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Verifying the Effect of the Response Displacement on wavelet transform-based capacity curve evaluation

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Abstract

After a building is built, it may be damaged during natural disasters such as an earthquake. Even if the building did not collapse, there could still be damage hidden behind other building components such as ceilings or partitions. With these unseen damages, the structure's lateral load resisting capacity integrity could be compromised, resulting in it having a higher probability of incurring severe damage in future events. One approach to detect such cases could be to implement "Structure Health Monitoring" techniques, where real-time building observation systems continuously monitor a building's lateral load carrying characteristics in real time.

One structural health monitoring method recently proposed for use in Japan by Kusunoki et al. [1] is to derive a building's capacity curve using acceleration data from earthquake events. The capacity curve expresses the relation between force demand and displacement response of the structure and is essentially the backbone curve of the hysteretic response. By analyzing the response capacity curve, one could estimate the extent of damage within the building based on the degree of inelastic response. The capacity curve is derived by (i) decomposing the acceleration data using the discrete wavelet transform method, (ii) calculating the resultant relative displacement response for each decomposed signal, (iii) removing the decomposed components which correspond to higher-order mode effects or long period noise component, (iv) recombining the remaining signal components, (v) simplifying the building's response into that of an equivalent single-degree-of-freedom (SDoF) system, and (vi) extracting the resulting backbone curve of the representative total acceleration (in place of force demand) versus relative displacement hysteretic response.

One issue with using acceleration data in the described approach is that residual displacement information cannot be captured, which could cause the peak displacement response to be underestimated. An alternative approach could be to explicitly use sensors capable of measuring a building's displacement or inter-story drift. In this paper, the potential to use the building's displacement response data directly into the capacity curve-based evaluation method instead of calculating it from acceleration response was investigated. This was done by examining results from two numerical single-degree-of-freedom (SDoF) systems; one elastic and the other inelastic. The traditional approach using only acceleration data by Kusunoki et al. [1] and that using the displacement response explicity were both considered.

Based on the comparison results for the elastic SDoF case, using building displacement response data explicitly causes the representative total acceleration versus relative displacement relationship at each decomposition level to have almost identical slope. While the slope matches the predominant period of the structure, this resulted in there being less indicators for selecting which decomposed signal components could be removed. Similar issues were also observed for the inelastic SDoF case.

Keywords: Structure Health Monitoring; Response Displacement Data; Discrete Wavelet-transform; Capacity curve

1. Introduction

Seismic design of buildings in seismically active countries has always been an important topic. Through the evolution of seismic design of all over the world, current seismic design philosophies and technologies applied in practice plays an important role in protecting many lives and property from various earthquakes. However, no matter how well a building is seismically designed, it may still incur damage during seismic events. In the event of a large earthquake, it is possible that large cracks and buckling/fracture of reinforcing bars occur in the structural elements even if the building did not collapse. Some structural damage may even occur during small earthquakes, particularly if it had been designed to a lower base shear coefficient. Due to such damage, the structure's lateral load resisting capacity integrity could be compromised, resulting in it having a higher probability of incurring severe damage in future events.

In order to assess the residual capacity of the structural system following an earthquake, much research aimed at qualifying the building's structural health based on the damage evaluation had been conducted. Currently, methods such as "disaster degree classification judgment [2]" or "emergency risk degree judgment [3]" are often used for confirming the safety of a building immediately after an earthquake in Japan. However, these methods involve visual inspection by engineers and therefore the outcome may be subjective. In addition, there are various other issues such as the availability of resources (e.g. number of engineers and inspection time) and the possibility of hidden damage (e.g. hidden behind non-structural elements).

Another approach to detect such damage could be to implement "Structure Health Monitoring" techniques, where real-time building observation systems continuously monitor a building's lateral load carrying characteristics. One structural health monitoring method recently proposed by Kusunoki et al. [1] and improved by Pan et al. [4] is to derive a building's capacity curve using acceleration data from earthquake events. The capacity curve expresses the relation between force demand and displacement response of the structure and is essentially the backbone curve of the hysteretic response. By analyzing the response capacity curve, one could estimate the extent of damage within the building based on the degree of inelastic response.

