^2} - \lambda^4\bar{Y}(\bar{X}) = 0, \tag{11}$$

where, H = total height of the wall, $\alpha^4 = \frac{G_e^* n_e l^2 H^2}{EI}$, $\lambda^4 = \frac{m\omega^2 H^4}{EI}$.

Considering two different co-ordinate system for the non-uniform coupled shear walls with viscoelastic dampers, Eq. (11) yields,

$$\frac{d^4\bar{Y}_z(\bar{X}_z)}{d\bar{X}_z^{\,4}} - \alpha_z^4 \frac{d^2\bar{Y}_z(\bar{X}_z)}{d\bar{X}_z^2} - \lambda_z^4\bar{Y}_z(\bar{X}_z) = 0, \tag{12}$$

where, z = 1, 2 considering lower and upper wall respectively.

2.2 Boundary conditions and solution methodology

The general solutions of the differential equations of Eq. (12) can be expressed as

$$\left.\begin{aligned} \bar{Y}_1(\bar{X}_1) &= C_1' \cosh a_1\bar{X}_1 + C_2' \sinh a_1\bar{X}_1 + C_3' \cos a_2\bar{X}_1 + C_4' \sin a_2\bar{X}_1 \\ \bar{Y}_2(\bar{X}_2) &= C_5' \cosh a_3\bar{X}_2 + C_6' \sinh a_3\bar{X}_2 + C_7' \cos a_4\bar{X}_2 + C_8' \sin a_4\bar{X}_2 \end{aligned}\right\} \tag{13}$$

The boundary conditions required to determine all the constants of Eq. (13) are follows:

(i) At $x = 0$ (fixed base),

$$\bar{Y}_1(0) = 0, \quad \frac{d\bar{Y}_1(0)}{dx_1} = 0. \tag{14}$$

(ii) At $x_1 = H_1$ (junction),

$$\left.\begin{aligned} &\bar{Y}_1(R_1) = \bar{Y}_2(0), \quad \left.\frac{d\bar{Y}_1}{d\bar{X}_1}\right|_{x_1=R_1} = \left.\frac{d\bar{Y}_2}{d\bar{X}_2}\right|_{x_2=0}, \\ &EI_1\left.\frac{d^2\bar{Y}_1}{d\bar{X}_1^2}\right|_{x_1=R_1} = EI_2\left.\frac{d^2\bar{Y}_2}{d\bar{X}_2^2}\right|_{x_2=0} - \int_0^{R_2} q_2(\bar{X}_2)d\bar{X}_2(l_1 - l_2), \\ &\left.\frac{dM_1}{d\bar{X}_1}\right|_{x_1=R_1} = \left.\frac{dM_2}{d\bar{X}_2}\right|_{x_2=0}. \end{aligned}\right\} \tag{15}$$

(iii) At $x_2 = H_2$ (roof),

$$\left.\frac{d^2\bar{Y}_2}{d\bar{X}_2^2}\right|_{x_2=R_2} = 0, \quad \left.\frac{dM_2}{d\bar{X}_2}\right|_{x_2=R_2} = 0, \tag{16}$$

where, H_1, H_2 = height of the lower and upper wall respectively, I_1, I_2 = moment of inertia of lower and upper wall respectively, l_1, l_2 = distance between centerline of lower and upper walls respectively, M_1, M_2 = moment in lower and upper wall respectively, $R_1 = H_1/H$ and $R_2 = H_2/H$.

Using these eight boundary conditions, Eq. (13) leads to a set of algebraic equations in the form of **AC** = 0, where **A** is the coefficient of matrix and **C** is the vector of constants. Here, the determinant of the coefficient matrix |**A**| must be equal to zero in order to obtain non-trivial solutions. From this, a characteristic equation will result, which can be solved numerically to obtain the values of λ corresponding to each mode of vibration. For the case of non-uniform coupled shear walls with viscoelastic dampers, the frequency parameter, λ is the complex frequency parameter. Therefore, the natural frequency, ω_i and damping ratio, ζ_i can be computed [9] as

$$\omega_i = \sqrt{\frac{\mathrm{Re}(\lambda_i^4)EI}{mH^4}}, \quad \zeta_i = \frac{\mathrm{Im}(\lambda_i^4)}{2\,\mathrm{Re}(\lambda_i^4)}. \tag{17}$$

Here, *i* is the mode of vibration.

