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Response Values by MDOF using Rocking Free Motion and Coupled System

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Abstract

The response values by Multi-Degree-of-Freedom (MDOF) Model using Rocking Free Motion and Coupled System in non-linear analysis are described. The rocking free motion are usually discussed in dynamic soil-structure interaction.

The equation of damped free motion of rocking for the rigid body is described in the following equation (Abst. 1). In this model, m is the mass, B(=2b) is the width of the rigid body, H (=2h) is the height and g is the gravity acceleration. In equation (Abst.1), I is the moment of inertia at the ground surface, c_I is the damping coefficient, k_I is the rocking stiffness, θ''= ($d^2\theta/dt^2$) is the response acceleration of angle, θ' (= $d\theta/dt$) is the response velocity of angle and q is the angle. M (=$k_I\,\theta$) is the internal moment of the rigid body, the relationship between M and θ is assumed with the positive rocking stiffness and the negative one.

$I\theta''+c_I\,\theta'+k_I\,\theta = 0$ (Abst. 1)

In this non-linear analysis for rocking, the relationship of $M-\theta$ is assumed. The rocking stiffness of the ground k_I (>0) is also assumed when θ is around zero from $-\beta$ to β and M is decreased to be zero after M is almost mgb until θ is almost b/h. The initial point is when θ is some value and the velocity is zero.

The functions of velocity are defined as the differential equations at time t of the above functions of angle.

Because the stiffness (M/θ) is negative to be ($-mgh$), the complementary functions include the hyperbolic functions (sinh, cosh) and constant b/h. The parameters, damping factor h_1, circular frequency ω_1, constant A_1, circular frequency ω_{n1}, constant B_1 when θ is more than β ($\theta>\beta$) and others are defined by the initial values $\theta=\beta_0$ and the velocity is zero. Some of the parameters, h_1 or ω_1, are defined by imaginary number i (= $\sqrt{-1}$) because the stiffness is negative. For example, when the stiffness k_I (= $-mgh$) is negative, the circular frequency ω_1 should be square root of (k_I/I) ($\omega_1=\sqrt{k_I/I}=\sqrt{-mgh/I}=i\sqrt{mgh/I}=i\sqrt{g/h}$) which could be defined by i. Not only ω_1 or ω_3 but also h_1 or h_3 are defined by i because the damping coefficient c_I in equation (Abst. 1) is a constant real number ($h_1\,\omega_1=c/(2m)=h_0\,\omega_0=h_2\,\omega_2$), even if the stiffness was changed.

In the conclusion, when the stiffness is negative and the coupled system is maintained, the response moment and the response angle were calculated by the hyperbolic functions.

Keywords: Response Values, Rocking Motion, MDOF, Hyperbolic Function, Negative Stiffness, Coupled System

1. Introduction

The response values by Multi-Degree-of-Freedom (MDOF) Model using Rocking Free Motion and Coupled System in non-linear analysis are described. The rocking free motion are usually discussed in dynamic soil-structure interaction.

The equation of damped free motion of rocking for the rigid body is described in the following Eq. (1). In this model, m is the mass, B(=2b) is the width of the rigid body, H (=2h) is the height and g is the gravity acceleration. In Eq. (1), I is the moment of inertia at the ground surface, c_I is the damping coefficient, k_I is the rocking stiffness, θ''= ($d^2\theta/d^2t$) is the response acceleration of angle, θ' (= $d\theta/dt$) is the response velocity of angle and q is the angle. M (=$k_I\,\theta$) is the internal moment of the rigid body, the relationship between M and θ is assumed with the positive rocking stiffness and the negative one.

