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Torsional response evaluation method for reinforced concrete frames based on 3D nonlinear finite element analysis

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Abstract

Current structural designs limit the eccentricity obtained from the elastic rigidity of concrete members, enabling the suppression of damage due to torsion. However, the eccentricity cannot consider the effect of plasticization of the members. In particular, the rigidity significantly changes in a reinforced concrete structure due to cracks in the concrete, causing yielding of the rebars, etc.; accordingly, the torsional response of the frame may change sequentially. To obtain more effective designs against eccentricity, it is important to understand the change in torsional response in the plastic region during the design stage. Thus, in this study we have developed a method for torsional response evaluation in the nonlinear region for application to structural design.

In this study, we conducted a detailed analysis of the response of each member during torsional deformation of the frame, based on three-dimensional nonlinear finite element (FE) analysis, and we considered the change of member rigidity in the plastic region and the accompanying movement of the center of rigidity. As a result, when torsional deformation of the frame occurred, column members were deformed simultaneously in two horizontal directions, confirming that the member performance deteriorates compared to the case where monodirectional deformation is exhibited. In addition, as the position of the center of rigidity changes due to the change in member rigidity that occurs during performance deterioration, the values of the eccentric distance and the elastic radius also fluctuate, and it was confirmed that the torsional response of the frame also changes according to these fluctuations. From this, it was found that to evaluate the nonlinear torsional response of the frame, it is important to evaluate the rigidity of the members, considering the deformation state of the columns receiving a horizontal bi-directional input. Therefore, in the torsional response evaluation method developed in this study, a “displacement / yield strength curve” was proposed and used as a tool for evaluating the member rigidity of a column that receives horizontal bi-directional input. By performing a calculation of the eccentric distance and elastic radius from the center of rigidity based on this curve, it was possible to obtain the degree of torsion in the frame.

Finally, the effectiveness of the proposed method was examined comparing the torsional response of the frame obtained by applying the proposed evaluation method with the results of the FE analysis, for a virtual multi-span frame with eccentricity. The comparison showed that the results obtained by the proposed method were equivalent to the FE analysis results. From this, it was confirmed that the proposed method is useful as a structural design tool for evaluation of the nonlinear torsional response of full-scale frames.

Keywords: R/C frame; Eccentricity; Plasticization; Torsional response evaluation; Finite element analysis

1. Introduction

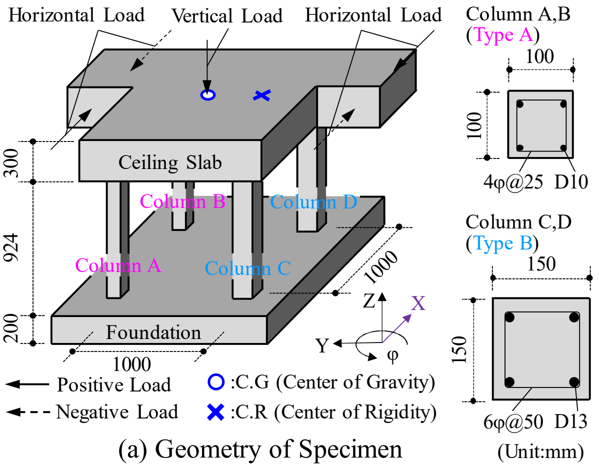
Eccentricity is calculated from the elastic rigidity of members, not considering the influence of rigidity changes that accompany member plasticization. It is important to understand the influence that these rigidity changes have on the torsional response of buildings where the sequential rigidity of reinforced concrete (RC) members change with cracks, crushing, or yielding of rebars. Thus, the establishment of a traceable evaluation method of torsional response in the nonlinear region is necessary.

In this context, the authors have proposed an approximative iterative calculation method for predicting nonlinear torsional response using finite element (FE) analysis for a single-story frame with a single span[1]. However, in this method the calculation is rather complex, and the calculation results change based on convergence condition and the number of iterations. Additionally, the restoring force characteristics of the member were used in the calculation process, but the bi-directional deformation of the column members due to torsional deformation of the frame was not considered. Previous research has shown that the structural design performance of column members deformed in two directions in a frame under torsional response does not meet expectations[2]. The displacement / yield strength curve has been proposed as a solution to this problem[3]. It is considered that restoring force characteristics under bi-directional deformation can be built, and a response evaluation that is closer to actual phenomena can be conducted by using this curve.

The objective of the present research was to develop a simpler and more practical method of predicting the nonlinear torsional response of RC structures with eccentricity. First, we created a nonlinear 3D FE analysis model using the online torsional response experiments conducted by Mizoguchi et al.[4]. Then, we used this model to implement a parametric analysis utilizing the eccentricity as a parameter, and discussed the influence of member plasticization on the torsion characteristics in the frame. Furthermore, we formulated an algorithm for predicting nonlinear torsional response, and confirmed the validity of the present method by comparing it to calculated results based on the torsional response obtained from the evaluated FE analysis and the proposed algorithm.

