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A Simple Method to Identify Torsional Flexibility in Buildings without performing Detailed Structural Analysis

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Abstract

Torsion in buildings arises from *torsional eccentricity* and *torsional flexibility*. Torsional *eccentricity* is reflected by *normalised eccentricity ratio (e/B)* and torsional *flexibility* by *natural period ratio* ($\tau = T_\theta / T$), the ratio of uncoupled Torsional Natural Period (T_θ) to uncoupled Translational Natural Period (T). τ depicts the mass and stiffness distributions in plan, and hence can be expressed also as ratio ($\tau = r_m/r_K$) of radius of gyration (r_m) of *mass* and radius of gyration (r_K) of *stiffness*. Buildings with τ*>1* are called *torsionally flexible buildings*, and those with τ*<1, torsionally stiff buildings*. In torsionally flexible buildings, the first natural mode of oscillation is the torsional mode of oscillation.

This paper presents a *simple, approximate method*, which helps to identify torsional flexibility in a building, while proportioning the building, even before detailed structural analysis is performed. This method is based on basic principles of mechanics, and uses the *Parallel Axis Theorem* employed in calculation of *Moment of Inertia* of cross-sections of members. The steps involved are (Fig.1): (1) consider one typical storey of the building, and assume rigid diaphragm action and uniform distribution of mass; (2) estimate r_m using plan geometry; (3) estimate r_K of all vertical elements (columns and/or walls) together in a storey using *translational* K_θ and *torsional stiffnesses K* of individual structural elements, considering influence of flexibility of adjoining beams and columns at top and bottom; in the estimation of stiffness, effects are considered of both *flexural and shear deformations;* and (4) estimate τ. The method is validated using *3D modal analysis*. Buildings with a spectrum of structural configurations are considered, which include effects of: (i) plan shape of lateral structural elements, (ii) plan aspect ratio (L/B = 1 to 3, where L is the longitudinal plan dimension) of the building, (iii) height to lateral dimension (*H/B* = 0.25 to 7.50) ratio of the building, and (iv) structural walls along one and two principal plan directions. Modal analyses are performed and τ (= T_θ/T) estimated. The results show that the proposed simple method captures accurately the torsional flexibility in buildings, and can be used for initial proportioning of both RC frame buildings and RC structural wall buildings.

Keywords: Torsional flexibility, 3D modal analysis, plan aspect ratio and height to lateral dimension ratio.

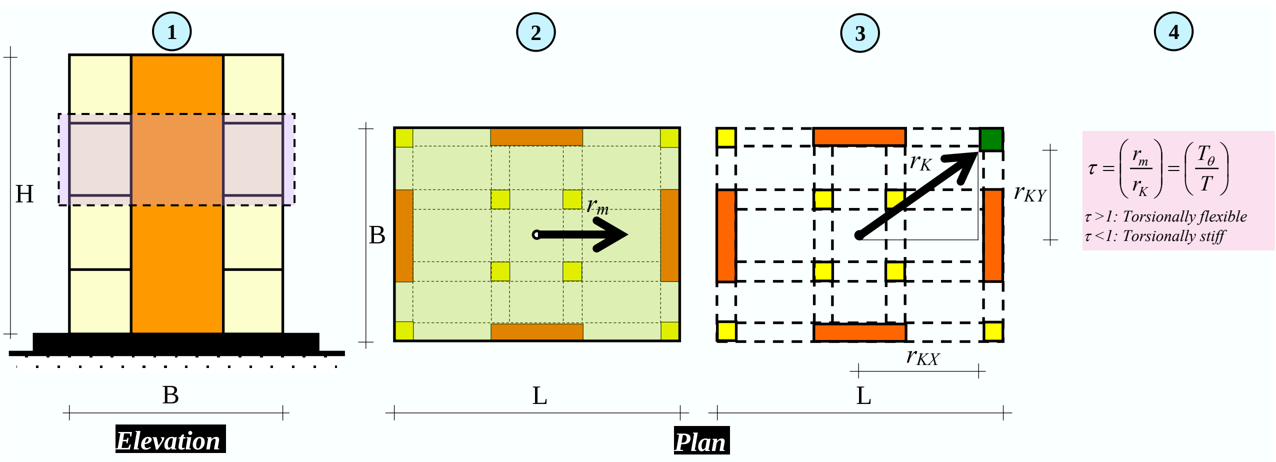

Fig. 1 – Procedure to identify torsional flexibility in buildings by the proposed simple, approximate method

1. Introduction

Under earthquake shaking, torsional effects are induced in buildings due to *torsional eccentricity* and *torsional flexibility*. Torsional eccentricity arises due to unsymmetrical distribution in mass, stiffness and/or strength, and torsional flexibility due to poor geometric structural configuration in plan. *Normalised torsional eccentricity* (*e/B*, where B is the building lateral dimension) and *natural period ratio* (τ) are the two critical structural parameters addresses these effects in buildings. Usually, *e/B* was focused on in most studies in the past, while τ was not. This study focuses on torsional effects arising from torsional flexibility. Here, τ is defined as the ratio of uncoupled (pure) *torsional natural period* T_θ to uncoupled (pure) *translational natural period T*, given by:

$$\tau = \frac{T_\theta}{T} = \frac{r_m}{r_K} . \tag{1}$$

where r_m and r_K represents radius of gyration of *mass* and radius of gyration of *stiffness*. Hence, τ depicts the *mass and stiffness distributions* in plan. Buildings with $\tau>1$ are called *torsionally flexible*, and those with $\tau<1$ *torsionally stiff*. Torsionally flexible buildings have their *first natural mode of oscillation* to be the *torsional mode of oscillation.* In this mode, large percentage of mass participates in torsional response rather than in the translational modes along each principal plan direction. Structural elements of such buildings have been found to have low torsional resistance to resist earthquake induced torsional inertia force effects. They can be symmetrical or unsymmetrical; this feature is present in buildings with central structural core wall, large plan aspect ratio or poor plan geometry. Also, equivalent static approach does not capture the true behaviour as it does not consider the torsional mode. Usually, such buildings demand large deformation capacities in lateral structural elements along the periphery as noticed in catastrophic failures in past earthquakes, *e.g.,* partial collapse of a building with structural core wall, where the deformation capacity of structural walls is not utilized properly (Fig.2a) [1], damage to corner column result in entire upper storey collapse (Fig.2b) [2] and complete damage of building with large plan aspect ratio (Fig.2c) [3]. Designing such buildings for amplified displacement demand results in poor performance, and some codes (such as *Eurocode* and *India*) explicitly avoid design of such buildings. This can be done in the preliminary design stage itself, by choosing a good structural configuration, but, the codes do not explicitly provide any procedure to capture torsional flexibility in buildings. Hence, it is essential to have a procedure that identifies torsional flexibility in buildings and the parameters influencing it, and avoid the same in practice.

