DAMPING RATIOS INFERRED FROM THE SEISMIC RESPONSE OF BUILDINGS

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Abstract

This paper summarizes an investigation that analyzed damping ratios inferred from 1335 seismic responses coming from 154 instrumented buildings in California. These values were inferred using a parametric system identification technique in the time domain and subjected to a series of reliability tests to ensure that only high-quality data was employed. A series of linear mixed-effects statistical models are evaluated in order to select the regressors that best explain the variance in the data. The influence of the building's fundamental period, height, aspect ratio, lateral-load resistant system, and material are examined, and a prediction equation for estimating the damping ratio of the fundamental mode is proposed. It is shown that the building height is the factor that best explains the variance in the data, with damping in the first translational mode decreasing as the building height increases. A series of more complex, multi-variate statistical models are also evaluated. Results show that, in terms of percentage of variance explained, there is little gain in favoring a more complex model over a simple height-dependent prediction equation. Therefore, a single prediction equation that relates damping in the fundamental mode with the building height is presented. The influence of the primary building material is investigated, finding no statistical difference between the damping ratios of steel and reinforced concrete buildings. Regarding lateral-load resistant systems, it is found that steel buildings with moment-resistant frames have, on average, higher damping ratios than buildings with steel braced frames. The effects of amplitude of the response – measured as the peak roof drift ratio – on damping are also examined. Results show that including the peak roof drift ratio as a regressor has little effect on improving the fit of the statistical model. Finally, damping in higher modes are analyzed, finding that damping increases with increasing modal frequency following an approximately linear trend.

Keywords: damping; system identification; damping of higher modes; earthquake records

1. Introduction

Estimation of a building's response to earthquake ground motions requires knowledge of the level of damping in the structure. Unlike other dynamic properties that can be calculated from the geometric and mechanical properties of the different components in a building, an engineer cannot directly calculate the level of damping that a building will experience during an earthquake. The complexity in its estimation arises because there are multiple sources of energy dissipation involved and, in general, very little is known about them separately – and even less when they are combined [1]. A common assumption employed in the seismic design of a building is that the damping forces are linearly proportional to relative velocities, in other words, it is assumed that damping acts in a linear viscous manner. Although there is no physical reason behind it, this assumption greatly simplifies the equations of motion of the problem [2] and, more importantly, multiple studies have shown that it produces acceptable results [e.g., 3–6]. Therefore, when employing a modal analysis with a linear viscous damping model, specifying the level of damping reduces to knowing which values should be assigned to the different modal damping ratios.

In the literature, many different recommendations have been given through the years for estimating damping ratios. During the late 1950s and through the 1960s, a first generation of damping recommendations was published [7–9]. These recommendations were based on static load tests of individual components and connections rather than on full buildings, and, according to their authors, it involved a great deal of engineering judgement [10]. As sensor technology progressed and accelerometers became widely available, new damping recommendations were developed based on the measured response of instrumented buildings [e.g., 11–18]. A thorough examination of these recommendations [19] revealed that they share at least one of the following problems: (1) They have been derived from databases of damping ratios inferred from buildings subjected to low-amplitude motions such as forced, wind, or ambient vibrations. It is well documented that, at very low amplitudes, the level of damping varies with the amplitude of the building response [e.g., 20–22] and therefore values inferred under low-amplitude motions should not be employed in the seismic analysis of buildings. (2) The methods employed to infer damping ratios do not provide reliable results. In some cases, large portions of the data come from methods known to produce biased results, such as the half-power bandwidth method. And (3) the statistical methods employed in the analysis of the damping data are not adequate. For example, multiple damping measurements are performed per building, but the regressions are not adjusted to take this into consideration.

This paper summarizes an investigation whose objective is to provide new, reliable, damping recommendations for the seismic analysis of buildings. A database was constructed after the analysis of 1335 seismic responses coming from 154 buildings and 117 earthquakes. The damping data was inferred employing a single system identification technique and a series of reliability screening tests was conducted to ensure the quality of the damping values. A comprehensive statistical analysis of the data was then performed to assess the influence of the building height, aspect ratio, fundamental period, primary structural material, lateral load-resistant system, and amplitude of the response, on the damping ratio of the fundamental mode. Damping in higher modes was also studied and the variation of modal damping ratio with modal frequency was examined. Finally, this paper provides a series of modal damping recommendations for the seismic analysis of buildings.

