EXPERIMENT ON TORSIONAL RESPONSE OF SYMMETRIC STRUCTURE INDUCED BY *Q–Δ* RESONANCE

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Abstract

There are few published studies on torsional response of a symmetric structure, possibly owing to a low perceived risk. However, a symmetric structure could experience torsional vibration due to the torque generated by the inertial force (*Q*) perpendicular to the horizontal displacement (*Δ*), because of geometric nonlinearity. The authors' research group has called this the "*Q–Δ* effect," and there are multiple resonance conditions of this torsional vibration. If a building satisfies one of these resonance conditions, a strong ground motion in two translational directions could induce the torsional response. In this study, we conducted an experiment to validate this *Q–Δ* resonance using a symmetric single-layer specimen with no eccentricity.

First, resonance conditions were derived for the setup of the validation experiment. The equations of motion and vibration were based on our previous studies. In this study, we considered the input ground motion of two superposed sine waves, with frequencies equal to the two translational modes' natural frequencies. With respect to the torsional response, we derived four resonance conditions for the torsional mode's natural frequency.

A specimen consisting of a slab, four columns, and two weights on the slab was fabricated focusing on one of the four resonance conditions in the torsional mode. The natural frequency in the torsional mode could be changed by moving the two weights on the slab. Adjusting the moment of inertia around the vertical axis passing through the center of gravity of the slab, ten patterns of natural frequencies were obtained from the one specimen.

To understand the specimen's structural and vibration characteristics, we conducted horizontal loading and free vibration experiments. All the experiments were conducted in two translational and one torsional directions. The vibration characteristics of the specimen were identified from the results of the free vibration experiments.

Using a uniaxial shaking table, we conducted shaking table experiments, in which the input acceleration was two superposed sine waves. We first verified that the torsional response was very small when the specimen was excited in translational mode directions. In addition, we verified that the vibration characteristics of the specimens did not change after shaking table tests for each pattern of the specimen configuration. The validation experiments were performed by setting the specimen in the 45-degree direction from the principal axis (translational mode) direction on the shaking table.

The translational response even when the natural period in the torsional mode was changed, the. The amplitude of the torsional response, however, did change, and it increased further when the torsional mode's natural frequency was closer to the theoretically predicted resonance frequency. This result confirms that the torsional vibration of a symmetric structure is excited by an inertial force perpendicular to the horizontal displacement. However, the experimental result was larger than the simulated response based on a theoretical model. Future studies will use a specimen more suitable for this analysis and revise the prediction models to account for the second-order mode.

Keywords: Q–Δ effect; Torsional resonance; Shaking table test; Geometric nonlinearity; Symmetric structure

1. Introduction

Numerous studies have been conducted on torsional response of structures [1], but few of them have considered a symmetric structure, possibly because of a low perceived risk. In fact, the Building Standards Law of Japan provides for considering a torsional vibration only for asymmetric structures above a certain minimum eccentricity. However, a symmetric structure could experience torsional vibration due to the torque generated by the inertial force (*Q*) perpendicular to the horizontal displacement (*Δ*), because of geometric nonlinearity

(Fig. 1). The authors' research group [2] named this the "*Q–Δ* effect," and showed that there are multiple resonance conditions of this torsional vibration. If a building satisfies one of these resonance conditions, strong ground motion in two translational directions could induce a torsional response. In this study, we conduct an experiment to validate this *Q–Δ* resonance using a symmetric, single-layer specimen with no eccentricity.

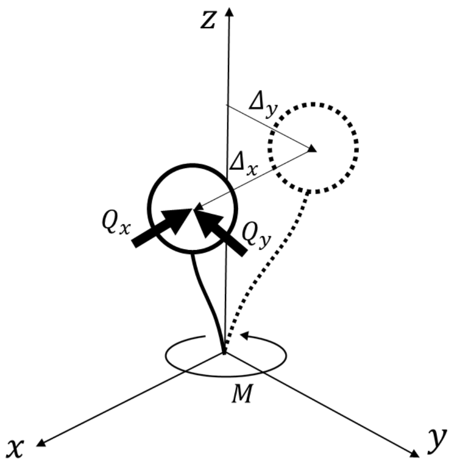


Fig. 1 – Torque generated by the *Q–Δ* effect

2. Derivation of resonance conditions

Resonance conditions were derived to set up the validation experiment. The vibration equations and derivation of resonance conditions were based on our previous studies [2, 3].

We considered a symmetric, single-story structure (Fig. 2). Relating that structure to the representation in Fig. 1, the origin represents the structure's center of gravity, and the horizontal displacement of the top slab is represented by the distance in the *x*- and *y*-axis directions and the rotation angle θ around the *z* axis, in an anti-clockwise direction from the *x* axis. With a naïve inference using a primitive model that columns can be replaced by linear springs in the x, y, and θ directions [2], the torque around the vertical axis generated at the center of the building foundation by the inertial force perpendicular to the horizontal displacement is represented as M (Fig. 1):

$$M = m(\ddot{x}_0 + \ddot{x})y - m(\ddot{y}_0 + \ddot{y})x \tag{1}$$

where the counterclockwise direction is defined as positive. In reality, the torque is generated due to geometric nonlinearity in the stiffness of the columns [3, 4].