For estimating τ using Eq. (1), r_m and r_K are required, which in turn require the estimation of *translational mass m*, *torsional mass* I_θ (otherwise known as *mass moment of inertia*), *translational stiffness K* and *torsional stiffness* K_θ. It is easier to estimate *seismic mass or mass moment of inertia* (by lumping the mass at each node), but a bit involved to calculate K and K_θ, because they vary all along the height of the building. There are analytical methods for the estimation of K but not of K_θ. Usually, K is estimated directly using simple methods giving *closed form expressions*, which may be applied *directly* or a few requiring response quantities from *basic structural analyses*. In the former method, *lateral stiffnesses* K_j are used of *individual vertical structural element* or *sub-assemblage* (which includes *beams* and *columns* on top and bottom of the storey considered); it directly accounts for *beam flexibility* to represent the *real building characteristics*. K of each storey is calculated as the *sum of lateral stiffness* K_j *of individual elements or sub-assemblage* [4, 5, 6, and 7]. Most methods assume same end rotations at beams and columns of each storey,

Fig. 2 – Damage observed in torsionally flexible buildings: (a) *2001 Bhuj Earthquake,* (b) and (c) *1985 Mexico City Earthquake* [1, 2 and 3]

i.e., the points of zero moments lie at their mid-lengths ([4] and [5]) and some consider variation in its locations at bottom stories [6]; beams and columns are terminated at their mid-length to constitute the sub-assemblage. Sometimes, these sub-assemblages are isolated from each storey along height at mid point of the cross section of beams [5]. In the calculation of K, the rotational degrees of freedom are eliminated by static condensation of the equilibrium equations [4]. Some analytical methods do not discriminate between exterior and interior sub-assemblages, while a few accounts for variation in boundary conditions at bottom and top storeys implicitly using modification factors [8]. Alternatively, the changes in these storeys are considered all along the height of the building explicitly [5]. But all these methods are applicable only to moment frame buildings or to structural wall buildings, but not generic enough to be applicable to all buildings [7 and 8]. Also, these methods give estimate of K at each storey of building in the preliminary design stage, without using the results of detailed structural analysis; a simple spread sheet modeling would suffice.

In the latter method, the results of simple structural analysis are used in the estimation of lateral stiffness of each storey using closed form expressions. A few *methods* convert an ideal planar frame into a single cantilever along with rotational restraints provided at each floor due to beams. But, these methods require prior assumption of lateral force distribution along the height of building [6]. Instead, a continuum *model* can be used to account for lateral stiffness variation of buildings, using flexural and shear components of the total lateral stiffness at each storey, in comparison to the numerical study. Additional correction factor is used to account for variation in lateral force distribution. But, the continuum model exactly estimates the lateral stiffness of each storey closer to real buildings, specifically in *moment frame and structural wall buildings* [8]. A few additional methods completely rely on structural analysis to estimate the lateral stiffness of each storey, which includes: (i) an n-storey frame connected with n springs in series, where the lateral stiffness of each storey except the first is estimated using equivalent storey stiffness (including the storey considered) and storey stiffness of all storeys below this storey; or (ii) restraining the adjacent bottom storey and applying a unit lateral deformation at the top. Methods (i) and (ii) require additional analyses. Hence, storey stiffness is estimated based on equivalent static approach using the ratio of total storey shear and the inter-storey displacement, which requires only a simple structural analysis; but, the storey stiffness varies with the change in lateral force distribution. Ultimately, to avoid this discrepancy, lateral stiffness of each storey is estimated based on the fundamental translational modal responses, which includes fundamental mode shape, its corresponding natural period and seismic mass present in each storey. Also, this method requires only single step structural analysis considering the dynamic characteristics of buildings and the solution is simple and unique; it does not require any change in lateral force distribution [7].

It is evident that most models used in practice focus on the estimation of the K of the building, because K was required to estimate the building lateral displacement or to find the stiffness irregularity along the building height. But, it is not representing the three dimensional characteristics of the building. Hence, in addition to translation, the rotational characteristics of buildings should be considered. In another method, a lumped mass stick model was considered with both translational and rotational characteristics through simple closed-form expressions using cross sectional properties; also, it accounted for both flexural and shear deformations. Using these equivalent properties, numerical studies were conducted with the stick model to estimate the buildings natural periods. Moreover, it is applicable to buildings that are unsymmetrical in plan and vertically irregular. But, the method fails to account for flexibility of beams at each floor. In addition, with rigid diaphragm and additional rocking restraints applied, this method highly overestimated responses [9]. Also, the calculation of seismic mass or mass moment of inertia was not clarified, and hence may not be applicable to all buildings. In all, there is one method to identify *torsional flexibility* in buildings in the preliminary design stage, which is simple.

Therefore, this paper presents a simple procedure to identify torsionally flexible buildings. Additionally, in the estimation of K and K_θ, four different analytical methods are presented to arrive at closed form expressions that may be used. The results are validated with 3D modal analyses *(3D MA).* Finally, a parametric study is conducted using 3D MA to identify parameters that influence torsional flexibility, especially in symmetric buildings and possible solutions to avoid these effects. This study is limited to elastic structural analysis and does not consider any inelastic action.

2. Proposed Procedure and Approximate Methods

Torsionally flexible buildings are identified by the following procedure using Eq. (1):
(1) Estimate the *radius of gyration of mass* r_m;
(2) Estimate the *radius of gyration of stiffness* r_K; *and*
(3) Compute the *Natural Period Ratio* τ using Eq. (1).
Both *single-storey* and *multi-storey* buildings can be studied by this method. The above *procedure* is explained in detail in the sections below.

