NON-CONVERGENT METHOD FOR EVALUATION OF DAMAGE AND STRENGTHENING EFFECTS IN MULTI-STOREY BUILDINGS

G. Jekic(1), V. Shendova(2), A. Zlateski(3)

(1) *Asst. Prof., Ph.D., Institute of Earthquake Engineering and Engineering Seismology (IZIIS), Ss. Cyril and Methodius University in Skopje, Republic of North Macedonia, jekic@iziis.ukim.edu.mk*

(2) *Prof., Ph.D., Institute of Earthquake Engineering and Engineering Seismology (IZIIS), Ss. Cyril and Methodius University in Skopje, Republic of North Macedonia, veronika@iziis.ukim.edu.mk*

(3) *Research Assistant, M.Sc., Institute of Earthquake Engineering and Engineering Seismology (IZIIS), Ss. Cyril and Methodius University in Skopje, Republic of North Macedonia, azlate@iziis.ukim.edu.mk*

Abstract

A development of the "initial stiffness alter ratio" (*ISAR*) of conventional high-rise buildings is presented. The proposed methodology has been developed to provide accurate structural health estimation in case of the highest limitation in operational modal analysis - identification of an input set of one modal eigenpair, consisting of one modal frequency and the corresponding mode shape, which is usually the first eigenpair. The approach provides locating and estimating structural damages or strengthening effects in buildings. The methodology is based on identifying initial stiffness of structures and its deviation over a certain time period. The *ISAR* method is based on results obtained by operational modal analysis of miultistorey buildings as a non-convergent which means that the accuracy does not depend on the number of eigenpairs included in the structural health state estimation process. The non-convergence of the proposed method excludes the need and the difficulties of identifying multiple eigenpairs, required in any convergent structural diagnosis approach. Identification of only one eigenpair is considered as sufficient. The calculation of *ISAR* is a straight-forward numerical procedure for comparison between the parameters of undamaged, damaged and repaired or strengthened structures. Theoretically the range of possible *ISAR* value stretches from -1 to + infinity. A positive value of *ISAR* represents a strengthened structure, a value between -1 and 0 represents a damaged structure and an *ISAR* value -1 represents a total stiffness deterioration and collapse of the structure. The accuracy and the reliability of the proposed structural health assessment approach has been verified through numerical and experimental test. The numerical test consists of comparison of the results of two simple lumped mass dynamical models with same stiffness properties, different storey mass distribution and same storey damage simulation. The accuracy and the reliability of *ISAR* and its limitations in the numerical test are elaborated in detail. The experimental test consists of *ISAR* accuracy evaluation on the structure of a scaled laboratory model of a traditional masonry building, tested on a shaking table in the Institute of Earthquake Engineering and Engineering Seismology (IZIIS) in Skopje. The results have shown that the proposed *ISAR*, based on the first modal eigenpair, can be applied as an instant damage or strengthening evaluation method in structural health monitoring of multistorey buildings, providing the location and severity of structural damage, or the achieved effects of strengthening techniques applied on real structures.

Keywords: stiffness alter ratio; pseudo-stiffness; structural health monitoring; eigen frequency; mode shape

1. Introduction

The process of Structural Health Monitoring (SHM) covers taking measures, registering and analyzing data that relates to the structure's behavior over a measured time frame and concludes whether the behavior had been within the predictions from the structural design process, initial state right after being constructed, or that the structure is completely operational and non-life or material goods threatening. SHM is used to estimate the present state of the structure alongside with potential for further exploitation, or eventual need for repair, by determining the effective structure's behavior pattern or possible structural damages. SHM strategies are generally classified in four different levels, depending on the information about the structural damage they can provide [1, 2]: level 1 (structural damage ascertaining), level 2 (level 1 + structural damage location), level 3 (level 2 + structural damage severity), level 4 (level 3 + prediction of the remaining structure's lifetime).

The SHM approach includes on site registering the structure's behavior by using multiple electronic equipment - sensors, data transmission and analysis and extraction of the damage or strengthening sensitive aspects of the data. Considering in situ experimental testing, ambient and forced vibration testing methods prevailed, being applicable for all types of structures, components and materials [3]. Detecting structural damages typically warrants the initial state and the altered state of the structures, to be compared. The availability of these comparisons provides tracing sequential changes of the damage sensitive structural parameters and estimating eventual damages and their severity.

