# INELASTIC SEISMIC RESPONSE OF STRUCTURAL SYSTEMS HAVING DIFFERENT HYSTERESIS BEHAVIORS

M.A. Hussain[(1)], S.C. Dutta[(2)]

[(1)] *Doctoral Student, Indian Institute of Technology (ISM) Dhanbad, Dhanbad, India, ahsaan.18dr0079@cve.iitism.ac.in*
[(2)] *Professor, Indian Institute of Technology (ISM) Dhanbad, Dhanbad, India, sekhar@iitism.ac.in*

## *Abstract*

A number of hysteresis models are available to analyze the inelastic behavior of structures under repetitive loading. All these models show different degree of preciseness and closeness to various available experimental results. However, these experimental results are limited to a predefined displacement path, a low number of cycles of loading, and loading rate. In this context, the present paper aims to study the inelastic seismic response of low rise buildings using different hysteresis models under a set of real earthquake loading. Four hysteresis models were used, ranging from simple elastoplastic to complex strength and stiffness degrading ones, reflecting four different types of hysteresis behavior. Seismic response of structures having 1, 3, 5, and 8 stories covering small, medium, and long-period structures is investigated. The primary inelastic responses of the structure in terms of ductility demand/story drift demand were presented in this study. Seismic responses were analyzed step-by-step integration using Newmark's β-γ method. Responses due to twenty far fault ground motions were expressed in terms of the mean and standard deviation to understand and minimize any case-specific effect of individual ground motion. Results show that the strength degrading hysteresis behavior predicts a relatively higher seismic response. Such behavior of strength degradation model may be due to degradation in strength, reducing the size of hysteresis loops, and also being devoid of lengthening the natural periods of structures due to the absence of stiffness degradation. Hence, simple strength degrading hysteresis behavior is found to exhibits conservative results and may be used to predict the maximum lateral inelastic demand of the structural system.

*Keywords: hysteresis models; inelastic seismic response; low rise structures.*

## 1. Introduction

Present seismic design philosophy suggests the structures should be designed such that it behaves elastically under the combination of dead load, live load, and for moderate earthquake loading. However, structures are strategically allowed to undergo the inelastic deformation while subjected to severe earthquake shaking. This is so because a severe earthquake may or may not come during the life period of building and, hence, the money spent on remaining elastic during such shaking is not justified. Controlled damage is allowed in such cases to avoid casualties, collapse, and striking a balance between safety and economy. Such inelastic deformations are allowed corresponding to various limit states like immediate occupancy, life safety, collapse prevention, etc. Hence, structures are allowed to undergo inelastic excursion according to different building codes such as Indian seismic code [1] and ASCE/SEI 7-05[2]. Further, most of the studies [e.g., 3–6] so far made on inelastic behavior of structure considered a simple bi-linear hysteresis model. However, in reality, the concrete members exhibit considerable stiffness and strength degradation during each cycle of loading, as observed in different experimental literature [e.g., 7–11]. Even the steel members [12] may also exhibit such deterioration because of local buckling under reversal cyclic loading. Hence, based on various experimental results, many force-displacement relationships or hysteresis models [e.g., 12–16] have been developed. Such hysteresis models developed by different researchers have been validated with the available experimental results and show the various degrees of closeness to it. However, all such experimental studies are based on the limited number of cycles of load and well-defined displacement history. On the other hand, earthquake motion is random, in which there are large numbers of cycles of loading acts on the structure as compared to the experimental ones. Hence, it is important to study the response due to the hysteresis models with special characteristic features under the real ground motion to realize the effect of these features separately and also in a combined way.

Several studies are available on the issue of the effect of different hysteresis behaviors on structures. For example, Loh and Oh [17] studied the effect of different hysteresis behaviors on single degree of freedom (SDOF) system, though the given literature [17] considered different force-displacement models, however, it had not considered the actual behavior of reinforced concrete member, i.e., incorporating the simultaneous occurrence of degradation of strength and stiffness. This is true for some more studies too [e.g., 18–21]. Further, a study was done by Huang and Foutch [23], which considered the stiffness and strength degradation model. However, in the given literature [23] authors used abnormally high degradation factor of 10%, 25%, and 40% resulting in unrealistic responses. Moreover, Dutta *et al.* [24] had done an exhaustive study based on different experimental data available and found that the average degradation of strength may vary in the range of 2-9%. Hence it has been advised in that literature to consider 5% of strength degradation due to each yielding. The same value of strength degradation factor ($\delta$) equal to 5% is adopted in the present study. In this context, the present study attempts to see the sensitivity of the SDOF system to various hysteresis models with the consideration of strength and stiffness degradation features separately and simultaneously by taking into account of the rational value of strength degradation. The natural period of the SDOF system is varied over a feasible range of 0.25-2.5 sec. Four hysteresis models are considered in this paper, which are namely, elastoplastic, stiffness degrading, strength degrading, and strength and stiffness degrading models. Such models help in developing insight into the influence of the phenomena of stiffness and strength degradation separately as well as in combined form. The study was extended further to observe the behavior of 3 stories, 5 stories, and 8 stories system using simple stick models with fundamental periods of 0.5sec, 1 sec, and 2 sec respectively. This part of the study helps to observe the validity of the response features for multistory systems and to develop further insight into the inelastic behavior of such systems due to different hysteresis models. The simplified systems idealized as stick models are shown below in Fig.1.