If the columns are assumed to be an elastic Euler column and the natural circular frequency and damping factor in each direction are represented by $\omega_x, \omega_y, \omega_\theta, \xi_x, \xi_y$, and ξ_θ, and moment of inertia and radius of gyration by I and r_θ, respectively, the structure generates displacement (x, y, θ) in response to ground acceleration $(\ddot{x}_0, \ddot{y}_0, 0)$, with translational and rotational vibration equations as follows [3, 4]:

$$\ddot{x} + 2\xi_x\omega_x\dot{x} + \omega_x^2 x = -\ddot{x}_0 - \frac{1}{4}(\omega_x^2 - \omega_y^2)y\theta \tag{2}$$

$$\ddot{y} + 2\xi_y\omega_y\dot{y} + \omega_y^2 y = -\ddot{y}_0 - \frac{1}{4}(\omega_x^2 - \omega_y^2)x\theta \tag{3}$$

$$\ddot{\theta} + 2\xi_\theta\omega_\theta\dot{\theta} + \omega_\theta^2\theta = -\frac{1}{4r_\theta^2}(\omega_x^2 - \omega_y^2)xy \tag{4}$$

The right-hand sides of Eqs. (2) to (4) constitute the geometric nonlinearity terms.

The input ground acceleration was assumed to be a double sine wave (two superposed sine waves) with frequencies equal to the natural frequencies of the two translational modes.

$$\ddot{x}_0 = \sin p_x t + \sin p_y t \quad (5)$$

$$\ddot{y}_0 = \sin p_x t + \sin p_y t \quad (6)$$

From these responses, four resonance conditions for the translational and rotational directions were derived. The resonance conditions in each direction are described below.

Translational directions:

$$\omega_x = p_x \quad (7)$$

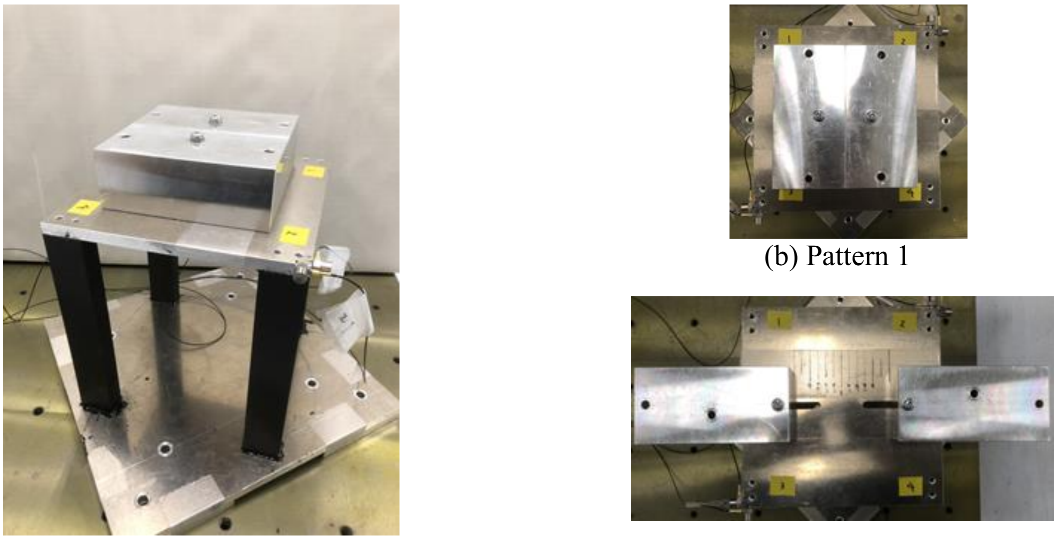
$$\omega_y = p_y \quad (8)$$

Rotational direction:

$$\omega_\theta = \begin{cases} 2p_x \\ p_x + p_y \\ |p_x - p_y| \\ 2p_y \end{cases} \quad (9)$$

3. Design and fabrication of the specimen

We designed and fabricated a symmetric, single-layer specimen with no eccentricity for the vibration experiment (Fig. 2). The specimen consisted of a slab, four columns, and two weights on the slab (Table 1), allowing investigation of each of the four resonance conditions in the torsional mode. In the study, we focused on the condition $\omega_\theta = p_x + p_y$, the torsional response which can appear and increase remarkably in real buildings.



(a) General view
(b) Pattern 1
(c) Pattern 10

Fig. 2 – Specimen

We designed the specimen such that the natural frequency in the torsional mode could be altered by moving the two weights on the slab. Adjusting the moment of inertia around the vertical axis passing through the center of gravity of the slab, ten different natural frequencies were realized using the same specimen components. We denoted the weight configuration with the weights placed closest to the center, and hence the highest natural frequency, "Pattern 1." Patterns 2 to 5 correspond to the weights being moved outward along tapped holes; for Patterns 6 to 10, the weights were rotated 90°. Four accelerometers were attached at two diagonally opposite corners of the slab. Table 2 shows each pattern's physical parameters.