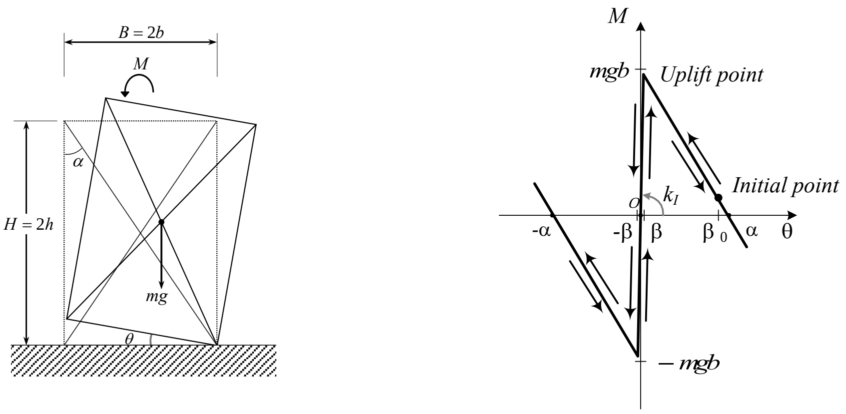
In the conclusion, when the stiffness is negative, the response moment and the response angle were calculated by the hyperbolic functions.

2. MDOF using Rocking Free Motion and Coupled System

2.1 Non-Linear Time History Response Analysis for Rocking

Generally, the equation of damped free motion of rocking in Fig. 1 was described in Eq. (1). In Fig. 1, m was the mass, B(=2b) was the width of the rigid body, H (=2h) was the height and g was the gravity acceleration. In Eq. (1), according to Ref. [1], I was the moment of inertia at the ground surface ($I = m\,h^2$), c_I was the damping coefficient, k_I was the rocking stiffness, $\ddot{\theta}$ was the response acceleration of angle, $\dot{\theta}$ was the response velocity of angle, h_I was the length of the moment arm by external forces and θ was the angle. M (= $k_I\,\theta$) was the internal moment of the rigid body, the relationship between M and θ was shown in Fig. 1 (b) [1]. When k_I was assumed to be too large at $\theta = 0$ and M was assumed to be 0, mgb or $-mgb$, the mathematical solutions for each θ are very difficult.

$$I\ddot{\theta} + c_I\dot{\theta} + k_I\theta = -m\ddot{y}h_I \tag{1}$$



(a) Rigid Body in Motion (b) Relationship of M and θ

The ground was rigid. α was the angle of the width to the height (b/h).

Fig. 1 - Rigid Body in Motion and the Relationship between the internal moment M of the centre of the gravity and the rotation angle of rocking θ when the ground was rigid

If the start of analysis has some horizontal displacement without velocity, and during the motion there were no external forces, the analysis should be simple. Therefore, in Fig. 1(b), in this non-linear time history response analysis for rocking, the relationship of $M - \theta$ was assumed, the rocking stiffness of the ground k_I (>0) was also assumed when θ was around zero from $-\beta$ to β and M was decreased to be zero after M was almost mgb until θ was almost α. The initial point was when θ was some value and the velocity was zero. The relationship of $M - \theta$ and the equation of damped free motion of rocking were as follows;

$$\beta < \theta \quad : \quad M = mgh\,(\alpha - \theta) \tag{2}$$

$$-\beta \le \theta \le \beta \quad : \quad M = k_I\,\theta \tag{3}$$

$$\theta < -\beta \quad : \quad M = -mgh\,(\alpha + \theta) \tag{4}$$

$$I\ddot{\theta} + c_I\dot{\theta} + M = 0 \tag{5}$$

These complementary functions at time t were known. Normally, to solve Eq. (5) for Eq. (3), the solution $\theta = C\,e^{pt}$ should be considered. To solve Eq. (2), after the characteristic equation using p doesn't have the constant α, the solution $\theta - \alpha \equiv \theta_\alpha = C\,e^{pt}$ should be considered. Thus, the response velocity $\dot{\theta}_\alpha$ = d(θ_α)/dt = d(θ-α)/dt = dθ/dt = $\dot{\theta}$ and the response acceleration $\ddot{\theta}_\alpha$ = d^2(θ_α)/dt = d^2(θ-α) /d^2t = d$^2\theta$/d^2t = $\ddot{\theta}$ were not dependent on α. Eq. (2) has the negative stiffness. Therefore, these characteristic equation using p was as follow;

$$p^2 + 2h_1\omega_1 p - \omega_1^2 = 0 \tag{6}$$

Damping factor h_1 and circular frequency ω_1 were real numbers.