2. Overview of the online torsional response experiment by Mizoguchi et al.

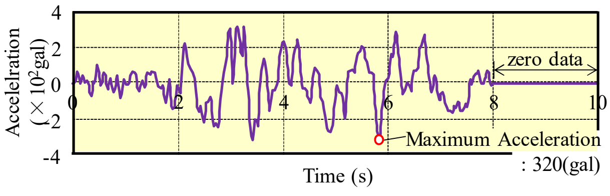
The experiment overview is shown in Fig.1. The specimen is a four-column RC frame composed of a ceiling slab, column members, and foundation slab. The two column members on the left (column members A and B) and on the right (column members C and D) are types A and B, respectively, for the column member cross section. For this reason, the frame exhibits rigidity eccentricity in the Y-direction, and the eccentricity at this point is R_{eX}=0.54. Additionally, the present experiment is based on an online response system, and the pressurized beam for response deformation is attached to the ceiling slab. The pressurized beam section for



(a) Geometry of Specimen

(b) Material Properties

Concrete	Steel			
Compressive Strength (N/mm^2)	Yield Strength (N/mm^2)			
	D13	D10	6φ	4φ
25.9	351.8	361.6	337.1	304.8



(c)Input Seismic Wave

Fig. 1 – Experiment Overview

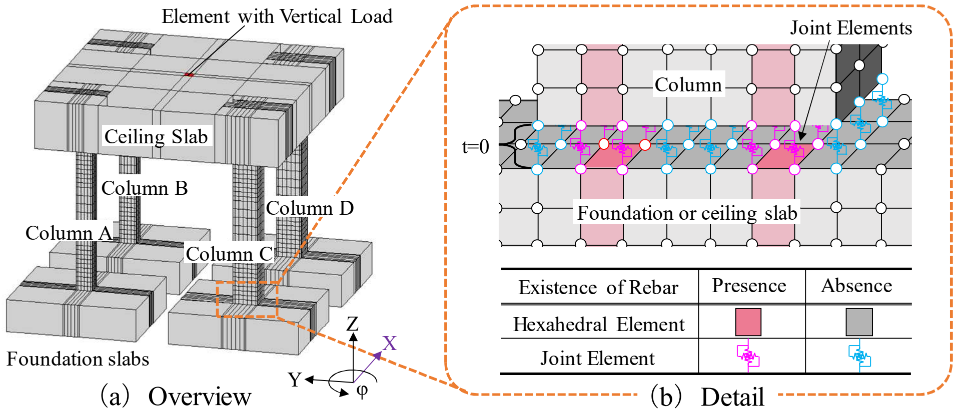
the calculation of rotating inertial mass is not included, and the ceiling slab has the dimensions of 150 cm×150 cm.

The east-west component in the Hachinohe Port during the 1968 Tokachioki earthquake was used as the input seismic wave, and the wave totaling 10 s (maximum acceleration range of 8 s plus 2 s of zero data) was corrected, providing a maximum acceleration of 320 gal. This wave was input in the X-direction of the specimen. Horizontal pressurization was conducted by two actuators set up on the pressurized beam, and the response deformation and torsion angle were forced on the center of gravity. In the vertical direction, a constant load (2.2×10^2 kN) was controlled to act on the specimen using an actuator set up on the center of the ceiling slab. The damping coefficient of the response calculations was set to zero in the present experiment.

3. Overview of the 3D nonlinear FE analysis

3.1 FE analysis model overview

A mesh division of FE Analysis model is shown in Fig.2. Concrete was as a hexahedral element, and the rebar was modeled in a distributed manner. The ceiling and foundation slabs were set as rigid bodies, and the axial force was considered as a weight on the element set in the center of the ceiling slab. Additionally, joint elements representing discrete cracks were added in the head and bottom of the column members to consider the pullout behavior of the longitudinal bars in the column members.



(a) Overview (b) Detail

Fig. 2 – Mesh Division

The constitutive laws of the materials are shown in Fig.3. The modified Ahmad model[5] was used for the compression side of concrete, the model by Izumo et al.[6] set at C = 1.0 was used for the tension stiffening model, and the JSCE Standard Specifications equation[7], which factors in the fracture energy G_F, was used for the tensile softening domain. A model where shear resistance changes in response to strain in the orthogonal direction of cracks[8] was used as a shear transfer model after cracking. Rebar was set as a

