NUMERICAL MODEL FOR PANEL ZONE WITH STEEL SQUARE HOLLOW SECTION UNDER MULTI-DIRECTIONAL LOADINGS

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Abstract

In order to ensure sufficient energy dissipation capacity under severe earthquake, steel moment frames are usually designed to achieve an entire beam-hinging collapse mechanism. On the basis of this viewpoint, the criterion to achieve strong column-weak beam has been adopted in seismic design codes of several countries in seismic area.

Steel moment frames which consist of wide flange beams and square hollow section columns with through diaphragms are often used for low to middle rise building structures in Japan. As panel zone at beam-to-column connection usually has the same cross section as the column below it directly, it is highly probable that the panel zone yields under severe earthquake as well as beams. For the purpose of examining the influence of elasto-plastic behavior of panel zones on seismic response of moment frames, a lot of numerical studies have been conducted by subjecting plane frames to uni-directional ground motion. However, there are few researches which are concerned with the seismic response of 3D frames with square hollow section columns and panel zones under bi-directional ground motion.

On the other hand, axial force, bi-directional shear forces and bi-axial bending moments act on the panel zones, under bi-directional ground motion. Besides the influence of shear force and axial force, the influence of bending moment can't be negligible on the elasto-plastic behavior of panel zone with large aspect ratio or under bi-directional loading. However, numerical model for panel zone, considering the elasto-plastic behavior under bi-direction shear forces and bi-axial bending moments, has not been proposed yet. Moreover, the influence of panel zones on steel moment frames subjected to bi-directional ground motion is still not completely understood.

Based on the background above, this paper proposes a novel numerical model for steel square hollow section panel zone with through diaphragms. This model consists of multi-spring components, which are built up of elasto-plastic springs with degrees of freedom regarding of axial deformation and shear deformation. Thereby, the model is able to consider the elasto-plastic behavior under correlation between axial force, bi-direction shear forces and bi-axial bending moments. Further the method to adopt the panel zone model into the 3D frame analysis program which has been developed by the authors is also described in this paper.

To verify the validity in the proposed numerical model for panel zone, the analysis results of the proposed model are compared to the results of finite element method analysis and experiments from previous researches, targeting single panel zones and partial cruciform frames which consist of beams, columns and panel zone. It is clarified by the comparisons that the analysis results of the proposed model are well corresponding to the finite element method analysis results or experimental results. In addition, the analysis result shows that the proposed model is possible to express the differences of plasticity state of springs in each multi-spring component with varied input directions or aspect ratios of panel zone.

Keywords: Steel structure, Beam-to-column connection, Input direction, Numerical model, Multi spring model

1. Introduction

In order to ensure sufficient energy dissipation capacity under severe earthquake, steel moment frames are usually designed to achieve an entire beam-hinging collapse mechanism. On the basis of this viewpoint, the strong column weak beam criterion has been adopted in seismic design codes of several countries in seismic area.

In Japan, steel moment frames, which consist of wide flange beams and square hollow section columns with through diaphragms, are often used for low to middle rise building structures. As panel zone of beam-to-column connection (hereinafter called panel) usually has the same cross section as the column below it directly, it is highly probable that panel yields under severe earthquake as well as beam and column [1].

From previous research, it is well known that panel has excellent energy dissipation capacity and superior deformation capacity [2-3]. And by designing panel to yield first, damage to columns and beams can be reduced. Therefore, the concept of allowing panels to yield have been accepted by recent design provisions.

A lot of numerical studies have been conducted to investigate the influence of elasto-plastic behavior of panels on seismic response of steel moment frames. Most of those researches focus on in-plane behavior of plane frames under uni-directional ground motion [4-11], only a few researches focus on seismic response of 3D frames, with square hollow section columns and panels, under bi-directional ground motion [12]. While moment frame, which is mentioned above, is subjected to bi-directional ground motion, axial force, bi-directional shear force and bi-axial bending moment will act on the panels. Usually the shear force has a dominant influence on the elasto-plastic behavior of panel. But according to reference [13,14], in case of panel with large aspect ratio or under bi-directional loading, the influence of bending moment cannot be ignored as well as axial force and shear force. However, the numerical analysis model for panels in reference [12] considers the elasto-plastic behavior only under shear force, and a numerical model for panel, which can consider the elasto-plastic behavior under bi-direction shear forces and bi-axial bending moments, has not been proposed yet. Thereby, the influence of panels on steel moment frames subjected to bi-directional ground motion is still incompletely understood.

Based on the background mentioned above, this paper proposes a novel numerical analysis model for steel square hollow section panel with through diaphragms under and describes the method to introduce this panel model to 3D frame model that proposed by the authors [15]. Furthermore, to verify the validity in the proposed numerical model for panel, the analysis results of the proposed model are compared to the results of finite element method analysis and experiments from previous researches.

2. Method of numerical analysis

2.1 Fundamental assumptions

The analysis method in this paper is based on our earlier research [15]. The fundamental assumptions are as below.

1) 3D model is adopted. Each nodal point has six degrees of freedom, three correspond to translational displacement along axes and other three correspond to rotational displacement around axes.

2) Each structural member is taken to be a simplified model with bar element, of which only degrees of freedom at both ends are taken into account.

3) About the modeling of beam-to-column connection, beam elements and column elements are connected to panel element rigidly as shown in Fig.1(b).