The solutions p_1, p_2 of Eq. (6) were

$$p_1 = -h_1\omega_1 + \omega_1\sqrt{h_1^2 + 1} \tag{7}$$

$$p_2 = -h_1\omega_1 - \omega_1\sqrt{h_1^2 + 1} \tag{8}$$

In Eq. (7) and (8), $h_1\omega_1$ was always smaller than $\omega_1\sqrt{h_1^2+1}$, ($h_1\omega_1 < \omega_1\sqrt{h_1^2+1}$). Therefore, always p_2 was negative and p_1 was positive ($p_2 < 0 < p_1$). Because p_1 was positive, the complementary function $\theta = C_1 e^{p_1 t} + C_2 e^{p_2 t}$ were described in Eq. (9), (10) and (11) using hyperbolic functions (sinh, cosh) and constant α or trigonometric functions (sin, cos). In Eq. (9) and Eq. (11), the constant α were necessary to maintain the equilibrium for moment, according to Eq. (2) and Eq. (4), respectively.

$$\beta < \theta \quad : \quad \theta = \alpha + \mathrm{e}^{-h_1\omega_1 t}(A_1\cosh\omega_{n1}t + B_1\sinh\omega_{n1}t) \tag{9}$$

$$\text{where} \quad \omega_{n1} = \omega_1\sqrt{h_1^2 + 1}$$

$$-\beta \le \theta \le \beta \quad : \quad \theta = \mathrm{e}^{-h_0\omega_0 t}(A_0\cos\omega_{n0}\,t + B_0\sin\omega_{n0}\,t) \tag{10}$$

$$\text{where} \quad \omega_{n0} = \omega_0\sqrt{1 - h_0^2}$$

$$\theta < -\beta \quad : \qquad \theta = -\alpha + e^{-h_2\,\omega_2\,t}(A_2 \cosh \omega_{n2} t + B_2 \sinh \omega_{n2} t) \tag{11}$$

$$\text{where} \qquad \omega_{n2} = \omega_2 \sqrt{h_2^2 + 1}$$

The undetermined coefficients, A_1, B_1 and others, were calculated from the initial values and the stiffness respectively. The functions of velocity and acceleration were defined as the differential equations at time t of the above functions of displacement.

When θ was close to β, after the initial point, the interval time in the analysis was 0.0005 (sec) (=2,000Hz) in Eq. (6). After the time when θ was β, the initial values of θ and the velocity $\dot{\theta}$ in Eq. (3) were defined to continue the analysis.

According to Eq. (3), the period T_I during grounding given by the positive stiffness k_I was considered to be 0.1 (sec). ($T_I = 2\pi\sqrt{I/k_I}$ = 0.1 (sec))

By the way, on the above analyses, all parameters were real numbers because the characteristic equation of Eq. (6) was setup for the negative stiffness, while the other characteristic equation well known as Eq. (12) was also setup for the positive stiffness, where the solutions were Eq. (13), according to the values of θ.

$$p^2 + 2h_0\omega_0 p + \omega_0^2 = 0 \tag{12}$$

$$p_{1,2} = -h_0\omega_0 \pm \omega_0\sqrt{1-h_0^2} \tag{13}$$

In negative stiffness, the circular frequency ω_1 (= square root of (k_I/I) ($\omega_1 = \sqrt{k_I/I} = \sqrt{-(mgh/I)} = i\sqrt{mgh/I} = i\sqrt{g/h}$) maybe described using imaginary unit i (= $\sqrt{-1}$). The damping factor h_1 for the negative stiffness maybe also described using i , because the damping coefficient c_I which was a constant real number ($h_1\,\omega_1 = c_I/(2I) = h_0\,\omega_0 = h_2\,\omega_2$) through analyses. The moment of inertia at the ground surface I was still $m\,h^2$ ($I = m\,h^2$). Finally, during the above analyses, all numbers could be real numbers without imaginary numbers by using Eq. (3) and the other characteristic equation well known.