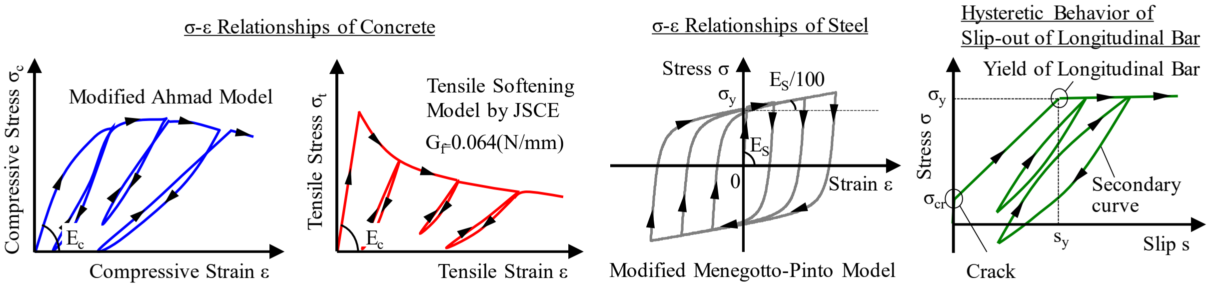


Fig. 3 – Stress－Strain Relationships of Materials

bilinear type, and the stiffness after yielding was set as 1/100th of the initial stiffness. The modified Menegotto-Pinto model proposed by Ciampi et al.[9] was used for hysteresis characteristics. The stress (σ) – slip (s) relationship of the joint elements used to express the pullout behavior of the longitudinal bars was included as well. A sufficiently large stiffness was set prior to reaching cracking strength, and the stiffness following cracking was set as a linear interpolation with yield strength. The yield strength of slip s_y was determined from the proposed equation by Mishima et al.[10] . The shear transfer characteristics after cracking were the same as with concrete.

3.2 Analysis results

The seismic wave from Fig.1 was input and a seismic response analysis was conducted. The natural period for the torsionally-coupled X-directional translational mode was T = 0.178 s as a result of the eigenvalue analysis. Damping was additionally set as proportional to the initial stiffness for the analysis, and a relatively small primary natural period of 0.3% was set because the conditions were similar to those in the experiments, where damping coefficient was zero. The time history response analysis results are shown in Fig.4. The analysis mostly simulated experimental tendencies, and we confirmed that the torsional response can be reproduced by using the presented model. Therefore, this model method was also used for subsequent analysis models.

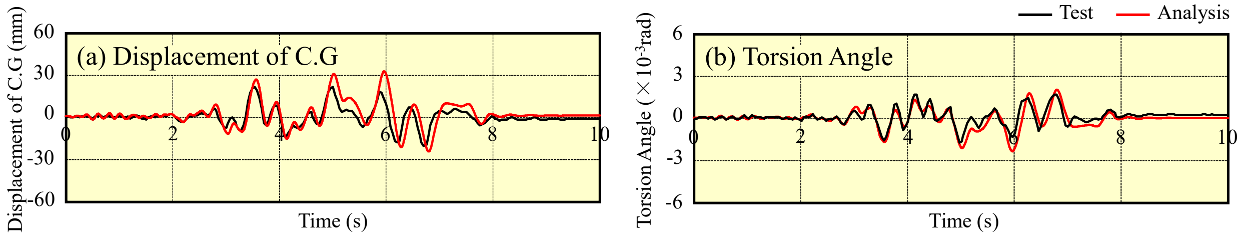
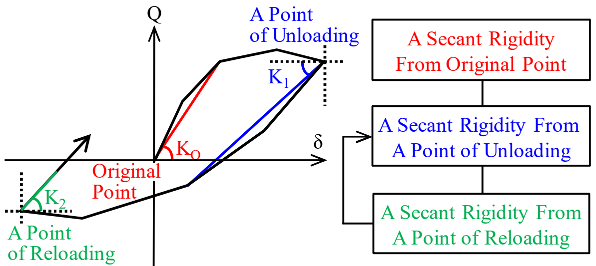


Fig. 4 – Analysis Results.

4. Analysis of torsion characteristics with parametric analysis

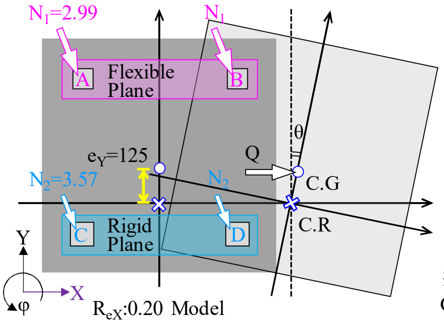
The center of rigidity shifted due to changes in rigidity accompanying member plasticization in the plastic domain; the torsion characteristics of the frame can subsequently change as well. The present section characterizes the influence of member plasticization on torsion characteristics of the frame by implementing analyses where eccentricity is set as a parameter.

Yabuki et al. proposed an algorithm that calculates the center of rigidity accompanying changes in member rigidity[11] . An overview of the member rigidity evaluation in this algorithm is shown in Fig.5. The present method evaluates member rigidity based on secant rigidity for simplification purposes, and the original point location used to evaluate secant rigidity was changed each time an unloading or reloading point was experienced. This was applied to each member response in a given step, and the center of rigidity at that time was calculated using the evaluated rigidity.



(a) Q-δ Relationship of Members
(b) Flow of Rigidity Evaluation

Fig. 5 – Overview of Member Rigidity Evaluation



Analysis Parameters

N_1 (N/mm²)	N_2 (N/mm²)
1.50 ～4.48	4.32 ～2.91
R_{eX}	e_Y (mm)
0.00 ～0.40	0.0 ～249

※ Elastic Radius is Common. (=623mm)

Fig. 6 – Overview of the Analysis Model

An overview of the analysis model is shown in Fig.6. The specimens from Mizoguchi et al. were used, and it was produced an eccentricity of the weight by applying a vertical load on each column member. The eccentricity R_{eX} was set from 0.00 to 0.40 at increments of 0.05, for a total of nine models. All models were set so that the axial force ratio of each column member was set as below 0.4. Materials whose characteristics where the same as an elastic body ("elastic model") and those whose characteristics were the same as a plastic body ("plastic model") were prepared to evaluate the influence from member plasticization. The present investigation focused on the frame response under monodirectional pressurization. Pressurization was set on the single point of the center of gravity, and a forced deformation was applied in the X-direction up to a story deformation angle R_{story} = 1/50. The frame in each model was generally set up so that it could rotate clockwise. Planes that exhibit high deformation due to torsional addition are herein defined as "flexible plane," and those that exhibit low deformation as "rigid plane."

