MODELLING INHERENT DAMPING IN INELASTIC TIME HISTORY ANALYSIS

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Abstract

In a nonlinear dynamic analysis, the most popular way of representing the inherent damping in a structure is by adopting the classical Rayleigh damping model. Although many studies have identified issues with this model, it remains the most popular choice in the currently available nonlinear time-history analysis software. The popularity of the model is mainly attributed to the mathematical and programming convenience. A new paradigm for modelling damping at the element level was proposed by the authors in Puthanpurayil et al. [14-16]. This approach introduces six different damping models to seismic nonlinear dynamic analysis. This paper compiles the responses of the different models compared to give users a rational basis for selecting an appropriate model for use in a non-linear time-history analyses. Results of both single ground motion studies and Incremental Dynamic analysis (IDA) have been used to highlight the uncertainty associated with modelling inherent damping in non-linear time-history analyses. For comparison purposes, more traditional damping models, including Rayleigh damping and Global Modal damping have been included in the comparisons. A recommended approach to model inherent damping in nonlinear time domain analysis is presented.

Keywords: Inherent damping, Inelastic dynamic analysis, Rayleigh damping, Modal damping, in-structure damping

1. Introduction

In a nonlinear dynamic analysis, the term *damping* is used as a synonym for energy dissipation that is not explicitly captured by the hysteretic responses of the structural elements. Therefore, in an inelastic dynamic analysis, damping represents the phenomenon of energy dissipation that is *un-modelled* [1]. This *un-modelled* dissipation may include cracking of non-structural components, yielding and cracking of gravity columns or beams, failure of gravity connections, cracking of partition walls etc. [1]. These mechanisms contributing to the overall *un-modelled* dissipation phenomenon can be large and it is virtually impossible to model these processes explicitly. Therefore, in a nonlinear dynamic analysis, in order to avoid the explicit modelling of these small physical mechanisms, an empirical approach is adopted by adding a phenomenological mathematical model to the classical system equilibrium equations. This phenomenological mathematical model representing the *un-modelled* dissipation tries to mimic the observed phenomenon ([2-3]). For an accurate depiction of the physical process, an ideal empirical mathematical model representing damping should have the following attributes:

(a) no appearance of unrealistic forces/moments associated with the damping phenomenon as the analysis progresses

(b) ease of implementation in an existing commercial software framework

(c) no explicit increase in the computational time due to the choice of the damping model.

The presence of unrealistic damping forces has been shown to give considerable inaccuracies in displacements and internal forces whereas the other two attributes are more related to the practical utility of the model from a commercial implementation point of view.

Keeping the above attributes as the datum, the present paper consolidates the existing and the advanced elemental damping models proposed by the authors. The paper also consolidates some of the numerical results published elsewhere and identifies the most suitable damping model for a nonlinear dynamic analysis according to the authors.

2. Existing damping models used in current practice

The classical, or the most popular, approach to modelling the *un-modelled* dissipation in inelastic seismic analyses is by the use of the Rayleigh damping model. In the truest sense, Rayleigh damping was formulated mainly for elastic analysis by Lord Rayleigh in 1887 [4] and is mathematically convenient especially in the domain of elastic modal analysis. According to Rayleigh, the damping coefficient matrix $\mathbf{C}$ maybe assumed as,

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K} \qquad (1)$$

where α and β are damping proportionality constants evaluated as a function of the system frequencies using a preconceived damping ratio. **M** and **K** are the mass and stiffness matrices of the structure. In the 1960s this approach was accepted on the assumption, that the damping actions (forces and moments) were small in comparison with the inertial and structural member actions at the joints of the structure. Therefore if the damping model was not precise it would not have a marked effect on the structural response [5].

There are different phenomenological approaches adopted when Rayleigh damping is incorporated into the inelastic analysis scenario. From an implementation point of view most commercial software seems to incorporate Rayleigh damping by either by using the initial stiffness matrix or by using the tangent stiffness matrix.

If initial stiffness matrix is used then eq. (1) becomes,

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}_{initial} \qquad (2)$$

$\boldsymbol{K}_{initial}$ is the stiffness matrix computed at the beginning of the analysis. $\alpha$and β are also computed at the beginning of the analysis and remain constant throughout the analysis. If the tangent stiffness matrix is used then eq. (1) becomes,

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}_{tangent} \quad (3)$$

where $\boldsymbol{K}_{tangent}$ is the current stiffness matrix and is constantly updated as the analysis proceeds with time. The parameters. α and β are computed at the beginning of the analysis and remain constant throughout the analysis. It must be noted that if the damping matrix is used as a tangent damping matrix in a non-linear analysis then the damping actions will exhibit hysteresis as the stiffness matrix has hysteresis. If the damping matrix is used as a secant damping matrix this hysteresis does not occur.

