



BASIC STUDY ON STRAIN HARDENING COEFFICIENT AFFECTING MAXIMUM STRENGTH OF SQUARE CFT BEAM-COLUMNS

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Abstract

For the structural design of concrete filled steel tubular (CFT) structures, it is important to appropriately formulate the restoring force characteristics of beam-column members. In Japan, the skeleton curve of CFT beam-columns is modeled by three broken lines (i.e., the tri-linear model) that represent the relationship between the bending moment M and the rotational angle R [1]. The bending strength M_u at the second break-point is the ultimate bending strength of short columns, and the value of α_u in $\alpha_u K_e$ which represents the third stiffness after M_u is set to values between 0.001 and 0.01, where K_e is the initial stiffness. The third stiffness has been shown to be a safety evaluation by comparing it with results of experiments. In Reference [2], a sensitivity analysis was carried out about the slenderness ratio, axial force ratio, material characteristics, residual stress, initial deflection, etc., which affect the performance in bending strength of steel H-shaped cross-section members. However, the effect on the bending strength of CFT members was not been clarified. Therefore, this paper concerns the effect of strain hardening coefficients on the relationship between the maximum bending moment and the slenderness ratio by taking axial force ratio, material strength, and cross-sectional dimensions of the steel tube as parameters. And a comparison with hollow steel tube members is carried out. It is found that as the strain hardening coefficient E_s/E increases, the value of M/M_{pc} increases. However, as the slenderness ratio increases, the value of M/M_{pc} becomes almost the same even though the strain hardening coefficients E_s/E are different. Compared with the case of the hollow steel tube, the effect of the strain hardening coefficient E_s/E on the relation between M/M_{pc} and $\bar{\lambda}$ for the CFT member has the same tendency as for the case of the hollow steel tube. However, for the case of the hollow steel tube, the value of M/M_{pc} is more susceptible to varying by E_s/E than for the case of the CFT member. In addition, the values of the length-thickness ratio l_k/D (in which l_k is the effective length) when $M/M_{pc}=1$ with different axial force ratio are shown as a function of the strain hardening coefficient E_s/E .

Keywords: concrete filled steel tubular structures, restoring force characteristics, strain hardening coefficient

1. Introduction

For the structural design of concrete filled steel tubular (CFT) structures, it is important to appropriately formulate the restoring force characteristics of beam-column members. In Japan, the skeleton curve of a CFT beam-column is modeled by three broken lines (i.e., the tri-linear model shown in Fig. 1) which represents the relationship between the bending moment M and the rotational angle R as outlined in the Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures [1] (referred to here as CFT Recommendations). The bending strength M_u at the second break point is the ultimate bending strength of short columns, and the value of α_u in $\alpha_u K_e$ which represents the third stiffness after M_u is typically set to be a value of about 0.001 to 0.01, where K_e is the initial stiffness. The third stiffness has been shown to be a safety evaluation by comparing it with experimental results. In Reference [2], a sensitivity analysis was carried out to determine how the slenderness ratio, axial force ratio, material characteristics, residual stress, initial deflection, etc., affect the performance in bending strength of steel H-shaped cross-section members. However, the effect on the bending strength of CFT members has not yet been clarified.

This paper concerns the effect of strain hardening coefficients on the relationship between the maximum bending moment and slenderness ratio when taking axial force ratio, material strength, and cross-



sectional dimensions of the steel tube as parameters. And a comparison with hollow steel tube members is conducted.

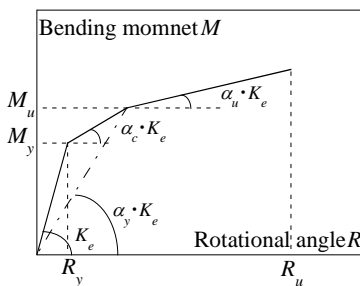
2. Analysis

2.1 Analytical model

In this paper, beam-columns with one end fixed and the other being free is considered in the analytical model. As seismic actions are significant for countries that experience regular earthquake, such as in Japan, the beam-column here is subjected to a constant axial force N and a lateral force H as shown in Fig. 2(a), where L is the length of beam-column, Δ is the lateral deflection, and R is the rotational angle which is equal to Δ/L . In Fig. 2(b), D is the width of the cross-section, and t is the thickness of the steel tube.