In the above studies, the predominant mode of building response was firstly extracted before obtaining the capacity curve. To extract only the predominant response for high-rise buildings, the discrete wavelet transform method, which decomposes a signal into multiple wavelets of varying widths and amplitudes and acts as a high and low-pass filter, can be used to remove higher-mode information and noise components that may affect the safety evaluation. After that, to obtain the building's displacement response, double integration is applied only to the wavelets selected as the main deformation mode. However, one issue caused by applying the method with using only acceleration data is that residual displacement information cannot be captured, which could cause the peak displacement response and the response during future events to be underestimated.

In this paper, the potential to use a building's measured displacement response data directly into the capacity curve-based evaluation method instead of calculating it from acceleration response was investigated. This was done by examining results from two numerical single-degree-of-freedom (SDoF) systems; one elastic and the other inelastic.

2. Capacity curve obtained from the acceleration data

In this chapter, the contents of discrete wavelet transform method and the derivation of the capacity curve from acceleration data is explained.

2.1 Wavelet transform method

The wavelet transform method is a time - frequency analysis method for mathematically assessing the degree of similarity between a signal f_0 and a mother wavelet [5]. In the first decomposition level, f_0 is decomposed into a high-pass signal g_1, which is the composition of wavelets of a fixed width and shape with the mother wavelet but with different amplitudes and time shifts, and a residual signal f_1.

$$f_0 = g_1 + f_1 \tag{1}$$

In the next decomposition level, f_1 is then decomposed into g_2 and f_2. If n decompositions were applied, f_0 is decomposed as shown in Eq. (2). The high-pass signal of the i^{th} decomposition level g_i is hereby termed the i^{th} rank, and f_n remains its orthogonality property even if decomposing is repeated.

$$f_0 = g_1 + g_2 + g_3 + \cdots + g_{n-1} + g_n + f_n \tag{2}$$

The maximum number of decompositions which can be performed is determined by Eq. (3), which depends on the length of the original data (N).

$$n = log_2 N \tag{3}$$

In the discrete wavelet transform, the width $\Delta_{\hat{t}}$ on the time axis of the window and the width Δ_f on the frequency axis of the window have an uncertainty relationship expressed by Eq. (4).

$$2\Delta_{\hat{t}} \cdot 2\Delta_f \geq 2 \tag{4}$$

Here, when the time increment of the original signal $f(x)$ is Δ_t, the time increment $\Delta_{t,i}$ of rank i (g_i) is defined as Eq. (5).

$$\Delta_{t,i} = \Delta_t * 2^i \tag{5}$$

The Nyquist frequency $\Delta_{f,i}$ of f_0 is defined as Eq. (6).

$$\Delta_{f,i} = \frac{1}{2\Delta_t * 2^i} \tag{6}$$

Therefore, wavelet transform is one method of time-frequency analysis that always satisfies the uncertainty relationship through Eq. (5) and Eq. (6). The wavelet transform method has the significant feature of being able to decompose signals in the time domain and capture localized peaks in the signal which overcomes the limitation of using conventional fast Fourier transform techniques.

2.2 Predominant rank selection

In previous research conducted by Kusunoki et al. [1], the ranks selected to estimate the appropriate ranks that capture the main predominant mode is termed the "predominant ranks", and this research will follow the name from previous research.

To get the meaningful capacity curve of the building properly, it is better to have the first mode pure information, without higher-mode information, of the building.[6] To do so, the first mode extraction from the measured response is performed by using the wavelet transform considering the selected predominant ranks of the building.[7]

Selection of the predominant ranks follows the following steps [8]:

(i) The rank with the largest cyclic area, which represents the largest dissipated energy during vibration, is selected as the temporary predominant mode rank.

(ii) When the representative linear slope of the adjacent rank is similar to that of the temporary predominant rank, it is added to the temporary rank from (i), and the process is repeated for the next adjacent rank.

(iii) The backbone curve of the hysteretic loop of the final rank combination is extracted from the measured response and used as the capacity curve.

For instance, Fig.1 shows the transfer function used from previous research conducted by Kusunoki et al. [1], where the Nyquist period of each rank (inverse of Nyquist frequency from Eq. (6)) of the signal considered was indicated using dashed lines. Here, the first mode predominant period was around 1.45 s, and anything past 2.5 s was due to long period noise components.