Rayleigh damping remains the most popular choice for modelling damping in nonlinear analysis mainly due to its familiarity, computational efficiency (it uses the already computed **M** and **K** matrices) and ease of implementation in a commercial software platform. However, its extension to nonlinearity as outlined above does not possess any physical reasoning. In the case of a nonlinear analysis, the mathematical convenience the Rayleigh damping possessed in a linear modal analysis no longer exists ([1], [6-7]).

Past studies, including Crisp [8], have shown that the most important shortcoming in the use of Rayleigh damping in inelastic analyses is the appearance of un-realistic damping forces/moments on the onset of yielding ([8-12]). Crisp observed damping moments of the order of values close to the yielding moments of the girders spanning into the joints in six storey and twelve storey frames when initial stiffness Rayleigh damping model was used. These observations recorded by Crisp (1980) has been further investigated and confirmed by numerous researchers (refer to [6, 13-18]).

Recorded literature, such as those listed above, suggests some modifications to the Rayleigh Damping model. These modifications have helped in improving the mathematical model (primarily by reducing the unrealistic forces). However, the majority of the improved methods still unable to be simply incorporated into existing analyses and often they require subjective limitations to the damping actions. Charney [14] suggested avoiding the Rayleigh model altogether.

In the realm of dynamic analysis, the Rayleigh model is not the only available damping model. More generic models are available in the literature. These include:

(a) Caughey damping [19] - a more generic series version of Rayleigh damping,

(b) Wilson and Penzien damping [20] - an efficient procedure to implement modal damping resulting in a damping matrix similar to that obtained by Caughey,

(c) Non-viscous damping models or convolution format damping models [21], and

(d) Frequency independent damping model [22] etc.

Again, to the knowledge of the authors, except for the Wilson and Penzien model which was introduced into Ruaumoko in 1978 [23] and used for Crisp's 1980 studies little effort has been shown for the extension of any of these models into the domain of nonlinear seismic analysis. This is mainly due to the complications involved in implementing them in an existing commercial software platform along with the computational and parametrization demands posed by these models.

Avoiding the Rayleigh model altogether and replacing it by a different mathematical formulation is considered the best approach to model inherent damping for seismic analysis [14]. However, to date, most of the research effort seems to be focused on empirically attempting to correct the Rayleigh formulation and adapt it to nonlinear scenario. Though models other than Rayleigh exist, their application is also very limited due to the reasons already discussed. Thus, a totally new approach to modelling damping is desired.

Puthanpurayil et al [15-17] introduced a new paradigm to modelling damping, by defining the damping at an element level and assembling the elemental damping matrices in the same way the mass and stiffness matrices are assembled. This paper presents a compilation of all those models and recommends a preferred modelling approach.

3.0 New proposed Models compiled in this study [12]

This section briefly compiles the new elemental damping models proposed by the authors (Puthanpurayil et al [15-17]). The elemental damping models are broadly classified based on the way the damping matrix is formed.

In the discrete elemental damping models, the elemental damping matrix is uses the semi-discretisation procedure as is undertaken for the continuum domain whereas in the continuum damping model, the dissipation function is introduced in the continuum level before the semi-discretisation procedure is done. The elemental damping matrix thus obtained in either approach is then assembled by the direct stiffness procedure similar to that for the mass and stiffness matrices.

3.1 Discrete elemental damping models

Two elemental models which are the elemental counterparts of the global models are described in this section. These are the

(a) Elemental Rayleigh damping (an elemental adaptation of the classical Rayleigh damping)
(b) Elemental Wilson-Penzien model (elemental adaptation of the classical Wilson-Penzien model).

A very brief theoretical overview of the elemental models is given. Additional information regarding the implementation of these models may be found in Puthanpurayil *et al.* ([15-17]).