First, we focused our investigations on the results of the plastic model R_{eX}: 0.20. Results of the center of rigidity calculation are shown in Fig.7. The center of rigidity approached the center of gravity with pressurization progress. The relationship between R_{story} and eccentric distance (e_Y) / elastic radius (r_{eX}) is shown in Fig.8. The concrete cracks of each plane and the points of yielding occurrence in the longitudinal bars are shown in the same figure. The values considerably changed as a result of the occurrence of each event, and the eccentric distance decreased whereas the elastic radius increased. These results showed that the plastic model in the plastic domain has less torsion than the elastic model (Fig.9).

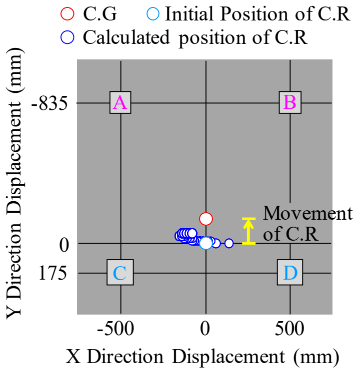


Fig. 7 – Center of Rigidity Calcula -tion Results

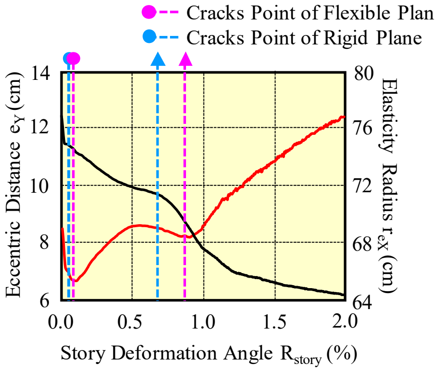


Fig. 8 – e_Y, r_{eX} − R_{story} Relationship

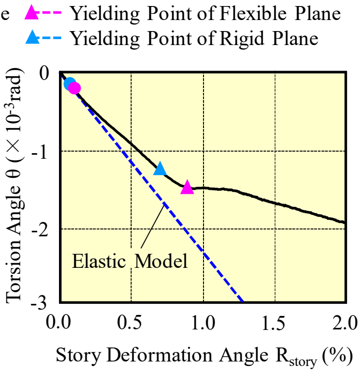


Fig. 9 – θ − R_{story} Relationship

The θ − R_{eX} and e_Y, r_{eX} − R_{eX} relationships of all models when R_{story} = 1/50 are shown in Figs.10 and 11. The degree of torsion increased compared to eccentricity for both the elastic and plastic models. The degree of torsion in the plastic model at this time was smaller than that in the elastic model. The counter-clockwise rotation of torsion in ReX: 0.00 and ReX: 0.05 of the plastic models should be noted. The eccentric distance values of both models were negative, which indicates that the positional relationship

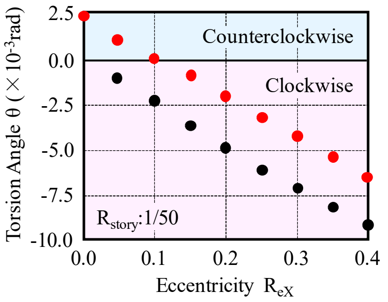


Fig. 10 – θ − R_{eX} Relationship

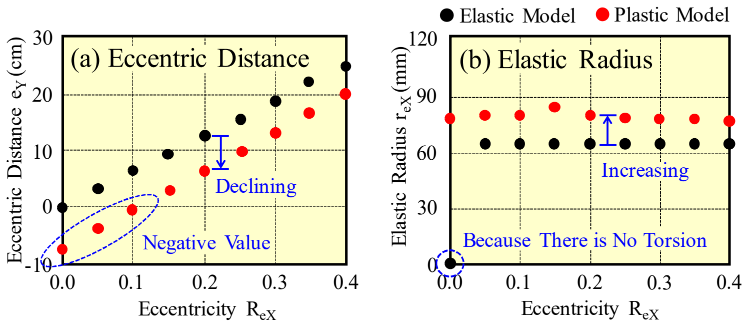


Fig. 11 – e_Y, r_{eX} − R_{eX} Relationship

the centers of gravity and rigidity has switched as those of the initial conditions. This phenomenon occurred due to the movement of the center of rigidity accompanying member plasticization, and this was the cause of the counter-clockwise rotation. Additionally, the elastic radius in the plastic model increased with regard to that in the elastic model, indicating that the torsion in the frame became more difficult as a result of plasticization. Elastic radius is expressed as the ratio between torsional and horizontal rigidities; thus, this result showed that torsion in the frame does not always become easier even with decreased member rigidity as a result of plasticization.