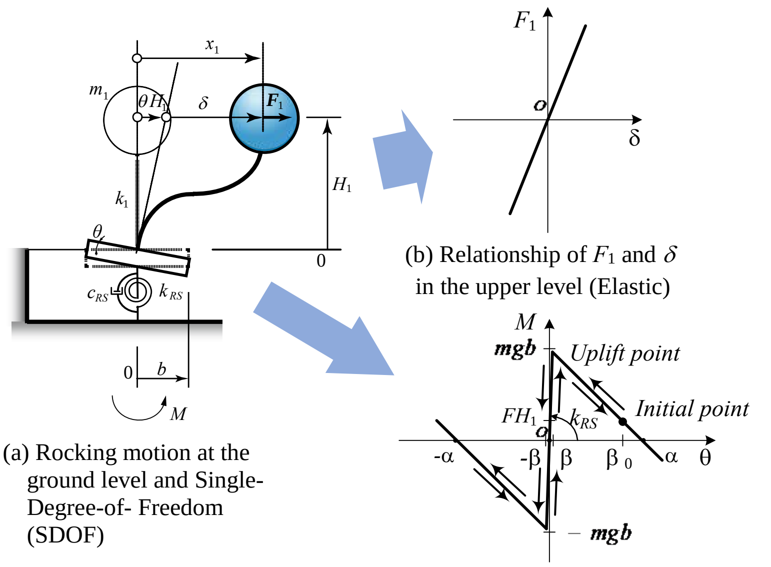
For example, an analysis when damping ratio h_0 in elastic area of rocking motion of MDOF was 0.4%, (h_0 = 0.4%), damping ratio h_1 in inelastic area was 0.1 (h_1 = 0.1) for negative stiffness or T_I = 0.1 (sec) in Fig. 6, the other values were follows ; ω_0 = 59.5 (rad./sec), ω_1 = 2.37 and $h_0\,\omega_0 = h_1\,\omega_1 = h_2\,\omega_2$ = 0.238 (1/sec). In this section, the values of h_1 and ω_1 were described without their units because they maybe had imaginary numbers.

2.2 MDOF using Rocking Free Motion and Coupled System

In MDOF analyses using Rocking Motion, the lower level has the non-linear relationship between moment and angle of Rocking and the upper level has the linear relationship between horizontal force and horizontal displacement in Single-Degree-of-Freedom (SDOF). The analyses maintain coupled system.

In analyses, x_1 is horizontal displacement of SDOF, δ is relative displacement at a storey in SDOF, θ is rocking angle, m_1 is mass of SDOF, I is moment of inertia (= $m_1\,H_1^2$), k_1 is stiffness of SDOF, k_{RS} is stiffness of rocking, c_1 is damping coefficient of SDOF(= $2h_1\sqrt{m_1 k_1}$), c_{RS} is damping coefficient of rocking (= $2h_{RS}\sqrt{I\,k_{RS}}$), $\ddot{y}$ is horizontal acceleration record on the ground surface, H_1 is height of SDOF, h_1, h_{RS} are damping ratios of SDOF and rocking respectively.

The relationship of horizontal force of SDOF F_1 and δ is elastic. In Eq. (14), the right side was assumed to be {0} for rocking free motion, and Eq. (14) was described using matrix of Eq. (17) which had external damping forces in free motion. As for Introduction, analyses which maintained coupled system meant calculating the solutions x_1, θ when any non-diagonal elements of stiffness matrix $[K]$ was not zero '0'. In these analyses, the external forces were 0, therefore, the solutions were calculated by the eigenvalue analysis.