- *Elemental Rayleigh damping model*

The elemental Rayleigh damping is a direct adaptation of the Global Rayleigh damping at the element level and is given as [15],

$$\boldsymbol{C}_e = \alpha_{eRD}\boldsymbol{M}_e + \beta_{eRD}\boldsymbol{K}_e \tag{4}$$

where α_{eRD} and β_{eRD} are the elemental damping coefficients. The main difference between the elemental Rayleigh damping and the classical Rayleigh damping (which is predominantly implemented at a global level) exists in the computation of the damping coefficients. In the elemental Rayleigh damping the coefficients are computed as,

$$\left.\begin{aligned} \alpha_{eRD} &= 2\xi_{eR1}\frac{\omega_e^i\omega_e^j}{\omega_e^i+\omega_e^j} \\ \beta_{eRD} &= 2\xi_{eR1}\frac{1}{\omega_e^i+\omega_e^j} \end{aligned}\right\} \tag{5}$$

where ω_e^i and ω_e^j are the i^{th} and the j^{th} elemental frequencies. ξ_{eR1} and ξ_{eR2} are elemental damping ratios which need to be parameterised as outlined in Puthanpurayil *et al*. [15].

- *Elemental Wilson-Penzien model*

Elemental Wilson-Penzien is the elemental version of the global Wilson-Penzien damping model proposed by Wilson and Penzien [20]. The elemental matrix after mathematical manipulation is given as,

$$\boldsymbol{C}_e = \Theta_e\Psi_e\Theta_e^T \tag{6}$$

where Θ_e is the mass normalized elemental deformation mode shape matrix. Ψ_e is a diagonal matrix with diagonal elements given by,

$$\psi_e^i = \frac{2\xi_{eWP}^i \, \omega_e^i}{M_{di}^i} \tag{7}$$

In the spirit of Wilson-Penzien, ξ_{eWP} was named as the elemental Wilson-Penzien damping ratio in Puthanpurayil et al. [15] and is often assumed to be equal for all elements. There is no restriction for ξ_{eWP} to be a constant for every element. If better parameterization methodologies are available then ξ_{eWP} can be treated as a variable for each mode of each of the elements comprising the global system. This shows the generality of the elemental Wilson-Penzien formulation [15]. This model is under testing for commercial applications.

3.2 Continuum damping models

The Euler Bernoulli beam continuum enhanced with the damping term is given as,

$$\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2} + F_{int}(x,t) + F_{ext}(x,t) + \frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right) = f(x,t) \tag{8}$$

where, $F_{\text{int}}(x,t)$ is the internal damping force caused by internal resistance and $F_{ext}(x,t)$ is the external damping force. $'x'$ refers to the spatial ordinate and $'t'$ refers to the time ordinate. $\rho, A(x), E$ & $I(x)$ refer to material density, geometric area, modulus of elasticity and second moment of area of the beam continuum. $f(x,t)$ is the externally applied load; $w(x,t)$ is the transverse displacement.

The external damping force is assumed as,

$$F_{ext}(x,t) = \gamma_{air}\frac{\partial w(x,t)}{\partial t} \tag{9}$$

γ_{air} is the external air damping coefficient which could be used to represent drag effects.

Kelvin Voigt, Time Hysteresis damping, Russell's damping and Extended Sorrentino models are differentiated by the way $F_{\text{int}}(x,t)$ is defined.

3.2.1 Spatially local continuum damping models [16]

In the local continuum model, the damping force at a point is the function of the response at the same point.

- *Kelvin Voigt damping*

In Kelvin Voigt damping the $F_{\text{int}}(x,t)$ is given as,

$$F_{int}(x,t) = \frac{\partial^2}{\partial x^2}\left(c_s I(x)\frac{\partial^3 w(x,t)}{\partial t \partial x^2}\right) \tag{10}$$

where c_s refers to the damping coefficient which converts strain rate into stress. The elemental Kelvin Voigt damping matrix is obtained by semi-discretization of equation (8) incorporating equations (9) and (10). The authors note that on semi-discretization the damping matrix obtained is proportional to the element stiffness matrix. This model may be viewed as a continuum version of the classical global Rayleigh damping model.

- *Time hysteresis damping*

The time hysteresis internal damping force maybe given as,

$$F_{\text{int}}(x,t) = EI(x)\int_0^t g(t-\tau)\frac{\partial}{\partial \tau}\left(\frac{\partial^4 w(x,\tau)}{\partial x^4}\right)d\tau \tag{11}$$

This model is temporally non-local and spatially local and on semi-discretization results in an integro differential equation.