2.1 Estimate radius r_m of gyration of mass

Radius r_m of gyration of mass is estimated using translational mass m and mass moment of inertia I_θ of lumped mass at each storey of building plan. Usually, it is estimated using available closed form expressions with an assumption of uniform mass distribution, for buildings with simple plan geometries like *square, rectangular and circular*. But, plan geometries of real buildings are complicated, and these expressions may not be applicable. Sometimes, the distribution is random of lateral structural elements in plan owing to architectural requirements or unsymmetrical building plan; in such cases, distribution of mass also may be non-uniform. Therefore, a generalised approach is required, which is applicable to buildings with different plan geometries (Fig.3). Primarily, the distributed mass in plan is lumped at the nodes at each floor, of: (i) slabs and beams, to the nodes in that floor, and (ii) vertical structural elements, one half to the node above that storey and the other half to the node below that storey. Usually, masses are lumped at their center of mass CM of building plan at each floor to idealise a building along the height of the building considering only translational degrees of freedom. But in real buildings, it is important to consider its rotational characteristics. Hence, in the estimation of mass moment of inertia, it is significant to consider the distribution of mass in plan. The steps involved in the estimation of r_m are (Fig.3):

(i) Compute *seismic mass of storey i* lumped at *CM*, m_i:

$$m_i = \sum_{j=1}^{n_i} m_j \, , \tag{2}$$

(ii) Compute *seismic mass moment of inertia of storey i* about *CM*, $I_{\theta i}$:

$$I_{\theta i} = \sum_{j=1}^{n_i} I_{\theta j} + \sum_{j=1}^{n_i} m_j r_j^2 \, , \tag{3}$$

(iii) Estimate *radius of gyration of mass at storey i*, r_{mi}:

$$r_{mi} = \sqrt{\frac{I_{\theta i}}{m_i}} \, . \tag{4}$$

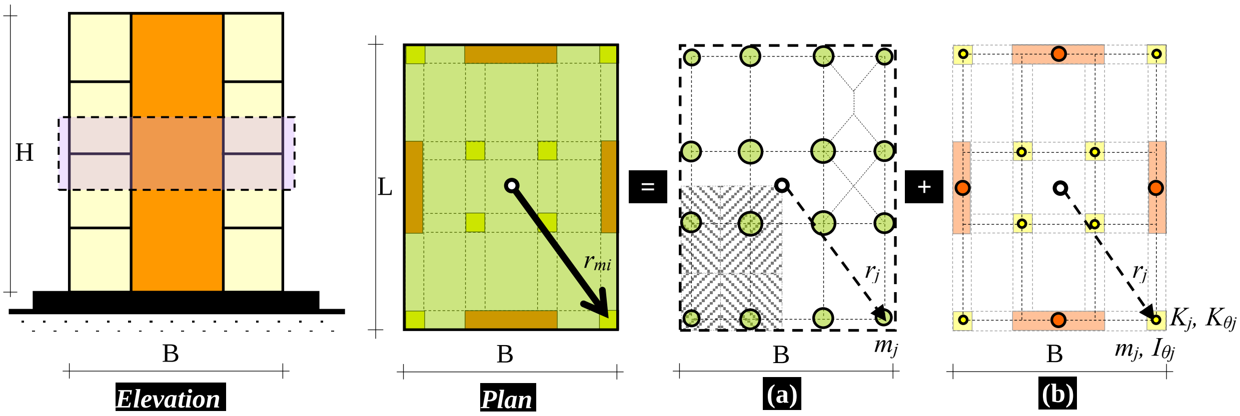

Fig. 3 – Estimation of r_{mi} using m_j and $I_{\theta j}$ by lumping mass at nodes *j* of each storey *i* including: (a) mass of slabs and beams, and (b) mass of vertical structural elements lumped at their centroid of cross-section

where m_j is the seismic mass lumped at node j of storey i, m_i the total seismic mass lumped at node j of storey i; n_i the total number of nodes j of storey i, $I_{\theta i}$ the seismic mass moment of inertia of storey i about CM, $I_{\theta j}$ mass moment of inertia of vertical element at node j of storey i about its centroid, r_j the distance from CM of seismic mass m_j lumped at node j of storey i.

2.2 Estimate radius r_K of gyration of stiffness

Radius r_K of gyration of stiffness is estimated using *translational* K and *torsional* K_θ *stiffnesses* of lateral load resisting structural elements (beams, columns and structural walls). It defines the geometric distribution of lateral load resisting elements in plan (similar to mass distribution of these elements given in Fig.3b). Four analytical methods *(AM)* are examined here to estimate translational and torsional stiffnesses of lateral load resisting elements, namely (Fig.4, 5 and 6):

(i) AM1: vertical elements in storey i fixed at their ends without including beams flexibility;
(ii) AM2: vertical elements in storey i have beams and columns connected whose far ends are fixed;
(iii) AM3: vertical elements in storey i have beams and columns connected but are truncated at their points of zero moments (adapted from [6] with a few modifications); and
(iv) AM4: idealised stick model with mass lumped at each storey, which requires results of 3D modal analysis of multi-storey building in focus.

Methods AM2 and AM3 are applicable to structural elements at interior location of middle storey (Fig.4 and 5), and modified for structural elements at ground and top storeys. AM1 assumes the joint rotation in adjoining beams to be zero *(i.e., flexurally stiff)* having complete shear building behaviour; whereas AM2 and AM3 consider the rotational flexibility of adjoining beams and make assumptions like: (a) at each storey beams and columns undergo same rotation, (b) points of zero moments occur at their mid-length, (c) adjoining beams (spanning in both directions) have same cross-sectional properties at top and bottom of the storey, and (d) columns sizes are uniform along the height, unrestrained at top against translational when estimating translational stiffness (Fig.4) and totally restrained at far ends of columns when estimating torsional stiffness, ignoring warping deformations (Fig.5). Also, while estimating translational stiffness of element i, the rotational degree of freedom is eliminated by static condensation. AM4 is an idealised stick model (Fig.6) having: (i) masses lumped at each storey (at *CM*) representing equivalent mass and mass moment of inertia of multi-storey building, and (ii) stiffnesses at each storey representing equivalent translational and torsional stiffness of multi-storey building. Simple closed form expressions are derived for AM1, AM2, AM3 and AM4, wherein AM4 uses natural frequencies and mode shape vectors (of fundamental translational and torsional modes) from eigen analysis of multi-storey buildings in the said expressions. All models consider flexural and shear deformations of structural elements, and axial deformations are ignored. The steps involved in the estimation of r_K at storey i are:

(i) Compute translational stiffness K_i of storey i

The translational stiffness K_j of any vertical element of storey i located at (x_j, y_j) in building plan is estimated using AM1, AM2 and AM3 (Table 1). The vertical elements are columns or structural walls; structural walls are considered as *equivalent columns*. The translational stiffness of storey i along any principal direction (*X or Y*) is obtained as the sum of translational stiffness of all vertical structural elements in the same direction given by:

$$K_i = \sum_{j=1}^{n_{Vi}} K_j \ , \tag{5}$$

In AM4, the translational stiffness K_i of storey i is estimated using fundamental translational natural frequency ω and the corresponding mode shape ϕ of a multi-storey building as (Fig.6):

$$K_i = \left[\frac{\omega^2 \sum_{i=1}^{n} m_i \phi_i}{[\phi_i - \phi_{i-1}]} \right], \tag{6}$$

where K_j is the translational stiffness of vertical lateral load resisting element at node j of storey i; n_{Vi} number of vertical lateral load resisting element at storey i. ϕ_i the mode shape vector of storey i, and n the total number of storeys in the multi-storey building.