Structural dynamic principles include the dynamic properties of the structure - the eigen frequencies and the corresponding mode shapes, which dependent on the structure's mass and stiffness distribution. The changes of mass distribution are rare, and in almost all cases, insignificant over the period of structure's exploitation. The change in stiffness distribution is impact-full on the structural health state and significantly impacts the dynamic properties and structural behavior under dynamic excitations.

Structural damage and strengthening result in stiffness changes of the bearing structural components. The change in eigen frequencies indicates alterations in the initial stiffness. However, when observed independently, their practical applications have their limits. Detecting small shifts in frequencies caused by damage in buildings, especially on higher levels of the structures, requires severe damage. For instance, critical damages of vital structural elements result with less than 5% of frequency shifts [4]. Local spatial deviation in stiffness, especially on upper floors, cannot be determined by the eigen frequency value of the lower modes because it is a general global property of the structure. However, higher eigen frequencies, which are usually impossible to be identified through operational modal analysis, are associated with local responses [2]. When the SHM process is based only on eigen frequency shifts, it can barely exceed any higher level of SHM than the level 1. Furthermore, various environmental factors (e.g., temperature) can change natural frequency of structures without any damage in the structure [5]. It remains difficult to determine even the damage location just by observing only the changes of modal frequencies [6, 7].

It has been concluded that the shifts in mode shapes are slightly sensitive to damage, but the uncertainties are present [8, 9]. A particular case is the presumption is the case in which all the storeys of an observed building suffer the same damage or strengthening (same percentage of stiffness decrease or increase), all the identified mode shapes of the damaged or strengthened structure do not differ from the ones of the structure in its initial state. Mode shape changes-based stiffness alter identification approaches do not detect stiffness shifts in this situation.

2. Storey Pseudo-Stiffness - Theory and Application

2.1 Definition of the Storey Pseudo-Stiffness

In systems with multiple degrees of freedom in one direction, the stiffness matrix $[K]$ (Eq. (1)) is obtained applying identified and then mass-normalized mode vectors, grouped in a mode shapes matrix $[\Phi]$ and squared angular frequency matrix $[\Lambda]$ containing squared angular frequency values as main diagonal members, and zeros as non-main diagonal members. Mode shape vectors must be normalized in a form providing the product $[\Phi]^T[M][\Phi]$ to be an identity matrix $[I]$. The mode shape matrix $[\Phi]$ contains the column vectors $\{\phi\}_i$ which are a normalized mode shape of every mode in the observed direction.

$$[K] \cong ([\Phi][\Lambda]^{-1}[\Phi]^T)^{-1} \tag{1}$$

To obtain the exact storey stiffness values by direct application of Eq. (1), all the system's modal eigenpairs (one eigen frequency in a certain direction and the corresponding mode shape) in one direction must be identified. Identifying higher modes is practically impossible applying system identification methods for operational modal analysis. Basically, only the first, and eventually the second eigenpair can be identified reliably. The storey stiffness obtained has a lower numerical value than the actual storey stiffness. The introduced pseudo-stiffness matrix $[\bar{K}]$ contains parameters that differ from actual stiffness. However, the pseudo-stiffness and the actual stiffness are closely related. The order of the storey pseudo-stiffness matrix is the number of considered mode shapes in the analysis. The pseudo-stiffness matrix is obtained by using Moore and Penrose inverse matrix [10], which is a generalized inverse matrix. In this case the number of degrees of freedom is higher than the identified eigenpairs (the mode shape vector matrix $[\Phi]$ is not a square matrix). The storey stiffness of the n-storey structure is estimated from the pseudo-stiffness matrix $[\bar{K}]$ multiplying by a generalized relative displacement vector $\{j\}_s$ (Eq. (2)) [11]. The matrix of generalized relative displacement column vectors $[j]$ is presented in Eq. (3).

$$\bar{k}_s = \{j\}_s^T[\bar{K}]\{j\}_s \tag{2}$$

$$[j] = [\{j\}_n \quad \{j\}_{n-1} \quad \{j\}_{n-2} \quad \cdots \quad \{j\}_1] = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{3}$$