2. System Identification

System identification is a tool to infer the properties of a system from it measured output and (in some cases) input signals. For earthquake engineering purposes, the system corresponds to a building, the input signal corresponds to the acceleration recorded at the base of the building, and the output signals correspond to the building's response measured as the acceleration recorded at different locations. In the literature, there are several system identification techniques that have been applied to infer the dynamic properties of buildings subjected to earthquakes [e.g.,3,6,23–25]. In this investigation, a modal minimization in time domain method was implemented. This technique, adapted from [3], consists in finding the parameters of a mathematical model of the building that will best reproduce the recorded outputs for a given ground motion. For this purpose, the following objective function was considered:

$$J(\Theta) = \frac{\sum_{j=1}^{N_s}\sum_{i=1}^{\tau}\left[\ddot{u}_j(i\Delta t) - \hat{\ddot{u}}_j(i\Delta t)\right]^2}{\sum_{k}^{\tau}\left[\ddot{u}_j(k\Delta t)\right]^2} \quad (1)$$

where $\ddot{u}_j(t)$ and $\hat{\ddot{u}}_j(t)$ correspond to the recorded and predicted relative acceleration, respectively, at the location of the j-th sensor at time t; N_s is the number of sensors in the building; Δt is the time step of the records; τ is the number of points in the signal; and Θ is the set of parameters to be optimized from the mathematical model of the building.

The building was considered to be linear, elastic, and with a fixed base. Damping was assumed to be of the linear viscous type, and the damping matrix was assumed to be classical. For a ground motion acceleration at the base $\ddot{u}_g(t)$, the predicted relative acceleration at the location of the j-th sensor in the building, $\hat{\ddot{u}}_j(t)$ was calculated using modal superposition as:

$$\hat{\ddot{u}}_j(t) = \sum_{n=1}^{N_m}\Gamma_n\phi_{jn}\ddot{D}_n(t) \quad (2)$$

where N_m is the number of modes considered in the analysis; Γ_n is the modal participation factor of the n-th mode, ϕ_{jn} is the mode shape of the n-th mode evaluated at the location of the j-th sensor; and $\ddot{D}_n(t)$ is the response of a single-degree-of-freedom system with period T_n and damping ratio ξ_n (equals to that of the n-th mode of the building) subjected to the ground motion acceleration $\ddot{u}_g(t)$. If the building is assumed to be at rest at time $t = 0$, the set of parameters to be optimized for this model are:

$$\Theta = \left(\begin{Bmatrix} T_1 \\ \vdots \\ T_{N_m}\end{Bmatrix}; \begin{Bmatrix} \xi_1 \\ \vdots \\ \xi_{N_m}\end{Bmatrix}; \begin{bmatrix} \Gamma_1\phi_{11} & \cdots & \Gamma_{N_m}\phi_{1N_m} \\ \vdots & \ddots & \vdots \\ \Gamma_1\phi_{N_s1} & \cdots & \Gamma_{N_m}\phi_{N_sN_m}\end{bmatrix}\right) \quad (3)$$

please note that, given that the building is assumed to be fixed at its base, the set of parameters inferred by the system identification method do not correspond to the properties of the superstructure alone, but those of a *replacement fixed-base structure* capable of reproducing the recorded structural response. In other words, the effects of soil-structure interaction are included in the inferred parameters. In soil-structure interaction literature, these parameters are often called the *effective* dynamic properties of the building [26].

Once the system identification method converged, the identified modal damping ratios ξ_n were subjected to a series of reliability screening tests. The purpose of these tests was to discard values that were not deemed reliable. Details of these tests can be found in reference [27]. Additional details about the system identification method employed and the optimization process can be found in reference [5].