(ii) Compute torsional stiffness $K_{\theta i}$ of storey i

In the estimation of torsional stiffness $K_{\theta j}$ of vertical structural elements using AM1, AM2 and AM3, the ends are assumed to be restrained from warping. The torsional stiffness of any vertical element of storey i located (x_j, y_j) in building plan is estimated using Table 1. The torsional stiffness of storey i is obtained as the sum of torsional stiffnesses of all vertical structural elements, as:

$$K_{\theta i} = \sum_{j=1}^{n_{Vi}} K_{\theta j} + \sum_{j=1}^{n_{VXi}} K_{Xj} y_j^2 + \sum_{j=1}^{n_{VYi}} K_{Yj} x_j^2 \,, \tag{7}$$

In AM4, the torsional stiffness $K_{\theta i}$ of storey i is estimated using the fundamental torsional natural frequency ω_θ and their corresponding mode shape ϕ_θ of a multi-storey building as:

$$K_{\theta i} = \left[\frac{\omega_\theta^2 \sum_{i=1}^{n} I_{\theta i} \phi_{\theta i}}{\lfloor \phi_{\theta i} - \phi_{\theta (i-1)} \rfloor} \right], \tag{8}$$

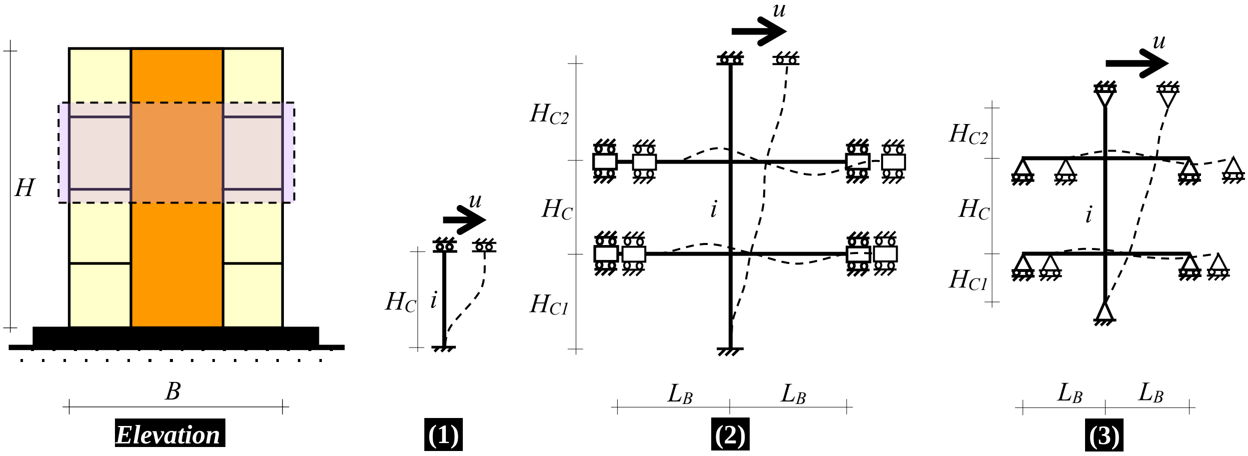

Fig. 4 – Building elevation with *analytical methods* (1), (2) and (3) taken from *middle storey i of the interior frame* in the estimation of *translational stiffness*

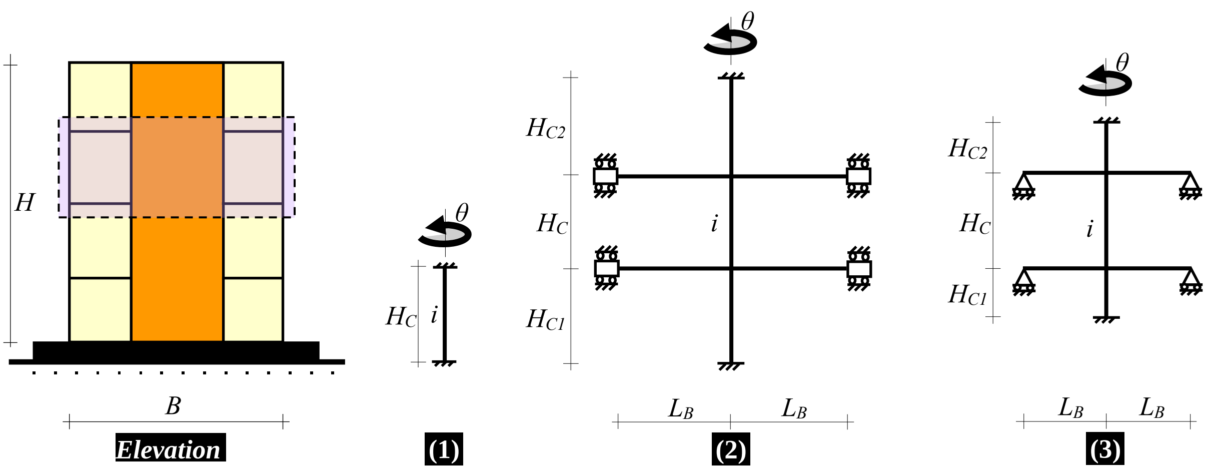

Fig. 5 – Building elevation with *analytical methods* (1), (2) and (3) taken from *middle storey level of the interior frame* in the estimation of *torsional stiffness*

Table 1 – *Translational* and *torsional stiffnesses* of elements located at storey *i* using different AM