4) Column elements and beam elements consist of an elastic component and two multi-spring components (hereinafter called MS component(s)) [15]. Only axial and flexural deformations are considered in the MS component for columns and beams.

5) Panel elements consist of two elastic components and three modified MS components, which are introduced in Section 2.2.

6) Springs of MS components are assumed to have bilinear restoring force characteristics.

7) Weight and mass are concentrated on nodal points.

8) Geometrical nonlinearity is not considered.

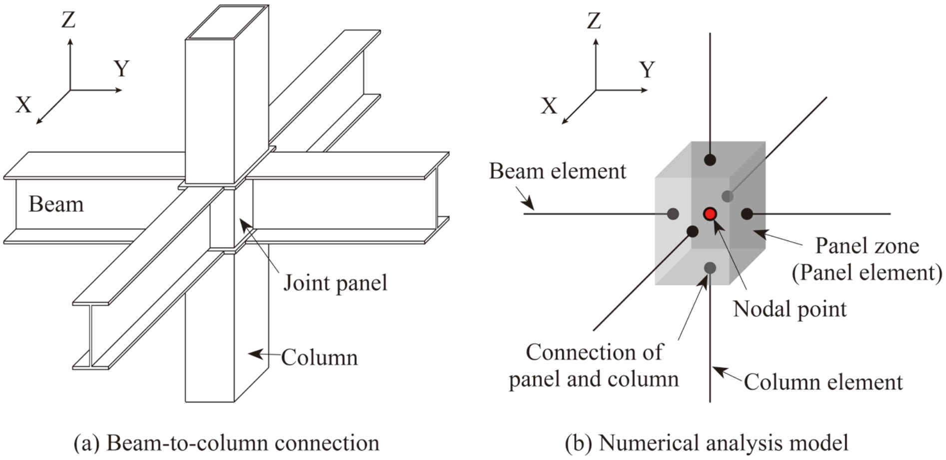


Fig. 1 – Numerical analysis model of beam-to-column connection

2.2 Details of panel element

This section describes the basic concept of numerical analysis method for square hollow section panels with through diaphragms. As the influence of bending moment is not negligible for panels with large aspect ratio or under bi-directional loading [12,13], modified MS components are used to consider the elasto-plastic behavior of panel under axial force, bi-directional shear force and bi-axial bending moment. Each panel element consists of two elastic components and three modified MS components, which are placed in series as shown in Fig. 2.

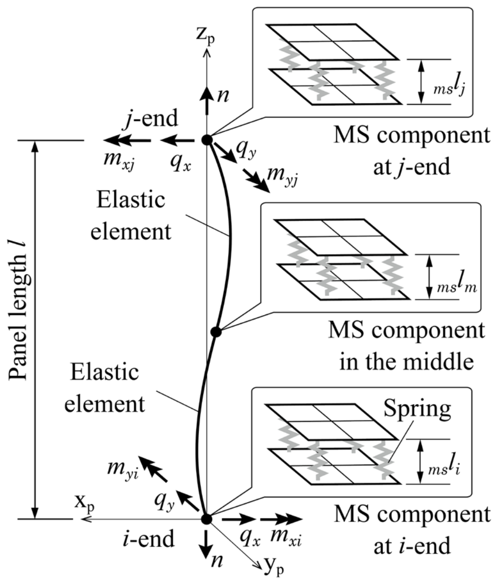
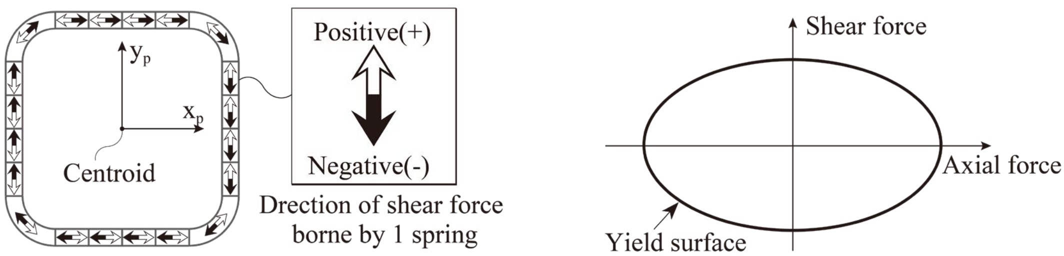


Fig. 2 – Details of panel element

A MS component consists of several elasto-plastic springs, which are arranged along the cross-section. Generally, each spring has only one degree of freedom in the axial direction, and the MS component can consider the elasto-plastic behavior under axial force and bi-axial bending moment.

For the purpose of considering the elasto-plastic behavior of panel under shear force, two degrees of freedom, in both axial and shear direction, are introduced to all springs of the modified MS component. As the thickness of square hollow section panel is thin enough that the shear force in the thickness direction can be ignored, each spring bears only one direction of shear force along the circumferential direction as shown in Fig. 3(a). Spring yields while the combined force reach yield surface, which is based on von Mises yield criterion, as shown in Figure 3(b). Note that the disposition of springs for square hollow section panel is the same as square hollow section columns in reference [15].