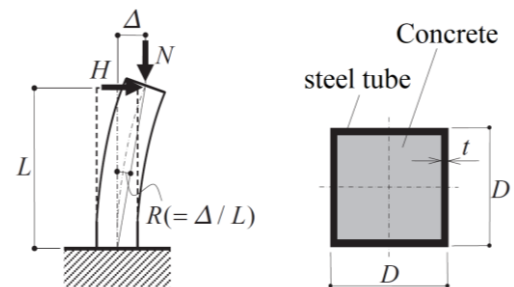
2.2 Analytical method

The relationship between the lateral force H and the lateral deflection Δ is obtained by the Column Deflection Curve method (CDC method for short), as outlined in the CFT Recommendations. Firstly, the relationship between the bending moment M and the curvature ϕ is calculated under the assumption that the cross-section does not change after deformation, and the deflection is quite small. Then, based on the $M-\phi$ relationship, the relationship between the lateral force H and the rotational angle R is obtained. Second-order effects are also considered in the CDC method. In this paper, the maximum strain ε_{max} and the maximum stress σ_u are predetermined. The calculation is not terminated until either the strain or the stress reaches a maximum value.



K_e : initial stiffness
 M_u : ultimate bending moment
 M_y : yield bending moment
 α_y : decrease ratio of stiffness
 α_u : the third stiffness coefficient

$$\alpha_c = \frac{M_u - M_y}{M_u - M_y}$$



(a) Loading conditions

(b) Cross section

Fig. 1 Skeleton curve for CFT beam-columns

Fig. 2 Analytical model

2.3 Stress-Strain relation

2.3.1 Concrete

The stress-strain relation for concrete used in this analysis is expressed by Eq. (1), which was originally outlined in the State-of-the-Art Report on High-Strength Concrete edited by AIJ in 1991 [3]. This current stress-strain model was proposed by Fafitis and Shah. The stress σ is constant when it reaches the compressive stress of concrete σ_B . Note that the tensile stress of concrete is neglected in this paper. The stress-strain curve of concrete is shown in Fig. 3(a).

$$\sigma = \begin{cases} 1 - \left(1 - \frac{\varepsilon}{\varepsilon_m}\right)^a & (\varepsilon < \varepsilon_m) \\ \sigma = \sigma_B & (\varepsilon \geq \varepsilon_m) \end{cases} \quad (1)$$



In Eq. (1), the index a , the elastic modulus of concrete E_c and the strain at the compressive strength ε_m are calculated by Eqs. (2), (3) and (4), respectively.

$$a = \frac{E_c \cdot \varepsilon_m}{c \sigma_B} \quad (2)$$

$$\varepsilon_m = 0.93 c \sigma_B^{1/4} \cdot 10^{-3} \quad (3)$$

$$E_c = (3.32 \times \sqrt{c \sigma_B} + 6.9) \times 10^3 \quad (4)$$

2.3.2 Steel tube

To investigate the effect of strain hardening, the stress-strain relationship of a steel tube is assumed to be a bilinear model consisting of two broken lines, as shown in Fig. 3(b). The strain hardening coefficient is expressed by E_s/E , in which E is the initial stiffness and E_s is the stiffness in the second branch (referred to as the second stiffness).