3. Verification of the Analytical Study

The governing equation of motion for non-uniform coupled shear walls with viscoelastic dampers is used to evaluate the periods and mode shapes for the first three modes of vibration. For verification, a comparison is carried out between the results obtained from the analytical solution and that from FEM.

3.1 Verification of the period of vibration and mode shape

The modal properties are solved in MATLAB [15] for the first three modes of non-uniform coupled shear walls with viscoelastic dampers using the proposed analytical solution. A finite element model of 21-story coupled shear walls is used to verify the periods of vibration and mode shapes of the system in this analysis, where the walls have a constant story height of 3 m and an abrupt change in cross-section at the 11th-story. Walls are modeled using an equivalent center-line model with rigid offsets to simulate the wall width, and the Kelvin-Voigt model is used for numerical modeling of dampers. The properties of the walls and material considered for the verification are given in Table 1. A commercially available viscoelastic damper, which consisted of ISD-111H viscoelastic material, is used to verify the periods of vibration and mode shapes whose properties are given in Table 2. The frequency-dependent properties of the viscoelastic material can be obtained from the data derived from the manufacturer-specified properties, adopted from the previous study [9].

The results from the analytical solution are then compared with the results obtained using FEM, which is illustrated in Table 3. The normalized mode shapes for the first three modes of non-uniform coupled shear walls with viscoelastic dampers are shown in Fig. 2. The results from both the analytical solution and FEM show good agreement.

Table 1 – Parameters of the walls for verification of analytical solution

H (m)	**H_1 (m)**	**H_2 (m)**	**Lower wall size (mm^2)**	**Upper wall size (mm^2)**	**l_1 (m)**	**l_2 (m)**	**M (kg/m)**	**E (GPa)**
63	33	30	8400×300	5400×300	13.2	10.2	67957.87	27.8

Table 2 – Parameters of viscoelastic dampers for verification of analytical solution

N	**A_{ve} (mm^2)**	**t_{ve} (mm)**	**G' (kPa)**	**H**	**k_d (kN/mm)**	**k_c (kN/mm)**	**k_s (kN/mm)**
18	240,800	5	130.35	0.8095	113	56.5	16.5

Table 3 – Verification of periods for the first three modes of vibration

Periods of vibration, T (s)						
Story	Mode 1		Mode 2		Mode 3	
	Analytical	FEM	Analytical	FEM	Analytical	FEM
21	0.600	0.631	0.200	0.211	0.088	0.093

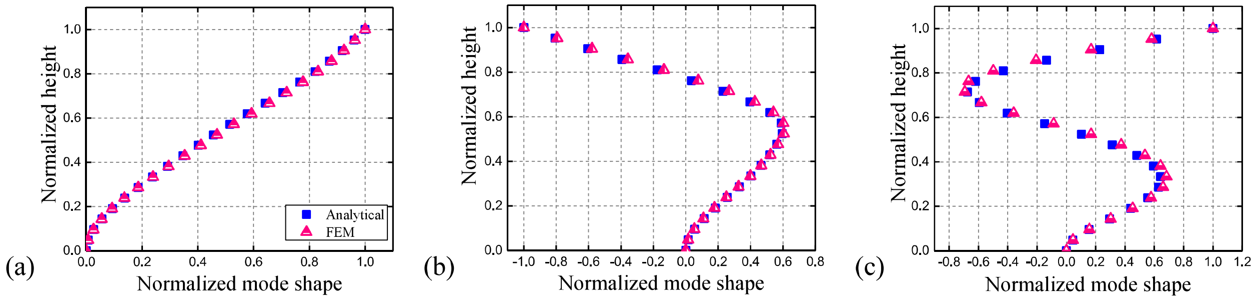

Fig. 2 – Comparison between results from the analytical solution and FEM for the normalized mode shapes (a) Mode 1; (b) Mode 2; (c) Mode 3.