The matrix $[\bar{K}]$ is calculated by applying Eq. (1) considering the pseudo-inversion of the matrices $[\Phi]$ and $[\Phi]^T$ as non-square matrices of normalized identified mode shapes and the matrix $[\Lambda]$ containing the squared angular frequencies corresponding to the identified mode shapes. Finally, the storey pseudo-stiffness value (Eq. (2)) is introduced as a parameter which is closely related to the corresponding storey real stiffness. The parameter $\bar{k}_s$ represents the s^{th} storey pseudo-stiffness of an order equal to the number of identified eigenpairs in one direction and its value is lower than the actual stiffness. Increasing the storey pseudo-stiffness order, its value converges to the actual storey stiffness – a storey pseudo-stiffness value of higher order represents a closer value of the storey pseudo-stiffness to the exact value of the actual storey stiffness. In other words, in case when all modal frequencies and mode shapes in one direction (translational or rotational) are identified, the n^{th} order storey pseudo-stiffness obtained is equal to the actual storey stiffness, which means that the highest possible order is the number of degrees of freedom in the corresponding direction.

2.2 Application of the First Order Storey Pseudo-Stiffness in Structural Health Estimation

2.2.1 Evaluation of First Order Storey Pseudo-stiffness

The main objective of the "first order storey pseudo-stiffness" structural health estimation approach is the ability to accurately obtain the storey stiffness before and after the event that caused structural damage or strengthening using only one eigenpair - ω_i and $\{\phi\}_i$ for a particular direction. This is beneficial for practical applications since in in-situ operational modal analysis only the first few modal parameters can be reliably identified due to a noise contamination in the measurements [12]. The set with the highest signal-to-noise (S/N) ratio is chosen (of the first translational mode in each direction, including torsion). The first order pseudo-stiffness matrix can be expressed by Eq. (4) (adequate to Eq. (1)).

$$[\bar{K}]_i = pinv(\{\phi\}_i^T)\omega_i^2 pinv(\{\phi\}_i) \tag{4}$$

where ω_i^2 is the squared chosen angular frequency of the structure; $pinv(\{\phi\}_i^T)$ and $pinv(\{\phi\}_i)$ are pseudoinverse of the normalized mode shape vector that corresponds to the chosen angular frequency. The first order pseudo-stiffness of any storey, obtained applying Eq. (2), has two essential properties [13]:

- the square root of $\bar{k}_s$ is equal to the absolute value of the sum of mode shape values of the observed storey and the soreys above it, multiplied by a ratio of proportion ψ_i, constant for all the storeys for the actual structural health state (Eq. (5)),

$$\sqrt{\bar{k}_{si}} = \psi_i \left|\sum_{j=s}^{n} \phi_{ji}\right| \qquad \psi_i = const \tag{5}$$

- the sum of $\bar{k}_s$values for all the stories represents a uniformly distributed storey stiffness for which an imaginary structure's eigen frequencies are equal or very close to the frequency ω_i of the actual structure with the same mass distribution (Eq. (6)).

$$k_{uni} = \sum_{s=1}^{n} \bar{k}_{si} \tag{6}$$

In Fig. 1(a), two dynamic models of building structures are presented. The model on the left represents the actual structure with the actual values of storey stiffness (k_1, k_2,…k_n) and actual lumped mass distribution (m_1, m_2,…m_n). On the right, the model of the corresponding imaginary structure with the uniformly distributed storey stiffness (k_{uni}) is presented. Both structures have the same or very similar eigen frequency for the same lumped mass distribution.

2.2.2 Storey stiffness estimation

Theoretically an infinite number of damage combinations can cause changes of the structure's eigen frequency for a certain percentage. For instance, the first eigen frequency is decreased to a lower value after damage occurrence in the first storey, or in a higher storey with a larger stiffness loss, or due to damage occurrence in several stories. In another words, many possible damage combinations can cause the same frequency drop in a multistorey building. What makes any damage combination unique and recognizable is the corresponding mode shape shifts. As a repercussion to that, it is obvious that any eigenpair, or mode shape, combined with the corresponding eigen frequency, is a unique feature of only one structural damage combination.

The modal parameters can be identified by applying any proposed system identification technique. Modal frequencies can be identified precisely. However, the mode shapes are often identified as non-dimensional numerical values in "initial form", depending on the applied system identification method. For further application, the mode shapes values need to be normalized.

