IMPROVING EARTHQUAKE BEHAVIOUR OF 5-STOREY RC MOMENT FRAME BUILDINGS THROUGH MEMBER SIZE PROPORTIONING

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Abstract

Reinforced concrete (RC) moment frame buildings up to five storeys tall constitute over 95% of the RC building stock. In such buildings, the construction cost is economised by adopting the same formwork in the construction of all beams in all storeys and similarly of all columns in all storeys. It is needs to be clarified if such design option (with columns in all storeys proportioned to be of the same size and beams in all storeys to be of the same size) is beneficial or detrimental to good seismic resistance. This paper examines the seismic behaviour of such buildings, using Nonlinear Static Analyses (NSA) and Incremental Dynamic Analyses (IDA). Alongside *three* other design options also are examined (with column size different in all storeys even though beam size is same, beam size different in all storeys even though column size is same, and all column and beam sizes are different in all storeys). For each design option, analytical expressions are provided for proportioning the stiffness of members for improved seismic performance of buildings up to 5 storeys; this is compared with the traditional design option with no guidance on proportioning stiffness and strength of members of such buildings. The seismic performances are compared of buildings proportioned using the suggested expressions based on the results of NSA and IDA. The best design option is the one with varying beam and column sizes along the five storeys; the other design options are ranked in the order of their seismic performance.

Keywords: Seismic Design; Low-rise Buildings; Stiffness; Plastic Rotation; Nonlinear Analysis

1. Introduction

Moment Frames (*MF*) are widely used as the *Lateral Load Resisting System* in low-rise RC buildings. In this system, contractors economise the cost of formwork by using the same sets of formwork in all storeys when casting beams and columns. This option (Option 1: 1A with fixed column bases and 1B with pinned column bases) with beams and columns of the same size in all storeys may lead to detrimental seismic behaviour (*e.g.,* concentration of plastic rotation in beams and large inter-storey drift demands in one storey). These detrimental effects in buildings (with member sizes proportioned using existing approach) can be observed on performing *Nonlinear Static Analysis* (*NSA*) and *Nonlinear Time History Analysis* (*NTHA*). Given this, it is necessary identify alternate ways of proportioning member sizes in *MF* buildings to overcome the said limitations; three options of proportioning member sizes in *MF* buildings are investigated (Options 2A-4A of fixed column bases and options 2B-4B of pinned column bases, as given in Table 1). In this paper, the efficiency of each option is quantified by assessing their static and dynamic behaviour. Also, closed-form expressions are presented to determine member sizes of beams and columns in each storey with the aim of ensuring that the modal mass participation of the fundamental lateral mode to be at least around 90%.

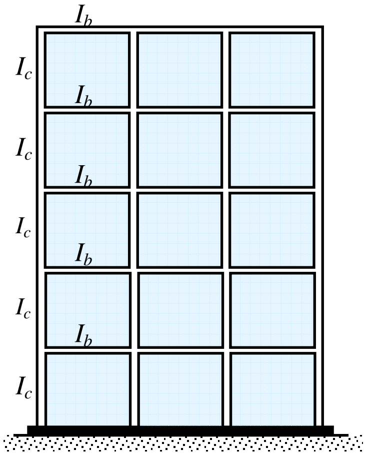

Fig 1 – A typical moment frame building with *uniform* size of beams and columns

Table 1 – Different options for proportioning member sizes in RC moment frame buildings

Storey	*Options*							
	***1A**: Fixed Base*		***2A**: Fixed Base*		***3A**: Fixed Base*		***4A**: Fixed Base*	
	***1B**: Pinned Base*		***2B**: Pinned Base*		***3B**: Pinned Base*		***4B**: Pinned Base*	
	Beam	*Column*	*Beam*	*Column*	*Beam*	*Column*	*Beam*	*Column*
5	I_b	I_c	I_b	I_{c5}	I_{b5}	I_c	I_{b5}	I_{c5}
4	I_b	I_c	I_b	I_{c4}	I_{b4}	I_c	I_{b4}	I_{c4}
3	I_b	I_c	I_b	I_{c3}	I_{b3}	I_c	I_{b3}	I_{c3}
2	I_b	I_c	I_b	I_{c2}	I_{b2}	I_c	I_{b2}	I_{c2}
1	I_b	I_c	I_b	I_{c1}	I_{b1}	I_c	I_{b1}	I_{c1}

Note: I_b and I_c represent second moment of area of beam and column sections respectively

2. Proportioning Member Sizes in MF Buildings

2.1 $\{\phi\}_1$ with Desired M_1^*

The sizes of members influence the fundamental lateral translational natural period T_1, the associated mode shape $\{\phi\}_1$ of oscillation, and the associated *Mass Participation Factor* M_1^*. In general, higher M_1^* is desirable. Hence, when proportioning members in a building and determining their sizes, M_1^* is desired to be more than about 90%. Such a requirement may be difficult to achieve always in buildings with beams of same size in all storeys and likewise columns (Options 1 in Table 1), but likely in buildings with members proportioned by the remaining 3 options (Options 2-4 in Table 1). To proportion member sizes in a building such that the M_1^* is more than the desired value, closed-form expressions are derived to: (a) identify $\{\phi\}_1$, and (b) estimate the member sizes of *MF* buildings as a function of identified $\{\phi\}_1$. For instance, M_1^* of $\{\phi\}_1$ of an *N*-storey *MF* building with uniform storey height, and seismic mass lumped equally at all storeys, is given as:

$$M_1^* = \left(\frac{1}{N}\right)\left[\frac{\left[N-(1-\alpha)\left(\frac{1-(1-\alpha)^N}{\alpha}\right)\right]^2}{N-2(1-\alpha)\left(\frac{1-(1-\alpha)^N}{\alpha}\right)+r^2\left(\frac{1-(1-\alpha)^{2N}}{1-(1-\alpha)^2}\right)}\right], \tag{1}$$

where N is the number of storeys and α a non-dimensional parameter used to define $\{\phi\}_1$; Eq.(1) is shown graphically in Fig. (2) for a 5-storey building with uniform storey height and with equal seismic mass lumped at all storeys. The associated $\{\phi\}_1$ is given by Eq.(2); it is of flexure type, linear and shear-type if α <0, α =0, and α >0, respectively. The $2N$ mode shape coefficients of $\{\phi\}_1$ of the N-storey building corresponding to the *translational* Δ_i and *rotational* θ_i degrees of freedom at storey i of the N-storey building are given by:

$$\{\phi\}_1 = \begin{Bmatrix} \Delta_N \\ \theta_N \\ \vdots \\ \Delta_i \\ \theta_i \\ \vdots \\ \Delta_1 \\ \theta_1 \end{Bmatrix}_1 = \begin{Bmatrix} \left(1+\sum_{j=1}^{j=N-1}(1-\alpha)^j\right) \\ \theta_N \\ \vdots \\ \left(1+\sum_{j=1}^{j=i-1}(1-\alpha)^j\right) \\ \theta_i \\ \vdots \\ 1 \\ \theta_1 \end{Bmatrix}_1 . \quad (2)$$

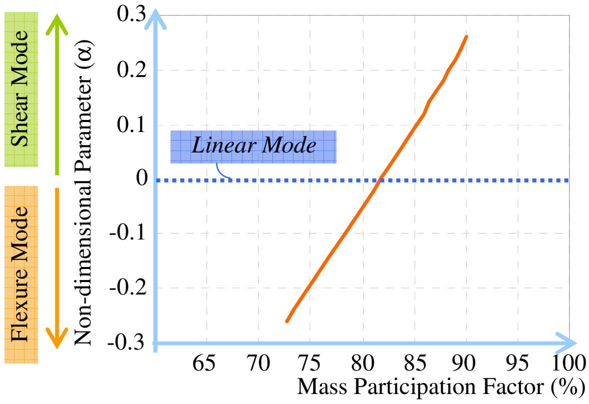

Fig. 2 – Change in M_1^* with α of a 5-storey building with uniform storey height and equal seismic weight in all storeys

2.2 Estimate of Member Sizes in 1-bay *N*-storey frame with Identified $\{\phi\}_1$

To begin with, closed-form expressions are derived for 1-bay 5-storey frame (Fig. 3). Details pertaining to the frame considered are: (a) L_c is centerline height of storey i and L_b center line length of the single bay, (b) I_{ci} and I_{bi} gross second moments of inertia of columns and beams in storey i, respectively, (c) κ_c and κ_b ratios of effective flexural rigidity to gross flexural rigidity of columns and beams, and (d) m_i lumped seismic mass at storey i. The governing equation of such a building are:

$$[K]-\omega^2[M]\{\phi\}_1 = \{0\}, \quad (3)$$

where $[M]$, $[K]$ and ω are lumped mass matrix (Eq.(4)), lateral translational stiffness matrix (Eq.(5)) for fixed column bases, and fundamental lateral translational circular frequency, respectively, where:

$$[M]=\begin{bmatrix} m_1 & 0 & 0 & \cdots & & & 0 & 0 \\ & 0 & 0 & \cdots & & & 0 & 0 \\ & & m_2 & \cdots & & & 0 & 0 \\ & & & \cdots & & & \cdots & \cdots \\ & & & & m_i & & 0 & 0 \\ & & & & & \cdots & \cdots & \cdots \\ & & & & & & m_5 & 0 \\ sym. & & & & & & & 0 \end{bmatrix}, \text{ and} \tag{4}$$

$$[K]=\begin{bmatrix} \sum_{j=1}^{2}\frac{12EI_{cj}\kappa_c}{L_c^3} & -\frac{6EI_{c1}\kappa_c}{L_c^2}+\frac{6EI_{c2}\kappa_c}{L_c^2} & -\frac{12EI_{c2}\kappa_c}{L_c^3} & \cdots & & & 0 & 0 \\ & \frac{6EI_{b1}\kappa_b}{L_b}+\sum_{j=1}^{2}\frac{4EI_{cj}\kappa_c}{L_c} & -\frac{6EI_{c2}\kappa_c}{L_c^2} & \cdots & & & 0 & 0 \\ & & \sum_{j=2}^{3}\frac{12EI_{cj}\kappa_c}{L_c^3} & \cdots & & & 0 & 0 \\ & & & \cdots & & & \cdots & \cdots \\ & & & & \sum_{j=i}^{i+1}\frac{12EI_{cj}\kappa_c}{L_j^3} & & 0 & 0 \\ & & & & & \cdots & \cdots & \cdots \\ & & & & & & \frac{12EI_{c5}\kappa_c}{L_c^3} & -\frac{6EI_{c5}\kappa_c}{L_c^2} \\ sym. & & & & & & & \frac{6EI_{b5}\kappa_b}{L_b}\ \frac{4EI_{c5}\kappa_c}{L_c} \end{bmatrix}. \tag{5}$$