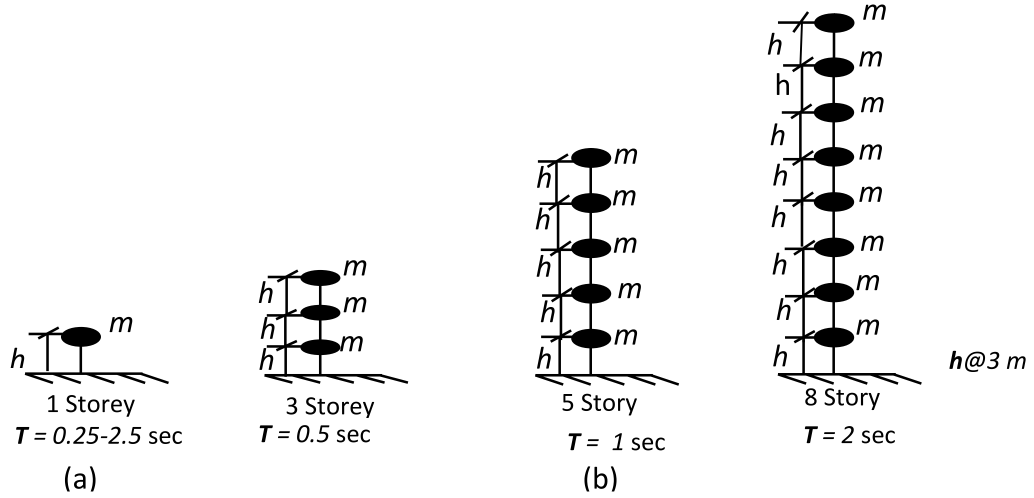



**Fig.1.** Idealized systems considered: (a) Single-story (SDOF) system; (b) Multistory (MDOF) system.

## 2. Methodology

If a multi-degree of freedom (MDOF) system is excited by seismic ground shaking, then the corresponding equation of motion can be written as follows in Eq. (1)

$$[\boldsymbol{m}]\{\ddot{x}\} + [\boldsymbol{c}]\{\dot{x}\} + \{f_s\} = -[\boldsymbol{m}]\{1\}\{\ddot{u}_g\} \tag{1}$$

where $[\boldsymbol{m}]$ is the lumped mass matrix of the system, $[\boldsymbol{c}]$ is damping coefficient matrix of the system, $\{f_s\}$ is restoring spring force vector, $\{x\}$ the relative displacement of lumped mass with respect to the ground, $\{1\}$ is the vector which has all the elements as 1, and $\ddot{u}_g$ is the ground accelerations along one of the principal axis of the structure. The dynamic response was computed using Newmark's β-γ method with step-by-step integration, and the modified Newton-Raphson technique was also incorporated to minimize the error accumulated at each step through iterations. The time step of integration was taken as smaller than *T/400,* as studied previously in the literature [13] for quick convergence, where *T* is the fundamental lateral period of the system. The damping matrix is constructed from the consideration of 5% of critical damping for each degree of freedom.

## 3. Hysteresis models

To study the influence of different hysteresis behaviors in computing the inelastic seismic response, four typical hysteresis models were considered. These hysteresis models are namely (a) Elastoplastic model (EP); (b) Strength degradation model (SD); (c) Stiffness degradation model (KD) only; (d) Strength and stiffness degradation model (SD+KD). The schematics diagrams of the models are shown in Fig. 2. Fig. 2 (a) shows an elastoplastic model, and it is intended to follow the perfectly elastoplastic behavior. The strength degradation model is an extension of an elastoplastic model. Fig. 2 (b) shows the schematic form of the strength degradation model. In this model, the strength is deteriorated by a fixed degradation factor $\delta$ of initial yield strength $F_{yo}$ during each inelastic excursion. So that yield strength at any instant comes to be $F_y = F_{yo}(1- n\delta)$, where $n$ is the number of yield excursion already taken place. As discussed earlier $\delta$ is chosen to be 0.05.