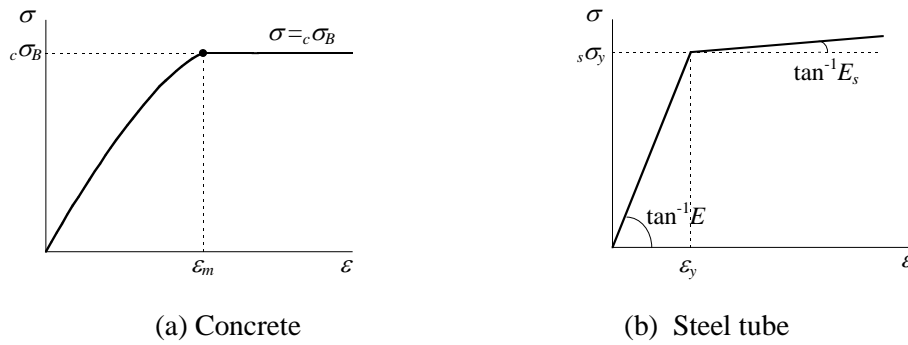


Fig. 3 Stress-strain Relationships

3. Results and discussion

3.1 Analytical parameters

The analytical parameters considered in this paper are shown as follows:

- 1) Axial force ratio $n = N/N_0$ ($N_0 = sA_s\sigma_y + cA_c\sigma_B$): 0.2~ 0.7.
- 2) Strain hardening coefficient E_s/E : 0.0005, 0.001, 0.002, 0.005 and 0.01
- 3) Width-thickness ratio D/t : 15, 20 and 30
- 4) Yield strength of steel tube $s\sigma_y$ (in units of N/mm^2): 235, 325 and 440
- 5) Compressive strength of concrete $c\sigma_B$ (in units of N/mm^2): 24, 36 and 60

The width of the cross section D was set to 300 mm. The slenderness ratio $\bar{\lambda}$ was set to the range 0 to unity, which was calculated by Eq. (5).

$$\bar{\lambda} = \frac{s\lambda}{\pi} \sqrt{\frac{s\sigma_y}{E}} \quad (5)$$

$$\lambda_s = \frac{l_k \cdot D}{s i} \quad (6)$$



In which, λ_s is the slenderness ratio of the steel tube calculated by Eq. (6). l_k is the effective length, and r_s is the radius of gyration for the steel tube.

3.2 Relation between M/M_{pc} and $\bar{\lambda}$

The relationship between M/M_{pc} and the slenderness ratio $\bar{\lambda}$ are shown in Figs. 4~7 where we have taken the strain hardening coefficient E_s/E as the investigative parameter. Therein, M is calculated by $H \times L$ where H is the lateral force and L is the length of beam-column. M_{pc} is the plastic resistance of the beam-column subjected to axial force and bending moment.

According to these figures, as the strain hardening coefficient E_s/E increases, the value of M/M_{pc} increases. However, as the slenderness ratio $\bar{\lambda}$ increases, the values of M/M_{pc} become almost the same even though the strain hardening coefficient E_s/E is different.

3.2.1 Comparison between the CFT member and the hollow steel tube

Figure 4 shows the relationship between M/M_{pc} and $\bar{\lambda}$ of the CFT member when the width-thickness ratio $D/t=15$, the yield stress of steel $\sigma_y=325\text{N/mm}^2$ and the compressive stress of concrete $\sigma_c=36\text{N/mm}^2$. Figs. 4(a), (b) and (c) show the cases for $n=0.2$, 0.4 and 0.6 respectively. According to Fig. 4, as the axial force ratio n increases, the effect on the value of M/M_{pc} due to the hardening coefficient E_s/E increases, i.e. the effect of the strain hardening coefficient E_s/E on the value of M/M_{pc} becomes more significant. The Fig. 5 shows the relationship between M/M_{pc} and $\bar{\lambda}$ for the hollow steel tube under the same conditions as Fig. 4.