3.2 Verification of the damping ratio

In this study, the damping is evaluated using discrete damping elements along with running the numerical free vibration decay method in PERFORM-3D [16]. The analysis is performed in PERFORM-3D using step-by-step integration through time using Newmark β method. The free vibration is conducted by applying the harmonic force input having the frequency equal to the natural frequency of that particular mode [17]. The harmonic input excitation, resulting in forced vibration response and free vibration response, is shown in Fig. 3 for the first mode of vibration. The computed damping ratios from the method of decay of motion using discrete elements and analytical solutions are given in Table 5, where j is the consecutive cycles.

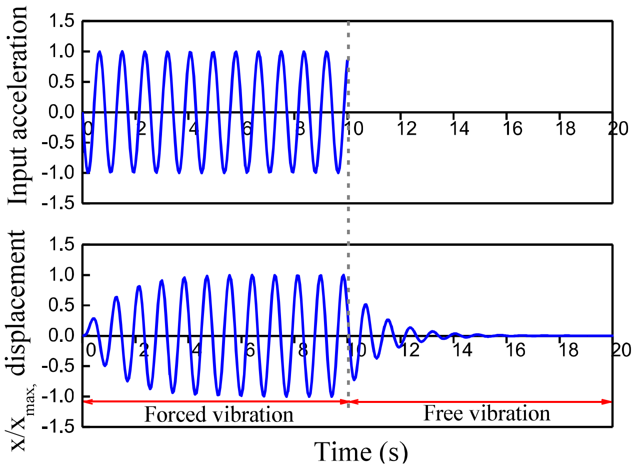

Fig. 3 – Harmonic excitation and resulting displacement response under forced and free vibration for the first mode.

Table 4 – Verification of (added) damping ratio for the first mode of vibration

Method	Equation	Damping ratio, ζ_1 (%)
FEM	$\zeta = \frac{1}{2\pi j} \ln \frac{u_i}{u_{i+j}}$	10.90
Analytical	$\zeta = \frac{Im(\lambda^4)}{2Re(\lambda^4)}$	12.54

4. Numerical Results and Applications

4.1 Results

The proposed analytical formulation presented in this study is used to observe the influence of changing height ratio, center-to-center distance ratio, the stiffness of the connecting element and slab stiffness on the modal properties for buildings with non-uniform coupled shear walls with viscoelastic dampers.

4.1.1 Effect of height ratio

Numerical analyses have been carried out for different height ratios of non-uniform coupled shear walls with viscoelastic dampers using MATLAB. In this study, 20-, 30-, and 40-story buildings are considered. The analyses results of the change in periods of vibration and damping ratios for the first mode with the changing height ratios, R_2/R_1 are shown in Fig. 4. It is evident that period and damping ratio also raise for the first mode of vibration with the increase in height ratio, which in turn gives rise to the flexibility of the structure.

4.1.2 Effect of center-to-center distance ratio

The varying modal properties, such as periods of vibration and damping ratios, have been investigated for changing center-to-center distance ratios, l_2/l_1. Three buildings with different heights are used in this study, considering the height ratio, $R_2/R_1 = 1$. It can be observed from Fig. 5 that with the increase in center-to-center distance ratios, where keeping center-to-center distances of lower walls constant, periods of vibration and damping ratios of buildings for the first mode exhibit declining nature.

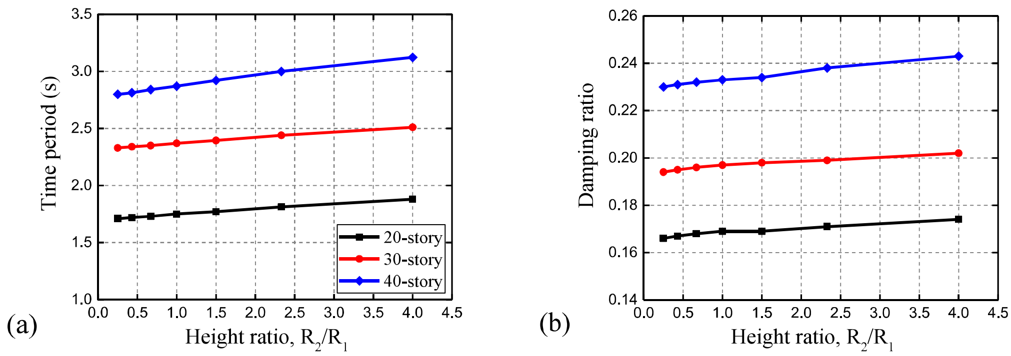

Fig. 4 – Effects of height ratio on the (a) time period and (b) damping ratio for the first mode of vibration of the buildings with different heights.