(a) Disposition of springs and direction of shear force borne by spring

(b) Yield surface of spring based on von Mises yield criterion

Fig. 3 – Details of springs within modified MS component

By introducing these two degrees of freedom to springs, modified MS components are able to consider the elasto-plastic behavior under axial force, bi-directional shear force and bi-axial bending moment. Here, bending moment in the middle is much smaller than bending moments at the end within a panel. Therefore, bending moment and flexural deformation of the modified MS component in the middle are assumed to be zero. And bending moments of the modified MS component at the ends is the same as the bending moments that act on the panel ends. Total length of three modified MS components is equal to the length of the panel, so the sum of axial and shear deformations of three modified MS components match the axial and shear deformation of the origin panel correspondingly. And only flexural deformation is considered for elastic components.

3. Stiffness equation of proposed panel element

3.1 Stiffness equation of modified MS component

This chapter describes stiffness equations of modified MS components and panel elements. As described in section 2.2, each spring of the modified MS components has two degrees of freedom, and tangent stiffness equation of the s-th spring is given by Eq. (1). Here, Δ denotes a small increment, the subscript s on the left side refers to the s-th spring, n and d_n denote the axial force and axial deformation, and q and d_q the shear force and shear deformation. While elastic limit is exceeded, the tangent stiffness matrix of spring follows the flow rule, and both kinematic hardening and isolated hardening of the yield surface can be considered.

$$\begin{Bmatrix} \Delta_s n \\ \Delta_s q \end{Bmatrix} = \begin{bmatrix} {}_s k_{11} & {}_s k_{12} \\ {}_s k_{21} & {}_s k_{22} \end{bmatrix} \begin{Bmatrix} \Delta_s d_n \\ \Delta_s d_q \end{Bmatrix} \tag{1}$$

Relationship between force increment vector $\{\Delta_{ms}\boldsymbol{f}\}$ of modified MS component and force increment vector of springs is given by Eq. (2), and relationship between deformation increment vector of modified MS component $\{\Delta_{ms}\boldsymbol{f}\}$ and deformation increment vector of springs is given by Eq. (3). Here, the subscript ms on the left side refers to MS component, q_x and d_{qx} denote the shear force and shear deformation along the x_p-axis, q_y and d_{qy} the shear force and shear deformation along the y_p-axis, m_x and θ_x the bending moment and rotation angle around the x_p-axis, m_y and θ_y the bending moment and rotation angle around the y_p-axis (see Fig.2).

$$\left\{\Delta_{ms}\boldsymbol{f}\right\} = \left\{\Delta_{ms}n \quad \Delta_{ms}q_x \quad \Delta_{ms}q_y \quad \Delta_{ms}m_x \quad \Delta_{ms}m_y\right\}^{\mathrm{T}} = \sum_{s=1}^{n}\left[{}_{s}\mathbf{T}\right]\begin{Bmatrix}\Delta_s n \\ \Delta_s q\end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix}\Delta_s d_n \\ \Delta_s d_q\end{Bmatrix} = \left[{}_{s}\mathbf{T}\right]^{\mathrm{T}}\left\{\Delta_{ms}d_n \quad \Delta_{ms}d_{qx} \quad \Delta_{ms}d_{qy} \quad \Delta_{ms}\theta_x \quad \Delta_{ms}\theta_y\right\}^{\mathrm{T}} = \left[{}_{s}\mathbf{T}\right]^{\mathrm{T}}\left\{\Delta_{ms}\boldsymbol{d}\right\} \tag{3}$$

[${}_{s}\mathbf{T}$] in Eq. (2) and Eq. (3) is given by Eq. (4). Here, ${}_{s}x$ and ${}_{s}y$ are coordinates of the s-th spring. ${}_{s}\theta$, which is the angle with x_p-axis, denotes the directions of shear force born by the s-th spring.

$$\left[{}_{s}\mathbf{T}\right]^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & {}_{s}y & -{}_{s}x \\ 0 & \cos{}_{s}\theta & \sin{}_{s}\theta & 0 & 0 \end{bmatrix} \tag{4}$$

From Eq. (1) to Eq. (4), stiffness equation of modified MS component is given by Eq. (5).

$$\left\{\Delta_{ms}\boldsymbol{f}\right\} = \sum_{s=1}^{n}\left[{}_{s}\mathbf{T}\right]\begin{bmatrix} {}_{s}k_{11} & {}_{s}k_{12} \\ {}_{s}k_{21} & {}_{s}k_{22} \end{bmatrix}\left[{}_{s}\mathbf{T}\right]^{\mathrm{T}}\left\{\Delta_{ms}\boldsymbol{d}\right\} \tag{5}$$

3.2 Flexibility equations of modified MS component and elastic component

As modified MS components and elastic components are place in series within a panel element, flexibility matrix of the whole panel element equals to the sum of flexibility matrixes of all those components. This section describes the flexibility equations of modified MS components and elastic components.