The first normalization is carried out in order the product $[\Phi]^T[M][\Phi]$ to be an identity matrix $[I]$. $\{\phi\}_{i(1)}$ represents the raw (non-normalized) form of the i^{th} identified mode shape, extracted from the system identification of the observed structure. The normalized form of the raw non-normalized mode shape $\{\phi\}_{i(2)}$ can be obtained applying Eq. (7) [14]. The first order pseudo-stiffness matrix $[\bar{K}]$ can be obtained applying Eq. (8), and the first order storey pseudo-stiffness $\bar{k}_i$ is obtained applying Eq. (2) for each storey.

$$\{\phi\}_{i(2)} = \frac{\{\phi\}_{i(1)}}{\sqrt{\{\phi\}_{i(1)}^T[M]\{\phi\}_{i(1)}}} \tag{7}$$

$$[\bar{K}]_i = pinv\left(\{\phi\}_{i(2)}^T\right)\omega_i^2 pinv\left(\{\phi\}_{i(2)}\right) \tag{8}$$

Calculation of each storey's actual stiffness can be performed by applying the "renormalized" mode shape vector $\{\phi\}_{i(3)}$, Renormalization of the mode shape vector has to be performed so that the expression in Eq. (9) is true. The second normalization can be performed applying Eq. (10). The renormalized mode shape is represented in Eq. (11).

$$\{\phi\}_{i(3)}^T\{\phi\}_{i(3)} = 1 \tag{9}$$

$$\{\phi\}_{i(3)} = \frac{\{\phi\}_{i(1)}}{\sqrt{\{\phi\}_{i(1)}^T\{\phi\}_{i(1)}}} = \frac{\{\phi\}_{i(2)}}{\sqrt{\{\phi\}_{i(2)}^T\{\phi\}_{i(2)}}} \tag{10}$$

$$\{\phi\}_{i(3)}^T = \{\phi_{ni(3)} \quad \phi_{n-1i(3)} \quad \cdots \quad \phi_{2i(3)} \quad \phi_{1i(3)}\}_i \tag{11}$$

The renormalized mode shape can then be applied for obtaining the "uniform lumped mass" (m_{uni}), for which, similarly to the equivalent uniformly distributed storey stiffness, an imaginary structure's eigen frequencies are equal or very close to the eigen frequency of the actual structure for the real storey stiffness distribution (Eq. (12)).

$$m_{uni} = \{\phi\}_{i(3)}^T[M]\{\phi\}_{i(3)} \tag{12}$$

In Fig. 1(b) two dynamical models of building structure are presented. The model on the left represents the actual structure with the actual storey stiffness (k_1, k_2,…k_n) and actual lumped mass distribution (m_1, m_2,…m_n). On the right, a model of the corresponding imaginary structure with the uniform lumped mass distribution (m_{uni}) is presented. Both models have the same or very similar modal eigen frequency for the same storey stiffness distribution.

When only the i^{th} eigen frequency of a particular structure and it's corresponding raw mode shape $\{\phi\}_{i(1)}$ are obtained applying system identification technique, the first order storey pseudo-stiffness and the value of the uniform mass distribution can be calculated. The real storey stiffness of the undamaged structure can be estimated by applying Eq. (13), considering that the values of the real lumped mass remain constant over time.

$$k_s \approx \frac{\bar{k}_{si}\sum_{j=s}^{n} m_j}{m_{uni}(n-s+1)\left|\Delta\phi_{si(3)}\sum_{j=s}^{n}\phi_{ji(3)}\right|} \tag{13}$$

$\Delta\phi_{si(3)}$ is the difference between the renormalized mode shape value of the observed storey s and the mode shape value of the storey beneath (s-1):

$$\Delta\phi_{si(3)} = (\phi_s - \phi_{s-1})_{i(3)} \quad (14)$$

The member k_s represents the s^{th} storey stiffness and n is the total number of storeys. The storey stiffness, obtained applying Eq. (13) is nearly equal to the exact value of the storey stiffness. Through numerical tests it has been concluded that the accuracy of the obtained storey stiffness depends inversely on the standard deviation of the mass distribution because the pseudoinverse of a rectangular matrix or a vector (the only identified mode shape vector) is an optimal statistical solution of an inconsistent least square system in a way that gives the minimum norm and therefore the closest solution [15]. Therefore, high variations in mass distribution can cause certain residual errors in the real storey stiffness calculation. The influence of these errors to the structural health estimation procedure's final result has been investigated and discussed further.