On semi-discretization, the elemental equation of motion is given as,

$$\boldsymbol{M}_e \ddot{\boldsymbol{d}}_e(t) + \boldsymbol{C}_{eair} \dot{\boldsymbol{d}}_e(t) + \boldsymbol{K}_e \int_0^t g(t-\tau) \dot{\boldsymbol{d}}_e(\tau)\, d\tau + \boldsymbol{K}_e \boldsymbol{d}_e(t) = \boldsymbol{f}_e(t) \quad (12)$$

where, $\mathbf{M}_e$ is the element mass matrix; $\mathbf{K}_e$ is the element stiffness matrix; $\mathbf{C}_{eair}$ is the element external damping matrix; $\ddot{\mathbf{d}}_e, \dot{\mathbf{d}}_e, \mathbf{d}_e$ are the element acceleration, velocity and displacement; $g(t-\tau)$ is the causal damping kernel function which is given as,

$$g(t) = \mu e^{-\mu t} \quad (13)$$

where, μ is a relaxation parameter. Adhikari [3] provides more details regarding this model.

Equation (12) is an integro-differential equation and cannot be solved using the classical methods of time integration. AAR method developed by the authors [24] is recommended for solving this equation in nonlinear dynamics.

3.2.2 Nonlocal continuum damping models

Spatially nonlocal damping models are models in which the damping force at a point is a function of the responses at all points in the domain. In the spatio-temporally nonlocal damping models, the damping force at a point is a function of responses at all points in the spatial domain along with response history.

- *Russell's damping model [17]*

Russell (1991) pioneered the development of nonlocal damping models by the development of a spatial hysteresis model. In the spatial hysteresis model, the internal damping term is described as a moment acting on a beam at a point 'x' due to the differential rotation of the beam at points ξ "near" x.

$$F_{\text{int}}(x,\xi,t) = -2\frac{\partial}{\partial x}\left\{\int_0^L h(x,\xi)\left[\frac{\partial^2 w(x,t)}{\partial x \partial t} - \frac{\partial^2 w(\xi,t)}{\partial x \partial t}\right] d\xi\right\} \quad (14)$$

On semi-discretization, the internal spatial hysteresis matrix is given as,

$$\boldsymbol{C}_i = 2\int_0^L \left[\frac{\partial \boldsymbol{N}^T(x)}{\partial x}\right]\left\{\int_0^L h(x,\xi)\left[\frac{\partial \boldsymbol{N}(x)}{\partial x} - \frac{\partial \boldsymbol{N}(\xi)}{\partial x}\right] d\xi\right\} dx \quad (15)$$

Where, $h(x,\xi)$ is the spatial kernel function and $\mathbf{N}(x)$ is the shape function.

The Russell spatial hysteresis model incorporated elemental equation of motion can be given as,

$$\boldsymbol{M}_e \ddot{\boldsymbol{d}}_e(t) + [\boldsymbol{C}_{eair} + \boldsymbol{C}_i]\dot{\boldsymbol{d}}_e(t) + \boldsymbol{K}_e \boldsymbol{d}_e(t) = \boldsymbol{f}_e(t) \quad (16)$$

In effect the Russell's model is a spatially nonlocal temporally local model which may be considered as a viscous model with spatially nonlocal terms. For more details on this see Banks and Inman [25], Puthanpurayil et al. [17].

- *Extended Sorrentino damping model (ESM) [17]*

ESM is a very generic model as it is a spatio-temporally nonlocal model. In the case of ESM,

$$F_{\text{int}}(x,t) = \frac{\partial^2}{\partial x^2}\left(\int_0^L \int_{-\infty}^t C_{\text{int}}(x,\xi,t-\tau)\frac{\partial^3 w(\xi,\tau)}{\partial \xi^2 \partial \tau} d\tau d\xi\right) \quad (17)$$

C_{int} is a spatio-temporal kernel function. Refer to Friswell *et al.* [26] and Puthanpurayil et al. [17] for more details.

4.0 Numerical study

This section presents a compilation of the performance of the elemental damping models in comparison to the global classical damping models based on a four-storey frame as described in Puthanpurayil et al. ([15-17]). First a single ground motion study is used to highlight the difference in response of the global models and the elemental models. Secondly a compilation of an IDA study is also presented [12].