3. Buildings Analyzed

The building database of the Center for Engineering Strong Motion Data [28] was employed. For any given earthquake in the database, the criteria employed for selecting buildings affected by that earthquake was the following: (1) The buildings did not experience significant damage during the earthquake, and therefore did not incur significantly into their inelastic range; and (2) the buildings did not have special earthquake-protection devices such as dampers or isolators. In total, 1335 seismic responses coming from 154 different buildings were analyzed. After passing the reliability screening tests, a database of 1037 high-quality damping ratios, coming from 144 buildings, was created. The data was classified by *building components.* In this investigation, a *building component* corresponds to one of the two principal perpendicular directions of a building. Table 1 shows a deaggregation by primary structural material and lateral force-resisting system of all the building components in the dataset. Please note that the total number of building components (261) is lower than twice the number of buildings (288) because, in some cases, a reliable value of damping ratio could not be obtained for a particular building direction.

Table 1 – Deaggregation of building components in the database per material and lateral force-resisting system

Primary material	Lateral system*	# Building components
Concrete	MF	17
Concrete	SW	77
Steel	MF	68
Steel	BF	41
Masonry	SW	14
Wood	MF	1
Wood	SW or BF	9
Mixed	MF	2
Mixed	SW or BF	32

(*) MF: Moment-resisting frames
BF: Braced frames
SW: Shear walls

4. Statistical Analysis of the Damping Data

4.1 Overview of the Statistical Model

A linear mixed-effects (LME) statistical model was employed to analyze the damping data. This kind of statistical models separate the variance of the data into within-building and between-building variability by employing two different types of factors: random effects and fixed effects. Random effects are those that affect individual buildings while fixed effects are those that influence multiple buildings. In this investigation, the "building component" factor that labeled each building (e.g., building B001 in EW direction) was considered a random effect while all other factors (e.g., building height, material, lateral system, etc.) were considered fixed effects. This step is crucial if statistical inference of the data is to be performed: given that there are multiple earthquakes recorded in each building, there are multiple observations of damping ratios per building component. Two or more data points coming from the same building component cannot be regarded as independent from each other. Consequently, independence of the observations, a key assumption of ordinary linear regression (OLR), is not met and therefore OLR should not be employed for statistical inference of the data. An LME model solves this issue by separating the variability.

The specific equations that define the LME model employed in this investigation are the following:

$$\ln(\xi_{ij}) = \beta_0 + \sum_{k=1}^{N} \beta_k X_{ki} + b_i + \varepsilon_{ij}$$
$$b_i \sim N(0, \sigma_b^2) \qquad \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2 \delta_s^2) \tag{4}$$

where ξ_{ij} is the j-th observation of the damping ratio of the i-th building; $\beta_0, \beta_1, \ldots, \beta_N$ are the regression coefficients of the fixed effects; X_{ki} are the fixed-effects regressors, b_i are the random effects, and ε_{ij} are the residuals. It is assumed that both the random effects and residuals are normally distributed with zero mean and variance σ_b^2 and $\sigma_\varepsilon^2 \delta_s^2$. δ_s is a parameter that allows to consider different variances between levels of categorical regressors (e.g. "concrete" and "steel" are different levels of the categorical regressor "material") and it represents the ratio of the standard deviations of the *S*-th and first levels. The natural logarithm of ξ_{ij}

was taken in order to satisfy the linearity assumption of the model. If only one categorical variable is present in the model, the following expression can be used to obtain an overall measure of the dispersion for the s-th level:

$$\sigma_{ln_s} = \sqrt{\sigma_b^2 + \sigma_\varepsilon^2 \delta_s^2} \tag{5}$$

The parameters of the statistical model ($\beta_0, \beta_1, \ldots, \beta_N, b_i, \sigma_b^2, \sigma_\varepsilon^2$, and δ_s^2) were obtained using the restricted maximum likelihood method (REML). The statistical inference of the model was based on the 95% confidence intervals of the regression coefficients of the fixed effects. These intervals were calculated using a normal approximation to the REML estimators, as recommended in [29] . All the statistic calculations were performed using the the *nlme* package of the statistical software *R* [30,31].