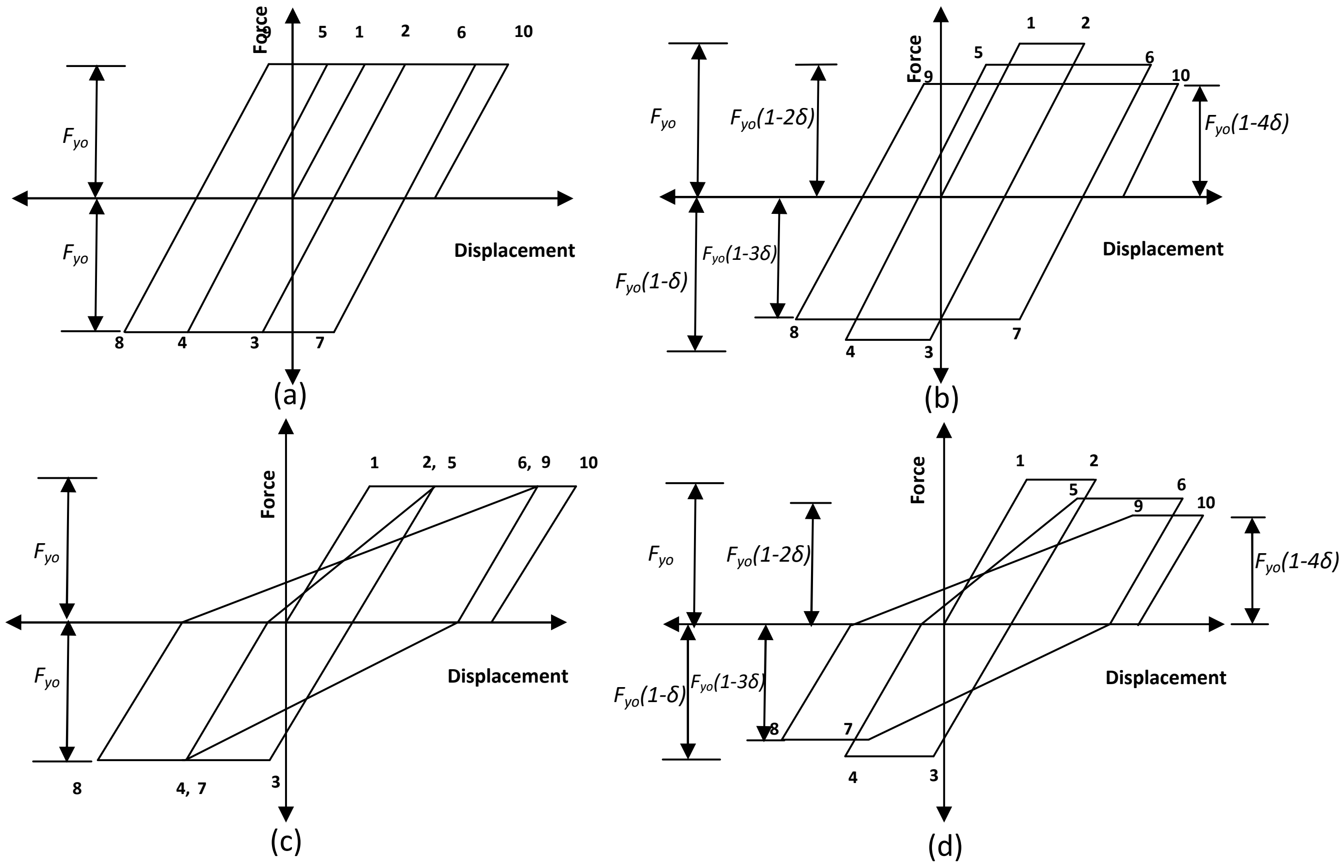



**Fig. 2.** Different Hysteresis Models: (a) Elastoplastic model; (b) Strength degradation model; (c) Stiffness degradation model; (d) Strength and Stiffness degradation model.

The stiffness degradation model is based on Takeda's model [14]. In this model, stiffness is calculated, such that elastic stiffness is equal to initial stiffness during all the unloading branches of the hysteresis cycle. On the other hand, during the loading, the elastic stiffness is dependent upon the accumulated inelastic strain in the previous cycle, as shown in Fig. 2 (c). Whereas the strength and stiffness degradation model is a combination of stiffness degrading and strength degrading models and adopted from a well-established literature Dutta and Das [13]. Fig. 2 (d) shows the strength and stiffness degradation model in a schematic form, with strength deteriorated by fixed degradation factor ($\delta$) of yield strength $F_{yo}$ at each inelastic excursion in addition to the stiffness degradation.