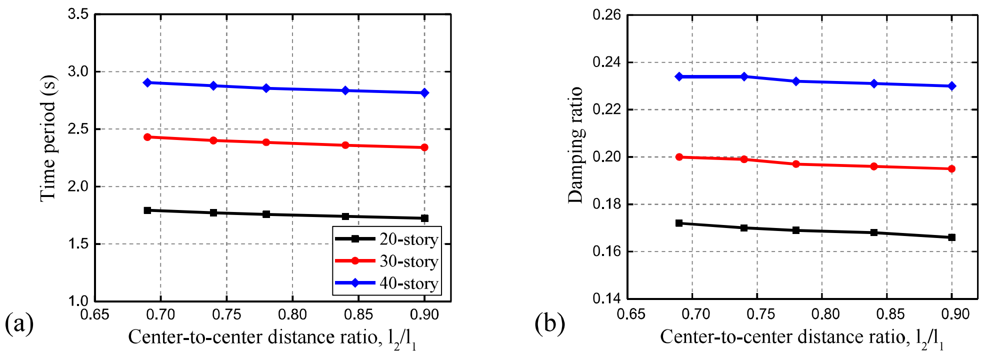

Fig. 5 – Effects of center-to-center distance ratio on the (a) time period and (b) damping ratio for the first mode of vibration of the buildings with different heights.

4.1.3 Effect of connecting element and slab stiffness

The stiffness of both the connecting element and slab are important parameters in studying the modal properties of such systems. Therefore, different values of the connection stiffness (k_c) in comparison with the stiffness of the damper (k_d) which yields to r_k = 1, 5, 10, 10,000 (rigid connection), and different values of the slab stiffness (k_s) in comparison with the stiffness of the damper (k_d) yielding to r_s = 0, 0.5, and 1 are considered for the analysis.

The results from the analyses considering negligible stiffness, r_s = 0 and center-to-center distance ratio constant for a 40-story building with different height ratios are shown in Fig. 6. It is clear that there is a significant influence of connection stiffness on both the periods of vibration and damping ratios with respect to the stiffness of the damper. For the case of r_k = 1, the values of the periods are larger than they are for the other cases, and also, the damping ratios reduce significantly. From Fig. 6(b), it can be observed that the damping ratios increase with the increase in the value of r_k. But no significant changes in damping ratio is observed with the change in height ratios of non-uniform coupled shear walls with viscoelastic dampers. On the other hand, Fig. 7 presents the analysis results, where rigid connection, r_k = 10,000 and a constant center-to-center distance ratio for 40-story building are considered. The results show that the periods of vibration and the damping ratios both decrease with the increase in the stiffness of the slab. For r_s = 1, the maximum possible damping ratio is lower than that is in the other cases.

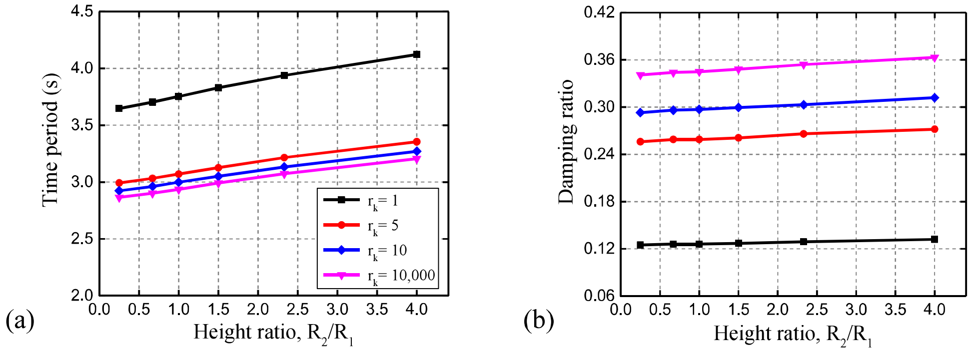

Fig. 6 – Effects of connecting element stiffness on the (a) time period and (b) damping ratio for the first mode of vibration of the 40-story building.