4.2 Regression Model Selection

There are multiple possible variables that could be included as fixed-effects regressors in the LME model. To select among them, we defined 4 areas where we wanted to explore possible correlations with damping ratio: (1) Variables related with soil-structure interaction; (2) amplitude of the response; (3) primary structural material; and (4) lateral load-resistant structural system. Our main objective in this selection process was to provide the simplest possible equation to predict damping ratio, that explained most of the observed variance in the data. To this end, the final selection of variables was done by measuring the percentage of variance by each possible regressor. The percentage of variance explained was calculated by computing the marginal R^2 statistic for LME models developed in reference [32].

Different investigations have found that there is a strong correlation between damping ratio of the first mode and the building height H, first translational period T, and aspect ratio AR (measured as the building height over its base dimension in a particular direction) [e.g.,16,18]. It has been argued that this correlation could be attributed to soil-structure interaction effects, as these parameters also govern the level of soil-structure interaction of the system [18,26,33]. These parameters, however, are highly colinear: taller buildings tend to have longer periods and larger aspect ratios, and therefore including more than one of them may not add relevant information to the model. To explore this, different models including one or more of these variables were explored. The mathematical description of these models and the percentage of variance explained by them is shown in Table 2. It was found that the building height is the variable that best explains the variance in the data, with an R^2 valu of 0.43. Combination of these variables were also explored but, as expected, it was found that adding more than one of these variables did not considerably improved the fit. Consequently, subsequent models only included the building height as a regressor.

Table 2 – LME regression models for variables related with soil-structure interaction

LME Regression Model	R^2
$\ln(\xi) = \beta_0 + \beta_1 \ln(H)$	0.43
$\ln(\xi) = \beta_0 + \beta_1 \ln(T)$	0.20
$\ln(\xi) = \beta_0 + \beta_1 \ln(AR)$	0.31
$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(T)$	0.46
$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(AR)$	0.43
$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(T) + \beta_3 \ln(AR)$	0.46

Next, the effect of including a measure of amplitude of the response in the statistical model was explored. For this study, we used the peak roof drift ratio (PRDR) as the preferred metric for amplitude. PRDR is defined as the peak displacement at the roof level relative to the ground level divided by the total height of the building.

Please note that including a structural response parameter in the equation adds complexity to the damping prediction equation as it would require iteration. PRDR was incorporated using the following model:

$$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(PRDR) \qquad R^2 = 0.47 \qquad (6)$$

A comparison of the R^2 values of this model and the one without the amplitude term shows that including PRDR only explains an additional 4% of the variance. Although it could be argued that this increment is not negligible, there is little gain from its inclusion, so we decided not to incorporate it in further models to avoid adding unnecessary complexity to the prediction equation.

The influence of the building primary structural was investigated next. First, the primary structural material was incorporated into the statistical model as:

$$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \sum_{k=2}^{M+1} [\beta_k D_{k-1}^{mat} + \beta_{k+M} D_{k-1}^{mat} \ln(H)] \qquad R^2 = 0.45 \qquad (7)$$

where D_k^{mat} is a dummy variable equal to 1 for buildings having the k-th level of the "material" categorical variable as their primary structural material, and equal to 0 otherwise; and k = 1, 2, …, M is the number of structural materials considered, minus 1. As can be seen in Equation (7), the model incorporating the primary building material into the prediction equation explains 45% of the data, that is, the primary structural material only accounts for an additional 2% of variance explained over the model without it. This suggests that the structural material is not a relevant variable for determining the level of damping ratio of a building.

The influence of the combined effect of the structural material and lateral load-resisting system (MLS) was then studied. To be able to draw meaningful conclusions about these variables, it is desirable to have at least 30 building components of each category. From Table 1, it can be seen that the following categories satisfy this requirement: Concrete-SW, Steel-MF, Steel-BF, and Mixed SW/BF. All the building components that did not fall into these categories were grouped and labeld as "Other". Consequently, the MLS categorical regressor was created, which can only take these 5 possible values. This variable was included into the statistical model as:

$$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \sum_{k=2}^{Q+1} [\beta_k D_{k-1}^{MLS} + \beta_{k+Q} D_{k-1}^{MLS} \ln(H)] \qquad R^2 = 0.49 \qquad (8)$$

As before, D_k^{MLS} is a dummy variable equal to 1 for buildings made of the k-th level of the "MLS" categorical variable, and equal to 0 otherwise; and k = 1, 2, …, Q is the number of combined material and lateral system categories, minus 1. Results show that this variable improves the fit from an R^2 of 0.43 of the model without this variable, to 0.49. This suggest that MLS is a better regressor than the material itself.