Firstly, the flexibility equation of modified MS component is given by Eq. (6).

$$\left\{\Delta_{ms}\boldsymbol{d}\right\} = \left[{}_{ms}\mathbf{c}\right]\left\{\Delta_{ms}\boldsymbol{f}\right\} = \left(\sum_{s=1}^{n}\left[{}_{s}\mathbf{T}\right]\begin{bmatrix} {}_{s}k_{11} & {}_{s}k_{12} \\ {}_{s}k_{21} & {}_{s}k_{22} \end{bmatrix}\left[{}_{s}\mathbf{T}\right]^{\mathrm{T}}\right)^{-1}\left\{\Delta_{ms}\boldsymbol{f}\right\} \tag{6}$$

Secondly, the flexibility equation of elastic components is given by Eq. (7). As all components are place in series, two elastic components can be represented by one equation as Eq. (7). And the flexibility matrix of elastic components is given by Eq. (8). Note that, the first term on the right side of Eq. (8) is the flexibility matrix about flexure deformation of the origin panel, and the second term is the sum of flexibility matrixes about flexure deformation of the MS components at both ends. Here, I is the second moment of area of square hollow section panel, E the Young's modulus, ${}_{s}A$ the cross-sectional area of the s-th spring, ${}_{ms}l$ the length of MS component (see Fig. 2).

$$\left\{\Delta_{e}\boldsymbol{d}\right\} = \begin{Bmatrix}\Delta_e\theta_{xi} \\ \Delta_e\theta_{yi} \\ \Delta_e\theta_{xj} \\ \Delta_e\theta_{yj}\end{Bmatrix} = \left[{}_{e}\mathbf{c}\right]\begin{Bmatrix}\Delta_e m_{xi} \\ \Delta_e m_{yi} \\ \Delta_e m_{xj} \\ \Delta_e m_{yj}\end{Bmatrix} = \left[{}_{e}\mathbf{c}\right]\left\{\Delta_{e}\boldsymbol{f}\right\} \tag{7}$$

$$\left[{}_{e}\mathbf{c}\right] = \begin{bmatrix} l/(3EI_x) & 0 & -l/(6EI_x) & 0 \\ 0 & l/(3EI_y) & 0 & -l/(6EI_y) \\ -l/(6EI_x) & 0 & l/(3EI_x) & 0 \\ 0 & -l/(6EI_y) & 0 & l/(3EI_y) \end{bmatrix} - \begin{bmatrix} \frac{{}_{ms}l_i}{\Sigma E_s A_s y^2} & 0 & 0 & 0 \\ 0 & \frac{{}_{ms}l_i}{\Sigma E_s A_s x^2} & 0 & 0 \\ 0 & 0 & \frac{{}_{ms}l_j}{\Sigma E_s A_s y^2} & 0 \\ 0 & 0 & 0 & \frac{{}_{ms}l_j}{\Sigma E_s A_s x^2} \end{bmatrix} \tag{8}$$

3.3 Flexibility equation of panel element

Deformation increment vector $\{\Delta \mathbf{d}\}$ of the whole panel element are equal to the sum of all components described above as shown in Eq. (9). Here, θ_{xi} and θ_{yi} are rotation angles at i-end, θ_{xj} and θ_{yj} are rotation angles at j-end. $[_{ms}\mathbf{T}_i]$, $[_{ms}\mathbf{T}_m]$, $[_{ms}\mathbf{T}_j]$, $[_e\mathbf{T}]$ are transformation matrixes for modified MS components and elastic components, in order to reconcile the degrees of freedom of each components and panel element.

$$\{\Delta \mathbf{d}\} = \left\{ \Delta d_n \quad \Delta\theta_{xi} \quad \Delta\theta_{yi} \quad \Delta\theta_{xj} \quad \Delta\theta_{yj} \right\}^{\mathrm{T}} = \left[_{ms}\mathbf{T}_i\right]\left\{\Delta_{ms}\boldsymbol{d}_i\right\} + \left[_{ms}\mathbf{T}_m\right]\left\{\Delta_{ms}\boldsymbol{d}_m\right\} + \left[_{ms}\mathbf{T}_j\right]\left\{\Delta_{ms}\boldsymbol{d}_j\right\} + \left[_e\mathbf{T}\right]\left\{\Delta_e\boldsymbol{d}\right\} \quad (9)$$

$$\left[_{ms}\mathbf{T}_i\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/l & 1 & 0 \\ 0 & 1/l & 0 & 0 & 1 \\ 0 & 0 & 1/l & 0 & 0 \\ 0 & -1/l & 0 & 0 & 0 \end{bmatrix}, \left[_{ms}\mathbf{T}_m\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/l & 0 & 0 \\ 0 & 1/l & 0 & 0 & 0 \\ 0 & 0 & 1/l & 0 & 0 \\ 0 & -1/l & 0 & 0 & 0 \end{bmatrix}, \left[_{ms}\mathbf{T}_j\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/l & 0 & 0 \\ 0 & 1/l & 0 & 0 & 0 \\ 0 & 0 & 1/l & 1 & 0 \\ 0 & -1/l & 0 & 0 & 1 \end{bmatrix}, \left[_e\mathbf{T}\right] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The flexibility equation of panel element can be obtained by substituting Eq. (6) and Eq. (7) into Eq. (9) as shown in Eq. (10). Note that $\{\Delta \boldsymbol{f}\}$ denotes the force increment vector of panel element as shown in Eq. (11), m_{xi} and m_{yi} are bending moments at i-end, m_{xj} and m_{yj} are bending moments at j-end. And $[\mathbf{k}]$, which is the invertible matrix of tangent flexibility matrix, is the tangent stiffness matrix of panel element.