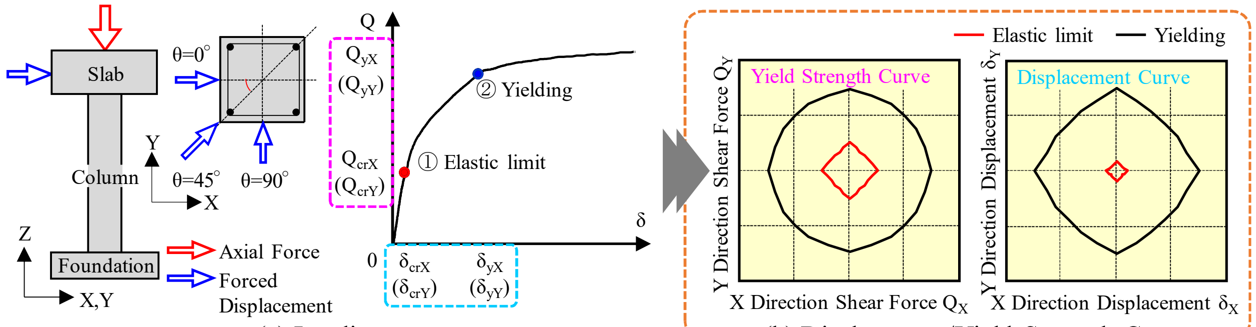
The results discussed indicate that torsion characteristics of the frame exhibit complex changes as a result of member plasticization, and this considerably contributes to the nonlinear torsional response. Thus, determining the event occurrence points for each member and accurately evaluating their accompanying changes in rigidity are important to understand the nonlinear torsional response of the frame.

5. Torsional response evaluation considering the nonlinear region

In the present section, a displacement / yield strength curve is proposed as a tool for evaluating the rigidity of members considering the bi-directional deformation conditions of column members in a frame under torsional response, and attempts are made to generalize this tool. Furthermore, a new nonlinear torsional response evaluation method based on the displacement / yield strength curve is discussed, and the effectiveness of the present evaluation method is confirmed by comparing the evaluation results of the virtual full-size frame and the FE analysis results.

5.1 Construction of the displacement / yield strength curve

Yokogawa et al. constructed a displacement / yield strength curve as a tool for evaluating the member response under bidirectional inputs during earthquakes[3] . This curve allows the evaluation of shear force (Q), and displacement (δ) at the time of member cracks and yielding, in an arbitrary displacement direction on the X-Y plane, being a suitable tool for evaluating the rigidity of column members in a frame under torsional response. The method for creating the displacement / yield strength curve is shown in Fig.12. The loading was varied in increments of 15° in the individual column member model, and a forced displacement up to a member angle R = 1/50 was implemented. The displacement / yield strength curve is an arrangement in the X-Y plane of the Q and δ at (1) elastic limit and (2) yielding. This curve is obtained from numerical analysis, but a generalized process is desirable for a simplified implementation of torsional response evaluation. The present section attempts to generalize the curve by conducting parametric analyses using the individual column member model. The subject of this analysis is the square cross-section column member with a preferential bending failure mode.



(a) Loading (b) Displacement/Yield Strength Curve

Fig. 12 – Creation Method of the Displacement / Yield Strength Curve

A generalized form of the displacement / yield strength curve determined from parametric analysis is shown in Fig.13. Influential factors from the calculated equations of the cracking and ultimate bending strengths of the column members according to RC standards[12] were chosen as parameters. The parameters were changed to investigate the influence of their variation on the displacement / yield strength curve and 34models were constructed. The analysis results and generalized form of each model are shown in Fig.13 (b) and (c). Each value of the analysis results was standardized by the X-directional Q and δ for the pressurized direction of 0°. The shapes of each curve were unified, and it was noted that the influence due to parameter changes is virtually nonexistent. The generalized forms were determined as to envelope the analysis results; the equations in the figure express the first quadrant of each curve. The restoring force characteristics for a displacement of the column member in a given direction can be constructed by using the generalized forms, and an evaluation of the rigidity of members based on this becomes possible.

(a)Analysis Parameters

Compressive Strength (N/mm²)	Yield Strength (N/mm²)	Tension Reinforcement Ratio (%)
18 ~27	295 345	0.41 ~2.53
Width ×Depth (mm×mm)	Height (mm)	Axial Force Ratio
100×100 ~700×700	924 ~3000	0.1 ~0.3

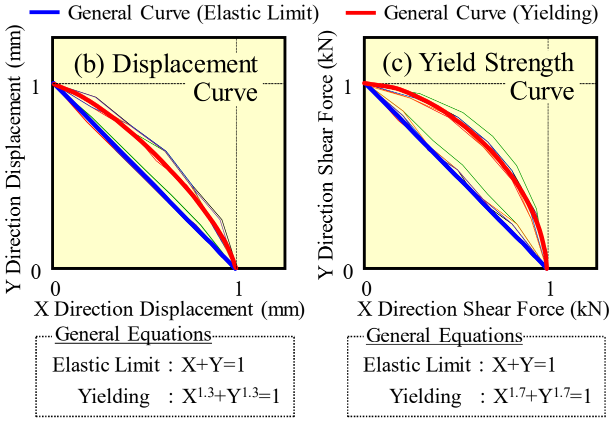


Fig. 13 – Generalization of the Displacement / Yield Strength Curve

5.2 Construction of a nonlinear torsional response evaluation method

An algorithm to evaluate the nonlinear torsional response is shown in Fig.14. The floor slab was assumed to be a rigid body in the present method. The α, β, and γ used for the calculation equations in the flow chart were set as $\alpha = 1+R_{eX}^2 \cdot L_{Y1}/e_Y$, $\beta = 1-R_{eX}^2 \cdot L_{Y2}/e_Y$, and $\gamma = R_{eX}^2 \cdot L_X/e_Y$.