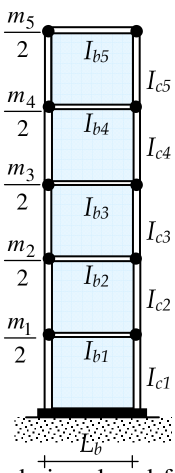

Fig. 3 –5-storey 1-bay frame considered to derive closed-form expressions to estimate member sizes

Table 2 lists the available equations and unknowns in Eq.(3) after substituting Eqs.(2), (4) and (5) in Eq.(3). In general, the number of available equations does not match with the number of unknowns. Hence, a few unknowns are the assumed to be given: I_b and I_{c1} in *Option* 2, I_{b5} and I_c in *Option* 3, and I_{b5}, I_{c5} and rotational components of mode shape are assumed proportional to translational component of mode shape (Eq. 6) in *Option* 4, i.e.,

$$\theta_i=\left(\frac{\Delta_i-\Delta_{i-1}}{L_c}\right)=\frac{(1-\alpha)^{(i-1)}}{L_c}. \tag{6}$$

Table 2 – Summary of available equations and unknowns in Eq. (3) for Options 2, 3, and 4

Unknown Quantity	*Option*			*Available Equation*	*Option*		
	2	**3**	**4**		**2**	**3**	**4**
Mode Shape: Rotational Component	5	5	1	Eigen Equation: Translational	5	5	5
Circular frequency: ω	1	1	1	Eigen Equation: Rotational	5	5	5
Moment of Inertia: I_{bi}	1	5	5	Assumed size of Beam	1	1	1
Moment of Inertia: I_{ci}	5	1	5	Assumed size of Column	1	1	1
Number of unknowns (Total)	**12**	**12**	**12**	*Available Equations* (Total)	**12**	**12**	**12**

2.2.1 *Size of Columns in Buildings with Uniform Size of Beams (Option 2)*

Sizes of beams and columns of the first storey are assumed in buildings with uniform beams in all storeys. To start the initial proportioning, the following are assumed: (a) the depth of beams to be between $L_b/10$ and $L_b/14$, and (b) the size of first storey columns (*pinned base*: $I_{c1} > 1.5I_b$ and *fixed base*: $I_{c1} < I_b$). The following are the steps to estimate sizes of columns in all storeys, excluding those in the first storey:

Step 1-1: Assume a value of θ_1 (say, $0.1/L_b$) and estimate the column size in storeys 2 to 5 using Eqs.(7) to (10), as:

$$I_{c2} = I_{c1}\left[\frac{(A_1+R)\theta_1 - A_2\dfrac{\Delta_1}{L_c} + P_1\left(A_3\theta_1 - \dfrac{2\Delta_1}{L_c}\right)}{\left(\dfrac{\Delta_2-\Delta_1}{L_c}\right)-\theta_1}\right]. \tag{7}$$

$$I_{c3} = I_{c1}\left[\frac{A_1\theta_1 - A_2\dfrac{\Delta_1}{L_c} + \left(\dfrac{2R}{L_c}\right)(\Delta_2-\Delta_1) + P_1\left(A_3\theta_1 - \dfrac{2\Delta_1}{L_c}\right)\left(3+P_2+R\left(\dfrac{I_{c1}}{I_{c2}}\right)\right)}{\left(\dfrac{\Delta_3+3\Delta_2-2\Delta_1}{L_c}\right)+\theta_1 - P_1\left(A_3\theta_1-\dfrac{2\Delta_1}{L_c}\right)\left(\dfrac{I_{c1}}{I_{c2}}\right)}\right]. \tag{8}$$

$$I_{c4} = I_{c1}\left[\frac{(A_1+R)\theta_1 - A_2\dfrac{\Delta_1}{L_c} + \left(\dfrac{2R}{L_c}\right)(\Delta_3-\Delta_2) + P_1\left(A_3\theta_1 - \dfrac{2\Delta_1}{L_c}\right)\left(3+3P_2+P_2P_3+RP_2\left(\dfrac{I_{c1}}{I_{c3}}\right)\right)}{\left(\dfrac{\Delta_4-3\Delta_3+4\Delta_2-2\Delta_1}{L_c}\right)-\theta_1 - P_1\left(A_3\theta_1-\dfrac{2\Delta_1}{L_c}\right)\left(P_2\left(\dfrac{I_{c1}}{I_{c3}}\right)-\left(\dfrac{I_{c1}}{I_{c2}}\right)\right)}\right]. \tag{9}$$

$$I_{c5} = I_{c1}\left[\frac{A_1\theta_1 - A_2\dfrac{\Delta_1}{L_c} + \left(\dfrac{2R}{L_c}\right)(\Delta_4-\Delta_3+\Delta_2-\Delta_1) + P_1\left(A_3\theta_1 - \dfrac{2\Delta_1}{L_c}\right)\left(3+3P_2+3P_2P_3+P_2P_3P_4+R\left(\dfrac{I_{c1}}{I_{c2}}\right)+RP_2P_3\left(\dfrac{I_{c1}}{I_{c4}}\right)\right)}{\left(\dfrac{\Delta_5-3\Delta_4+4\Delta_3-4\Delta_2+2\Delta_1}{L_c}\right)+\theta_1 - P_1\left(A_3\theta_1-\dfrac{2\Delta_1}{L_c}\right)\left(P_2P_3\left(\dfrac{I_{c1}}{I_{c4}}\right)-P_2\left(\dfrac{I_{c1}}{I_{c3}}\right)+\left(\dfrac{I_{c1}}{I_{c2}}\right)\right)}\right]. \tag{10}$$