$$\{\Delta \boldsymbol{d}\} = \left(\left[_{ms}\mathbf{T}_i\right]\left[_{ms}\mathbf{c}_i\right]\left[_{ms}\mathbf{T}_i\right]^{\mathrm{T}} + \left[_{ms}\mathbf{T}_m\right]\left[_{ms}\mathbf{c}_m\right]\left[_{ms}\mathbf{T}_m\right]^{\mathrm{T}} + \left[_{ms}\mathbf{T}_j\right]\left[_{ms}\mathbf{c}_j\right]\left[_{ms}\mathbf{T}_j\right]^{\mathrm{T}} + \left[_e\mathbf{T}\right]\left[_e\mathbf{c}\right]\left[_e\mathbf{T}\right]^{\mathrm{T}}\right)\{\Delta \boldsymbol{f}\} = [\mathbf{k}]^{-1}\{\Delta \boldsymbol{f}\} \quad (10)$$

$$\{\Delta \boldsymbol{f}\} = \left\{ \Delta n \quad \Delta m_{xi} \quad \Delta m_{yi} \quad \Delta m_{xj} \quad \Delta m_{yj} \right\}^{\mathrm{T}} \quad (11)$$

3.4 Reduction of degrees of freedom

According to Eq. (10), the panel element has five degrees of freedom. In order to reduce calculation load of the whole 3D frame model with this panel element, the through diaphragms at both ends of panel are assumed to be parallel. Thus, θ_{xi} and θ_{yi} of Eq. (9) is equal to θ_{xj} and θ_{yj} correspondingly, and the degrees of freedom of panel element can be reduced as shown in Eq. (12) and Eq. (13).

$$\begin{Bmatrix} \Delta_p m_x \\ \Delta_p m_y \\ \Delta_p n_z \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}\{\Delta \boldsymbol{f}\} = \left[\mathbf{T}_{ns}\right]\{\Delta \boldsymbol{f}\} \quad (12)$$

$$\{\Delta \boldsymbol{d}\} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{Bmatrix} \Delta_p \gamma_x \\ \Delta_p \gamma_y \\ \Delta_p d_z \end{Bmatrix} = \left[\mathbf{T}_{ns}\right]^{\mathrm{T}}\begin{Bmatrix} \Delta_p \gamma_x \\ \Delta_p \gamma_y \\ \Delta_p d_z \end{Bmatrix} \quad (13)$$

By substituting Eq. (10) and Eq. (12) into Eq. (13), the tangent stiffness equation of panel element is modified to have three degrees of freedom as Eq. (14).

$$\begin{Bmatrix} \Delta_p m_x \\ \Delta_p m_y \\ \Delta_p n_z \end{Bmatrix} = \left[\mathbf{T}_{ns}\right][\mathbf{k}]\left[\mathbf{T}_{ns}\right]^{\mathrm{T}}\begin{Bmatrix} \Delta_p \gamma_x \\ \Delta_p \gamma_y \\ \Delta_p d_z \end{Bmatrix} = \left[_p\mathbf{k}\right]\begin{Bmatrix} \Delta_p \gamma_x \\ \Delta_p \gamma_y \\ \Delta_p d_z \end{Bmatrix} \quad (14)$$

4. Introduction of panel element to 3D frame model

4.1 Stiffness equation of panel in the global coordinate system

This chapter describes the method to introduce the panel element into 3D frame model of reference [15] that is proposed by the authors. In the 3D frame model, there is a global coordinate system where the 3D model is based on, as well as separate element coordinate systems for every beam and column elements. The global coordinate system is fixed, but element coordinate system goes along with the corresponding element.

In order to introduce the panel element to the 3D model, panel element coordinate system, which has origin at nodal point of panel (see Fig.1(b)), is installed. Transformation between global coordinate system and panel element coordinate system is shown by Eq. (15). Here, $\{x_p\ y_p\ z_p\}^{\mathrm{T}}$ denotes an arbitrary vector in panel element coordinate system, $\{X\ Y\ Z\}^{\mathrm{T}}$ denotes the same vector in global coordinate system, [**T**] is the coordinate transformation matrix.

$$[\mathbf{T}]\begin{Bmatrix} x_p \\ y_p \\ z_p \end{Bmatrix} = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \tag{15}$$

In this paper, axes of panel element coordinate system are considered to have the same direction as the global coordinate system. Thus, [**T**] is given by Eq. (16) initially and updated every step as the nodal point of panel rotates.

$$\text{Initial }[\mathbf{T}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

By using the coordinate transformation matrix [**T**], the relationship between deformation increment vector in panel element coordinate system and global coordinate system is given by Eq. (17). Here, Γ_x, Γ_y and D_z denote the deformations of panel in the global coordinate system, Γ_x and Γ_y are the shear deformation angle around X and Y-axis correspondingly, D_z the deformation along Z-axis. According to Eq. (12), γ_x is the shear deformation angle around x_p-axis and γ_y is the shear deformation angle around y_p-axis. Therefore, a 3×3 matrix is adopted to change the order of deformations within a deformation increment vector, and Eq. (17) is finally transformed into Eq. (18).