All existing elemental and global damping models are compared in this section. Following abbreviations are used to identify different damping models included in the plots hereafter:

- Initial stiffness based global Rayleigh damping (ISRD)
- Tangent stiffness based global Rayleigh damping with constant coefficients (TSRD),
- Global Wilson-Penzien (GWP)
- Elemental Rayleigh damping with updated proportionality coefficients (ELRD)
- Elemental Wilson-Penzien model implemented as a constant damping matrix (EWP)
- Elemental Wilson-Penzien model implemented using the updated tangent stiffness matrix (UEWP)
- Elemental Kelvin Voigt (ELKV)
- Elemental Time hysteresis (ELTH)
- Elemental Russell model (ELR)
- Elemental Extended Sorrentino model (EESM).

4.1 Example 4 Storey Frame.

Figure 1.0 shows a four-story RC frame described in Arede [27], designed in accordance with Eurocode 8 (EC8) and Eurocode 2(EC2) is used for the study. The frame is designed for Ductility Class High assuming a PGA of 0.3g. The frame has a mass of 29800 kg at each first floor node and 29500 kg at all other nodes. The columns are 450 mm square and the beams are 300 mm wide by 450mm deep. Youngs Modulus was taken as E = $3.5\times10^{10}\ N/m^2$

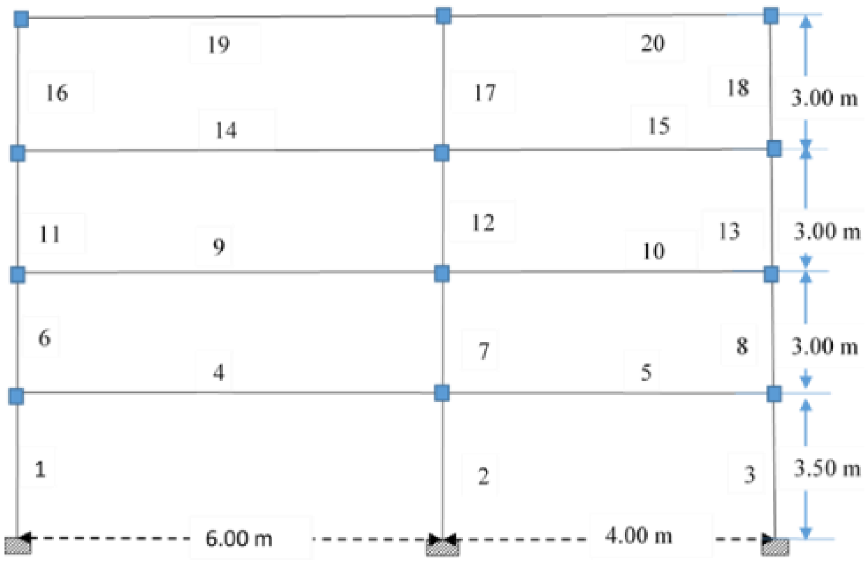

Figure 1.0 Four Storey Frame

4.2 Single ground motion study

Figure 2.0 represents the response of elastic structure and Figure 2.0 represents the response of inelastic structure. *It can be clearly seen that when the structure is elastic there is not much difference in the response between the different damping models.* When the structure enters its inelastic state, the response starts to show marked difference as outlined in Figure 3.0.