In Eqs.(7) to (10), constants R, P_1, P_2, P_3, P_4, A_1, A_2, and A_3 are given by:

$$\{R, P_i\} = \left\{\frac{3I_b\kappa_b L_c}{I_{c1}\kappa_c L_b}, \frac{\sum_{j=i+1}^{j=5} m_j\Delta_j}{\sum_{j=i}^{j=5} m_j\Delta_j} \text{ for } 2 < i \le 4\right\}, \tag{11}$$

$$\{P_1, A_1, A_2, A_3\} = \begin{cases} \dfrac{\sum_{i=2}^{i=5} m_i \Delta_i}{\sum_{i=1}^{i=5} m_i \Delta_i}, 2, 3, 1 \text{ for buildings with first storey column fixed at its base} \\ \dfrac{\sum_{i=2}^{i=5} m_i \Delta_i}{4\sum_{i=1}^{i=5} m_i \Delta_i}, 1.5, 1.5, 2 \text{ for buildings with first storey column pinned at its base} \end{cases} . \quad (12)$$

Step 1-2: Iterate θ_1 (assumed in **Step 1-1**) till Eq.(13) is satisfied. Then, re-estimate the size of columns using Eqs.(7) to (10) and θ_1 that satisfied Eq.(13).

$$\left[(A_1 + R)\theta_1 - A_2\Delta_1 + \left(\frac{2R}{L_c}\right)(\Delta_5 - \Delta_4 + \Delta_3 - \Delta_2) + P_1\left(A_3\theta_1 - \frac{2\Delta_1}{L_c}\right)\left(3 + 3P_2 + 3P_2P_3 + 3P_2P_3P_4 + R\frac{I_{c1}}{I_{c5}}P_2P_3P_4 + R\frac{I_{c1}}{I_{c3}}P_2\right)\right] = 0 \quad (13)$$

2.2.2 *Size of Beams in Buildings with Uniform Size of Columns (Option 3)*

Sizes of columns and of top storey beam are assumed in buildings with uniform columns in all storeys. To start the initial proportioning, the following are assumed: (a) the depth of beam in the top storey between $L_b/12$ and $L_b/14$, and (b) the size of columns ($I_c > 2.5 I_{b5}$). The following are the steps to estimate sizes of beams in all storeys, excluding that in the top storey:

Step 2-1: Estimate the size of beams in storeys 1 to 4 using Eqs. (14) to (17).

$$I_{b1} = \left(\frac{I_c \kappa_c}{\kappa_b}\right)\left(\frac{L_b}{3L_c}\right)\left[\frac{(3/L_c)(\Delta_2 - 0.5A_4\Delta_1) - \theta_2}{\theta_1} - A_5\right]. \quad (14)$$

$$I_{bi} = \left(\frac{I_c \kappa_c}{\kappa_b}\right)\left(\frac{L_b}{3L_c}\right)\left[\frac{(3/L_c)(\Delta_{i+1} - \Delta_{i-1}) - (\theta_{i+1} + \theta_{i-1})}{\theta_i} - 4\right] \quad \text{for } 2 < i < 4. \quad (15)$$

In Eqs.(14) and (15), $\theta_1, \theta_2, \theta_3, \theta_4$, and θ_5 are estimated as:

$$\theta_5 = \frac{\left(\frac{2}{L_c}\right)[(-A_6S_1 + S_2 - S_3 + S_4)(\Delta_5 - \Delta_4) + A_7\Delta_1 - 2\Delta_2 + 2\Delta_3 - \Delta_4] - \left(\frac{3}{L_c}\right)[(-A_6S_1 + S_2 - S_3 + S_4 - 1)(\Delta_5 - \Delta_4)]}{(-A_6S_1 + S_2 - S_3 + S_4) - (-A_6S_1 + S_2 - S_3 + S_4 - 1)(2 + R)}, \quad (16)$$

$$\theta_4 = \left(\frac{3}{L_c}\right)(\Delta_5 - \Delta_4) - (2 + R)\theta_5, \quad (17)$$

$$\theta_3 = (\theta_4 + \theta_5)(A_6S_1 - S_2 + S_3) - \left(\frac{2}{L_c}\right)[(A_6S_1 - S_2 + S_3)(\Delta_5 - \Delta_4) - A_7\Delta_1 + 2\Delta_2 - \Delta_3], \quad (18)$$

$$\theta_2 = (\theta_4 + \theta_5)(-A_6S_1 + S_2) - \left(\frac{2}{L_c}\right)[(-A_6S_1 + S_2)(\Delta_5 - \Delta_4) + A_7\Delta_1 - \Delta_2], \text{ and} \quad (19)$$

$$\theta_1 = (\theta_4 + \theta_5)(A_6S_1) - \left(\frac{2}{L_c}\right)[(A_6S_1)(\Delta_5 - \Delta_4) - A_6\Delta_1]. \quad (20)$$