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} \Delta_p\gamma_x \\ \Delta_p\gamma_y \\ \Delta_p d_z \end{Bmatrix} = [\mathbf{T}]^{\mathrm{T}}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} \Delta_p\Gamma_x \\ \Delta_p\Gamma_y \\ \Delta_p D_z \end{Bmatrix} \tag{17}$$

$$\begin{Bmatrix} \Delta_p\gamma_x \\ \Delta_p\gamma_y \\ \Delta_p d_z \end{Bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}[\mathbf{T}]^{\mathrm{T}}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} \Delta_p\Gamma_x \\ \Delta_p\Gamma_y \\ \Delta_p D_z \end{Bmatrix} = [\mathbf{T}']\begin{Bmatrix} \Delta_p\Gamma_x \\ \Delta_p\Gamma_y \\ \Delta_p D_z \end{Bmatrix} \tag{18}$$

As the transformation for force increment vector is contragredient to the transformation for deformation increment vector, the tangent stiffness equation of panel in global coordinate system is given by Eq. (19). Here, M is the bending moment and N is the force in the global coordinate system.

$$\begin{Bmatrix} \Delta_p M_x \\ \Delta_p M_y \\ \Delta_p N_z \end{Bmatrix} = [\mathbf{T}']^{\mathrm{T}}[{}_p\mathbf{k}][\mathbf{T}']\begin{Bmatrix} \Delta_p\Gamma_x \\ \Delta_p\Gamma_y \\ \Delta_p D_z \end{Bmatrix} \tag{19}$$

4.2 Relationship between nodal point and connection points within a panel element

According to assumption 1) from Section 2.1, each nodal point has six degrees of freedom. For nodal point with panel as shown in Fig. 1(b), other three degrees of freedom that correspond to the deformations of panel are added. And the degrees of freedom of connection point between panel and other elements, are considered as the subordinate variable of the nine degrees of freedom of nodal point with panel in this paper.

Firstly, deformation increment vector of an arbitrary point i in the panel element coordinate system is given by Eq. (20) with deformation increment vector of panel element. Here, the subscript i on the left side refers to the point i, ${}_iz_p$ is the z_p-axis coordinate of point i, $[{}_p\mathbf{T}_A]$ and $[{}_p\mathbf{T}_B]$ are transformation matrixes. Considering point i as the connection point on the surface of panel, B_{11} and B_{22} changes according to which surface plane that point i is belonged to. For the case that the normal direction of surface plane is parallel to x_p-axis, B_{11}=0 and B_{22}=1. B_{11}=1 and B_{22}=0 for the case of y_p-axis, B_{11}=0 and B_{22}=0 for the case of z_p-axis.

$$\begin{Bmatrix} \Delta_i d_x \\ \Delta_i d_y \\ \Delta_i d_z \\ \Delta_i \theta_x \\ \Delta_i \theta_y \\ \Delta_i \theta_z \end{Bmatrix} = \begin{bmatrix} {}_p\mathbf{T}_A \\ {}_p\mathbf{T}_B \end{bmatrix} \begin{Bmatrix} \Delta_p \gamma_x \\ \Delta_p \gamma_y \\ \Delta_p d_z \end{Bmatrix} \tag{20}$$

$$\left[{}_p\mathbf{T}_A\right] = \begin{bmatrix} 0 & {}_iz_p & 0 \\ -{}_iz_p & 0 & 0 \\ 0 & 0 & {}_iz_p / l \end{bmatrix}, \quad \left[{}_p\mathbf{T}_B\right] = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then, deformation increment vector of point i in the global coordinate system is given by Eq. (21). Here, the subscript 0 on the left side refers to the nodal point of panel element, D and Θ denotes the deformation and rotation in global coordinate system, (X, Y, Z) are coordinates in global coordinate system.

$$\begin{Bmatrix} \Delta_i D_x \\ \Delta_i D_y \\ \Delta_i D_z \\ \Delta_i \Theta_x \\ \Delta_i \Theta_y \\ \Delta_i \Theta_z \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{T}_R & \mathbf{T}\cdot{}_p\mathbf{T}_A\cdot\mathbf{T}' \\ \mathbf{0} & \mathbf{I} & \mathbf{T}\cdot{}_p\mathbf{T}_B\cdot\mathbf{T}' \end{bmatrix} \begin{Bmatrix} \Delta_0 D_x \\ \Delta_0 D_y \\ \Delta_0 D_z \\ \Delta_0 \Theta_x \\ \Delta_0 \Theta_y \\ \Delta_0 \Theta_z \\ \Delta_p \Gamma_x \\ \Delta_p \Gamma_y \\ \Delta_p D_z \end{Bmatrix} \tag{21}$$

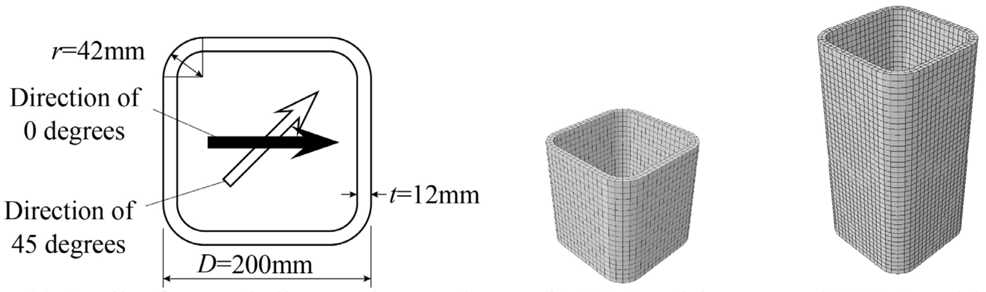
$$\left[\mathbf{I}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \left[\mathbf{0}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \left[\mathbf{T}_R\right] = \begin{bmatrix} 0 & ({}_iZ - {}_0Z) & -({}_iY - {}_0Y) \\ -({}_iZ - {}_0Z) & 0 & ({}_iX - {}_0X) \\ ({}_iY - {}_0Y) & -({}_iX - {}_0X) & 0 \end{bmatrix}$$

5. Validity of proposed numerical model

5.1 Comparation of proposed model and FEM

In order to verify validity of the proposed panel element, analysis results of proposed panel element are compared to results of finite element method (FEM) in this section.