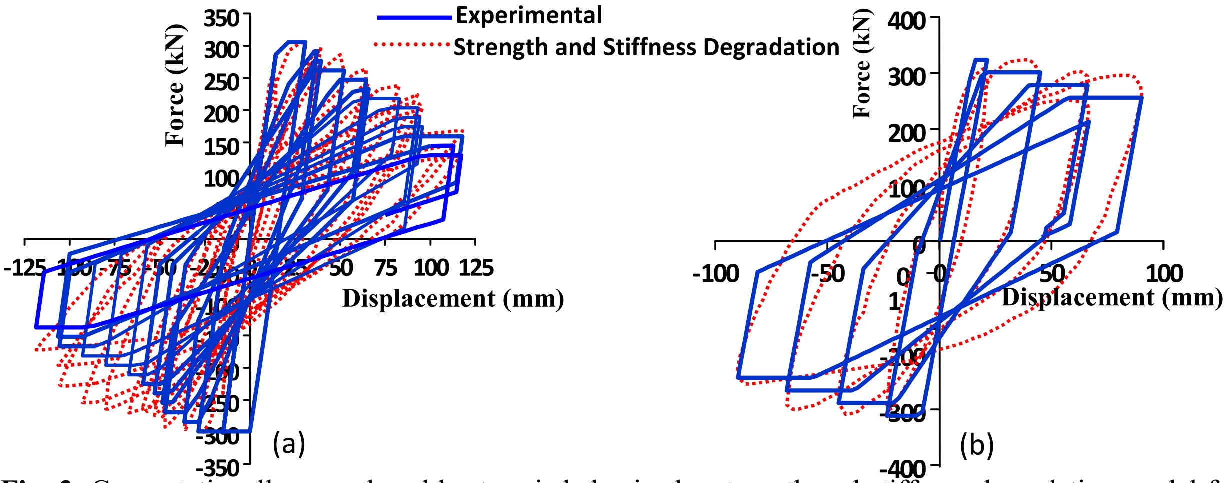



**Fig. 3.** Computationally reproduced hysteresis behavior by strength and stiffness degradation model from the literature [13]: (a) specimen C1-3, presented by Mo and Wang [10], (b) specimen U6, presented by Saatcioglu and Ozcebe [11].



The performances of these hysteresis models were compared with experimentally obtained cyclic load-displacement history [13]. The accuracy of the results is judged in the paper [13] to see how closely the models are tracing the loops and how closely they were representing the energy dissipated through the experimentally obtained load-displacement curve. The performances of all hysteresis models are shown through the sample results shown in Table 1 from the relevant literature [8-11]. Table 1 shows the accuracy level of different hysteresis models as obtained from the literature [13]. The strength and stiffness degrading model shows sufficient accuracy level in predicting load-displacement behavior as compared to the experimental results. The performance of this model in the sample form presented in Fig. 3 as studied in the literature [13].

**Table 1.** Accuracy of different hysteresis models with respect to observed experimental results as studied by Dutta and Das [13].

| Samples Source (Unit of Energy Dissipated) | Hysteresis Model | Energy Dissipated | % Error |
|---|---|---|---|
| Mo and Wang (2000)[10] Specimen – C1-3 (KN-mm) | Observed | 241789.281 | ------ |
| | EP | 869681.425 | 259.68 |
| | KD | 365579.596 | 51.20 |
| | SD | 580207.942 | 139.96 |
| | SD+KD | 251911.083 | 4.19 |
| Mo and Wang (2000)[10] Specimen – C2-3 (KN-mm) | Observed | 248824.908 | ------ |
| | EP | 705580.555 | 183.56 |
| | KD | 360869.337 | 45.03 |
| | SD | 426812.859 | 71.53 |
| | SD+KD | 219241.652 | 11.89 |
| Priestley and Benzoni (1996)[9] Column 1 (Kips-in) | Observed | 1264.858 | ------ |
| | EP | 2629.655 | 107.90 |
| | KD | 1378.281 | 8.97 |
| | SD | 1729.534 | 36.75 |
| | SD+KD | 1084.109 | 14.29 |
| Saatcioglu and Ozcebe (1989) [11] Specimen – U6 (KN-mm) | Observed | 131095.00 | ------ |
| | EP | 230183.90 | 75.59 |
| | KD | 177223.10 | 35.19 |
| | SD | 202184.30 | 54.23 |
| | SD+KD | 112315.90 | 14.32 |
| Brown and Jirsa (1971) [8] Specimen–88-35-RV10-60 (Kips-in) | Observed | 632.855 | ------ |
| | EP | 1203.123 | 90.11 |
| | KD | 1002.081 | 58.34 |
| | SD | 945.004 | 49.32 |
| | SD+KD | 581.346 | 8.14 |

Note: EP = Elastoplastic; KD = Stiffness Degradation; SD = Strength Degradation; SD+KD = Strength and Stiffness Degradation.

## 3. Ground motion

In this study, a total of twenty far faults ground motions were used to study the inelastic seismic response of structures based on the different hysteretic rules. Soil conditions in all the cases are firm and far away from causative faults. All such earthquake data are used from recent literature [25]. Further, the brief details of all twenty earthquakes are listed in Table 2, and corresponding response spectrums for all earthquakes are plotted to the same scale of PGA as shown in Fig. 4. All the response spectra broadly exhibit the expected nature.