(1) Eccentric distance e_Y, elastic radius r_{eX}, and eccentricity R_{eX} were calculated according to current regulations;

(2) The ultimate target story displacement δ_{aim} for the center of gravity was set;

(3) A displacement $(\delta_X - \delta_Y)$ / yield $(Q_X - Q_Y)$ curve for each column member was created;

(4) The restoring force characteristics $(Q - \delta)$ of each member were constructed from the intersection point of the created displacement / yield strength curve and the member displacement direction. The member displacement direction was expressed using the following equations:

$$\text{Column member of the flexible plane : } y=(\gamma/\alpha)\cdot x \tag{1}$$

$$\text{Column member of the rigid plane : } y=(\gamma/\beta)\cdot x \tag{2}$$

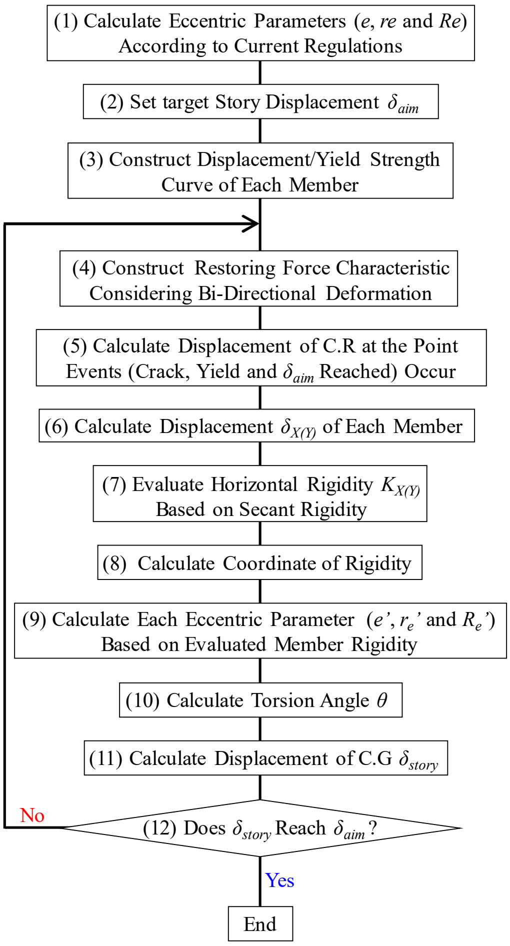
(5) The horizontal displacement $\delta_{C.R}$ of the center of rigidity at the time of proximal event occurrence was calculated among the event occurrence points of all members (either the cracking or yielding point).

$$\text{X-direction (flexible plane) : } \delta_{C.RX}=\delta_{cr}(y)_X/\alpha \tag{3}$$

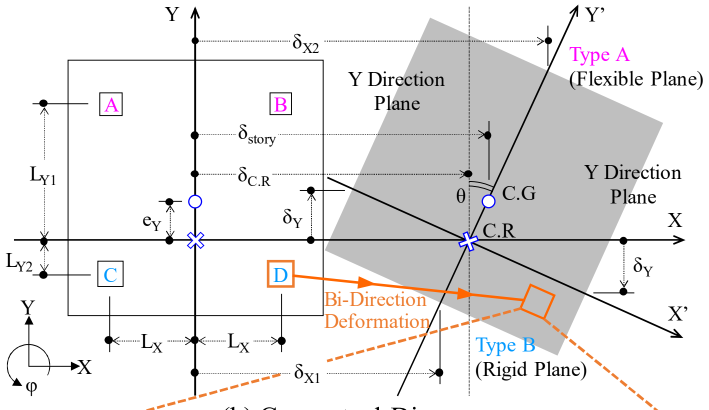
$$\text{X-direction (rigid plane) : } \delta_{C.RX}=\delta_{cr}(y)_X/\beta \tag{4}$$

$$\text{Y-direction (both flexible / rigid planes) : } \delta_{C.RY}=\delta_{cr}(y)_Y/\gamma \tag{5}$$