Details of panel are as shown in Fig. 4(a). Panel length l (200mm and 400mm) and input direction of shear force (0 and 45 degrees) are adopted as parameters.



(a) Details of square hollow section panel (b) FEM model (Panel length 200mm) (c) FEM model (Panel length 400mm)

Fig. 4 – Details of panel and FEM model

The same bilinear restoring force characteristics is applied to both proposed model and FEM model. Yield stress is 363N/mm^2, and strain hardening coefficient n is 0.01. The length of modified MS component at the ends is 10% of panel length. About the boundary condition, the bottom of panel is fixed, and the top is rotation restricted. Shear force is acted at the top of panel monotonically along the direction described above.

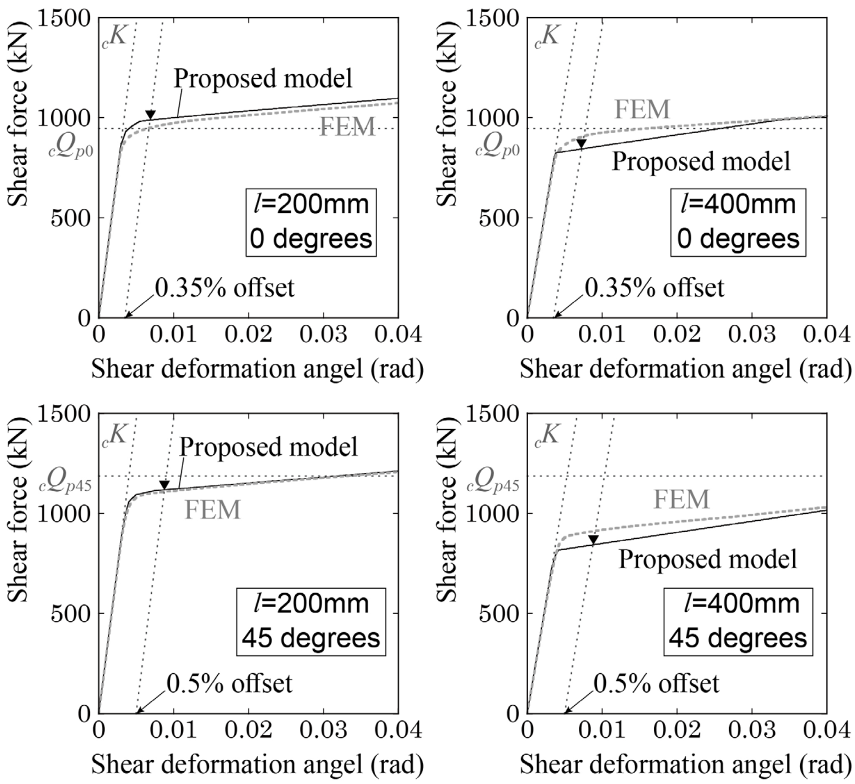


Fig. 5 – Relationships between shear force and shear deformation

Analysis results are shown in Fig. 5. Here, calculated elastic stiffness $_cK$, which considers both flexural and shear deformation of panel, and full plastic shear strength $_cQ_{p0}$ and $_cQ_{p45}$ according to reference [16] are added to Fig. 5 as comparison criteria.

From Fig. 5, the results of proposed model are well corresponding to FEM results. And the elastic stiffness of both numerical analyses are equal to the calculation value. However, focus on the cases that the panel length is 400mm, full plastic shear strength from analysis result that is shown by ▼ are much smaller than the calculation value specially under 45-degree input. This is because the calculation method for full plastic shear strength of panel according to reference [16] ignores the influence of bending moment, and the range of application for this method is that the aspect ratio of panel (l/D) is smaller than 1.6. Note that, ▼ in Fig. 5 shows the offset strength, which is equivalent to full plastic strength, according to reference [16].

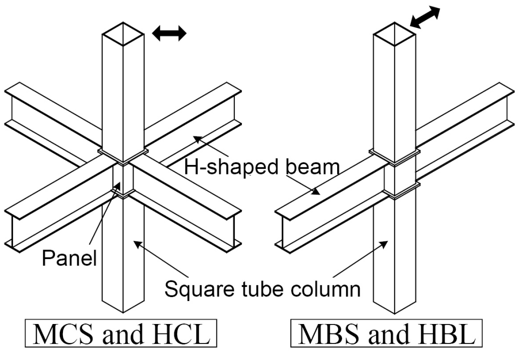
5.2 Comparation of proposed model and Loading test

In this section, results of proposed model are compared to loading tests of cruciform frames that consist of wide flange beams and square hollow section columns with through diaphragms [17]. Details of cruciform frame specimens are shown in Table 1 and Fig. 6. The bottom of specimen is connected as pin support and beam ends are connected as roller supports. Horizontal force acts on the top of specimen along the direction as shown in Fig 6(a). Panel length and loading direction are taken as parameters.