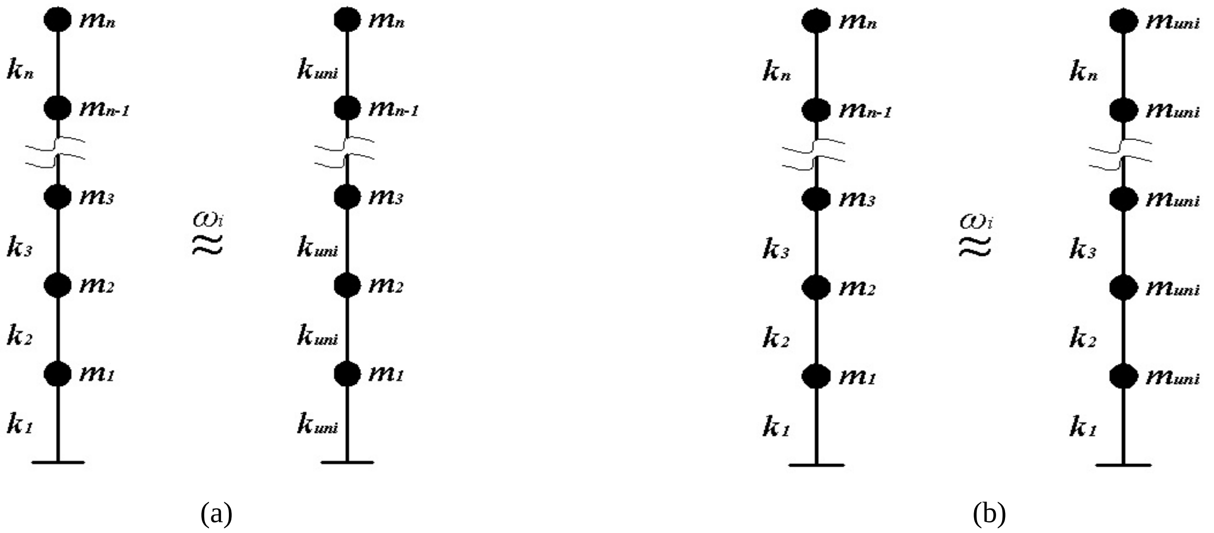

Fig. 1 – Dynamic model of real structure and imaginary structure: (a) - uniform storey stiffness, (b) - uniform storey mass

The storey stiffness estimation can be furthermore simplified to straight forward calculation, avoiding the need of renormalizing and summing the mode shape values by implementing the constant relationship from Eq. (5) into Eq. (13).

Considering that the storey pseudo-stiffness is a function of the storey stiffness distribution and the mode shape, for any structure with a specified mass and stiffness distribution, there is a constant numerical relationship among the mass, storey stiffness, mode shape, storey pseudo-stiffness and mode shape difference for every storey of the structure. The constant relationship, demonstrated in Eq. (5), is further developed in Eq. (15). Therefore, any storey's stiffness can be derived as a function of the square root of the storey pseudo-stiffness, mass distribution mode shape, multiplied by the constant ratio ψ_i (Eq. (16)).

$$\frac{\sqrt{\bar{k}_{si}}}{\left|\sum_{j=s}^{n} \phi_{ji(3)}^{u}\right|} = \frac{k_s m_{uni}(n-s+1)\left|\Delta\phi_{si(3)}\right|}{\sqrt{\bar{k}_{si}}\sum_{j=s}^{n} m_j} = \psi_i = const \quad (15)$$

$$k_s \approx \psi_i \frac{\sqrt{\bar{k}_{si}}\sum_{j=s}^{n} m_j}{m_{uni}(n-s+1)\left|\Delta\phi_{si(3)}\right|} \tag{16}$$

Considering that the ratio ψ_i is equal for every storey, the easiest way of deriving it is from the pseudo-stiffness and the mode shape of the top (n^{th}) storey. The stiffness of the top storey can be obtained applying Eq. (17). the ratio ψ_i can be derived applying Eq. (18).

$$k_n = \omega_i^2 m_n \left|\frac{\phi_{ni(3)}}{\Delta\phi_{ni(3)}}\right| = \psi_i \frac{m_n\sqrt{\bar{k}_{ni}}}{m_{uni}\left|\Delta\phi_{ni(3)}\right|} \tag{17}$$

$$\psi_i = \frac{\omega_i^2\left|\phi_{ni(3)}\right| m_{uni}}{\sqrt{\bar{k}_{ni}}} \tag{18}$$

The relationship among the mode shape values is always constant for specified state of the structure, regardless of the normalization applied. The storey stiffness can be obtained by implementing Eq. (18) into Eq. (16), which provides the mathematical expression of the storey stiffness as a function of mass, eigen frequency and storey pseudo-stiffness (Eq. (19)), where ε_s represents the residual error dependent on the lumped mass distribution.