A final model was investigated incorporating PRDR to the model defined by equations (8):

$$\ln(\xi) = \beta_0 + \beta_1 \ln(H) + \beta_2 \ln(PRDR) + \sum_{k=3}^{Q+2} [\beta_k D_{k-2}^{MLS} + \beta_{k+Q} D_{k-2}^{MLS} \ln(H)] \qquad R^2 = 0.53 \qquad (9)$$

The purpose of this model was not to provide a damping prediction formula, but to verify that the trends observed in the previous, simpler, statistical models were also valid when controlling by the amplitude of the response.

5. Results for the First Translational Mode

The following subsections summarize the main findings obtained using the different statistical models described above. Due to space limitations, the mathematical results that support these findings such as parameter values, variances, and confidence intervals, are not included. The interested reader can refer to [19] for these and other details not included in this paper.

The results of the previous section show that the building height is the single factor that explains most of the variance in the data. The statistical model that includes the building height as a single fixed-effect regressor has an R^2 of 0.43, and correspond to the simplest damping equation assessed in this investigation. Fig. 1 shows the variation of damping ratio of the fundamental mode with the building height. It can be seen that damping ratio decreases with increasing building height following a trend that is approximately hyperbolic. The figure also includes a curve with the predicted median, using the equation resulting from the statistical analysis, and its corresponding $\pm 1\sigma_{ln}$ prediction intervals. It can be seen that, for buildings taller than 21 meters, the median damping ratio is lower than 5%, the typical value recommended by most structural codes. Moreover, it can be seen that several tall buildings with heights greater than 200m have damping values lower than 2.5% – the damping ratio recommended by most guidelines for the seismic design of tall buildings.

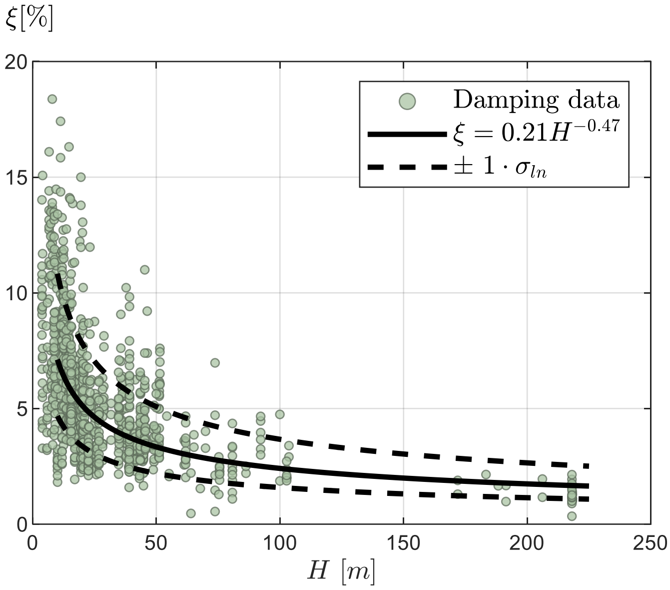

Fig. 1 – Damping ratio of the first mode as a function of the building height for all data, and results of the LME regression equation.

In Section 4.2 it was shown that separating the data by primary structural material did not significantly improve the fit. The posterior analysis of the coefficients of the statistical model shows that there is no statistical difference between damping ratios coming from reinforced concrete buildings and steel buildings. This can be seen in Fig. 2, which plots the values of damping ratios as a function of the building height, separating them by primary structural material. The figure shows no clear difference of damping ratios coming from one material or the other. Results show that the dispersion around the median values for steel buildings is 79% of that observed in reinforced concrete buildings. This can be seen in Fig. 2, where for any given value of the building height, the dispersion of reinforced concrete data is larger than the data coming from steel buildings. The large variability around the median values, especially for reinforced concrete buildings, is probably why no statistical difference was found between these two materials.