**Table 2.** Details of twenty earthquake ground motion used in this study.

| S.No. | Year | Earthquake | Station | Duration (in secs) | Magnitude ($M_w$) | Epicenteral Distance (km) |
|---|---|---|---|---|---|---|
| 1. | 1999 | Chi | CHY086 | 90.00 | 7.62 | 60.15 |
| 2. | 1999 | Chi | CHY036 | 90.00 | 7.62 | 44.02 |
| 3. | 1999 | Chi | CHY034 | 197.0 | 7.62 | 46.13 |
| 4. | 1952 | Kern Country | Taft | 54.16 | 7.36 | 43.49 |
| 5. | 1987 | Whittier Narrows | Lakewood, Del Amo Blvd | 29.76 | 7.28 | 22.68 |
| 6. | 1992 | Landers | Baker | 50.00 | 7.28 | 123.89 |
| 7. | 1992 | Landers | Cool water | 27.96 | 7.28 | 82.12 |
| 8. | 1989 | Loma Prieta | Presidio | 39.95 | 6.93 | 97.79 |
| 9. | 1989 | Loma Prieta | Hollister City Hall | 39.09 | 6.93 | 47.90 |
| 10. | 1994 | Northridge | Laguna | 32.00 | 6.69 | 42.28 |
| 11. | 1994 | Northridge | Terminal Island Fire Stn. 111 | 34.99 | 6.69 | 58.51 |
| 12. | 1994 | Northridge | Castaic Old Ridge | 40.00 | 6.69 | 40.68 |
| 13. | 1971 | San Fernando | Castaic Old Ridge Route | 30.00 | 6.61 | 25.36 |
| 14. | 1979 | Imperial-Valley | Calexico | 193.9 | 6.53 | 17.65 |
| 15. | 1979 | Imperial Valley | Cerro Prieto | 63.74 | 6.53 | 24.82 |
| 16. | 1979 | Imperial Valley | Delta | 99.99 | 6.53 | 33.73 |
| 17. | 1992 | Big Bear | Desert Hot Spr. (New Fire Stn.) | 60.00 | 6.46 | 40.46 |
| 18. | 1966 | Parkfield | Parkfield | 16.00 | 6.19 | 75.99 |
| 19. | 1986 | N. Palm Springs | Temecula | 40.00 | 6.06 | 74.95 |
| 20. | 1987 | Whittier- Narrows | Tarzana | 39.99 | 5.99 | 43.52 |

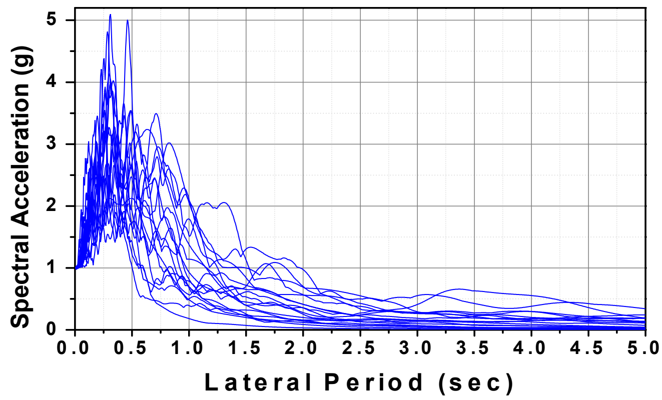



**Fig. 4.** Response Spectrum for twenty earthquakes as considered in this study.

## 4. Results and discussion

This section has been presented in the form of two subsections consisting of discussion on response exhibited by single-story (i.e., SDOF) and multistory (i.e., MDOF) systems, respectively.

### 4.1 Single story systems

For studying the response of single-story systems, ductility demand was calculated over a range of feasible lateral periods with different response reduction factors. Fig. 5 shows the variation of the ductility demand with the lateral period of a system having different response reduction factors for four types of hysteresis models. The plots are made of mean response obtained from all ground motions in the form of bar charts. The plots also exhibit a mean ± standard deviation for each bar to have an idea about the level of variation that occurs due to ground motion. It can be seen from the given figure that as the lateral period, *T* is increasing; different hysteresis models were exhibiting similar results. Further, consideration of the strength degradation model gives more conservative results as compared to ones obtained from other models and can be used for all practical purposes.