AM	Translational Stiffness K_j of element at node *j*	Torsional Stiffness $K_{\theta j}$ of element at node *j*
1	$\frac{12EI_C}{H_C^3}\left(\frac{1}{1+\beta_C}\right)$	$\frac{GJ}{H_C}$
2	$\left[\frac{EI_C}{H_C^3(1+\beta_C)}\right]\left[12-\left(\frac{A_1}{A_2}\right)\right]$ where, $A_1=36\left[3(3+2\beta_C)+(p_1)(4+\beta_{B1})\left(\frac{H_C}{L_B}\right)\left(\frac{I_B}{I_C}\right)\left(\frac{1+\beta_C}{1+\beta_{B1}}\right)\right]$; $A_2=\left\{\begin{array}{l}2(5+2\beta_C)(4+\beta_C)-(2-\beta_C)^2+(p_2)(13+4\beta_C)(4+\beta_{B1})\left(\frac{H_C}{L_B}\right)\left(\frac{I_B}{I_C}\right)\left(\frac{1+\beta_C}{1+\beta_{B1}}\right)\\+(p_3)^2(4+\beta_{B1})^2\left(\frac{H_C}{L_B}\right)^2\left(\frac{I_B}{I_C}\right)^2\left(\frac{1+\beta_C}{1+\beta_{B1}}\right)^2\end{array}\right.$ and p_1, p_2, p_3 are constants (= 4, 2, 2 for *interior columns* and 2, 1, 1 for *exterior columns*).	$\frac{2GJ}{H_C}+\frac{p_2EI_B(4+\beta_{B2})}{L_B(1+\beta_{B2})}$
3	$\left[\frac{EI_C}{H_C^3(1+\beta_C)}\right]\left[A_3-\left(\frac{6H_C(6H_CA_7+A_4A_6)+A_4(6H_CA_6+A_4A_5)}{A_5A_7-A_6^2}\right)\right]$ where, $A_3=12+\left(\frac{B_1}{C}\right)-\left(\frac{B_3^2}{B_6C}\right)$; $A_4=-6H_C+\left(\frac{B_2}{C}\right)-\left(\frac{B_3B_5}{B_6C}\right)$; $A_5=H_C^2(4+\beta_C)+\left(\frac{I_4+I_5+I_6}{C}\right)$; $A_6=H_C^2(2-\beta_C)$; $A_7=H_C^2(4+\beta_C)+\left(\frac{B_4+I_7+I_8}{C}\right)-\left(\frac{B_5^2}{B_6C}\right)$; $B_1=B_6=\left[\frac{EI_C}{H_{C2}^3(1+\beta_{C2})}\right]\left[12-\left(\frac{36}{4+\beta_{C2}}\right)\right]$; $B_2=\left[\frac{EI_C}{H_{C2}^3(1+\beta_{C2})}\right]\left[6H_{C2}-\left(\frac{6H_{C2}(2-\beta_{C2})}{4+\beta_{C2}}\right)\right]$; $B_3=\left[\frac{EI_C}{H_{C2}^3(1+\beta_{C2})}\right]\left[-12+\left(\frac{36}{4+\beta_{C2}}\right)\right]$; $B_4=\left[\frac{EI_C}{H_{C2}^3(1+\beta_{C2})}\right]\left[H_{C2}^2(4+\beta_{C2})-\left(\frac{H_{C2}^2(2-\beta_{C2})^2}{4+\beta_{C2}}\right)\right]$; $B_5=\left[\frac{EI_C}{H_{C2}^3(1+\beta_{C2})}\right]\left[-6H_{C2}+\left(\frac{6H_{C2}(2-\beta_{C2})}{4+\beta_{C2}}\right)\right]$; $C=\left[\frac{EI_C}{H_C^3(1+\beta_C)}\right]$; $I_4=\left[\frac{EI_C}{H_{C1}^3(1+\beta_{C1})}\right]\left[H_{C1}^2(4+\beta_{C1})-\left(\frac{H_{C1}^2(2-\beta_{C1})^2}{4+\beta_{C1}}\right)\right]$; and $I_5=I_6=I_7=I_8=\left[\frac{EI_B}{L_B^3(1+\beta_{B1})}\right]\left[L_B^2(4+\beta_{B1})-\left(\frac{L_B^2(2-\beta_{B1})^2}{4+\beta_{B1}}\right)\right]$, where $I_5=I_7=0$ for *exterior columns*.	$GJ\left(\frac{1}{H_C}+\frac{1}{H_{C2}}\right)+\frac{p_2EI_B}{L_B(1+\beta_{B2})}\left[(4+\beta_{B2})-\left(\frac{(2-\beta_{B2})^2}{(4+\beta_{B2})}\right)\right]$

Note:

$\beta_C=2(1+v)\left(\frac{D_C}{H_C}\right)^2$; $\beta_{C1}=2(1+v)\left(\frac{D_C}{H_{C1}}\right)^2$; $\beta_{C2}=2(1+v)\left(\frac{D_C}{H_{C2}}\right)^2$; $\beta_{B1}=2(1+v)\left(\frac{D_B}{L_B}\right)^2$; and $\beta_{B2}=2(1+v)\left(\frac{B_B}{L_B}\right)^2$.

E is the modulus of elasticity; *G* the shear modulus; *v* the poisson's ratio; H_C the height of column at storey *i*; H_{C1} and H_{C2} the height of column at top and bottom of storey *i*; D_C the depth of column; L_B the length of beam; D_B the depth of beam; D_B the width of beam; I_B and I_C the second moment of area of cross-section of beams and columns; and *J* the torsional constant of cross-section.

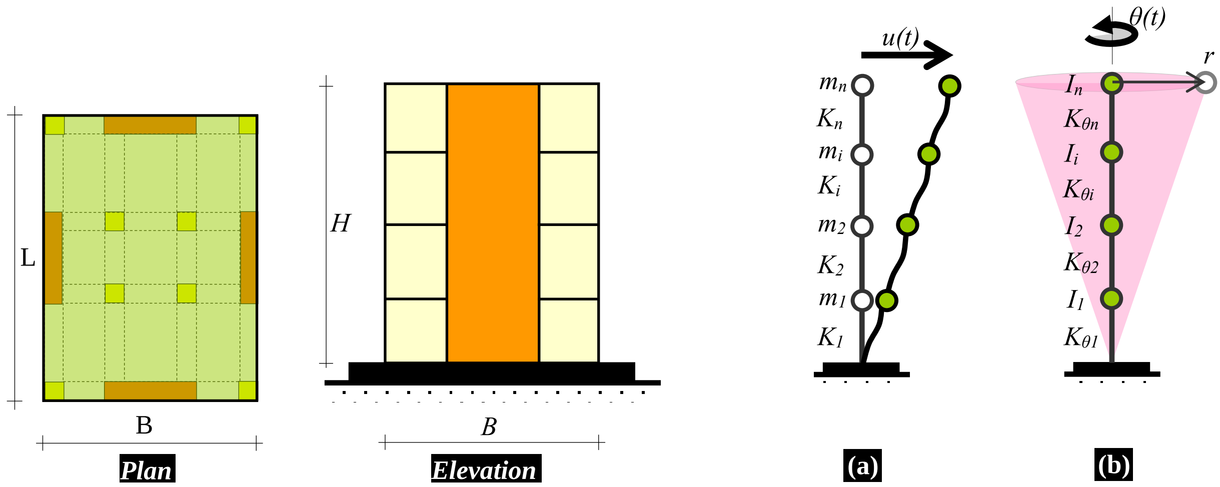

Fig. 6 – Building elevation and plan with a stick model (AM4) having mass lumped at each storey showing: (a) translational and (b) torsional modes

(iii) Calculate the *radius r_{Ki} of gyration of stiffness* at storey i

The *radius r_{Ki} of gyration of stiffness* of storey i in any principal plan direction (*X or Y*) is given by:

$$r_{Ki} = \sqrt{\frac{K_{\theta i}}{K_i}} \; . \tag{9}$$

where K_{Xj} and K_{Yj} are the translational stiffnesses of vertical lateral load resisting element at node j of storey i along X- and Y-directions; n_{VXi} and n_{VYi} the numbers of vertical lateral load resisting element at storey i along X- and Y-direction; x_j and y_j the locations of element i measured from the CM; $K_{\theta i}$ the torsional stiffness of storey i about CM, $K_{\theta j}$ the torsional stiffness of vertical structural element located at node j of storey i; and $\phi_{\theta i}$ the mode shape vector of storey i.