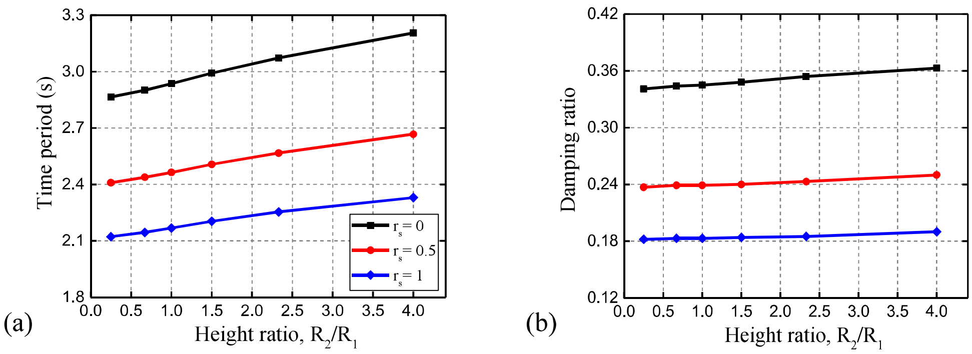

Fig. 7 – Effects of slab stiffness on the (a) time period and (b) damping ratio for the first mode of vibration of the 40-story building.

4.2 Selection of preliminary size of coupled shear walls

In this study, 20-, 30-, and 40-story buildings are analyzed to select the preliminary size of coupled shear walls. For the preliminary size selection, displacement of the w^{th} floor for the i^{th} mode is calculated [17] as

$$u_{wi} = \Gamma_i \phi_{wi} D_i, \tag{19}$$

where, Γ_i (modal participation factor) and ϕ_{wi} (i^{th} mode shape value at the w^{th} floor) from analytical solution and D_i (displacement spectrum ordinate) from site-specific spectra are obtained. The displacements and inter-story drift ratios for different wall depths for the 40-story building are shown in Fig. 8. As there is an abrupt change in the middle of the total height, a change in the inter-story drift ratio (IDR) is observed in the middle of the buildings (Fig. 8(b)). Similar responses are observed in the case of 20- and 30-story buildings, which are not shown here. For the damped structures, the damping modifier is applied to the elastic displacement spectrum for different levels of damping ratio, ζ. A commonly used expression of damping modifier presented in 1998 EC8 [18] is

$$\mathbf{R}_\zeta = \sqrt{\frac{0.07}{0.02+\zeta}} \tag{20}$$

4.3 Time history analysis for verification of the application

Time history analysis for 40-story non-uniform shear walls coupled with viscoelastic dampers is carried out using FEM to verify the analysis of the application of the proposed governing equation. Elastic beam-column elements are used here to represent the walls.

The case study building is located in Los Angeles, California, USA. A suite of 14 far-fault ground motions is used and scaled to match the 5% critically damped target spectra in this study (Fig. 8(a)). It is seen from Fig. 8(b, c) that the results (displacement and inter-story drift ratio (IDR)) based on the continuum method go along with the response spectrum analysis (RSA) using FEM, whereas time history analysis results show differences less than 12%, which is a tolerable error for the preliminary design of coupled shear walls.