(a) Rocking motion at the ground level and Single-Degree-of- Freedom (SDOF)

(b) Relationship of F_1 and δ in the upper level (Elastic)

(c) Relationship of moment M and angle θ of Rocking motion at the ground level (Inelastic)

Notes) $\alpha = b / H_1$

β is the angle of Uplift point when M less than mgb is divided by k_{RS}.

Fig. 2 - Rocking motion and Single-Degree-of-Freedom (SDOF)

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1(x_1 - \theta H_1) = -m_1\ddot{y} \\ I\ddot{\theta} + c_{RS}\dot{\theta} + k_{RS}\theta = -m_1\ddot{y}H_1 \end{cases} \tag{14}$$

$$x_1 = \theta H_1 + \delta \tag{15}$$

$$\beta < \theta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1H_1 \\ 0 & -m_1gH_1 \end{bmatrix}\begin{Bmatrix} x_1 - b \\ \theta - \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{16}$$

$$-\beta \le \theta \le \beta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1H_1 \\ 0 & k_{RS} \end{bmatrix}\begin{Bmatrix} x_1 \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{17}$$

$$\theta < -\beta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1H_1 \\ 0 & -m_1gH_1 \end{bmatrix}\begin{Bmatrix} x_1 + b \\ \theta + \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{18}$$

During inelastic phase, corresponding to Eq. (2) and (4), the analyses were executed using Eq. (16) and Eq. (18). Moreover, when the upper and the right element "- k_1 H_1" of stiffness matrix $[K]$ in Eq. (16) was also considered into the lower and the right element to be Eq. (19). These "- k_1 H_1" in Eq. (20) or (22) are considered to be Coupling Terms.

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1(x_1 - \theta H_1) = -m_1\ddot{y} \\ I\ddot{\theta} + c_{RS}\dot{\theta} - k_1 x_1 H_1 + k_{RS}\theta = -m_1\ddot{y}H_1 \end{cases} \quad (19)$$

$$\beta < \theta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 H_1 \\ -k_1 H_1 & -m_1 g H_1 \end{bmatrix}\begin{Bmatrix} x_1 - b \\ \theta - \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (20)$$

$$-\beta \le \theta \le \beta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 H_1 \\ -k_1 H_1 & k_{RS} \end{bmatrix}\begin{Bmatrix} x_1 \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (21)$$

$$\theta < -\beta \quad : \quad \begin{bmatrix} m_1 & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_{RS} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 H_1 \\ -k_1 H_1 & -m_1 g H_1 \end{bmatrix}\begin{Bmatrix} x_1 + b \\ \theta + \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (22)$$

3. Analysis and Results

In the analysis, while the damping ratio was a parameter, the response moment M and the response angle θ of the centre of mass were calculated and compared with the experimental values. In recent years, the experimental project of free-standing concrete column specimens by placing some situations of ground was produced, when the damped free motion given an initial value of the horizontal displacement was performed by the several corporations [3]. Specimens, height 3.7m, width 1.1m, the thickness 1.5m and the weight 104.9 kN in the free end. The height of the centre of mass was 1.743m. The specimens were placed on the ground, were inclined to the top displacement horizontally 0.375m and so on. During these experiments.

In Fig. 3, the Idealized relationship between M and θ for the rocking motion at the lower level shows the typical points which were the $M = mgb$ = 57.7kN･m was in y-axis and the angle of the width to the height (b/h) (=550/1,743 = 0.316 rad.) in x-axis. In experimental relationship, the Uplift point (M,θ) = (57.6kN･m, 0.0005rad.) and the Initial point (37kN･m, 0.113rad.) were proposed. The Initial point was located under the line of the Idealized relationship. These points were exchanged to the relationship between the shear coefficient (F/mg), which was shear force (F) divided by the weight (mg), and the horizontal displacement (x) of the centre of mass. For the idealized and experimental F/mg and x, the idealized point was (F/mg, x) = (0.315 - , 0.087cm) and the Initial point (0.202 - , 19.7cm).

As for the upper level SDOF, the period was 0.2 (sec) and damping coefficient h_{SDOF} = 5%.