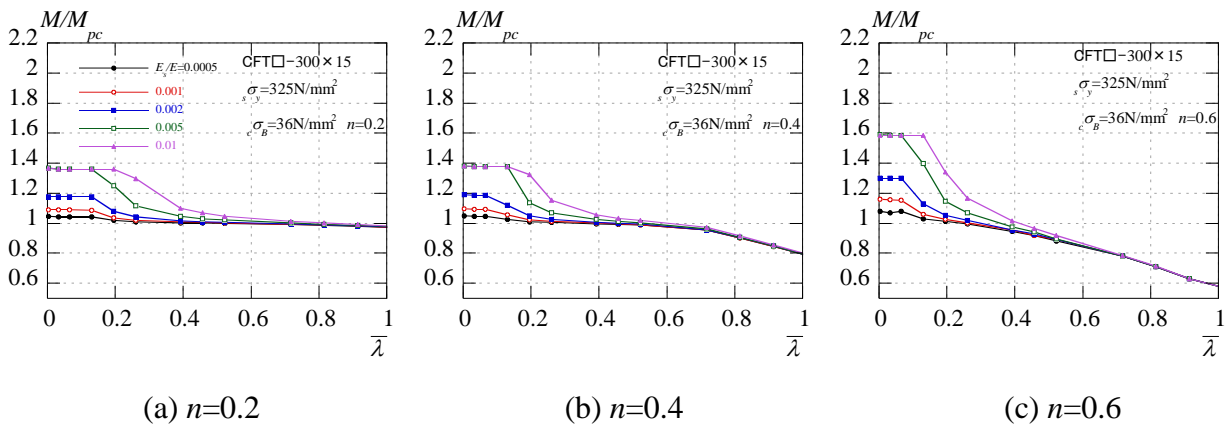


Fig. 4 Relation of M/M_{pc} and $\bar{\lambda}$ (Case of CFT member)

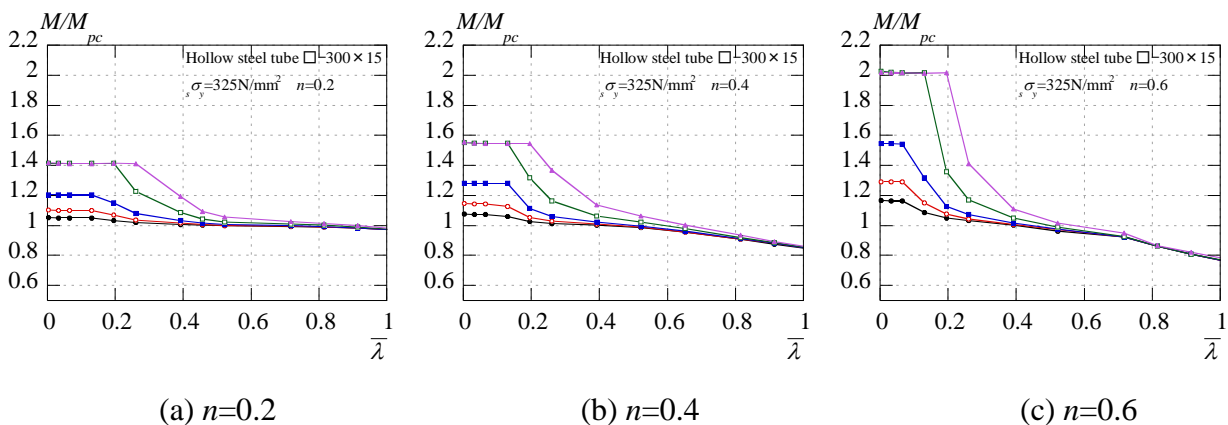


Fig. 5 Relation of M/M_{pc} and $\bar{\lambda}$ (Case of hollow steel tube)



Compared Figs. 4 and 5, it can be seen that the effect of the strain hardening coefficient E_s/E on the relationship between M/M_{pc} and $\bar{\lambda}$ for the CFT member has the same tendency as the case of the hollow steel tube. However, for the case of the hollow steel tube, the value of M/M_{pc} is more susceptible to varying by E_s/E than for the case of the CFT member.