$$k_s = \frac{\omega_i^2\sum_{j=s}^{n} m_j}{(n-s+1)}\left|\frac{\phi_{ni}}{\Delta\phi_{si}}\right|\sqrt{\frac{\bar{k}_{si}}{\bar{k}_{ni}}} * \varepsilon_s \tag{19}$$

2.2.3 Storey stiffness alter ratio

Finally, when both, the storey stiffness values of the damaged versus undamaged or repaired/strengthened versus undamaged structure are known, the initial stiffness alter ratio for the s^{th} storey ($ISAR_s$) can be obtained in general form (Eq. (20)):

$$ISAR_s = \frac{k_s^*}{k_s}\frac{\varepsilon_s^*}{\varepsilon_s} - 1 \tag{20}$$

where the asterisk (*) represents the property of an altered state of the structure. In calculation of the *ISAR*, k_s^* and k_s are equal or nearly equal to the storey stiffness of the altered and initial structure respectively. Both residual errors ε_s and ε_s^* originate from the same source - the mass distribution, which is considered as constant. Therefore $\varepsilon_s = \varepsilon_s^*$, so they cancel each other out. The procedure results in a reliable *ISAR* calculation.

By placing Eq. (19) into Eq. (20), the initial stiffness alter ratio can be expressed as in Eq. (21).

$$ISAR_s = \frac{\omega_i^{*2}}{\omega_i^2}\left|\frac{\phi_{ni}^*\Delta\phi_{si}}{\phi_{ni}\Delta\phi_{si}^*}\right|\sqrt{\frac{\bar{k}_{si}^*\bar{k}_{ni}}{\bar{k}_{si}\bar{k}_{ni}^*}} - 1 \tag{21}$$

where *n* represents the property of the top storey and *s* represents the corresponding parameter of the observed storey.

The parameters of the *ISAR* function from Eq. (21) are easily obtainable. One eigen frequency and the corresponding mode shape of the initial and altered structure can be easily identified by the system identification techniques, while the pseudo-stiffness values can be directly obtained by applying first Eq. (7), then Eq. (8) and finally Eq. (2). The range of possible *ISAR*-value stretches from -1 to +infinity (Fig. 2).

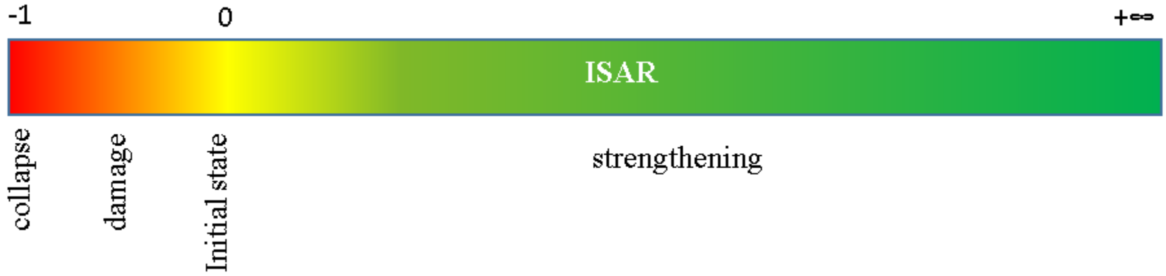

Fig. 2 – Theoretical range of possible *ISAR*-value

The characteristic values and subranges of *ISAR* are the following:

- *ISAR* = -1 (a state of total stiffness deterioration and collapse);
- -1 < *ISAR* < 0 (damaged structure);
- *ISAR* = 0 (structure in its initial state - undamaged);
- *ISAR* > 0 (strengthened structure).

In practical cases, the operational modal analysis clearly identifies only the first translational mode ($i = 1$) in every orthogonal direction of building structure. Therefore, the first eigenpair is the most suitable for a storey health estimation. The Eq. (22) can be applied for *ISAR* calculation since in the first transitional mode there is no phase difference in the vibration between any two DOFs.

$$ISAR_s = \frac{\omega_1^{*2}\phi_{n1}^{*}\Delta\phi_{s1}}{\omega_1^{2}\phi_{n1}\Delta\phi_{s1}^{*}}\sqrt{\frac{\bar{k}_{s1}^{*}\bar{k}_{n1}}{\bar{k}_{s1}\bar{k}_{n1}^{*}}} - 1 \qquad (22)$$

The accuracy of the proposed methodology for obtaining storey damage index has been tested. The testing results are further demonstrated and commented.