Table 1 – Main components of the specimen

Name	Material	Mass [kg]	Dimension [mm]	Number
Slab	A5052	1.06	200×200×10.0	1
Column	CR (A45)	0.21	33.0×20.0×200	4
Weight	A5052	1.15	135×67.5×48.0	2
Pedestal	A5052	2.40	300×300×10.0	1

Table 2 – Mechanical parameters of the specimen

Pattern	Distance between the center of gravity and the weight [mm]	Moment of inertia I $\times 10^{-3}$ [kg·m²]	Radius of gyration r_θ $\times 10^{-3}$ [m]
1	33.8	20.6	73.3
2	41.8	22.1	75.8
3	49.8	23.8	78.7
4	57.8	25.8	82.0
5	66.8	28.5	86.1
6	82.5	34.0	94.1
7	90.5	37.3	98.5
8	98.5	40.8	103
9	108	45.2	109
10	117	50.0	114

4. Evaluating the mechanical characteristics of the specimen

4.1 Linear Properties

4.1.1 Method for loading experiments

Figure 3 is a schematic of the horizontal loading experiment to investigate the force–displacement relation in the translational direction. We attached a load cell to the specimen and applied a horizontal force through the load cell, with the displacement measured by a laser displacement meter.

Figure 4 shows the schematic of the rotational loading experiment to investigate the torque–rotation angle relation. Two wires rotated the specimen: one winding a disk that was pulled directly, and the other winding a disk that pulled via a sheave. Two wire tensioning forces generated torque with no translational displacement. The rotation angle was calculated from the position of the laser light on the screen, reflected by a reflector on the specimen, using the following equation:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{a}{l}\right) \tag{10}$$

where a and l are the shift in the laser light position and the distance between the reflector and screen, respectively. To check the translational displacement's influence on the torque–rotation angle relation, a second rotational-direction experiment was performed following a forced translational displacement.

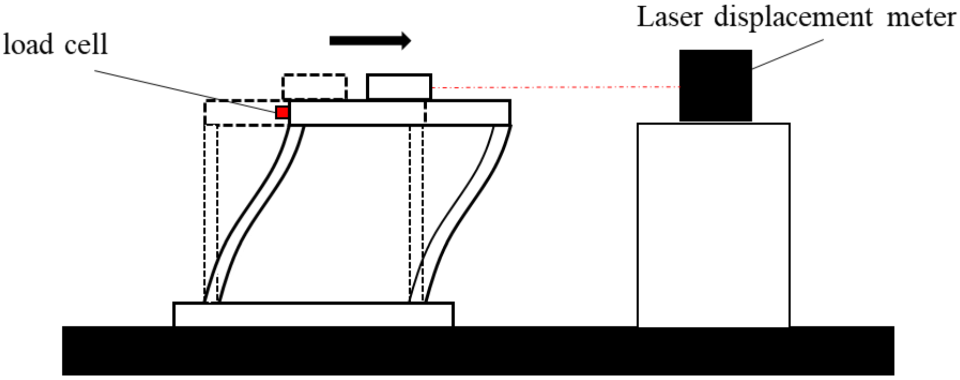


Fig. 3 – Horizontal loading experiment

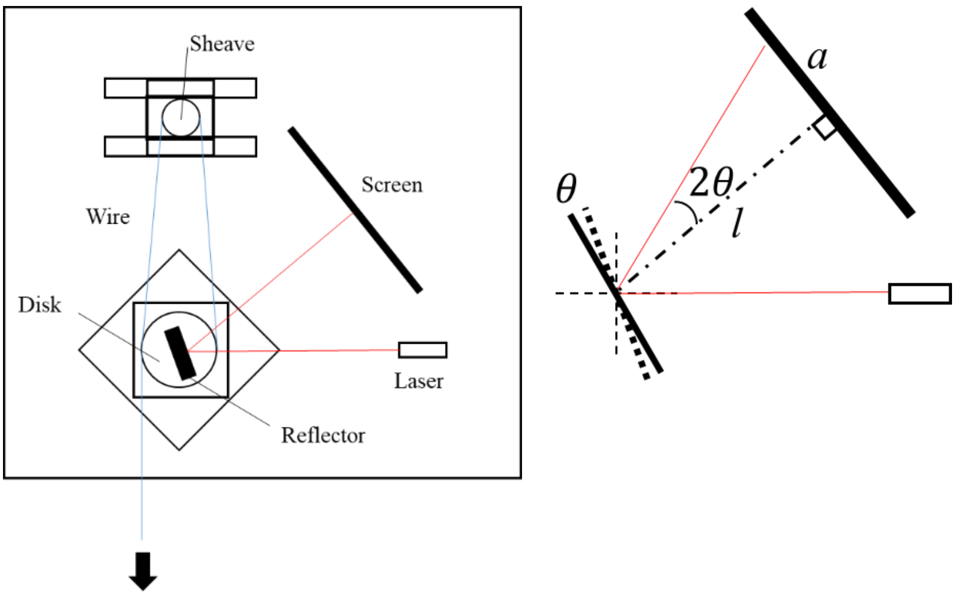
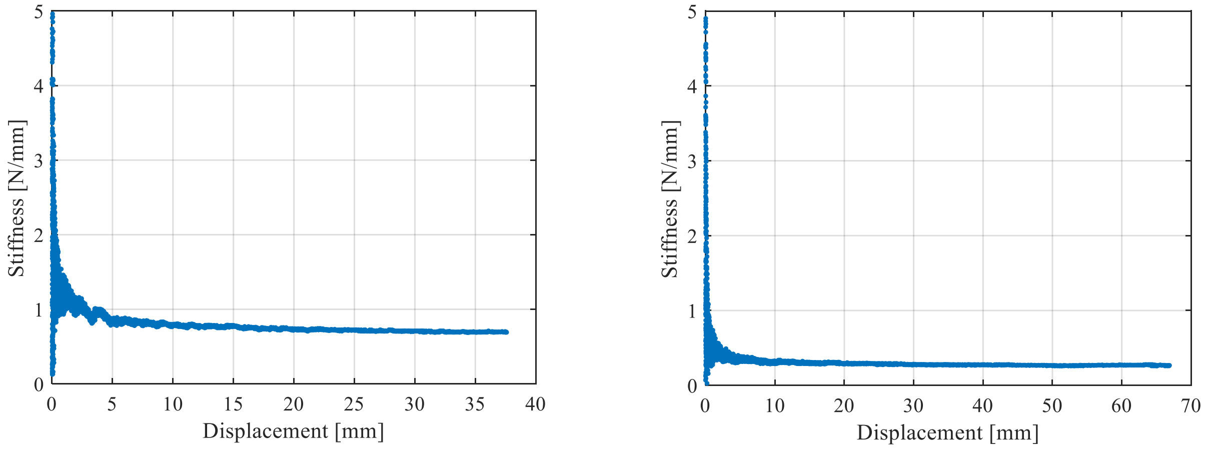