3.2.2 Comparison by the width-thickness ratio D/t

Figures 6(a), (b) and (c) show the relationship between M/M_{pc} and $\bar{\lambda}$ of the CFT member when the width-thickness ratios $D/t=15, 20$ and 30 respectively. The yield stress of steel $\sigma_y=325\text{N/mm}^2$ and the compressive stress of concrete $\sigma_B=36\text{N/mm}^2$. The axial force ratio n is 0.4. According to these figures, as the width-thickness ratio D/t increases, the effect of the strain hardening coefficient E_s/E on the values of M/M_{pc} becomes greater.

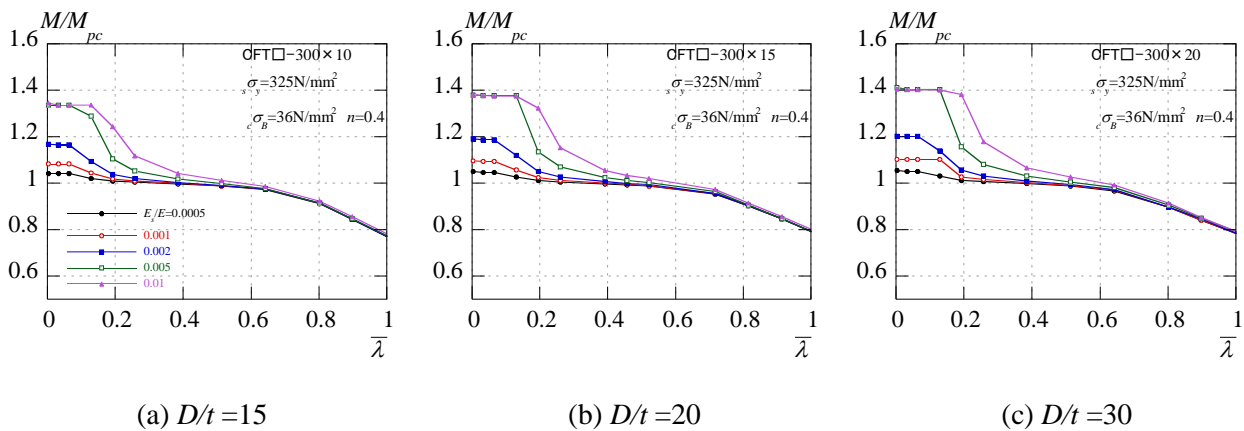


Fig. 6 Relationship between M/M_{pc} and $\bar{\lambda}$ (as a function of D/t)

3.2.3 Comparison by the stress of steel tube σ_y

Figures 7(a), (b) and (c) show the relationship between M/M_{pc} and $\bar{\lambda}$ of the CFT members when the yield stress of steel $\sigma_y=235\text{N/mm}^2, 325\text{N/mm}^2$ and 440N/mm^2 respectively. The width-thickness ratio $D/t=20$ and the compressive stress of concrete $\sigma_B=36\text{N/mm}^2$. The axial force ratio n is 0.4. According to these figures, as the yield stress of steel σ_y decreases, the effect of the strain hardening coefficient E_s/E on the value of M/M_{pc} becomes larger.