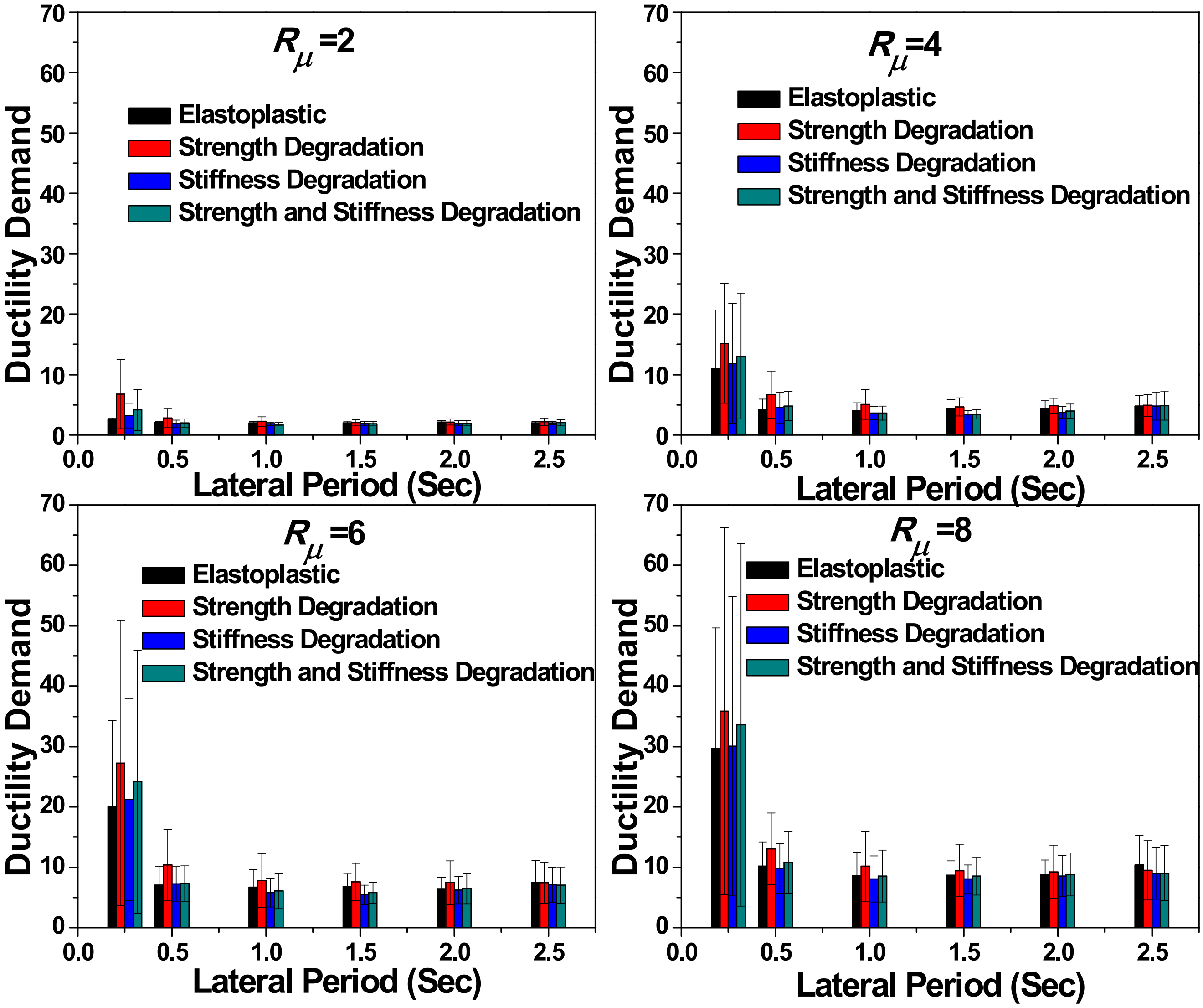



**Fig. 5.** Comparison of different hysteresis models using ductility demand for different degrees of response reduction factor.

It can also be inferred that as the degree of response reduction factor increases, responses due to various hysteresis models exhibit minimal variation. Moreover, the response of strength and stiffness degradation and stiffness degradation model seem to exhibit less ductility demand as compared to the model having strength degradation and elastoplastic model. Such behavior may be due to the degradation of stiffness resulting in period lengthening. However, from the above figure, it may be well understood that displacement, as well as ductility demand, is maximum if the strength degradation hysteresis model is used. Hence, performance-based design made based on demand obtained using this simple model will possibly lead to the safer side results, at least for crucial concrete structures.



Fig. 6 consists of four sets of graphs for different response reduction factors. Further, each set has four graphical windows, each of which corresponds to different hysteresis rules. Each graphical window shows the variation of normalized force with respect to normalized displacement corresponding to the east-west component of Northridge Old Ridge, 1994 earthquake. Normalized force is considered as the ratio of the lateral spring force at any instant to the yield lateral spring force considered for the given response reduction factor. Similarly, the normalized displacement of the mass is defined as the ratio of the instantaneous lateral displacement of the mass to the yield lateral displacement. This normalized displacement is actually the same as ductility demand. All such graphs are corresponding to the lateral period, $T$ = 0.5 sec of the SDOF system.

Elastoplastic | Strength Degradation | Stiffness Degradation | Strength and Stiffness Degradation