3. Validation of the Initial Stiffness Alter Ratio

3.1 *ISAR* accuracy test on numerical models

The efficiency and accuracy of *ISAR* has been tested on two numerical dynamical models of a five storey shear building in a form of five degrees of freedom lumped mass dynamical systems (Model I and Model II).

Model I is a dynamical model with randomly lumped mass and storey stiffness distribution for undamaged and damaged state. The structural damage has been simulated by reducing the storey stiffness. Numerical modal analysis for the both undamaged and damaged structural state has been conducted. In Model II the storey stiffness values for undamaged and damaged state are the same as in Model I, but with uniform lumped mass distribution (14 tons on each floor). The *ISAR* for every storey for the first and the second eigenpair has been obtained and compared with the percentage of stiffness loss due to the simulated damage.

Table 1 – Storey lumped mass and stiffness (undamaged and damaged state)

Storey No.	Model I - Lumped mass (tons)	Model II - Lumped mass (tons)	Storey stiffness k_s (kN/m) undamaged	Storey stiffness k_s* (kN/m) damaged
5	8	14	70000	50000
4	12	14	80000	70000
3	14	14	80000	60000
2	12	14	90000	60000
1	16	14	120000	80000

Table 2. Model I – Obtained *ISAR* values based on the first and the second eigenpair

Storey No.	Stiffnes reduction (%)	$ISAR_{(1)}$	$ISAR_{(2)}$
5	28.6	-0.2855	-0.2856
4	12.5	-0.1251	-0.1184
3	25.0	-0.2524	-0.3103
2	33.3	-0.3355	1.7833
1	33.3	-0.3370	-0.3758

Table 3. Model II – Obtained *ISAR* values based on the first and the second eigenpair

Storey No.	Stiffnes reduction (%)	$ISAR_{(1)}$	$ISAR_{(2)}$
5	28.6	-0.2867	-0.2858
4	12.5	-0.1243	-0.1249
3	25.0	-0.2500	-0.2501
2	33.3	-0.3331	-0.3334
1	33.3	-0.3335	-0.3333

From the results it is observed that the *ISAR* value, obtained from the first eigenpair ($ISAR_{(1)}$) matches the exact stiffness reduction, while *ISAR* value of the Model I, obtained from the second eigenpair ($ISAR_{(2)}$) is considered as accurate for the 5th storey, less accurate for the 4th story and unaccurate for the lower storeys, from which substantially inaccurate for the 2nd storey.

The results demonstrate that both *ISAR* values, based on both eigenpairs match the exact percentage of stiffness loss when the uniform storey mass m_{uni} is equal to the real lumped mass, as in Model II.

From the numerical tests of the *ISAR* accuracy, it is concluded that the obtained *ISAR* value based on the first eigenpair ($ISAR_{(1)}$) can accurately detect and quantify the storey damage since it is not sensitive to the variation of the floor mass distribution. On the other hand, the obtained *ISAR* value based on higher eigenpairs ($ISAR_{(i>1)}$) is sensitive to the variation of the floor mass distribution (which is a realistic case) and therefore it shows substantial errors in case of non-uniform mass distribution along the DOFs. Results from numerous numerical tests demonstrated that the *ISAR* value based on higher eigenpairs can be considered as reliable only for the storeys above the corresponding mode shape's highest braking point (the highest local maximum of the mode shape value). Since the first mode shape doesn't have a local maximum, the value of $ISAR_{(1)}$ is reliable all over the building's height.

3.2 Experimental accuracy test of *ISAR* on the laboratory model of a masonry structure

The experimental test of the accuracy and reliability of $ISAR_{(1)}$ was performed as a parallel research applying microtremor tests during an investigation of innovative materials for seismic strengthening of existing masonry buildings within the research project "Experimental Verification of Innovative Technique for Seismic Upgrading of Traditional Masonry Building" [16], realized by the Institute of Earthquake Engineering and Engineering Seismology (IZIIS) in Skopje, in collaboration with the RÖFIX Company, Austria and the SINTEK Company, Skopje in the period February-September 2013. The laboratory model was to a scale 1:2 of hypothetical 2-storey brick masonry building, constructed for a shaking table test (Fig. 3(a)).