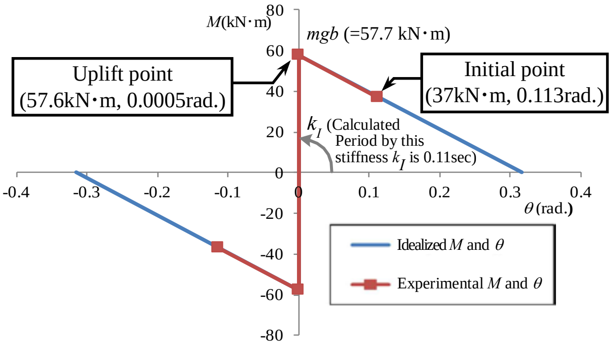
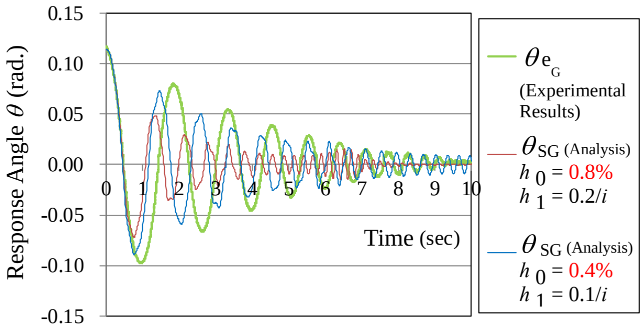


Fig. 3 - Relationship M - θ of Rocking Model

Fig. 4 shows the comparison of experimental results (θ_{eG}) and analysis results (θ_{SG}) when the number of coupling term "- $k_1 H_1$" of Matrix [K] in Eq. (14), (16), (17) and (18) was one. In analysis results, damping ratio h_0 in elastic area of rocking motion of MDOF is 0.8%,(h_0 = 0.8%), or damping ratio h_1 in inelastic area is 0.2/i (h_1 = 0.2/i). Another analysis were made when h_0 = 0.4%, or h_1 = 0.1/i .

In Fig. 4, Fig. 5, Fig.6 and this section 3., the imaginary number i was described in order to show that the analyses had negative stiffness.

In Fig. 4, the amplitude of response angle (θ_{SG}), when time was near 10 (sec) and h_0 = 0.4%, were close to the experimental results (θe_G) than θ_{SG} , when h_0 = 0.8%. And the response shear coefficients Co (= F_1 / $m_1 g$) of the upper level SDOF were calculated. The maximum Co were about 1.4 or 2.7. These values of Co were larger than the required value of Co more than 0.2 in the regulations.



Notes)

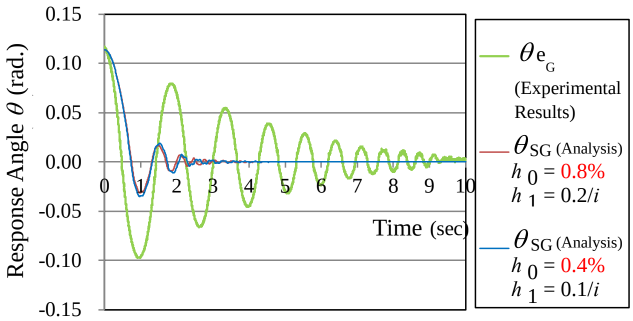
(1) $\theta_{SG} = x_1 / H_1$

(2) An analysis when Damping Ratio h_0 wan elastic area of Rocking Motion of MDOF was 0.8%,(h_0 = 0.8%), or Damping Ratio h_1 in inelastic area was 0.2/i (h_1 = 0.2/i). Another analysis when h_0 = 0.4%, or h_1 = 0.1/i .

(3) Both analyses when Damping Ratio of SDOF h_{SDOF} = 5%.

(4) Until 10(sec), the amplification of the analysis results (θ_{SG}) in h_0 = 0.4% were matching to the experimental results (θe_G).