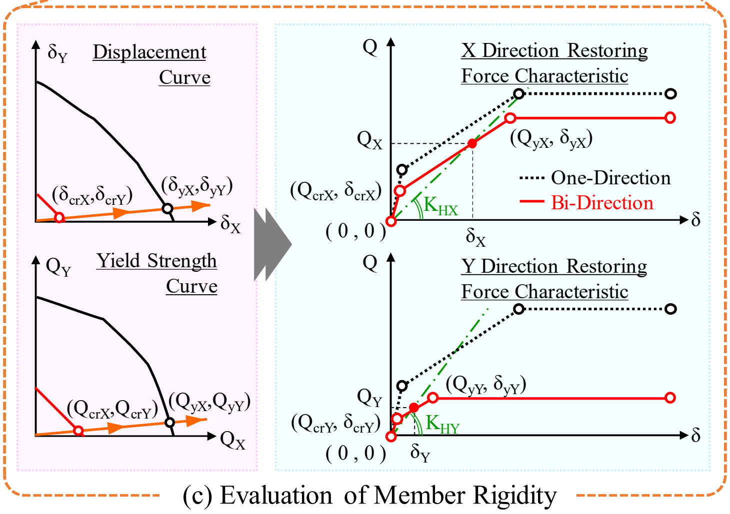
$$\delta_{C.R}=\min(\delta_{C.RX},\ \delta_{C.RY}) \tag{6}$$



(a) Evaluation Flow



(b) Conceptual Diagram



(c) Evaluation of Member Rigidity Considering Bi-Direction Deformation

Fig. 14 – Algorithm for Nonlinear Torsional Response Evaluation

(6) The horizontal displacement δ of each member at the time of horizontal deformation of the center of rigidity determined in Step (5) was calculated.

$$\text{X-direction (flexible plane)} : \delta_X = \delta_{C.R} \cdot \alpha \tag{7}$$

$$\text{X-direction (rigid plane)} : \delta_X = \delta_{C.R} \cdot \beta \tag{8}$$

$$\text{Y-direction (both flexible / rigid planes)} : \delta_Y = \delta_{C.R} \cdot \gamma \tag{9}$$

(7) Using the restoring force characteristics constructed in step (4), the horizontal rigidity K_H of each member was evaluated based on secant rigidity.

$$K_{HX(Y)} = Q_{X(Y)}/\delta_{X(Y)} \tag{10}$$

(8) The coordinates of rigidity C.R. (L_X, L_Y) were calculated from the evaluated horizontal rigidity;

(9) Each eccentricity parameter (e_Y', r_{eX}', R_{eX}') was calculated from the re-evaluated member rigidity and center of rigidity position;

(10) The torsion angle was calculated.

$$\theta = (R_{eX}'^{\,2} \cdot \delta_{C.R})/e_Y' \tag{11}$$

(11) The center of gravity displacement δ_{story} was calculated.

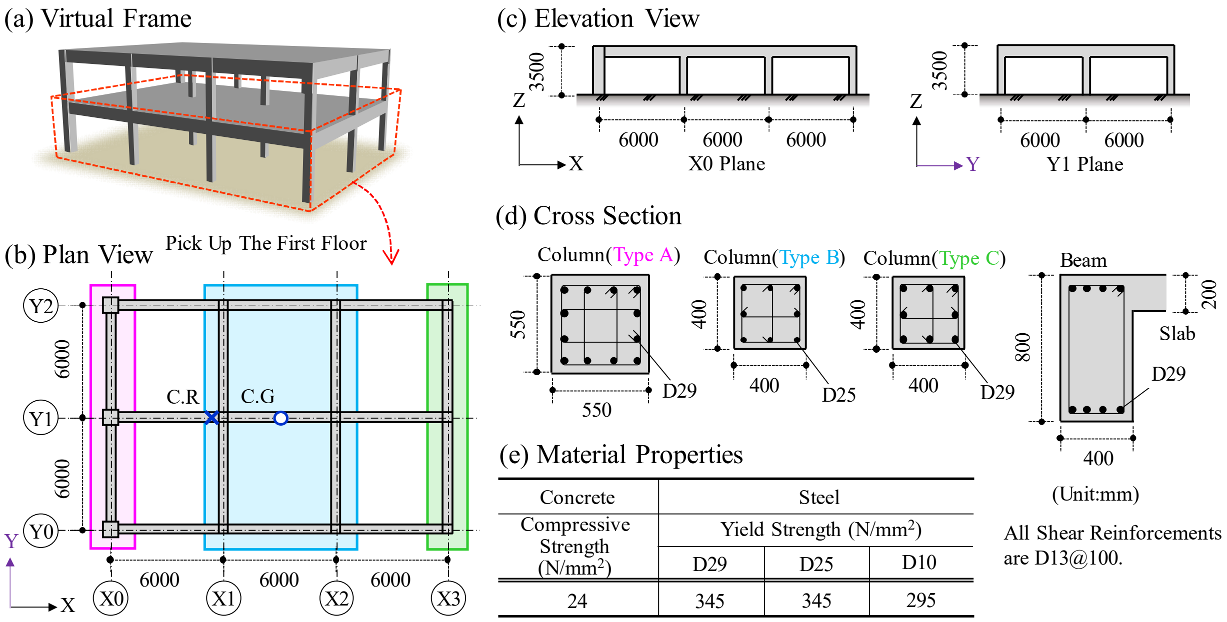
$$\delta_{story}=\delta_{C.R}+e_Y'\cdot\theta \tag{12}$$

(12) If the center of gravity displacement does not reach the objective displacement δ_{aim}, the evaluation returns to Step (4) and repeats Steps (4)－(11). The response evaluation is ended if the objective displacement is reached.