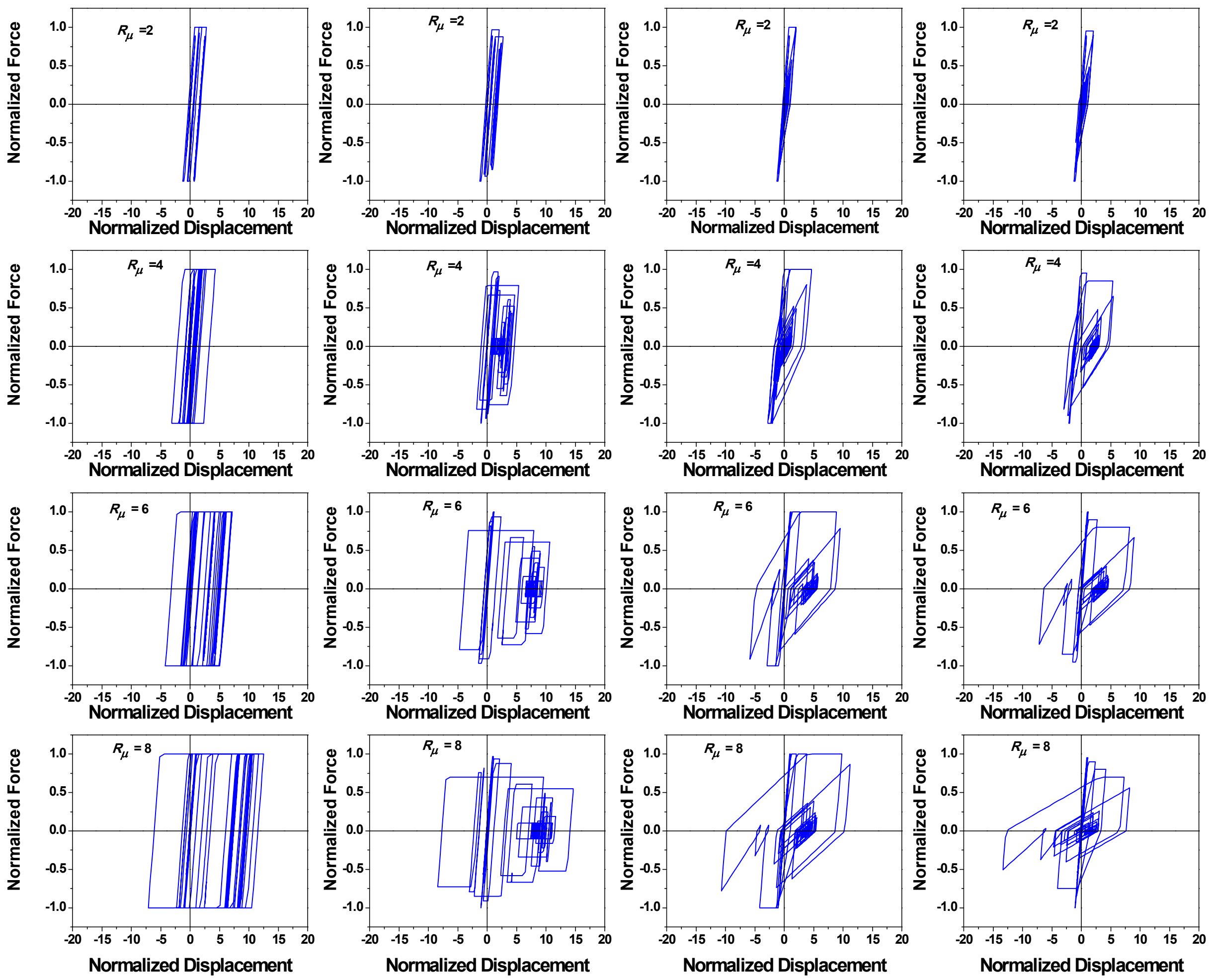



**Fig. 6.** Hysteresis behaviors for the SDOF system corresponding to four hysteresis models for the EW component of Northridge Old Ridge, 1994 earthquake and natural period, $T$ =0.5 sec.