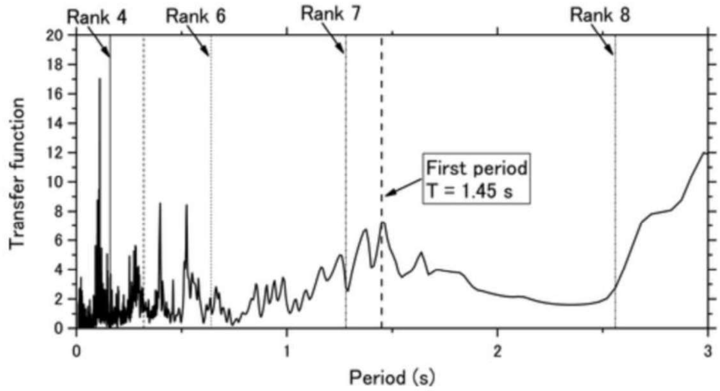

Fig. 1 – Example of Transfer function (from Kusunoki et al. [1])

The slope of the hysteretic response for all ranks were compared to determine the temporary predominant mode rank. Fig. 2 shows hysteretic behavior of ranks 5-10. Note here that "representative acceleration" is a proxy for the base shear and "representative displacement" is the approximate structural displacement at the height of effective mass. Calculation for these properties is described in the next section.

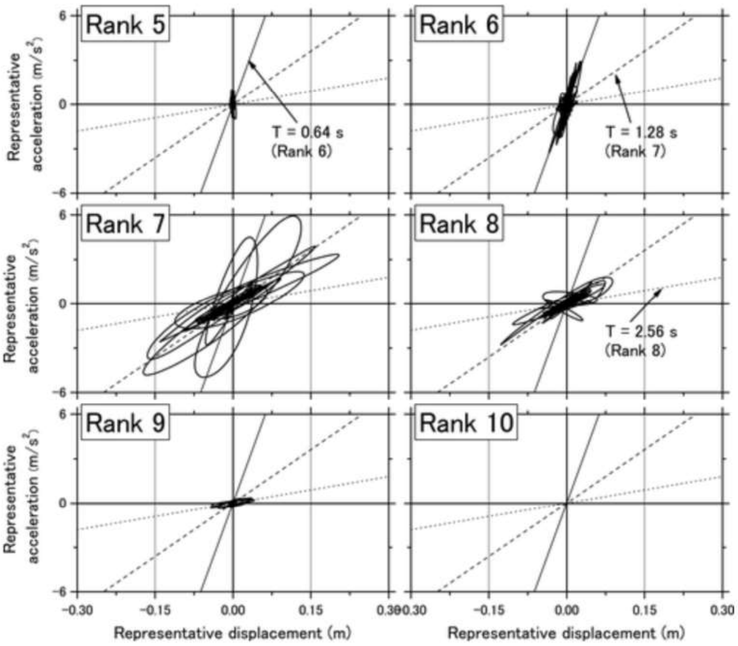

Fig. 2 – Example of hysteretic curve of wavelet transform ranks (from Kusunoki et al. [1])

From Fig. 2, rank 7 has been selected as temporary predominant mode rank. Then, ranks 6 and 8 were also selected in the final combination.

2.3 Capacity curve using wavelet transform

Fig.3 visualizes the relationship between measured acceleration and acceleration after applying the wavelet transform method, where $\ddot{X}_i$ is the absolute acceleration at floor i, $\ddot{x}_i$ is the relative acceleration at floor i, $\ddot{x}_0$ is the ground acceleration, ${}_1\ddot{x}_i$ is the relative acceleration for the first mode at floor i, and ${}_1\ddot{x}_0$ is the component of the ground acceleration with similar frequency content to the building's first mode.