2.3 Compute Natural Period Ratio τ

The *natural period ratio* τ_i of storey i along any principal plan direction (*X or Y*) is given by:

$$\tau_i = \frac{r_{mi}}{r_{Ki}} \; . \tag{10}$$

In all, the results of middle storey alone are presented, because observations at the ground and roof top are typical. Using the procedure proposed in *Sections 2.1-2.3*, buildings are proportioned in plan considering simple single-storey analytical methods *AM1, AM2 and AM3*, before performing the formal structural analysis and design. Torsionally flexible building can be avoided through proper geometric distribution of mass and stiffness. For buildings with uniform plan geometry, this is possible by tuning *number, size, shape and re-arrangement of lateral structural elements* in plan (by locating the structural elements along periphery). In buildings with uniform distribution of stiffness in plan, this is possible by tuning the choice of plan geometry (considering simple plan shapes and avoiding complex shapes) and by limiting plan aspect ratio. Alternatively, in AM4, buildings are modeled in available computer programs and the results of modal analysis used in closed form expression provided therein.

3. Validation with 3D Building Modal Analysis

Buildings with different plan configurations (Table 2) are considered to validate the proposed procedure to estimate τ using all four analytical methods. These include buildings with RC moment frames, with RC structural walls, and *combinations* of these two. In particular, these buildings are chosen to consider the effect of: (i) plan shape of vertical structural elements resisting lateral loads, (ii) plan aspect ratio (L/B = 1 to 3, where L is the longitudinal plan dimension) of the building, (iii) height to lateral dimension (*H/B*) ratio varying from 0.25 to 7.50, and (iv) structural walls along one, and along two principal plan directions. Apart from (i) and (ii), (iii) is considered to account for the effect *H/B*, as in multi-storey buildings. All these building configurations *(BC)* have *4m* bay length in each direction and *3m* floor height. Column sizes are taken as: (i) $500mm \times 500mm$ for square columns and (ii) $300mm \times 500mm$ for rectangular columns of frame buildings, and (iii) $400mm \times 400mm$ for structural-wall frame buildings. And, structural wall and beam sizes are taken as $4000mm \times 400mm$ and $300mm \times 400mm$. Dimensions of all structural elements are kept uniform throughout the height of building. The materials used are M30 grade concrete and Fe415 steel. Beam-column joints are assumed to be rigid. The effect of cracked section properties is not considered, because this study focuses on initial proportioning of buildings in plan. The member sizes chosen ensure the strong column-weak beam philosophy. The effect of masonry infill walls is not considered.

Commercial structural analysis software SAP2000 is used to model the buildings. Rectangular structural walls are modelled using 2D shell element, and flanged wall sections (*e.g., angle and hollow core structural walls*) using equivalent frame element having same dimensions. Slab thickness of 150mm is considered in all the stories to ensure diaphragm action without assigning any rigid diaphragm constraint, because the rigid diaphragm assumption in modelling of slab in computer program may lead to large errors in structural wall buildings [9]. Rigid zones are not modelled. Only self-weight is considered of structural components modelled in the numerical study; additional gravity and lateral loads are ignored. τ $(=r_m/r_K)$ is estimated using the four analytical methods and verified through (T_θ/T) from 3D Modal Analysis of the building using *SAP 2000*. But, the results of buildings with *5*- and *10*-storey having fixed bottom are shown for comparison with analytical methods (Table 3). Fig.7 shows normalised τ_N estimated from the ratio of 3D MA and different analytical methods. AM4 uses results from 3D MA and their mean values of τ are used for comparison. Following are the salient observations of this study (Tables 2 and 3, Figs.7 and 8):