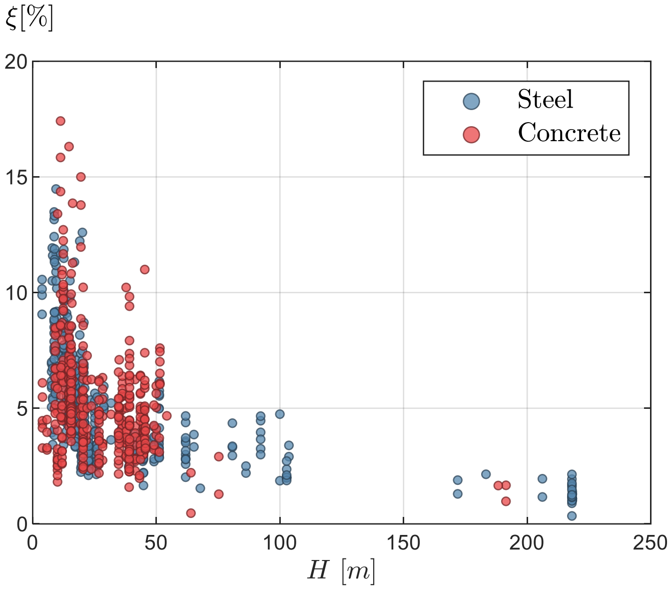

Fig. 2 – Damping ratio of the first mode as a function of the building height for steel and reinforced concrete data.

The influence of the combined material and lateral force-resisting system was investigated next by applying the LME regression model defined by equation (8). Results show that there is a statistically significant difference between the median damping ratios of steel moment frames (SMF) and steel braced frames (SBF). Fig. 3 shows the variation of damping ratios with building height for SMF and SBF structures, as well as their corresponding prediction equations with the coefficients resulting from the regression analysis. It can be seen that for any given value of the building height, SMF buildings have, on average, larger damping ratios than SBF buildings. This is consistent with findings from investigations made in buildings subjected to wind motions [34–36]. Some studies have argued that these differences arise because damping is higher in buildings where the contribution of shear deformations is large with respect to the contribution of flexural deformations [34]. In general, the first mode shape of buildings with moment frames can be related to lateral shear deformations, while buildings with braced frames can be related to cantilever flexural action. Another explanation for these observations can be attributed to soil-structure interaction: the centroid of the first mode shape of buildings with moment frames tend to be significantly lower than those with braced frames, which – other variables being equal – translates into lower contribution of rocking motion of moment frame buildings. Given that soil-structure interaction is primarily governed by the first mode, a lower contribution of rocking motion translates into a larger effective damping of the fundamental mode [33].

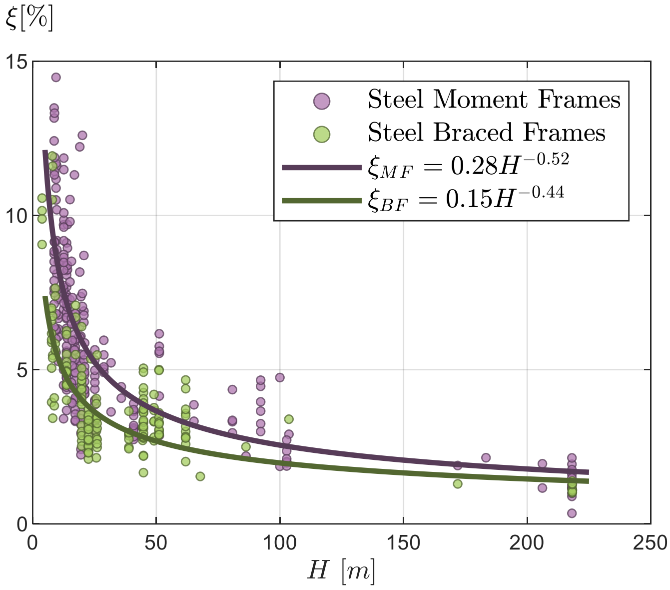

Fig. 3 – Damping ratio of the first mode as a function of the building height for steel moment frames and steel braced frames, and results of the LME regression equation.