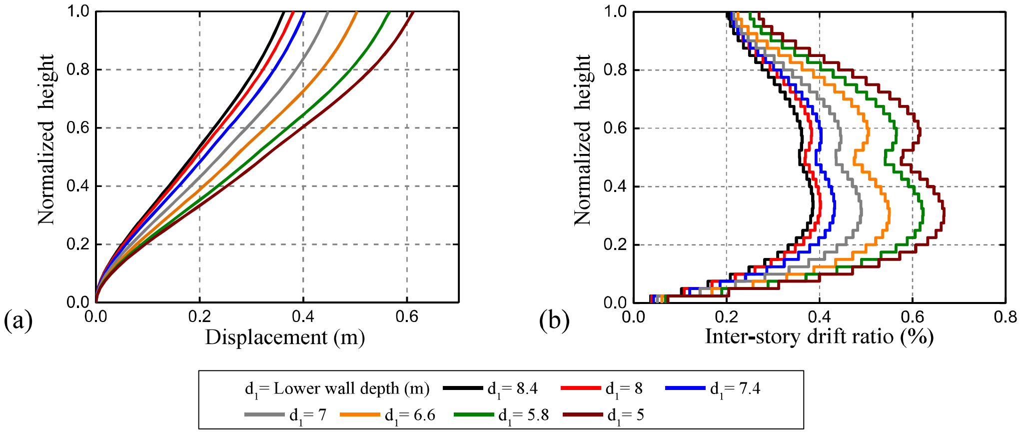

Fig. 8 – (a) Displacement and (b) inter-story drift ratio for different wall depth for 40-story building.

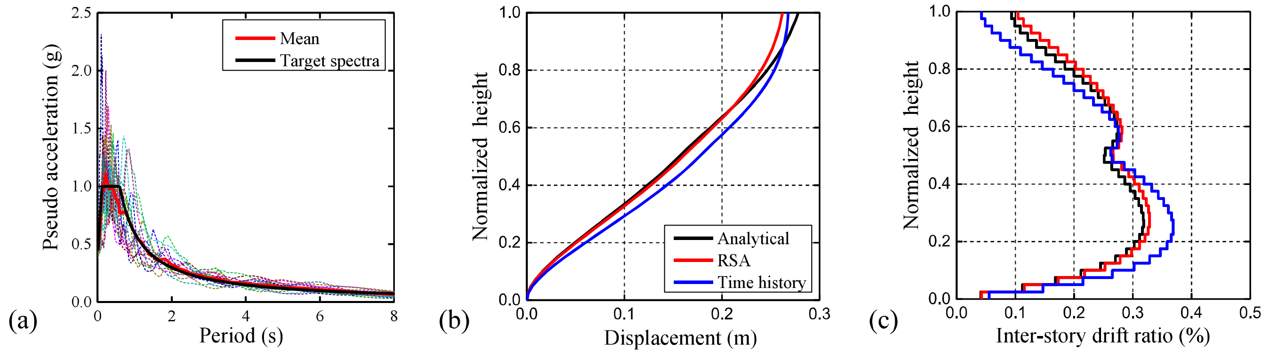

Fig. 9 – (a) 5%-damped target acceleration spectrum with the mean acceleration spectrum of selected scale ground motions; (b) displacement and (c) inter-story drift ratio of 40-story non-uniform coupled shear walls with viscoelastic dampers.

5. Conclusions

A governing equation of motion was obtained for non-uniform coupled shear walls with viscoelastic dampers, where the stiffness of the connecting elements and that of the slab directly above the damper were considered, but where the axial deformation of walls were not taken into account. The fourth-order differential governing equation of motion was solved numerically using eight appropriate boundary conditions.

The accuracy of the method was verified against the results obtained using FEM. The effects of different coupled walls and damper parameters such as height ratio, center-to-center distance ratio, connection stiffness, and slab stiffness on the periods of vibration and damping ratio were investigated. With the rise in height ratio, the period of vibration and damping ratio also increased for the first mode of vibration. On the contrary, periods of vibration and damping ratios exhibited a declining nature with the increase in center-to-center distance ratios. Connection stiffness and slab stiffness had a significant influence on the periods of vibration and damping ratios with respect to the height ratios. Additionally, the proposed analytical formulation was proven to be useful for selecting the preliminary size of the non-uniform coupled shear walls with viscoelastic dampers by observing the structural response. Finally, the response spectrum and time history analysis results from FEM were compared with the analytical results, which did not show any significant difference.

6. Acknowledgement

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