Fig. 4 – Rotational loading experiment

4.1.2 Experimental results

Figure 5 shows the horizontal loading experiment results. The vertical axis represents the secant stiffness calculated from the measured force and displacement. Data variation near zero displacement is the influence of noise when a small force is applied. In the range of small displacements up to 10 mm, the stiffness decreases along with the displacement in both x- and y-directions. On the other hand, when the displacement is more than 10 mm, the stiffness is relatively constant.

The rotational loading experiment results, with and without forced translational displacement, are shown in Figs. 6 and 7, respectively, with the vertical axis indicating the secant stiffness as in the translational loading experiment. The stiffness is relatively constant when the rotational angle exceeds 1° in all cases. When forced translational displacement is applied, the stiffness decreases by approximately 16%.



(a) *x* direction (b) *y* direction

Fig. 5 – Force–displacement relation

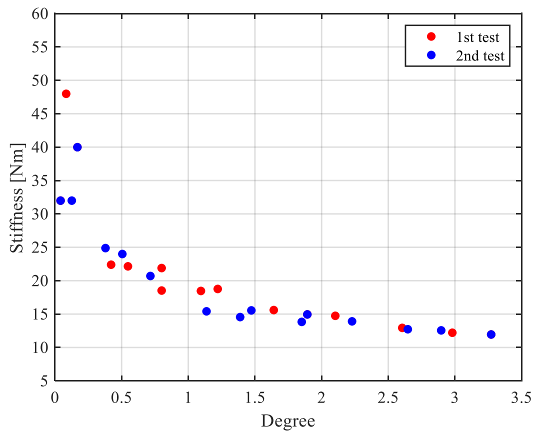
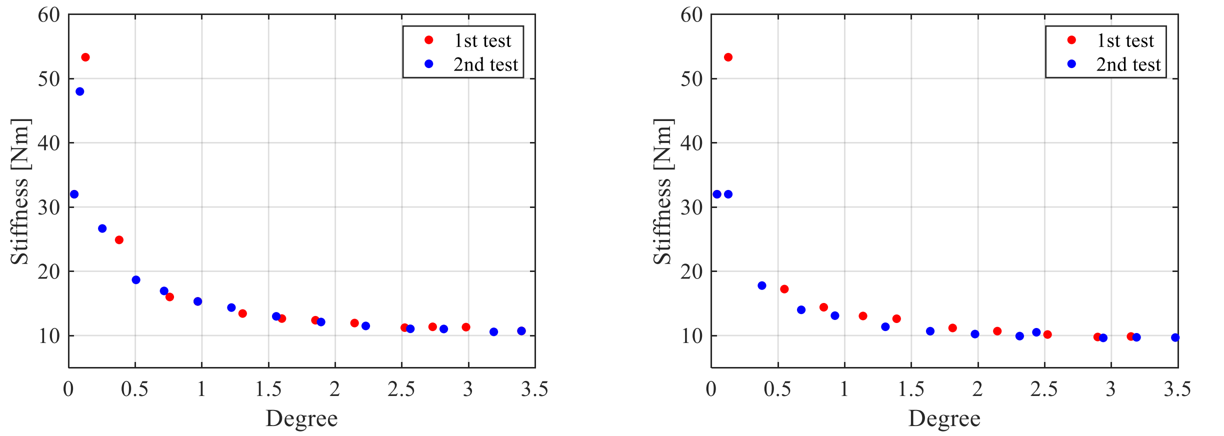