6. Damping Ratios in Higher Modes

Analyzing damping ratios of higher modes poses a much bigger challenge because the amount of building components with reliable damping ratios for higher modes is scarce. To this end, we selected a subset of building components and earthquakes where we were able to reliably infer the damping ratio of at least the third mode. This reduced the data to 119 seismic responses coming from 24 buildings and 46 earthquakes, for a total number of 567 reliable damping values. For each building component, the reliable damping ratios were plotted against their identified modal frequencies. The top row of Fig. 4 shows an example of this for 3 different building components. It can be seen that, in these cases, damping ratios tend to increase with increasing modal frequency and that this increment follows an approximately linear trend. This is why we passed an ordinary linear regression line to fit the data of each building, as can be seen in the same figure. To be able to compare the data comping from different buildings, each damping value was normalized by the estimate of the damping ratio of the first mode $\tilde{\xi}_1$ calculated using the linear regression of their building component, evaluated at the average of the identified frequencies $\tilde{f}_1$. The bottom row of Fig. 4 show the normalized values obtained for the three cases shown. Fig. 5 shows the variation of the normalized modal damping ratios with the normalized modal frequencies for all the dataset. It can be seen that modal damping ratios increase with increasing modal frequency, and that the increment can be approximated by the linear trend:

$$\frac{\xi_n}{\tilde{\xi}_1} = 0.92 + 0.12\frac{f_n}{\tilde{f}_1} \quad (10)$$

From these results, the following equation was proposed for damping in higher modes:

$$\xi_n(f_n) = \xi_1\left[1 + 0.12\left(\frac{f_n}{f_1} - 1\right)\right] \quad (11)$$

The authors have argued that the approximately linear increment of modal damping ratios with increasing modal frequencies observed in the data can be attributed to soil-structure interaction effects [33].