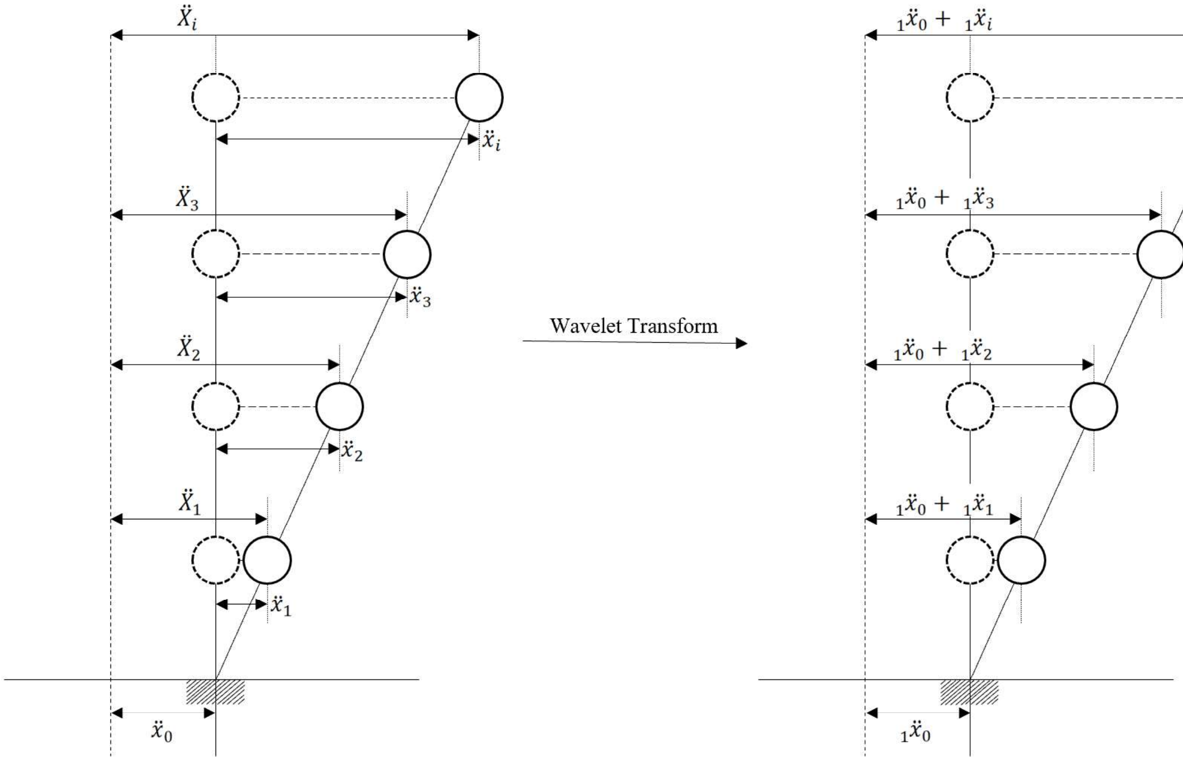

Fig. 3 – Measured acceleration and obtained acceleration for each rank after applying wavelet transform.

${}_1\ddot{x}_0$ and ${}_1\ddot{x}_0 + {}_1\ddot{x}_i$ can be obtained by applying wavelet transform to the ground acceleration $\ddot{x}_0$ and measured absolute acceleration of each floor $\ddot{X}_i$, respectively. The relative displacement can be obtained by applying Eq. (7).

$$ {}_1x_i = \iint ({}_1\ddot{x}_0 + {}_1\ddot{x}_i)\, dt^2 - \iint {}_1\ddot{x}_0\, dt^2 \tag{7} $$

Once the floor response of each Rank was obtained from applying the discrete wavelet transform to floor acceleration signals and applying Eq. (7), the response can be simplified into an equivalent SDoF response. Methods to do this is presented in Kusunoki et al. [1]. The equivalent SDoF total acceleration and relative displacement response are termed the "representative acceleration" and "representative displacement" response, respectively.

3. Case study on obtaining the capacity curve of an elastic SDoF structure

3.1 Case study details

In this section, numerical analysis of the SDoF linear model has been conducted to compare the case of using the displacement data obtained from the measured acceleration data and the case of using the measured displacement data. The north-south component (NS) of the JMA Kobe earthquake record was used as the input seismic wave. In addition, the single floor SDoF linear model was used to minimize other factors, such as mass ratio, building height, and the effects of second mode that may interfere with the verification. Brief information on the input wave, model and mother-wavelet used in this study are shown in Table 1.

Table 1 – Brief information of the input wave and SDoF models used in this study.