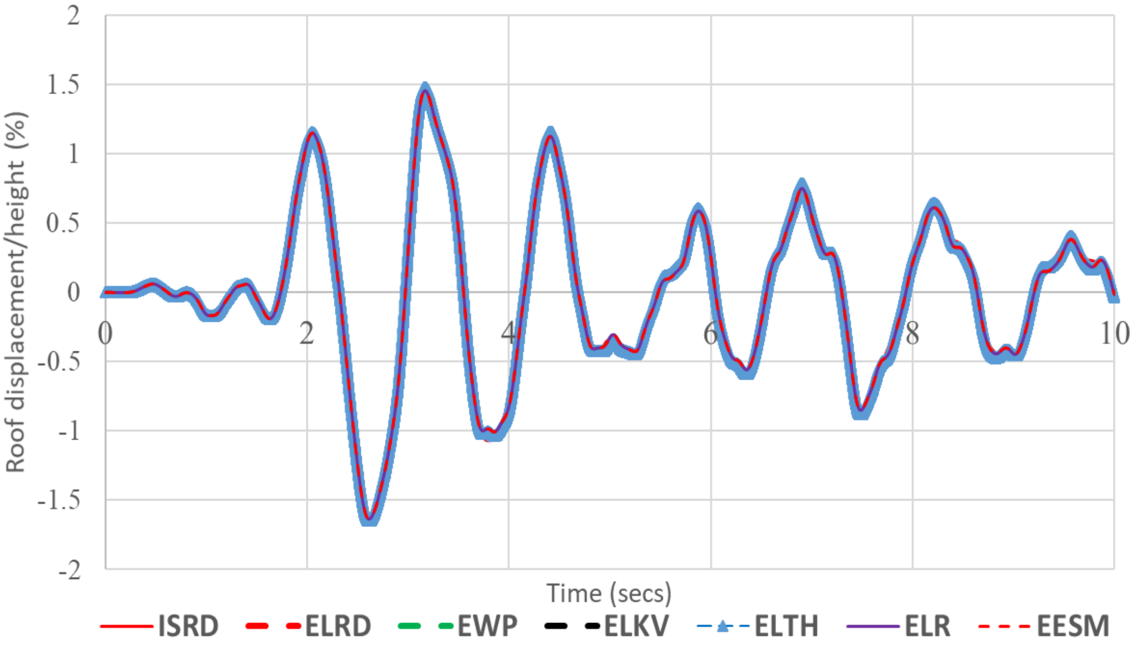

Figure 2.0 Single ground motion response -elastic structure

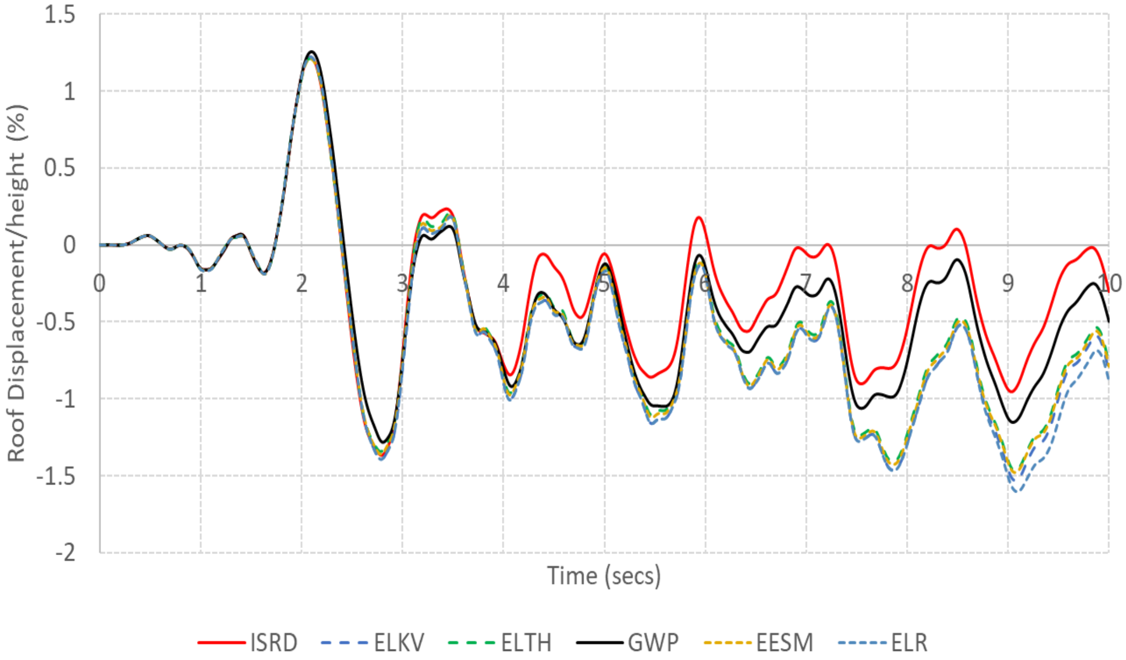

Figure 3.0 Single ground motion response -inelastic structure

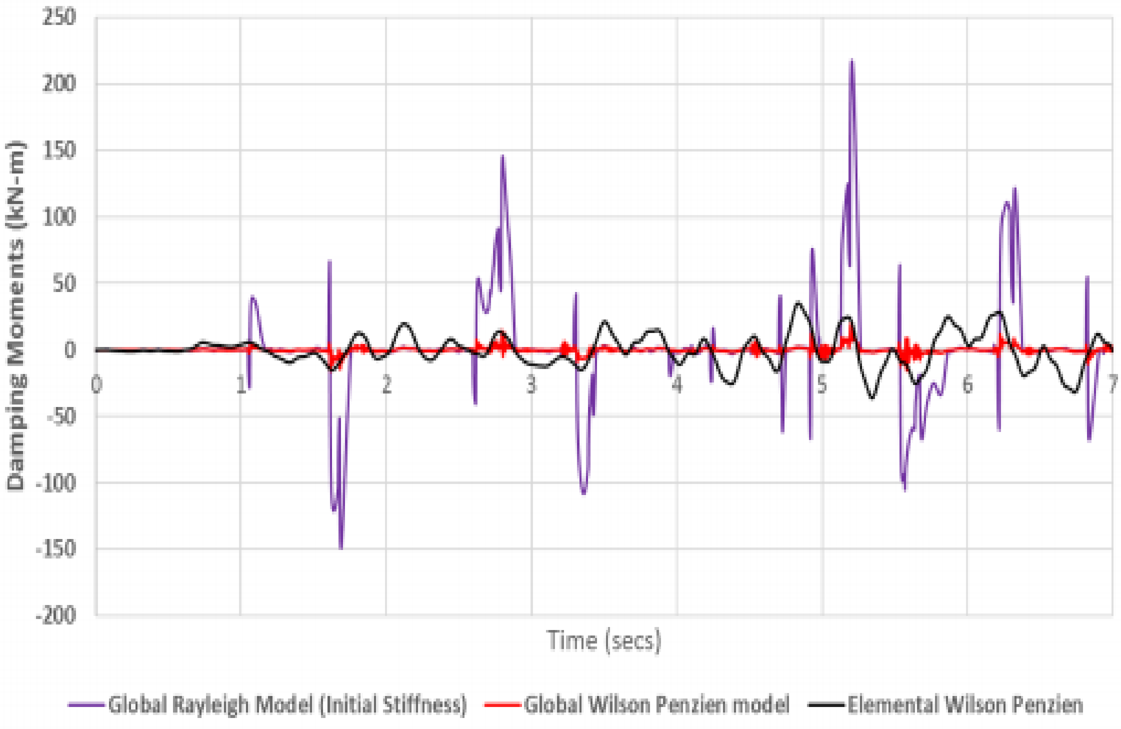

Figure 4.0 Damping moment plots [10]

Figure 4.0 represents a qualitative comparison of the damping moment plots between elemental and global models. Only the elemental Wilson Penzien is plotted for clarity. It can be clearly seen that, compared to the global Rayleigh damping, the Elemental Wilson Penzien gives much reduced damping moments. Very similar results are obtained for the other damping models.

4.2 Incremental dynamic analysis

Figure 5.0 illustrates the mean IDA curves for location independent peak interstory drift ratio as the engineering demand parameter (EDP). All the elemental damping models give noticeably higher drifts when compared to those of the global damping models.