In Eqs.(18) to (22), constants R, S_1, S_2, S_3, S_4, A_4, A_5, A_6, and A_7 are given by:

$$\{R, S_i\} = \left\{ \frac{3I_{b5}\kappa_b L_c}{I_c \kappa_c L_b}, \frac{\sum_{j=i}^{j=5} m_j \Delta_j}{m_5 \Delta_5} \text{ for } 2 < i < 4 \right\}, \text{ and} \quad (21)$$

$$\{S_1, A_4, A_5, A_6, A_7\} = \begin{cases} \dfrac{\sum_{i=1}^{i=5} m_i \Delta_i}{m_5 \Delta_5}, 0, 4, 1, 2 & \text{for buildings with first storey column fixed at its base} \\ \dfrac{4\sum_{i=1}^{i=5} m_i \Delta_i}{m_5 \Delta_5}, 1, 3.5, 0.5, 1.5 & \text{for buildings with first storey column pinned at its base} \end{cases} . \tag{22}$$

2.2.3 *Size of Members in Buildings with Non-Uniform Size of Columns and Beams (Option 4)*

Sizes of columns and beam of the top storey are assumed in buildings with non-uniform beams and columns in all storeys. To start the initial proportioning, the following are assumed: (a) the depth of beams in the top storey between $L_b/12$ and $L_b/14$, and (b) the size of columns in top storey ($I_c > 2.5 I_{b5}$). The following are the steps to estimate sizes of beams and columns in all storeys, excluding those in the top storey:

Step 3-1: Estimate size of beams and columns in storeys 1 to 4 using Eqs.(23) and (24), as:

$$I_{ci} = A_8 I_{c5} \left[\frac{\sum_{j=i}^{5} \Delta_j m_j}{\Delta_5 m_5} \right] \left(\frac{\theta_5}{\theta_i} \right) \left[\frac{\dfrac{I_{b5}\kappa_b}{L_b} + \dfrac{I_{c5}\kappa_c}{6L_c}\left[1 - \dfrac{\theta_4}{\theta_5}\right]}{\dfrac{I_{b5}\kappa_b}{L_b} + \left(\dfrac{I_{c5}\kappa_c}{6L_c}\right) A_9} \right], \text{ and} \tag{23}$$

$$I_{bi} = \frac{L_b}{\kappa_b} \left[\left\{ \left(\frac{I_{ci}}{I_{c5}} \right) A_{10} + \left(\frac{I_{c,i+1}}{I_{c5}} \frac{\theta_{(i+1)}}{\theta_i} \right) \right\} \left(\frac{I_{b5}\kappa_b}{L_b} \right) + \left[\left(\frac{\theta_{i+1}}{\theta_i} \right) \left(\frac{\theta_5 + \theta_4}{3\theta_5} \right) - \left(\frac{2}{3} \right) \right] \left(\frac{I_{c(i+1)}\kappa_c}{L_c} \right) + A_{11} \right]. \tag{24}$$

In Eqs. (23) and (24), A_8, A_9, A_{10}, and A_{11} are given by:

$$\{A_8, A_9, A_{10}, A_{11}\} = \begin{cases} 1, \left(1 + \dfrac{2\theta_4}{\theta_5} - \dfrac{3\theta_{i-1}}{\theta_i}\right), 1, \left(\dfrac{1}{3}\right)\left(\dfrac{\theta_4}{\theta_5} - \dfrac{\theta_{i-1}}{\theta_i}\right)\left(\dfrac{I_{ci}\kappa_c}{L_c}\right) & \text{for } i > 1 \\ 4, 2\left(\dfrac{\theta_4}{\theta_5} - 1\right), 0.5, \left(\dfrac{\theta_4 - \theta_5}{6\theta_5}\right)\left(\dfrac{I_{ci}\kappa_c}{L_c}\right) & \text{for } i = 1, \text{column pinned at its base} \\ 1, \left(1 + \dfrac{2\theta_4}{\theta_5}\right), 1, \left(\dfrac{\theta_4}{3\theta_5}\right)\left(\dfrac{I_{ci}\kappa_c}{L_c}\right) & \text{for } i = 1, \text{column fixed at its base} \end{cases} . \tag{25}$$

In Eqs.(23) to (25), θ_i is as defined in Eq.(6). A more generalised equation for this proportioning option is avaliable in the literature [1].

2.3 Estimate of Member Sizes of *M*-bay *N*-storey frame with Identified $\{\phi\}_1$

Replicate the member sizes obtained for 1-bay *N*-storey frame to arrive at the member sizes of *M*-bay *N*-storey frame. Thus, using Eqs.(6) to (25), size of members are identified for Options 2, 3 and 4, so that the *MF* buildings have the desired M_1^*.

3. Seismic Performance Assessment of Buildings with Different Proportioning Scheme

3.1 Study Building and Method of Analyses

Eight 5-storey RC MF buildings are considered. Of the 8 buildings, 4 have their column bases *fixed* and other 4 their column bases *pinned*. The four buildings with the same fixity of the column base are proportioned by one of the four options listed in Table 1. In all 8 buildings, the following are same:

(a) Geometry (in plan and elevation) of the building (Fig. 4),

(b) Design gravity loads on buildings (uniform floor dead and live loads of 8.38 and 2.39 kN/m^2, respectively),

(c) Seismic weight of each storey (= 6921 kN, this includes self weight of partitions & members and the contribution from design loads)
(d) Design lateral force for which buildings are designed (=10% of the seismic weight of the building),
(e) Design Code used to design members present in buildings ([2]),
(f) Grade of Concrete (*M35*) and reinforcement (*Fe415*) used for the design of members,
(g) Size of beams and columns present in the top-storey of buildings,
(h) Square is the cross-section of columns and 400mm is the width of beams,
(i) Longitudinal reinforcement present in the top part of the beam (B_T in Table 4) is twice the quantity present in the bottom part of the beam, and
(j) Beams have a plastic rotation capacity of 0.025rad ([3]).