Fig. 6 – Torque–rotational angle relation without forced translational displacement



(a) *x* direction (b) *y* direction

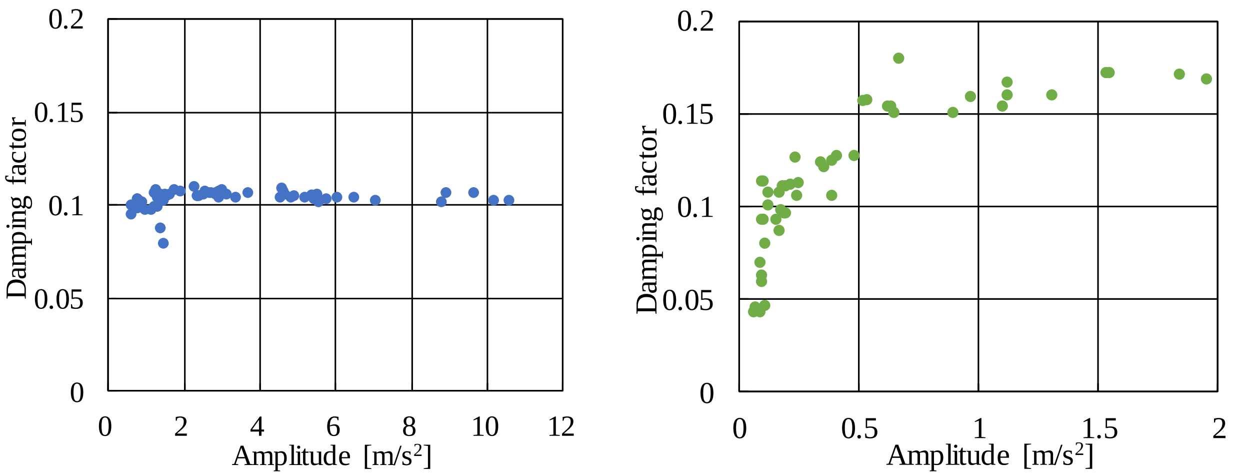
Fig. 7 – Torque–rotational angle relation with forced translational displacement

4.2 Amplitude dependency

4.2.1 Free vibration experiments

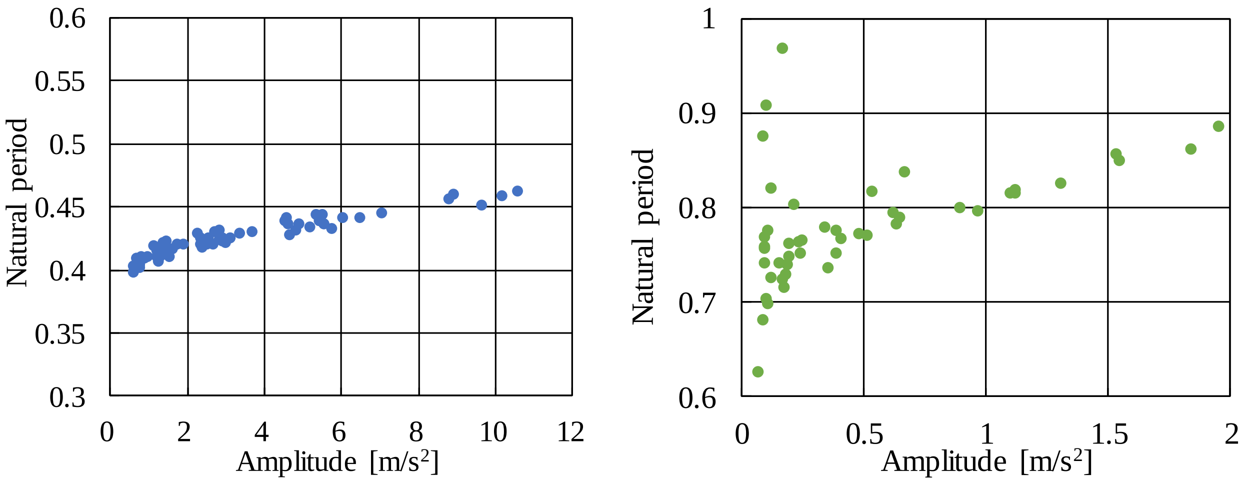
Free vibration experiments were performed in each direction to evaluate the specimen's amplitude dependency. In the two translational directions, the experiments were repeated twelve times, with initial displacements ranging from 10 to 50 mm. In the rotational direction, the experiments were performed for each weight configuration, with initial displacements of 5° and 10°.

The results confirm that the damping factor is almost constant and independent of both the acceleration and angular acceleration amplitude (Fig. 8), but the natural period increases with the amplitude in each direction (Fig. 9).