Table 2 – Configurations of buildings considered in this study

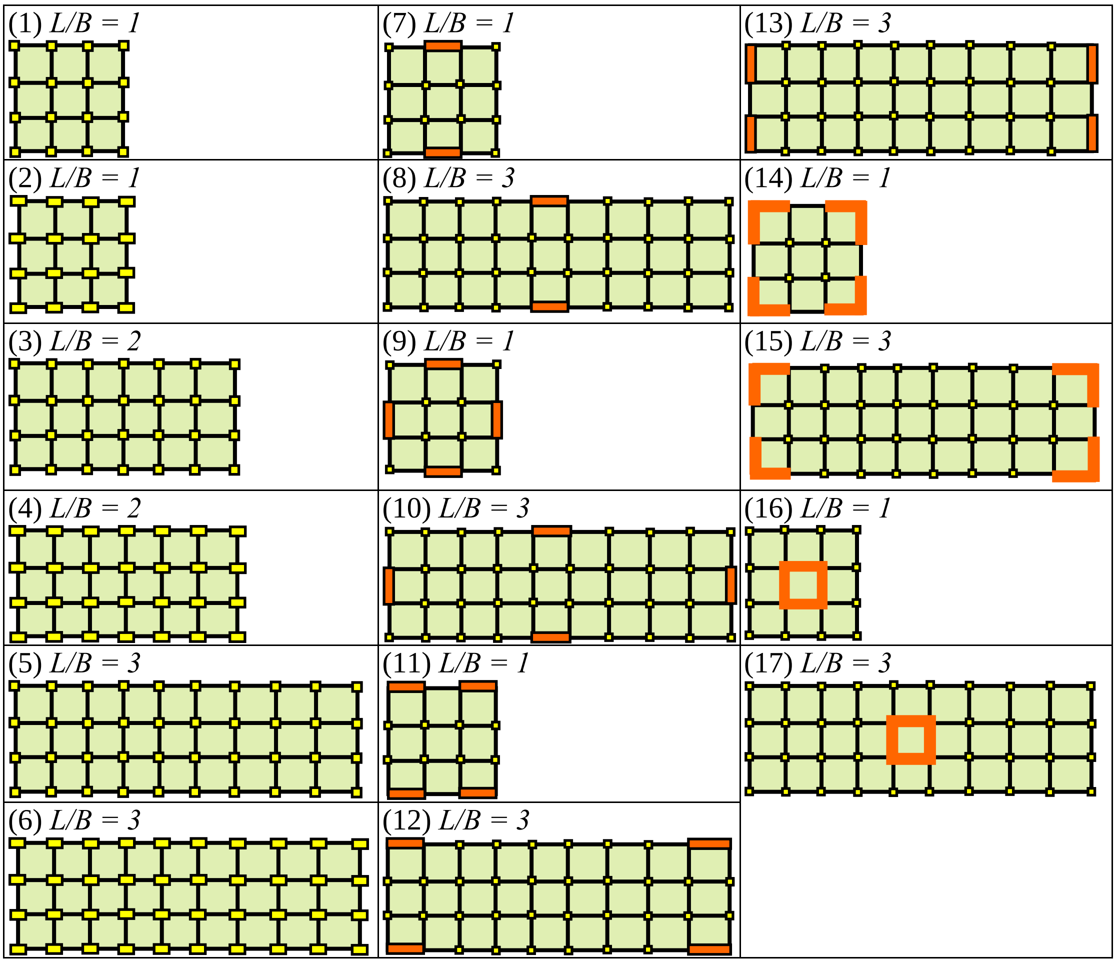

Table 3 – Comparison of results of τ based on proposed procedure using AM and 3D Modal Analysis

BC	AM1		AM2		AM3		Modal Analysis				
							AM4			3D MA	
	τ_X	τ_Y	τ_X	τ_Y	τ_X	τ_Y	No. of storey	τ_X	τ_Y	τ_X	τ_Y
1	0.91	0.91	0.85	0.85	0.83	0.83	5	0.92	0.92	**0.92**	**0.92**
							10	0.89	0.89	**0.90**	**0.90**
2	1.08	0.66	0.95	0.68	0.92	0.66	5	1.01	0.79	**1.02**	**0.79**
							10	0.96	0.78	**0.99**	**0.79**
3	0.92	0.92	0.91	0.89	0.90	0.87	5	0.97	0.93	**0.96**	**0.93**
							10	0.97	0.91	**0.97**	**0.92**
4	1.26	0.77	1.11	0.77	1.09	0.76	5	1.13	0.85	**1.14**	**0.85**
							10	1.11	0.83	**1.12**	**0.85**
5	0.94	0.94	0.94	0.92	0.94	0.91	5	1.00	0.95	**0.99**	**0.95**
							10	1.02	0.93	**1.01**	**0.94**
6	1.38	0.85	1.22	0.84	1.20	0.83	5	1.22	0.90	**1.22**	**0.90**
							10	1.20	0.88	**1.21**	**0.89**
7	0.96	0.23	0.96	0.19	0.95	0.25	5	0.93	0.39	**0.93**	**0.40**
							10	0.88	0.51	**0.88**	**0.51**
8	1.67	0.57	1.68	0.50	1.53	0.62	5	1.30	0.78	**1.30**	**0.74**
							10	1.17	0.84	**1.19**	**0.80**
9	0.70	0.70	0.69	0.69	0.70	0.70	5	0.72	0.72	**0.71**	**0.71**
							10	0.71	0.71	**0.71**	**0.71**
10	0.65	0.65	0.64	0.64	0.66	0.65	5	0.72	0.71	**0.72**	**0.70**
							10	0.80	0.76	**0.79**	**0.75**
11	1.02	0.20	1.02	0.16	1.01	0.21	5	0.99	0.32	**0.98**	**0.35**
							10	0.94	0.44	**0.94**	**0.46**
12	1.73	0.48	1.83	0.42	1.69	0.54	5	1.46	0.73	**1.45**	**0.70**
							10	1.27	0.84	**1.28**	**0.79**
13	0.17	0.70	0.14	0.70	0.20	0.71	5	0.39	0.71	**0.37**	**0.72**
							10	0.52	0.79	**0.52**	**0.78**
14	0.92	0.92	0.92	0.92	0.91	0.91	5	0.74	0.74	**0.73**	**0.73**
							10	0.74	0.74	**0.73**	**0.73**
15	0.59	0.59	0.59	0.59	0.59	0.59	5	0.62	0.58	**0.63**	**0.59**
							10	0.66	0.66	**0.67**	**0.66**
16	1.71	1.71	1.10	1.10	0.59	0.59	5	2.06	2.06	**2.19**	**2.19**
							10	0.89	0.89	**0.94**	**0.94**
17	2.54	2.54	2.08	2.08	1.19	1.18	5	2.05	2.00	**2.16**	**2.11**
							10	1.31	1.27	**1.38**	**1.33**

(i) *Effect of geometric distribution of mass using L/B:* With increase in L/B from 1 to 3, the torsional mode of the building shifted from third mode to second mode (*e.g.,* in cases *1 and 5*). Therefore, the effect of plan aspect ratio is critical;

(ii) *Effect of geometric distribution of stiffness:*

(a) *In RC Frame Buildings:* As observed in (i) above, for *L/B=1* by changing the shape of cross section from square to rectangle, the torsional mode of the building is shifted from third mode to second mode. The same is noticed even when plan aspect ratio *L/B* is changed from *1* to *2 or 3*, and cross-sections from *square to rectangle (e.g.,* in cases *1 to 6);*

(b) *In Structural Wall-Frame Buildings having structural walls oriented along one direction* (in the longitudinal plan direction *e.g.,* as in cases 7, 8, 11 and 12): Building becomes torsionally flexible (τ>1) with L/B changes from 1 to 3. But, with orientation of same structural wall along transverse plan direction *(e.g., as in cases 13),* the building becomes torsionally stiff. This philosophy is mostly adopted in buildings located in Chile. Comparison of results with analytical methods AM1-AM4 shows drastic variation in τ estimated in the direction perpendicular to direction of wall. Therefore, in the estimation of r_m, the contribution of these element lumped masses in I_θ along its major direction is ignored. Thus, the mean values of τ (using AM4) are in good agreement with *3D MA* though there is drastic variation along the height of building;

(c) *In Structural Wall-Frame Buildings having structural walls along both directions with walls located at periphery* (*e.g.,* as in cases 9, 10, 14 and 15): Buildings are always torsionally stiff (τ<1) even with L/B = 1 to 3 as evidenced in practice; and

(d) *In Structural Wall-Frame Buildings having structural core wall at center* (*e.g.,* as in cases 16 and 17): Buildings are always torsionally flexible (τ>1) and the effect gets worse with increase in L/B.