The sizes and longitudinal reinforcement in members in each of the 8 buildings are listed in Tables 3 and 4, respectively.

After the buildings are designed, their seismic performance is assessed using a representative *2D* frame (oriented along X-direction) of the buildings. The buildings are modelled and analyzed using commercial structural analysis software SAP 2000 (*version* 19) [4]. The key details of the building models are: (a) members are modeled using linear frame elements, (b) the ratio of *effective rigidity* to *gross rigidity* of beams and columns are 0.4 and 0.7, respectively, (c) beam-column joints are modeled using *elastic* elements with high rigidity, (d) seismic mass is lumped at the beam-column joints, (e) slabs are not modeled, and integral action of slab and beams is not considered when estimating flexural strength and stiffness of beams, (f) flexural yielding of beams and columns are considered, and all other modes of failure are precluded through capacity design and detailing of structural elements, (g) inelasticity in beams is modeled using lumped plastic hinges (M-θ) and hinges are assigned at a distance of 0.5D from the face of the column [5], (h) *Takeda model* is used to define hysteresis in beams [6], (i) inelasticity is modeled in columns using lumped plastic hinges (P-M-θ) hinges are assigned at a distance of 0.5D from the face of beam, and (j) *NTHA* is performed considering 5% *Rayleigh damping*.

Seismic performance of designed buildings is assessed using *NSA* and *NTHA*. In the said analyses, failure of building denotes the first instance of exhaustion of plastic rotation capacity (=0.025 rad) in beams. The lateral force-displacement response is estimated by pushing buildings laterally to failure using a load profile identical to the fundamental lateral translational mode shape of oscillation $\{\phi\}_1$ [7]. The dynamic seismic performance of buildings is assessed by subjecting each building to *seven* ground motions that are scaled to design intensity of shaking (=0.4g) (Table 5). This study uses the *spectral scaling* method to scale ground motions [8]. Further, the incremental dynamic seismic performance of each building is assessed by subjecting the buildings to three higher intensities of shaking (namely 0.6g, 0.8g, and 1.0g).

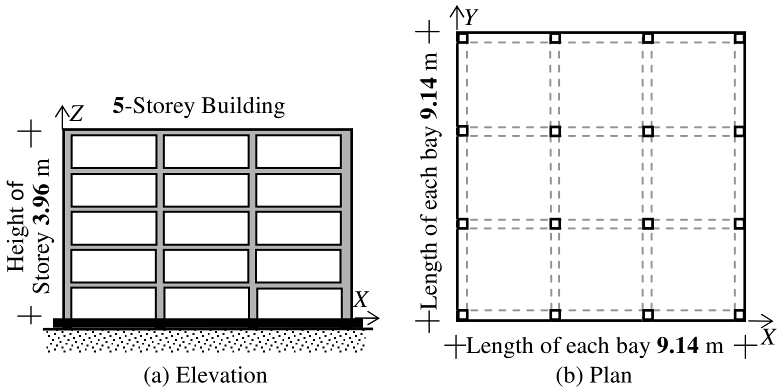

Fig. 4 – (a) Elevation and (b) Plan of 5-Storey building

Table 3 –Depth of members (mm) (*B* and *C* denotes *Beams* and *Columns*, respectively)

Storey	*Option* **1A**		*Option* **1B**		*Option* **2A**		*Option* **2B**		*Option* **3A**		*Option* **3B**		*Option* **4A**		*Option* **4B**	
	B	*C*	*B*	*C*	*B*	*C*	*B*	*C*	*B*	*C*	*B*	*C*	*B*	*C*	*B*	*C*
5						800		800	750		750		750	800	750	800
4						850		850	950		850		810	870	810	870
3	750	800	750	800	750	[#]850	750	[#]850	900	800	860	800	820	880	820	880
2						[#]850		[#]850	940		860		700	[#]880	790	[#]880
1						[#]850		[#]850	620		810		500	[#]880	690	[#]880

[#] Considering practicality, uniform size of columns is provided between the storey, which requires the maximum size of column, and the first storey; although the closed-form equations warrants smaller size of columns between the said storeys.

Table 4 – Longitudinal Reinforcement (%) present in (a) the top part of beams and (b) columns

Storey	*Option* **1A**		*Option* **1B**		*Option* **2A**		*Option* **2B**		*Option* **3A**		*Option* **3B**		*Option* **4A**		*Option* **4B**	
	B_T	*C*	B_T	*C*	B_T	*C*	B_T	*C*	B_T	*C*	B_T	*C*	B_T	*C*	B_T	*C*
5	1.19		1.21		1.17	1.23	1.21	1.23	0.96		1.04		1.11	1.23	1.12	1.23
4	1.44		1.50		1.43		1.56		1.00		1.19		1.32	1.95	1.38	1.95
3	1.68	1.92	1.85	1.92	1.65	1.70	1.85	1.70	1.20	1.92	1.38	1.92	1.56		1.67	
2	1.74		2.13		1.67		2.13		1.32		1.68		1.80	1.91	2.09	1.91
1	1.52		2.50		1.42		2.48		1.64		2.12		2.10		2.50	