Fig. 4 - Comparison of Experimental Results (θe_G) and Analysis Results (θ_{SG}) when the coupling term of Matrix [K] was one.



Notes)

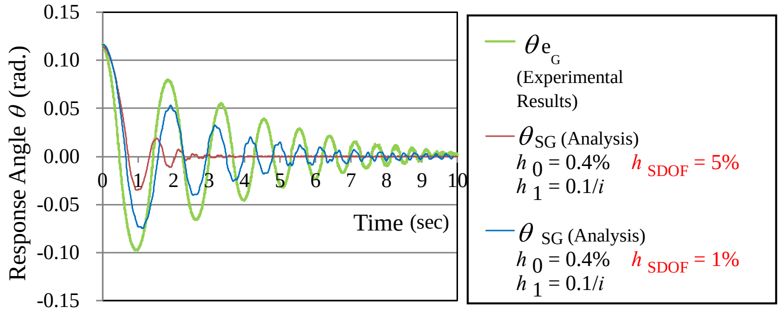
(1) $\theta_{SG} = x_1 / H_1$

(2) An analysis when Damping Ratio h_0 in elastic area of Rocking Motion of MDOF was 0.8%,(h_0 = 0.8%), or Damping Ratio h_1 in inelastic area was 0.2/i (h_1 = 0.2/i). Another analysis when h_0 = 0.4%, or h_1 = 0.1/i .

(3) Both analyses when Damping Ratio of SDOF h_{SDOF} = 5%.

(4) The amplification of the analysis results (θ_{SG}) in h_0 = 0.8% or 0.4% were not matching to the experimental results (θe_G).

Fig. 5 - Comparison of Experimental Results (θe_G) and Analysis Results (θ_{SG}) when the coupling terms of Matrix [K] were two.



Notes)

(1) $\theta_{SG} = x_1 / H_1$

(2) An analysis when Damping Ratio h_0 in elastic area of Rocking Motion of MDOF was 0.4%,(h_0 = 0.4%), or Damping Ratio h_1 in inelastic area was 0.1/i (h_1 = 0.1/i).

(3) Both analyses when Damping Ratio of SDOF h_{SDOF} = 5% or 1%.

(4) The amplification of the analysis results (θ_{SG}) in h_{SDOF} = 1% were matching to the experimental results (θe_G).

Fig. 6 - Comparison of Experimental Results (θe_G) and Analysis Results (θ_{SG}) when the coupling terms of Matrix [K] were two.

Fig. 5 shows the results when the number of coupling term "- k_1 H_1" of Matrix [K] in Eq. (19) ~ (22) was two. In Fig. 5, the analysis results were smaller than those when he number of coupling term was one.

Fig. 6 shows the results when the number of coupling term was two, h_0 = 0.4%, or h_1 = 0.1/i and damping coefficient h_{SDOF} of SDOF were 5% or 1%. In Fig. 6, the analysis results of h_{SDOF} =1% were close to the experimental results. The response shear coefficients Co of the upper level SDOF were also calculated. The maximum Co were about 0.90 or 0.64. These values of Co for h_{SDOF} =1% were closer to the required value of Co more than 0.2 in the regulations than those for h_{SDOF} = 5%.

4. Conclusions

In this paper, the time history analyses of rocking motion of MDOF Model using Coupled System in non-linear analysis were described when the stiffness during lift up is negative, the period during grounding was 0.1 (sec) and the SDOF Model for the upper structure was elastic. The main points of these results were as follows;

(1) The Rocking Model analyses when the upper structure was SDOF Model and the lower model was rocking motion, were executed in external damping free motions and non-linear analyses by eigenvalue analyses and others.

(2) According to the analyses results, the response shear coefficient "Co" for the upper structure SDOF were about 1.4 (-) or 2.7 (-) and about 0.64 (-). The Co was decreased because the number of coupling terms was changed from one to two.

(3) These dynamic analyses for Soil-Structure-Interaction are still needed.

5. References

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