5.3 Confirming the effectiveness of the present method

The present section constructs a virtual full-scale frame and confirms the effectiveness of the present evaluation method by comparing the response evaluation results of this frame with FE analysis results.

An overview of the virtual frame is shown in Fig.15. The evaluated subject was the first floor of a full-size two-story frame. The frame dimensions were 3 x 2 spans (longitudinal x span direction), and the three column member types were types A-C. All three have preferential bending failure modes, and the members were set up to provide large yield strengths, although type A has larger initial rigidity and yield strength than type B, and type C has the same rigidity as type B. The frame was set up so that the X0 plane was type A, the X1 / 2 planes were type B, and the X3 plane was type C. For this reason, the frame exhibited rigidity eccentricity in the X-direction, and the eccentricity at this point was $R_{eY}=0.42$.

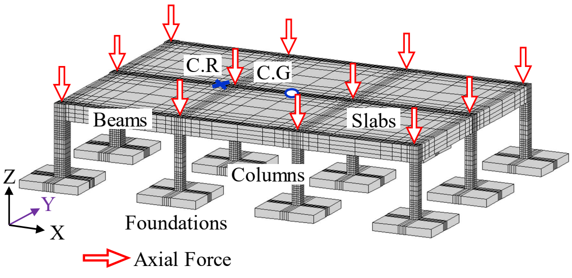


Concrete	Steel		
Compressive Strength (N/mm²)	Yield Strength (N/mm²)		
	D29	D25	D10
24	345	345	295

Fig. 15 – Overview of the Virtual Frame

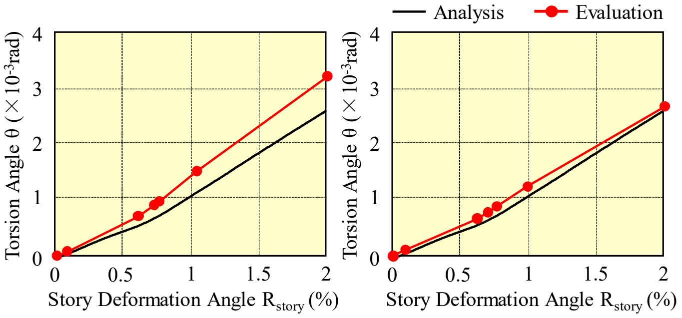
A mesh division is shown in Fig.16. The floor and foundation slabs, beams were set as rigid bodies to match the assumptions made in the torsional response evaluation method constructed in the previous section. Additionally, the weight of the second floor was applied as an added axial force on each of the column member heads. A forced displacement in the Y-direction was applied to the center of gravity up to a story displacement angle $R_{story}=1/50$ for the pressurization.

The evaluation results and the FE analysis of the virtual frame are shown in Fig.17(a). The evaluation results mostly corresponded to the analysis results despite some overestimation. Previous research[2] has shown that the torsional resistance mechanism of the frame is expressed as the sum of torsional resistance due to horizontal column member rigidity and the torsional resistance of the column member itself. Thus, as the torsional rigidity of the column member itself was not considered in the torsional



Slabs, Beams and Foundations are Rigid.

Fig. 16 – Mesh Division



(a) No Torsional Rigidity of Column Itself

(b) Considering Torsional Rigidity of Column Itself

Fig. 17 – Response Evaluation Results

rigidity calculation of the frame during the calculation process, there was an underestimation of the torsional resistance of the frame and subsequent overestimation of the torsion angle.

The evaluation results when the torsional rigidity of the column member itself was considered are shown in Fig.17(b). This torsional rigidity was determined from numerical analysis. The evaluation results corresponded well to the analysis results, and this showed that a higher accuracy evaluation is possible by considering the torsional rigidity of the column member itself. However, it is currently difficult to determine the torsional rigidity of the column member without numerical analyses. The present evaluation method was constructed with the intention of practical use; thus, the response behavior of the frame can be mostly evaluated without considering the torsional rigidity of the column member itself, and its evaluation results tend to be on the safe side. Therefore, the method is considered effective as an evaluation method with design use objectives, considering the torsional rigidity of the column itself is ignored under current regulations.

6. Conclusions

Three-dimensional nonlinear FE analyses were conducted using experiments of Mizoguchi et al., and the influence of member plasticization on torsion characteristics of a frame were determined. Additionally, an algorithm for evaluating the nonlinear torsional response was created, and comparisons between FE analysis results and evaluation results revealed the following findings:

(1) Changes in the torsional characteristics of a frame in the plastic domain can be evaluated by determining the changes in eccentric distance and elastic radius due to the occurrence of each event;

(2) The evaluation results from the proposed algorithm are favorable when compared to the analysis results, and we confirmed that nonlinear torsional response can be evaluated by using the present method. A future challenge is to confirm whether this method is applicable to multi-span frames and members with a shear fracture mode;

(3) A generalized displacement / yield strength curve was confirmed to be effective as a tool for obtaining the restoring force characteristics of a column member in a frame undergoing torsional response. Future work will generalize the displacement / yield strength curve of rectangular cross-section column members with a preferential bending failure mode and members with a preferential shear failure mode.

7. References

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