Table 5 – Characteristics of the suite of 7 ground motions considered in this study

No.	*Event*	*Station*	*Year*	M_w	*PGA* (g)	*Epicentral distance* (km)
1	Cape Mendocino	Eureka-Myrtle and West	1992	7.10	0.154	44.6
2		Fortuna-Blvd			0.116	23.6
3	Landers	Fire Station	1992	7.30	0.152	24.9
4		Palm Springs Airport			0.076	37.5
5		Desert Hot Spring			0.171	23.2
6	Northridge	Lake Hughes#1	1994	6.70	0.087	36.3
7		Downey-Co Maint Bldg			0.230	47.6

3.2 Response of Buildings

3.2.1 *Dynamic Characteristics of Buildings*

In buildings with fixed column bases, (a) difference persists between the targeted $\{\phi\}_1$ and $\{\phi\}_1$ of buildings and (b) *actual* M_1^* is less than *target* M_1^* (=90%) (Fig. 5(a)). In contrast, in buildings with pinned column bases, (a) $\{\phi\}_1$ of buildings match well with *target* $\{\phi\}_1$, and (b) *actual* M_1^* is either equal or more than the *target* M_1^* (Fig. 5(b)). Further, among buildings with fixed column bases, the maximum difference in: (a) the fundamental lateral translational periods (T_1) is 13%, and (b) M_1^* (Fig. 5) is 4.3%. Thus, the influence of base fixity is more significant on the dynamic characteristics of buildings than on member sizes (Fig. 5). Furthermore, buildings **3A** and **3B** (with members sized to have *uniform* column and *non-uniform* beams) have the smallest natural period among buildings with fixed and pinned column bases, respectively.

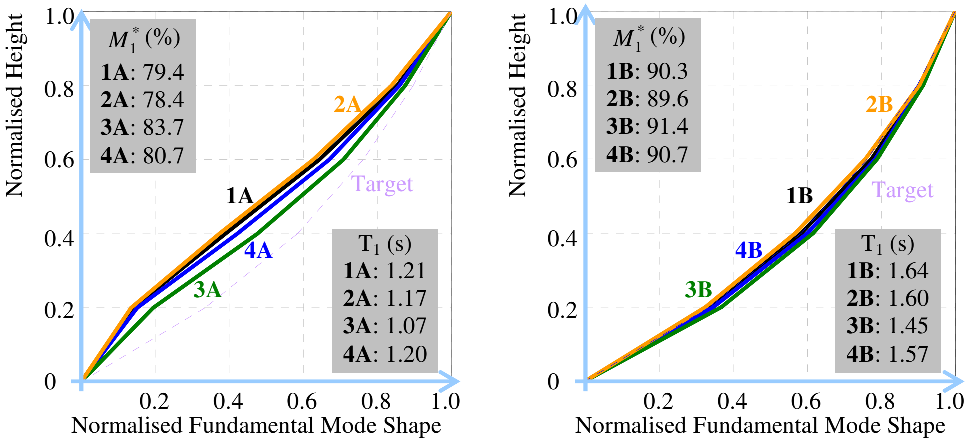

Fig. 5 – Dynamic characteristics of buildings in which first storey columns are (a) fixed at its base (b) pinned at its base

3.2.2 *Results: Nonlinear Static Analysis*

On an average, the lateral displacement capacity (Δ_u) of buildings with fixed column bases is 15% more than that of buildings with pinned column base (Fig. 6(a)). Following are the reasons for increased Δ_u of buildings with fixed column bases: (a) efficient utilization of plastic rotation capacity of beams (*i.e.*, $\theta_{pbdemand}/\theta_{pbcapacity}$) and (b) lesser instance of detrimental column yielding (Fig. 6(b)). Further, among buildings with same base fixity:

(a) Buildings **4A** and **4B** (with *Non-uniform Beams* (*NB*) and *Non-uniform Columns* (*UC*)) utilise the plastic rotation capacity in beams the most; hence, Δ_u is the largest in them of all buildings,
(b) Building **3B** (with *NB* and *uniform columns* (*UC*)) utilizes plastic rotation capacity in beams the least; hence, Δ_u is the least in it of all buildings,
(c) NO instance of detrimental column yielding is observed in Buildings **4A** and **4B**, and
(d) Detrimental column yielding (in 5 out of 7 cases) is noticed most in Buildings **3B** (with *NB* and *UC* size).