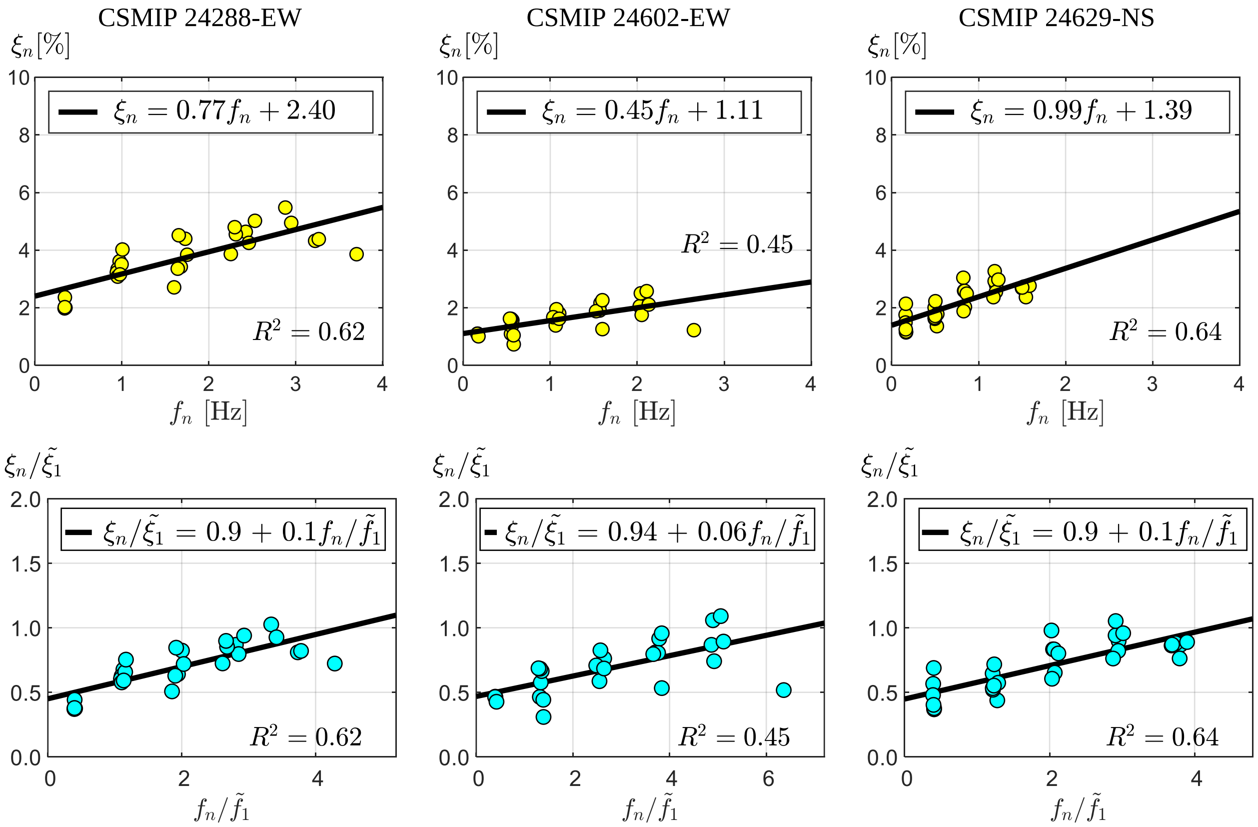

Fig. 4 – Top: Variation of modal damping ratios with modal frequency in 3 different buildings. Bottom: Values normalized by the estimates of the first mode.

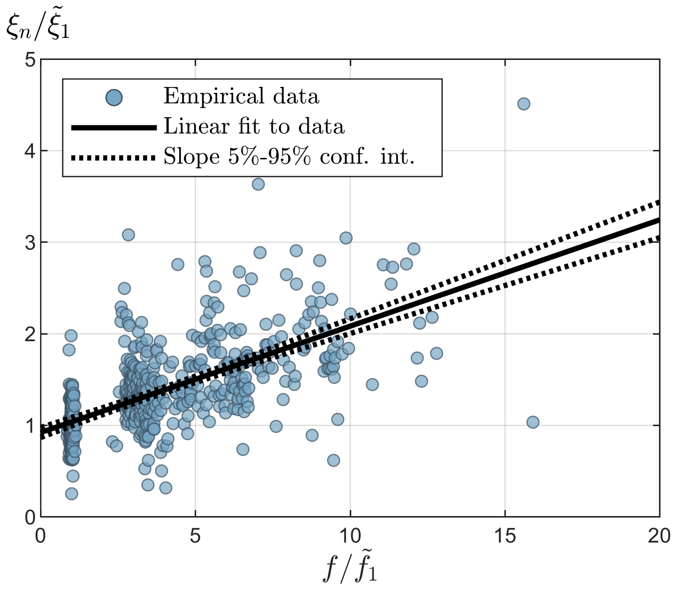

Fig. 5 – Variation of modal damping ratios with modal frequency for all buildings, normalized by the estimates of the first mode.

7. Summary and Conclusions

This paper summarized an investigation whose objective is to improve our understanding of damping in buildings responding elastically to earthquakes and to provide damping recommendations for the level of modal damping ratio that should be employed in the analysis of buildings when using an elastic, modal analysis. To this effect, 1335 seismic responses, coming from 154 different instrumented buildings in California, were analyzed employing a parametric system identification technique in time domain. The identified damping ratios were subjected to a series of reliability screening tests to ensure that only damping ratios deemed reliable were employed in the subsequent analyses, and a database of 1037 high-quality first mode damping ratios was conformed. The dataset was analyzed employing a series of linear mixed-effects statistical models. It was found that the building height is the single factor that explains most of the variance observed in the data, and a prediction equation based on the building height was proposed. It was found that including information about the building material does not significantly improve the fit of the statistical model and no statistical difference was found between damping ratios coming from reinforced concrete buildings and steel buildings. Results show that the lateral force-resisting system influences the damping ratio of the fundamental mode, finding that steel moment frames have, on average, higher damping ratios than steel braced frames. Damping in higher modes was also investigated. A subset of the buildings, consisting of those where multiple observations of damping ratios of at least the third mode was possible, was employed. It was found that modal damping ratios increase with increasing modal frequencies following an approximately linear trend. Based on these empirical results, a linear equation for the variation of damping with frequency was proposed. With this equation, a designer can specify damping in higher modes following an evidence-based trend.

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