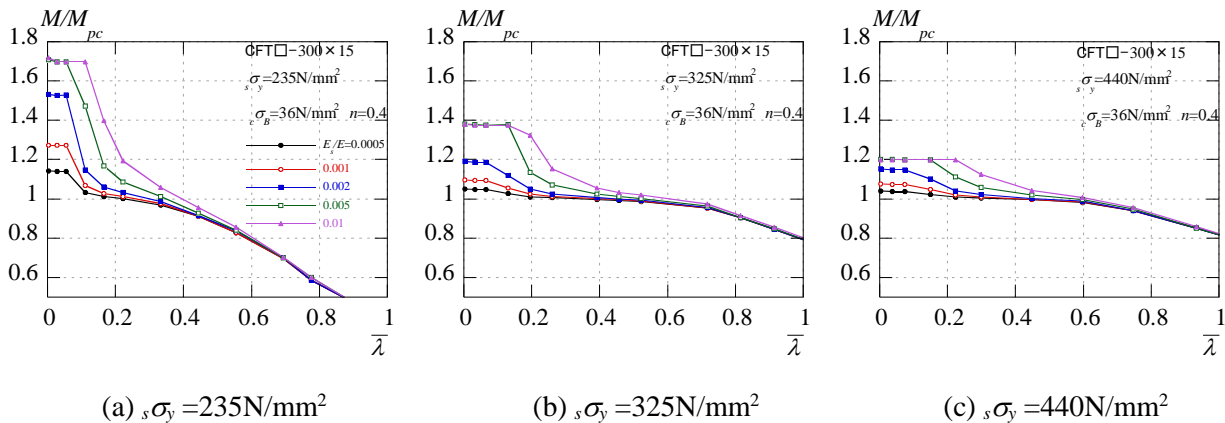


Fig. 7 Relation of M/M_{pc} and $\bar{\lambda}$ (as a function of σ_y)



3.2.4 Comparison by the compressive stress of concrete $c\sigma_B$

Figures 8(a), (b) and (c) show the relationship between M/M_{pc} and $\bar{\lambda}$ of the CFT member when the yield stress of steel $c\sigma_B = 24\text{N/mm}^2$, 36N/mm^2 and 60N/mm^2 respectively. The width-thickness ratio $D/t=20$ and the yield stress of steel $s\sigma_y = 325\text{N/mm}^2$. The axial force ratio $n=0.4$. According to these figures, the effect of the strain hardening coefficient E_s/E on the values of M/M_{pc} becomes greater as the compressive strength of concrete $c\sigma_B$ decreases.

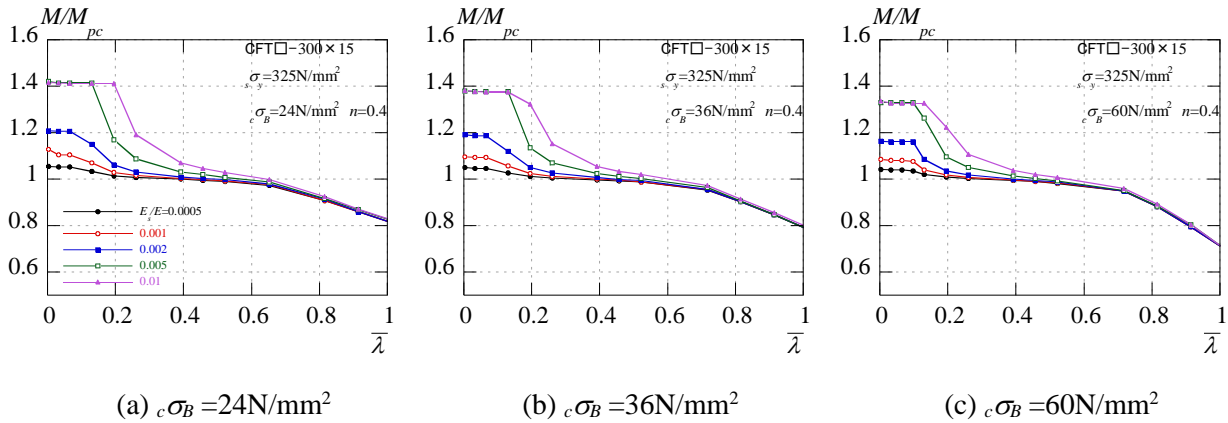


Fig. 8 Relation of M/M_{pc} and $\bar{\lambda}$ (as a function of $c\sigma_B$)

3.3 Length-thickness ratio l_k/D when $M/M_{pc}=1$ for different axial force ratio n

The values of the length-thickness ratio l_k/D (in which l_k is the effective length) when $M/M_{pc}=1$ with different axial force ratios n are shown in Fig. 9 by taking the strain hardening coefficient E_s/E as a parameter. Figs. (a) and (b) are the cases of the CFT member and the hollow steel tube respectively. According to Fig. 9, as the strain hardening coefficient E_s/E increases, the value of l_k/D increases when the axial force ratio n takes the same value. Compared with the case of the hollow steel tube, the values of l_k/D for the case of the CFT member are larger than for the case of hollow steel tube when the strain hardening coefficient E_s/E is set to be the same value.