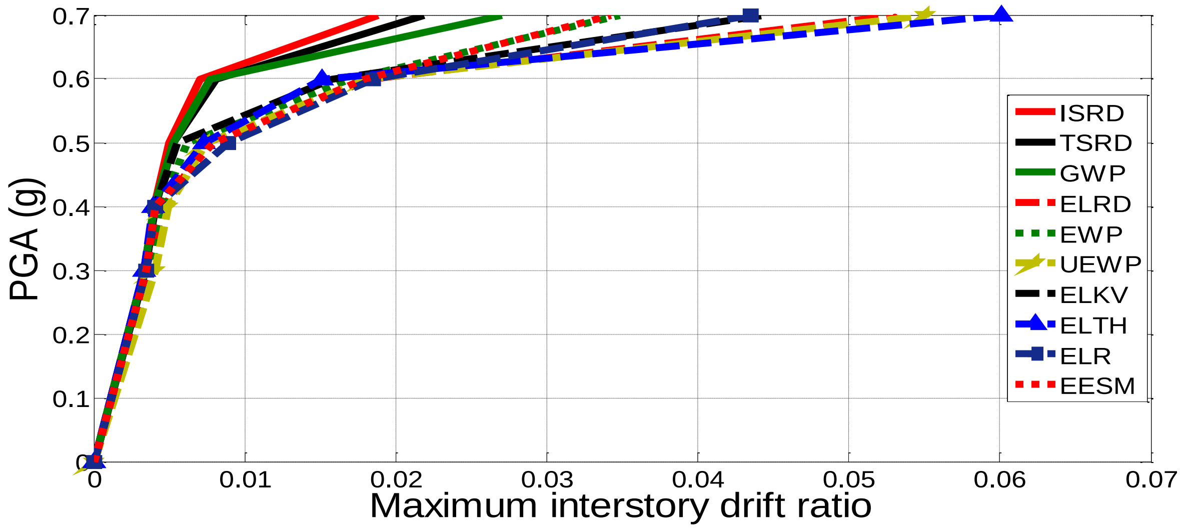

Figure 5.0 IDA results

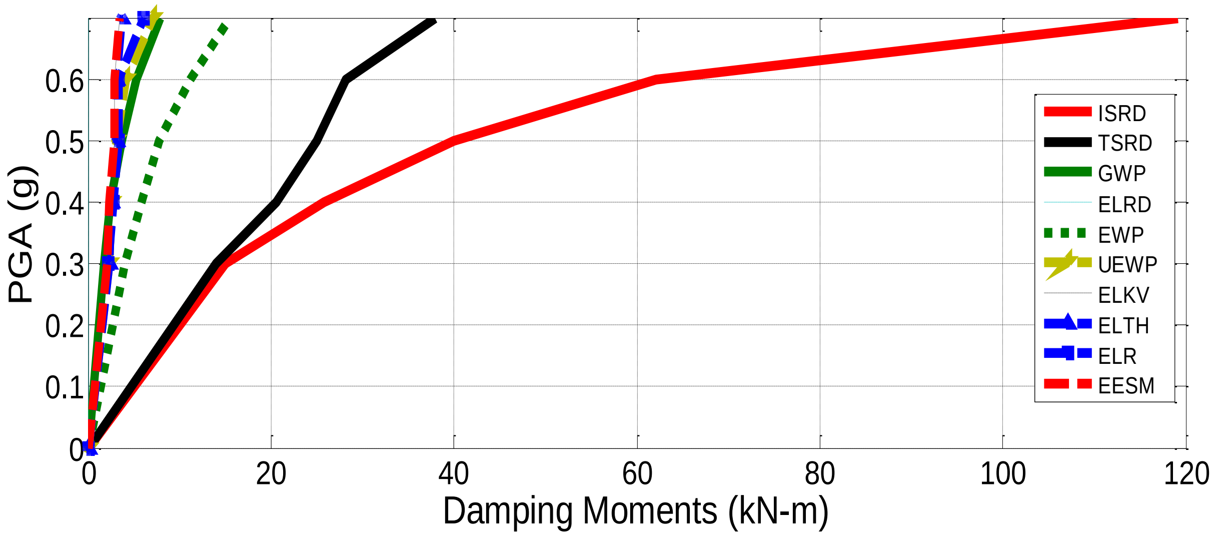

Figure 6.0 IDA results

Figure 6 illustrates the peak damping moments for all the models. The ISRD model gives the highest damping moment which is a higher moment than the yield moment of the frame members. All the elemental damping models produce relatively small damping moments.

It is difficult to get an explicit correlation between the appearance of damping moments and its effect on the peak responses of the structure; but it is shown here as well as in previous studies [8] that the lesser the damping moments the larger the structural response. As outlined in the introduction, an ideal damping model should offer no appearance of unwanted damping actions which compromise the responses along with a reduced computational demand say in comparison of that required for the Global Wilson-Penzien damping model. In that spirit, the elemental models tend to predict reliable responses with no appearance of unwanted actions, forces of moments. As a result, the authors believe that they produce more realistic responses with less untoward side effects. Explicit experimental evidence is needed to be undertaken in the near future to determine the damping coefficients required for the elemental damping models.

5.0 Conclusions

A compiled summary of the performance of the elemental damping models in comparison to global damping models published in literature is presented in this paper. The single ground motion study emphasises the fact that when the system is elastic all the models produce similar results and the changes only appear once the system goes inelastic. The IDA results presented indicates that the elemental damping models may perform much better than the global damping models as unwanted damping moments are reduced considerably. Also elemental damping models have reduced computational demand as the forces are computed at the element level and have the advantage of being able to adjust the damping coefficients as the elemental damping matrices reflect the occurrence of inelasticity in the members. Based on the results compiled herein, the updated elemental Wilson-Penzien model maybe treated as a preferred elemental damping model for the generic use in nonlinear time-history analyses. Nonlocal continuum models are also reccomended as they reflect the physics of dissipation but means of defining the required coefficients will require further investigation. More research needs to be done to confirm this initial insight of the preferred models.

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