(a) *x* direction (b) *y* direction

Fig. 8 – Amplitude–damping factor relation



(a) *x* direction (b) *y* direction

Fig. 9 – Amplitude–natural period relation

4.2.2 Identification of vibration characteristics

The specimen's vibration characteristics are identified from the results obtained by the free vibration experiments. The damping factor is evaluated from the average of the measurements. With respect to the natural period, a linear approximation $T = a_1 A_{\max} + a_2$ is applied for the relationship between the natural period and the amplitude. It is calculated by substituting the maximum acceleration assumed for $A_{\max}$; the predicted maximum accelerations in the shaking table experiments (described below) are 5.0 m/s^2 and 1.5 m/s^2 in the x and y directions, respectively. Data with an acceleration less than 0.5 m/s^2 were excluded in the above linear regression because the influence of noise was considered to be too large.

Table 3 shows the vibration characteristics in each direction. With respect to the rotational direction, we use the maximum angular acceleration, measured in the shaking table test described in the following section, to estimate the natural period and natural circular frequency. From the results in the two translational directions, the resonant circular frequency in the rotational direction is predicted by $\omega_\theta = 14.4 + 7.41 = 21.8$ rad/s. The natural period, therefore, is 0.288 s.

Table 3 – Vibration characteristics of the specimen

Vibration characteristics	*x* direction	*y* direction	*θ* direction
Natural period [s]	0.436	0.846	—
Damping factor	0.104	0.161	0.103
Natural circular frequency [rad/s]	14.4	7.41	—

5. Induced torsional vibration

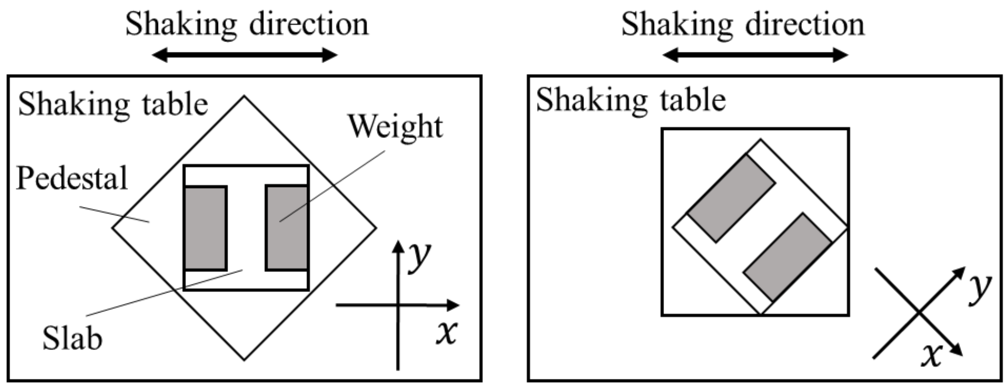
5.1 Method for shaking table experiments

To verify the *Q-Δ* resonance phenomenon, we conducted experiments using a uniaxial shaking table. The input acceleration is a double sine wave with circular frequencies in the two translational directions calculated in the previous section, i.e., $p_x = 14.4$ rad/s and $p_y = 7.41$ rad/s. The experiment consisted of three steps (described below), repeated for all ten weight configurations.

In Step 1, we verified the specimen did not have unintended eccentricity, i.e., the torsional response is very small when the specimen is excited in translational directions. To confirm the torsional vibration during the excitation in the x direction, the specimen was positioned and shaken as shown in Fig. 10(a). Similarly, for the y direction, the specimen was rotated through 90° in Fig. 10(a) and shaken.

In Step 2, the specimen was rotated through 45° and shaken as shown in Fig. 10(b) to measure the induced torsional vibration.

Step 3 repeated Step 1, to confirm that the vibration characteristics of the specimens did not change after the shaking table experiments.

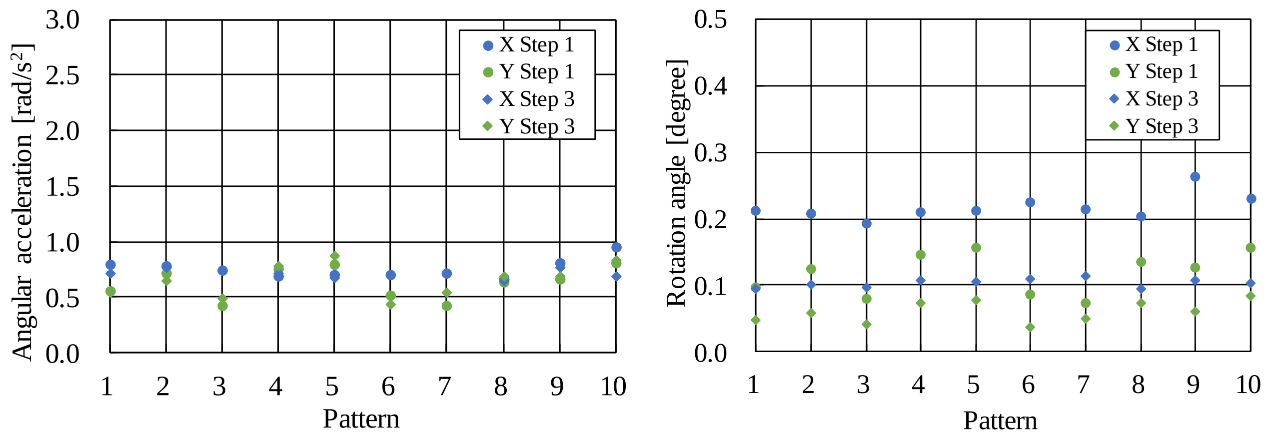


(a) Shaking in x direction (Steps 1 and 3) (b) Shaking in 45° direction (Step 2)