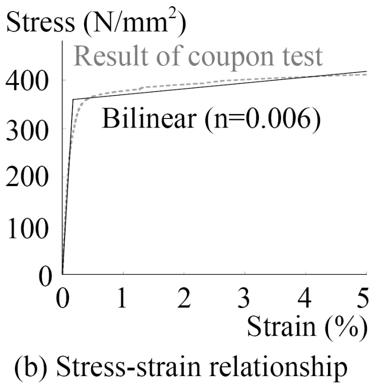
Bilinear restoring force characteristics is applied to all elements of numerical analysis model. Yield stress, which is obtained from coupon test as shown in Table 1, is adopted. Strain hardening coefficient n is 0.01 for beam, and 0.006 for column and panel as shown in Fig. 6(b). The length of modified MS component at the ends is 10% of panel length.

Table 1 – Details of members within cruciform frames [17]

Specimen	Beam (SN400B)	Column (BCR295)	Panel (BCR295)
MBS	H – 350×175×7×11	□ – 250×9	□ – 250×9 (σ_y=360N/mm^2)
MCS	(σ_y=286N/mm^2)	(σ_y=360N/mm^2)	
MBL	H – 500×200×10×16	□ – 250×9	
HCL	(σ_y=287N/mm^2)	(σ_y=360N/mm^2)	



(a) Cruciform frame specimens



(b) Stress-strain relationship of panel in cruciform frames

Fig. 6 – Details of cruciform frames and restoring force characteristics

Fig. 7 shows the results of loading tests and analyses, which is the relationships between the in-plane nodal moment and story drift angle. Note that, nodal moment is a product of horizontal force and specimen height. Here, results of Kuwahara's model for panel, which is only compatible with 0-degree input, by using the non-linear analysis program CLAP [18] are also shown in Fig. 7.

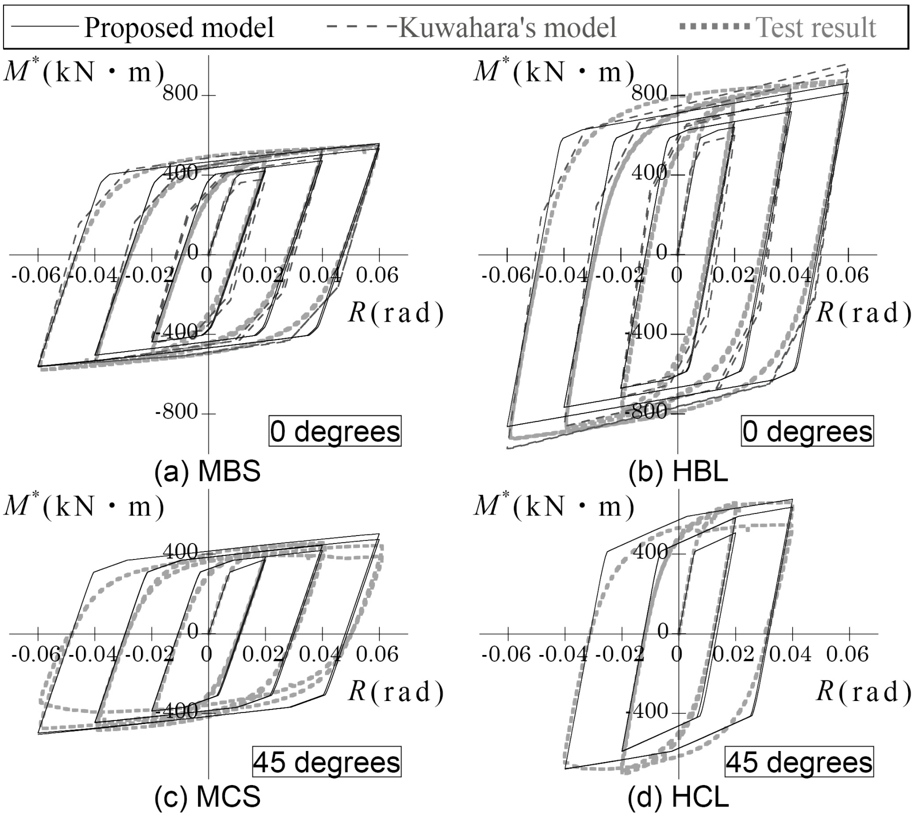


Fig. 7 – Relationships between in-plane nodal moment M^* and story drift angle R

From Fig.7, initial yield strength of proposed model is smaller than the result of loading test. Specially, the cases with large panel length and under 45-degree input, in which the bending moment at the end of panel is much larger, shows the largest difference between analysis and loading test. This is because the entire length of spring yields at the same time under axial stress while the modified MS component are subjected to large bending moment. On the order hand, while the panel of cruciform frame specimen yields under large bending moment, plastic zone evolve from the end step by step.

The tangent stiffness immediately after yielding of the proposed model may be smaller than test result, in case of large panel length or 45-degree input. However, as the plastic zone evolves, the difference between analysis and test results become smaller. Except the strength deterioration due to weld fracture, overall elasto-plastic behaviors of proposed model are corresponding well to test results. And the analytical accuracy of proposed model is almost the same as Kuwahara's model.

6. Conclusions

For the purpose of investigating the influence of panel on steel moment frames subjected to bi-directional ground motion, a novel numerical analysis model for steel square hollow section panel with through diaphragms is proposed. By comparing to FEM analysis results and loading test results from previous researches, the proposed model is able to express elasto-plastic behaviors of panel under axial force, bi-direction shear forces and bi-axial bending moments accurately.

7. Acknowledgements

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8. References

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