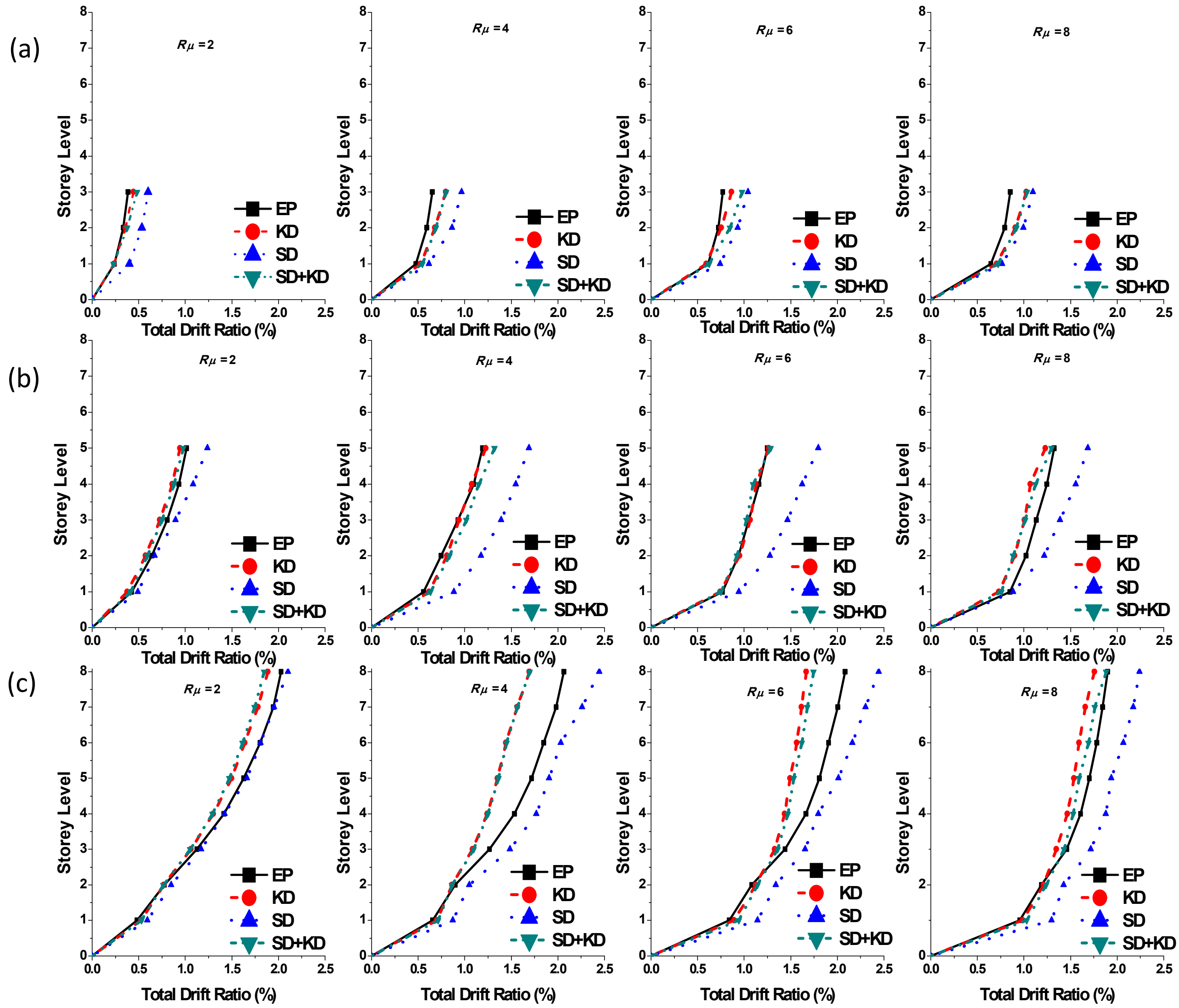
Further, from the above figure, it may be confirmed that the strength degradation model seems to have more ductility demand, which may be due to the reduction of lateral strength of the member. On the other hand, it can be seen that the degradation of stiffness, causing lengthening of the lateral period, which exhibits reduced ductility demand as compared to other hysteresis models. The plots corresponding to the stiffness and strength degradation is supposed to give an accurate prediction as being a more precise model. However, for the larger lateral period and response reduction factor elastoplastic model may be used as the difference in the responses due to various hysteresis models seem to be marginal.



## 4.2 Multistory systems

The response of 3, 5, and 8 stories systems were also studied using four hysteresis models. The inter-story drift is normalized by story height to obtain the story drift ratio. The story level is plotted along the ordinate and the mean of the total story drift ratio corresponding to the set of twenty ground motion in abscissa. For each of the systems, plots are available for $R_\mu = 2, 4, 6,$ and $8$. Fig. 7 presents such plots for 3 stories, 5 stories, and 8 stories systems. Likewise, the idealized single-story systems, strength degrading model exhibits the maximum response. So, it may be advised to carry out the design based on the larger response obtained for strength degrading model to be on the safer side, depending on the importance level of structure. Furthermore, Fig. 7 also shows that the difference in response for four different models for such a multistory system is marginal.




Note: EP = Elastoplastic; KD = Stiffness Degradation; SD = Strength Degradation; SD+KD = Strength and Stiffness Degradation.

**Fig.7.** Total drift ratio profile for multistory system: (a) 3 stories system; (b) 5 stories system; (c) 8 stories system.



## 5. Conclusions

The present study makes an effort to investigate the inelastic seismic demand attributed to different types of hysteresis behaviors for the idealized single and multistory systems. Nonlinear time history analysis was carried out for a set of twenty ground motion for different natural periods of structures to calculate the seismic demands. Following broad conclusions may be summarized based on the present study.

1) The study on the inelastic seismic behaviors of a single and multistory systems show that the use of strength degradation hysteresis model may be used because of its simplicity for obtaining the upper bound value of the inelastic demands.
2) The hysteresis models with stiffness degrading characteristics cause period lengthening resulting in a reduction in the inelastic demands.
3) For single-story systems, the stiffer period and the higher value of the response reduction factor show larger displacement and ductility demand. For multistory systems, the difference in response for four models exhibits marginal variation.
4) As the lateral period increases, the difference in response due to various hysteresis models becomes marginal. Hence, the simplest one, i.e., elastoplastic model may be used for all practical purposes, particularly to generate a preliminary idea about the inelastic demand.

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