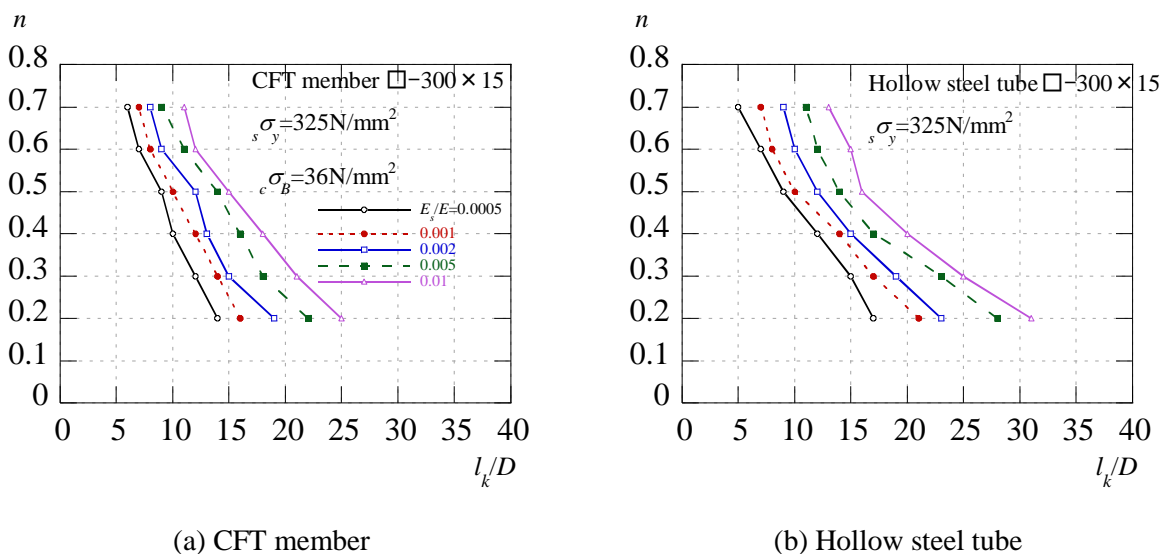


Fig. 9 Relationship of $n-l_k/D$ when $M/M_{pc}=1$



4. Conclusions

This paper investigated the effects of the strain hardening coefficient E_s/E on the relationship between the slenderness ratio $\bar{\lambda}$ and M/M_{pc} for different combinations of axial force ratio, material strength and cross-sectional dimensions of CFT beam-columns. The details are shown as follows:

- 1) As the strain hardening coefficient E_s/E increased, the value of M/M_{pc} increased. However, as the slenderness ratio $\bar{\lambda}$ increased, the value of M/M_{pc} became almost the same even though the strain hardening coefficients E_s/E were different.
- 2) The effect of the strain hardening coefficient E_s/E on the relationship between M/M_{pc} and $\bar{\lambda}$ for the CFT member had the same tendency as the case of the hollow steel tube. However, for the case of the hollow steel tube, the value of M/M_{pc} is more susceptible to varying by E_s/E than for the case of the CFT member.
- 3) As the width-thickness ratio D/t increased, the effect of the strain hardening coefficient E_s/E on the values of M/M_{pc} became larger.
- 4) As the yield stress of steel $s\sigma_y$ decreased, the effect of the strain hardening coefficient E_s/E on the values of M/M_{pc} became great.
- 5) The effect of the strain hardening coefficient E_s/E on the values of M/M_{pc} became greater as the compressive stress of concrete $c\sigma_B$ decreased.
- 6) The values of the length-thickness ratio l_k/D when $M/M_{pc}=1$ for the case of the CFT member were larger than for the case of the hollow steel tube when the strain hardening coefficient E_s/E were set to be the same value.

5. References

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