Fig. 10 – Position of the specimen

5.2 Experimental result

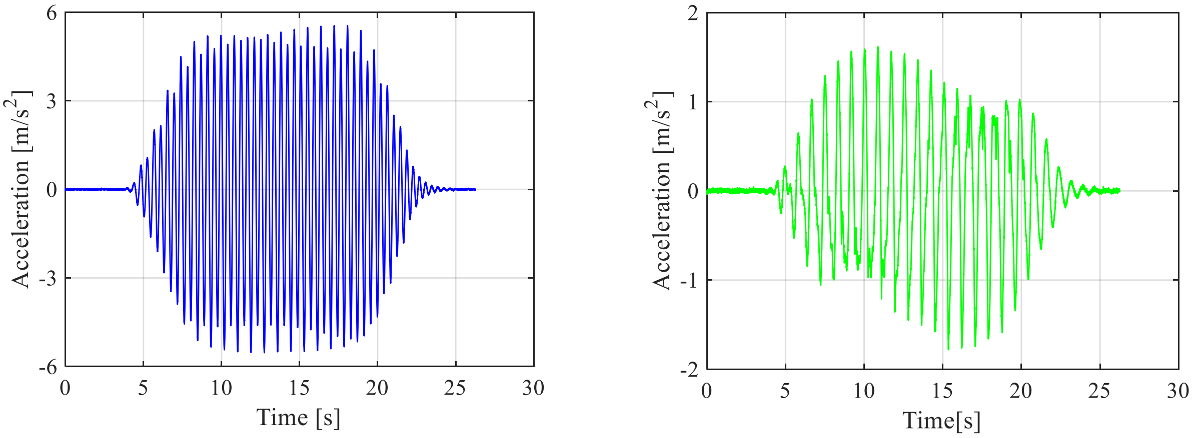
The results of Steps 1 and 3 are shown in Fig. 11. Rotation angles are calculated by integrating the angular accelerations twice. A slight torsional response occurred even when the specimen was subjected to translational excitation. This could be caused by initial eccentricity from the processing stage of the specimen and wobble of the shaking table in the direction perpendicular to the intended excitation. Because there is no large difference in the torsional response in each weight configuration, eccentricity is due, not to the asymmetric arrangement of the weights, but to the rubber columns. However, we considered that the torsional response magnitude was not large enough to affect the verification experiment of induced torsional response. Further, the responses in Steps 1 and 3 were almost equal, confirming that the damage to the specimen due to the excitation was negligible.



(a) Angular acceleration (b) Rotation angle

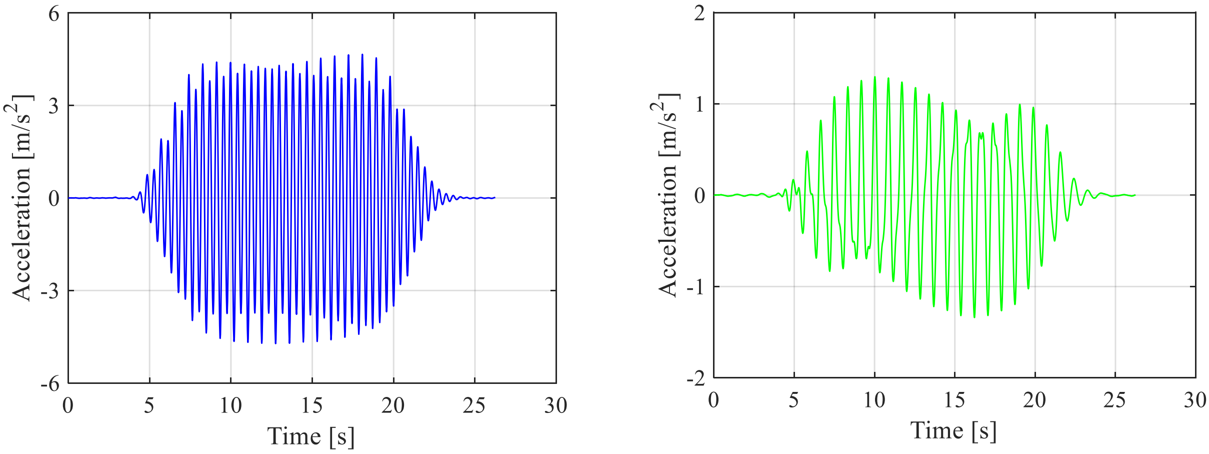
Fig. 11 – Torsional response during excitations in x and y-directions

Figure 12 shows the results of Step 2, the verification of induced torsional response. Firstly, the observed acceleration response time histories are shown in Fig. 12(a) and (b) for the two translational directions x and y, respectively. Figure 13 shows the acceleration response time histories simulated with the Newmark β method (β = 1/6), in which geometric nonlinearity terms in the translational directions, Eqs. (2) and (3), are neglected for simplicity. Comparing Figs. 12 and 13, the observed response is larger in both directions.