Mostly buildings up to 10 storeys tall are prone to first or second mode torsion. Therefore only 5- and 10-storey buildings *(having H/B = 1.25 and 2.50)* results are used for comparison with analytical methods. But, the results of buildings with other H/B ratio are given in Fig.8. Even then, buildings with moment frames and structural walls on one direction with large L/B have τ>1. Hence, code provisions must place an upper limit on L/B or must add structural walls perpendicular to longitudinal plan dimension (say L). Also, it is preferable to distribute the structural walls along the periphery of the building in both directions rather than confining it at center; and

(iii) *Effect of H/B Ratio:* In general, with increase in *H/B* ratio, buildings that are *torsionally flexible* become *torsionally stiff* and those *translationally stiff* become *translationally flexible* (*e.g.,* as in cases 16 & 17, Fig.8). Hence, the effect of torsion in buildings is dominant in mostly low-rise buildings up to 10 stories (*e.g.,* as in cases 2, 4, 6, 8, 12, 16 & 17, Fig.8). On the other hand, low-rise buildings are always stiffer in translation. Hence, the effect of *H/B* is critical.

The *Analytical Methods* are able to predict the *3D Modal Analysis Response* of RC moment frame and structural wall dominant buildings. AM1 to AM3 do not make good prediction of buildings with structural walls in one direction or structural wall core at center. Even then, mean values of τ based on AM4 show similar result as 3D MA, though there is drastic variation of τ along its height. In all, ~5% error is observed between 3D MA (of 5- and 10-storey building) and mean values of τ based on AM4 for RC frame and structural wall dominant buildings. And, ~9% is noticed in 5-storey buildings with structural walls in one direction and core walls, which reduces to ~6% in comparison with 10-storey buildings. On the whole, the observations remain same by all four analytical methods in single- and multi-storey buildings, though there is variation in their magnitude. Even then, AM1 to AM3 are simple and effective to identify torsional flexibility in buildings without any software-based structural analysis, irrespective of fixity at the base of the buildings. AM4 closely predicts 3D MA response as it considers the deformed configuration of building to consider the stiffness variation along its height. Hence, it shall be applied to different types of buildings. Also, it must ensure that more mass participation *(≥70%)* in translation and torsion. Even though 3D MA directly provides τ (=T_θ/T), AM4 shows variation of τ *(=* r_m/r_K*)* along the height of buildings considering both the distribution in mass and stiffness. Hence, code provision should include this parameter τ in defining both torsional irregularity (τ>1, as defined in Indian code) and vertical irregularity in buildings.

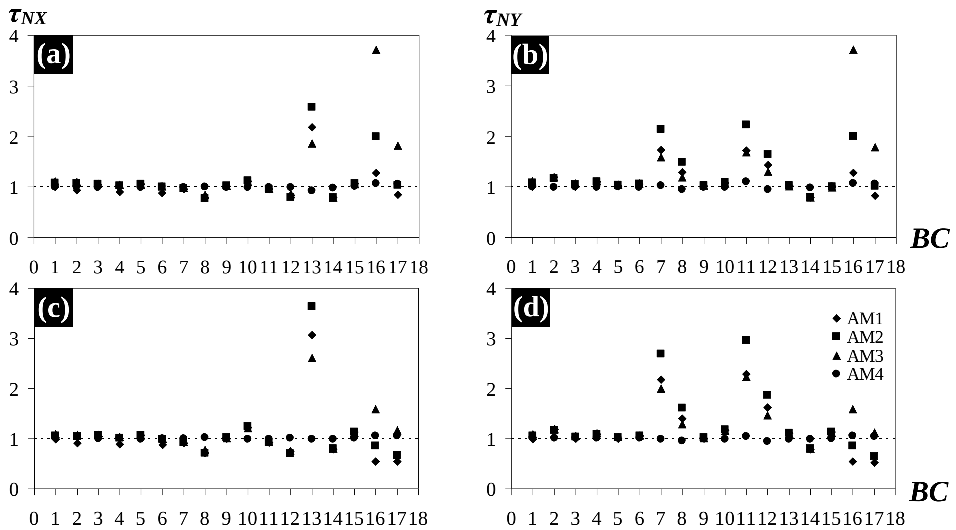

Fig. 7 –Normalised τ_N (=$\tau_{3D\ MA}/\tau_{AM}$) estimated from the ratio 3D MA and four analytical methods: (a) & (b) for 5-storey, and (c) & (d) for 10-storey buildings along X- and Y-directions

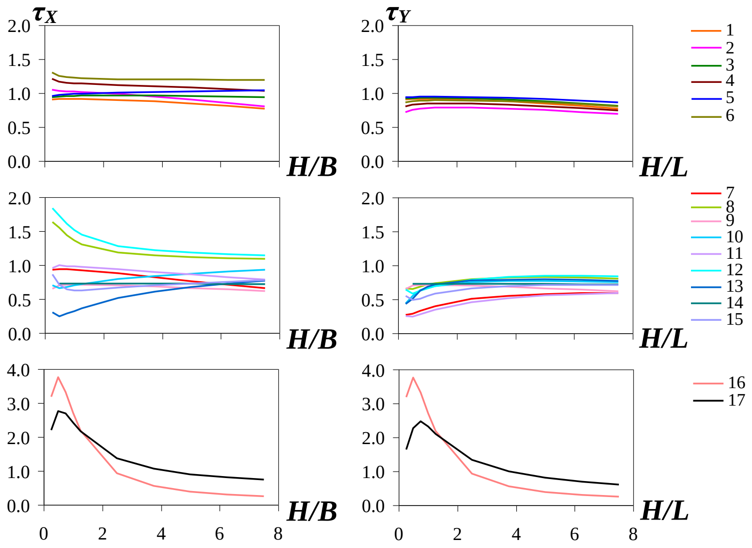

Fig. 8 – Effect of H/B and H/L on τ based on 3D MA along X- and Y-directions

4. Conclusions

Torsionally flexible buildings showed brittle damage or collapse under earthquake shaking. Hence, most building code provisions explicitly avoid design of these buildings in initial design stage. A simple procedure (using hand-calculation based single-storey analytical methods AM1, AM2 and AM3) is proposed (using a single parameter, $\tau = r_m/r_K$ *or* T_θ/T) to identify torsionally flexible buildings *(i.e.,* $\tau>1$*)* when initially proportioning buildings; this is valid for buildings with uniform mass and stiffness along their height. And, for buildings with variation in stiffness along their height, an alternate simple procedure (using a lumped stick model in analytical method AM4) is suggested, which considers the fundamental translational and torsional mode shapes, and their corresponding natural frequencies. The results of analytical methods are validated with 3D Modal Analysis; AM4 closely predicts the response and can be used for buildings with any distribution of stiffness and mass along the height. Influences of various critical parameters are identified and solutions suggested to make the buildings from being *torsionally flexible* ($\tau>1$) to being *torsionally stiff* ($\tau<1$). Also, recommendation is made to include the parameter τ in design codes to identify buildings that are torsionally irregular and vertically irregular considering the contributions of mass and stiffness along with the rotational characteristics of buildings.

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