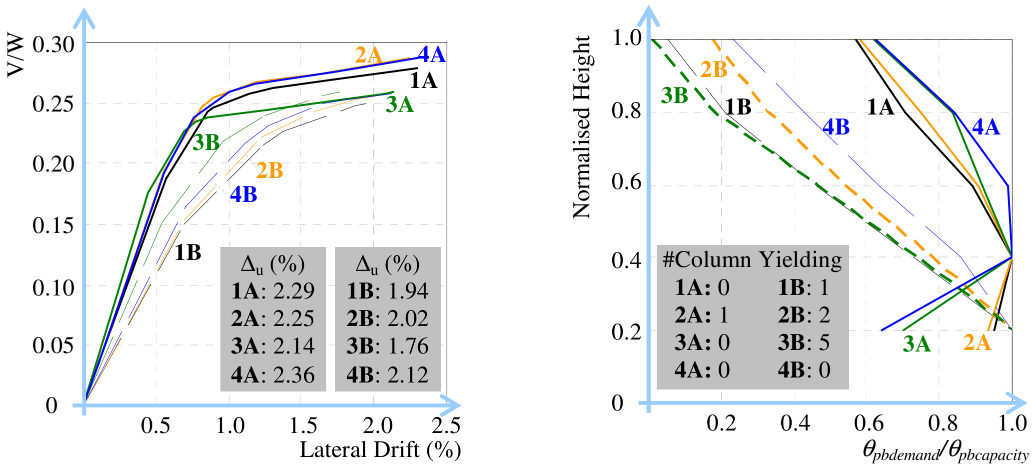

Fig. 6 – (a) Lateral force-displacement response of buildings (b) ratio of plastic rotation demand to plastic rotation capacity of beams (*inset*: number of columns that yields)

3.2.3 *Results: Nonlinear Time History Analysis*

All buildings, with the exception of building **1B**, resist all seven ground motions scaled to design intensity of shaking (Table 5). In buildings with column bases pinned, the number of GMs resisted by Building **1B** drastically falls from 6 to 1 as the intensity of shaking increases to *0.6g*; also, the instance of column yielding increase from 1 to 7 at this level of shaking. This suggests that adopting both columns and beams of uniform size is NOT a good option when proportioning members of buildings with pinned column bases.

In contrast to buildings with column bases pinned, buildings with column bases fixed resist: (a) intensity of shaking of 1.0g, and (b) the impose ground shaking with fewer instances of column yielding (Table 5). Further, in the study buildings: (a) buildings **4A** and **2A** resist most number of ground motions of higher intensities (0.6g, 0.8g and 1.0g), (b) building **1A** resisted only 2 ground motions scaled to a intensity of 1.0g, and (c) building **3A** resisted the most instances of column yielding. One reason for the better performance of buildings **4A** and **2A** is the efficient utilization of plastic rotation capacity in members along the building height (Fig. 7).

Table 5 – Number of Ground Motions (GMs) (a) sustained by the building and (b) that leads to yielding of columns

	GMs Sustained								*GMs that lead to Column Yielding*							
Intensity of shaking	**1A**	**2A**	**3A**	**4A**	**1B**	**2B**	**3B**	**4B**	**1A**	**2A**	**3A**	**4A**	**1B**	**2B**	**3B**	**4B**
Design level: 0.4*g*	7	7	7	7	6	7	7	7	0	0	1	0	1	7	1	0
0.6*g*	7	7	6	7	1	2	6	6	0	0	5	0	7	7	6	6
0.8*g*	6	6	5	6	0	1	2	1	1	2	6	2	7	7	7	7
1.0*g*	2	6	4	6					2	5	7	4				

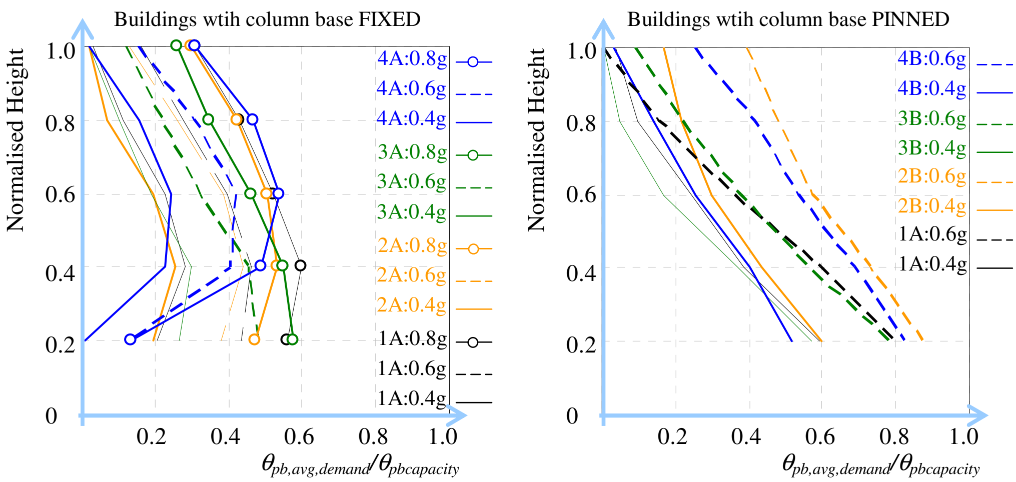

Fig. 7 – Variation of the ratio of average plastic rotation demand to the plastic rotation capacity of beams along the building height when buildings are subjected to increasing intensity of shaking

4. Conclusion

Proportioning size of members in buildings influences the seismic performance of buildings; also, degree of fixity of column bases plays an important role. This paper presents *closed-form expressions* to estimate the member sizes in buildings such that the building has a desired mass participation factor associated with the fundamental lateral translational mode of oscillation. Of the different options for proportioning the members, under strong earthquake ground shaking, buildings with:

(a) *non-uniform* beams and *non-uniform* columns exhibit *best* seismic response, and those with

(b) *uniform* beams and *non-uniform* columns are the s*econd best*,

Also, compared to buildings with pinned column bases, the buildings with fixed column bases perform better when resisting strong earthquake shaking. Further, in buildings with pinned column bases, buildings with *uniform* beams and *uniform* columns exhibit the *poorest* seismic performance.

5. Reference

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