On the roof plate an additional concrete slab with a thickness of 12 cm is constructed with a unique function of additional mass on the roof panel. The model's lumped mass before testing, after testing and after retrofitting, remains constant (12.7 tons on the roof slab and 15.1 tons on the first storey's slab). Both storeys were constructed with the same bearing masonry walls, expected to possess the same storey stiffness.

Testing of the 1:2 scaled model was carried out using equipment which, operating as an integral system, had to provide the following functions: generation of programmed motions, measurement and recording the characteristic values of excitation, the dynamic behavior, data processing and presentation. The shaking table mechanism enabled programmed generation of translational vibrations in both horizontal and vertical direction.

The shaking table tests of the model required special testing program consisting of several test phases, considering the expected information about the dynamic behavior of the prototype and the effectiveness and justification of applied strengthening method and technology. The same testing procedure was applied for original and for retrofitted model, consisting of two main phases [16]:

- tests for definition of dynamic characteristics of the model, before and after performing seismic tests at each phase, in order to check stiffness degradation, caused by micro or macro cracks developed during the tests;
- seismic testing by selected earthquake records until the model becomes heavily damaged.

The model has been subjected in its W-E direction to three characteristic earthquake effects (El Centro, 1940; Petrovac N-S, 1979, and Northridge, 1994). The dynamic properties of initial, damaged and retrofitted model were checked by ambient vibration technique before being subjected to earthquake excitation.

The model was retrofitted by the innovative technique that was originally developed, and for this particular case designed and applied (Fig. 3(b)). From the comparison of results, it was concluded that the repair and retrofitting slightly increased not only the damaged model's stiffness, but also the initial model's stiffness.

(a) (b)

Fig. 3 – Laboratory model – initial state before testing (a); retrofitting process between two tests (b)

The obtained *ISAR*-value has been compared with a previously verified damage detection method – a storey damage index (*SDI*) [12]. Table 4 demonstrates the comparison between the $ISAR_{(1)}$ and $SDI_{(1)}$.

Table 4 – Comparison between $SDI_{(1)}$ and $ISAR_{(1)}$ on the laboratory model

	damaged vs. initial		**retrofitted vs. initial**	
Storey	***SDI***	***ISAR***	***SDI***	***ISAR***
2	0.17	-0.17	-0.37	0.37
1	0.41	-0.42	-0.36	0.35

4. Conclusions

The purpose of the presented research was realized by the development of a structural health monitoring methodology based on in-situ operational modal analysis of high-rise buildings. The research was focused on a development of a non-convergent methodology for precise detection, localization and quantification of damages or strengthening effects in high-rise buildings by identifying one eigenpair. Applying the proposed methodology, the limitations of convergent methods have been overcome. Changes in eigen frequencies and mode shapes are indicators of damage or strengthening without precise details if analyzed independently one from another. The eigen frequency may drop or rise equally for different combinations of damage or strengthening of the structure (by intensity and location), but each combination of stiffness changes is followed by a unique change in mode shapes. Generally, the mode shape, combined with the corresponding eigen frequency, is a unique feature of only one storey stiffness distribution. Accordingly, the comparison of the combinations of eigen frequencies and mode shapes, identified by two consecutive observations of a structure, has a potential of a reliable indicator of damage or strengthening. The *ISAR*-ratio provides a reliable evaluation of changes in storey stiffness when it is based on the identification of the first transient eigenpairs ($ISAR_{(1)}$). However, negligible difference between the calculated and the actual ratio is possible. $ISAR_{(1)}$ possesses considerable potential for wide practical application taking into account the fact that the greatest effective modal mass of the objects from the high-build structure, and consequently damages, is a component of the first translatory vibration mode, defined by the first eigenpair in translatory direction. Also, the first eigenpair in translatory direction is the simplest for identification in operational modal analysis. The advantage of the *ISAR*-based structural health estimation approach is that *ISAR* in its final mathematical form depends only on the parameters of the floor of interest and the top floor, avoiding the dependence on the parameters on the other floors, which makes it suitable for quick and simple structural health estimation, as well as computer software programming for a continuous and automated structural health monitoring process of a third level – ascertaining structural damages, as so as estimating their location and severity. The presented methodology, based on time synchronized registrations of ambient vibrations, enables the diagnosis of the current state of the building structure based on time-de-aggregated measurements. The first measurement, performed on an initial state of a building structure, is a reference for defining all possible successive states of the structure complementary to the results of the later operational modal analysis.

5. References

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