(a) *x* direction (b) *y* direction

Fig. 12 – Observed acceleration response time histories



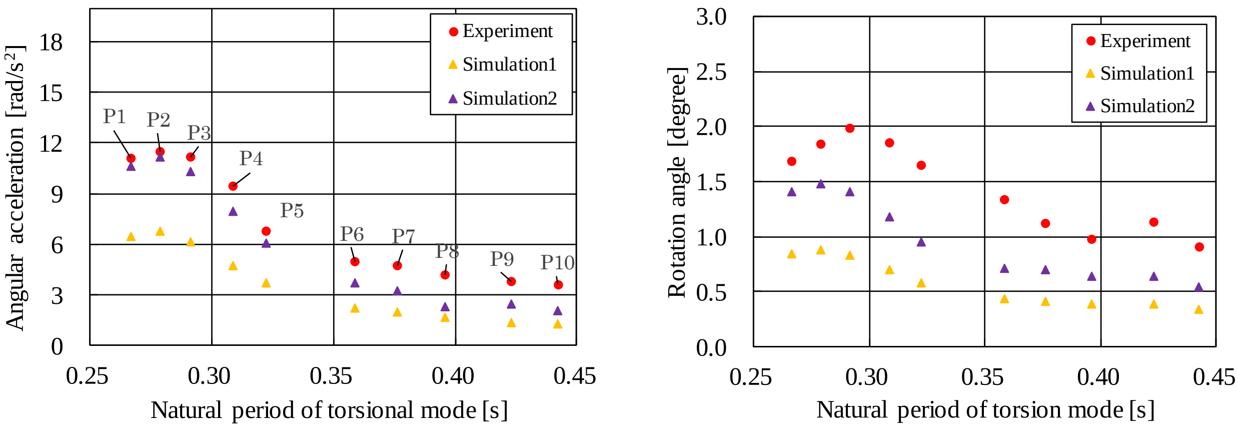
(a) *x* direction (b) *y* direction

Fig. 13 – Simulated acceleration response time histories

Finally, the results of the induced torsional response are shown in Fig. 14. The horizontal axis represents the natural period in the rotational direction for each weight pattern, which is calculated from the linear approximation formula evaluated in Section 4 and the stiffness reduction described in Section 3. We observe that the torsional response changes with the natural period change in the rotational direction; note also that the response in the translational direction does not change, even if the natural period changes in the rotational direction. At the predicted resonance point, the torsional response increased significantly to a maximum value, confirming that torsional response is induced in a symmetric structure due to the Q–Δ resonance. However, in the case of the angular acceleration response, in the time history analysis (Simulation 1) the simulated response is smaller than the observed response. This could be because the damping factor of the numerical model is smaller than the calculated damping factor in the translational mode. In this study, the damping factor was calculated from the results of unidirectional free vibration, but in the verification experiment it oscillated in two directions. It was confirmed that the stiffness was reduced by displacement in two directions. The damping factor was, therefore, re-identified based on the experimental response. The simulation was performed, incrementing the damping factor by 0.005 to find the value that minimizes the root mean square error of the effective value of the translational acceleration. The corrected damping factors are shown in Table 4. A second time history analysis of the torsional response (Simulation 2) was based on the re-identified damping factor and showed a much closer agreement with the observed angular acceleration. However, although there was also some improvement in the rotation angle compared to Simulation 1, the Simulation 2 values were still smaller than the experimental responses. This could be due to the initial eccentricity of the specimen or because the damping factor is small in the rotation direction. Although this exercise was successful in verifying the torsional vibration excitation, there were many factors causing divergences from the prediction model. Future studies will need to fabricate specimens from more suitable materials.

Table 4 – Damping factor

Direction	Initially identified	Re-identified
x	0.104	0.085
y	0.161	0.110



(a) Maximum angular acceleration (b) Maximum rotation angle

Fig. 14 – Maximum torsional response during excitations in 45° direction

6. Conclusion

In this study, we conducted experiments to validate *Q-Δ* resonance, where a torsional response is induced by the inertial force perpendicular to the horizontal displacement. We derived the resonance conditions' torsional response with an input ground motion consisting of two sine waves. Focusing on one of the four resonance conditions, a single-story specimen was fabricated, and experiments were performed to confirm its mechanical and vibration characteristics. Horizontal loading experiments were performed to confirm the specimen's linear property; the stiffness decreased along the small range of displacement in both translational and rotational directions, but it became constant when the displacement exceeded a certain level. It was also observed that the stiffness in the rotational direction reduced when the specimen was subjected to a forced displacement. Free vibration experiments were performed to identify the specimen's vibration characteristics. As a result, we confirmed that the damping factors were constant regardless of the vibration amplitude, and the natural period increased with the amplitude. The verification experiment of induced torsional response confirmed that the torsional response increased at the predicted resonance point and reached a maximum. This verifies that the torsional response is induced by the *Q–Δ* effect, even in a symmetric structure. Future studies will use a specimen more suitable for this analysis and revise the prediction models to account for the second-order mode.

Acknowledgements

This work was financially supported by JSPS KAKENHI Grant Number 16H04455 and SECOM Science and Technology Foundation.

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