Input wave	**Region**	**Maximum acceleration**	**Magnitude**	**Time increment**
Kobe NS	Hyogo	818 cm/sec^2	6	0.02sec
Story	**Mass Ratio**	**Damping coefficient (h)**	**Predominant period**	**Mother wavelet**
1F	100%	5%	0.366 sec	Symlet 10

For single degree of freedom (SDoF) structures, the maximum floor number is 1 (i.e. i = 1). Accordingly, the representative acceleration and representative displacement are simplified as shown in Eq. (8) and (9).

$$representative\ acceleration = {}_1\ddot{x}_1 + {}_1\ddot{x}_0 \tag{8}$$

$$represenstative\ displacement = {}_1x_1 = \iint ({}_1\ddot{x}_0 + {}_1\ddot{x}_1)dt^2 - \iint {}_1\ddot{x}_0\, dt^2 \tag{9}$$

Based on the analysis results, the hysteresis curve and the transfer function of the SDoF linear model were obtained as shown in Fig.4 and Fig.5 for each. The Nyquist period of each rank and predominant period of the model are indicated using dashed lines.

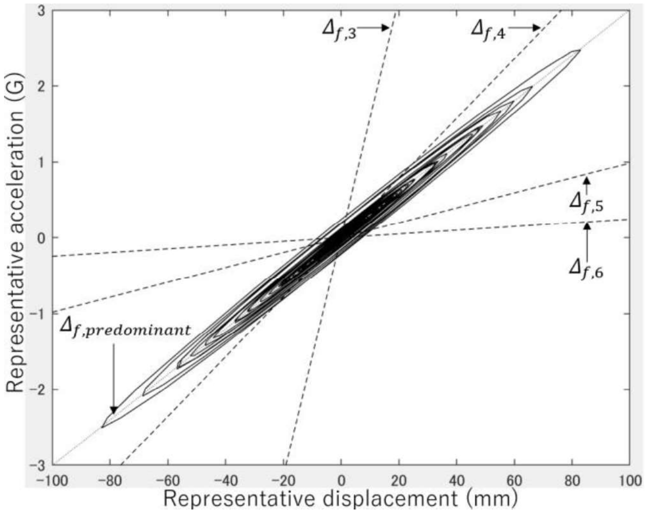

Fig. 1 – Hysteresis curve of elastic SDoF linear model

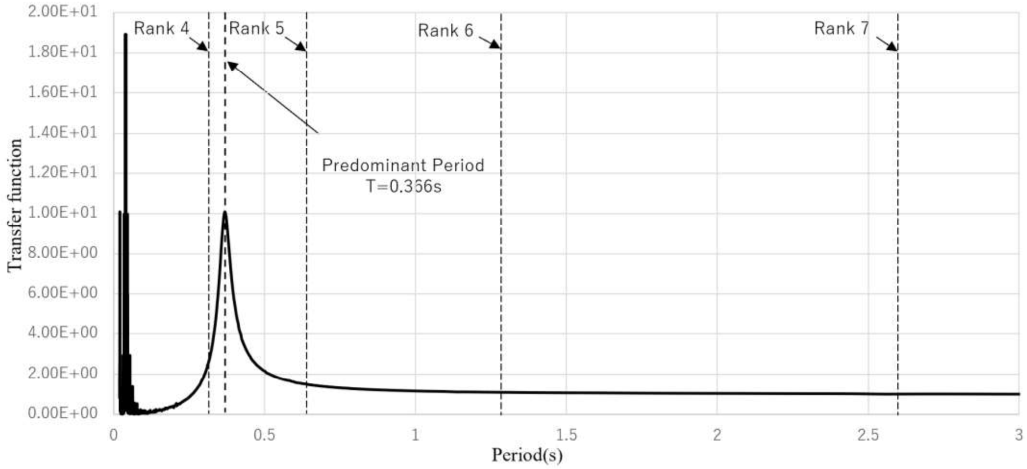

Fig. 5 – Transfer function of elastic SDoF linear model

3.2 Predominant rank selection

In this section, hysteretic response for each rank was derived using two methods; (i) applying the wavelet transform to acceleration data then double integrating to obtain displacement following Eq. (9), and (ii) applying the wavelet transform to both acceleration and displacement data. Fig.6 shows the results of extracted capacity curve by each method. The capacity curve of the black line shows the capacity curve obtained by applying double integral to the acceleration data used in the existing research. On the other hand, the red line is the result obtained by using both acceleration data and displacement data. Meanwhile, the period from each rank obtained from the best-fit slope of each rank is shown in Fig. 7 along with the Nyquist frequencies.

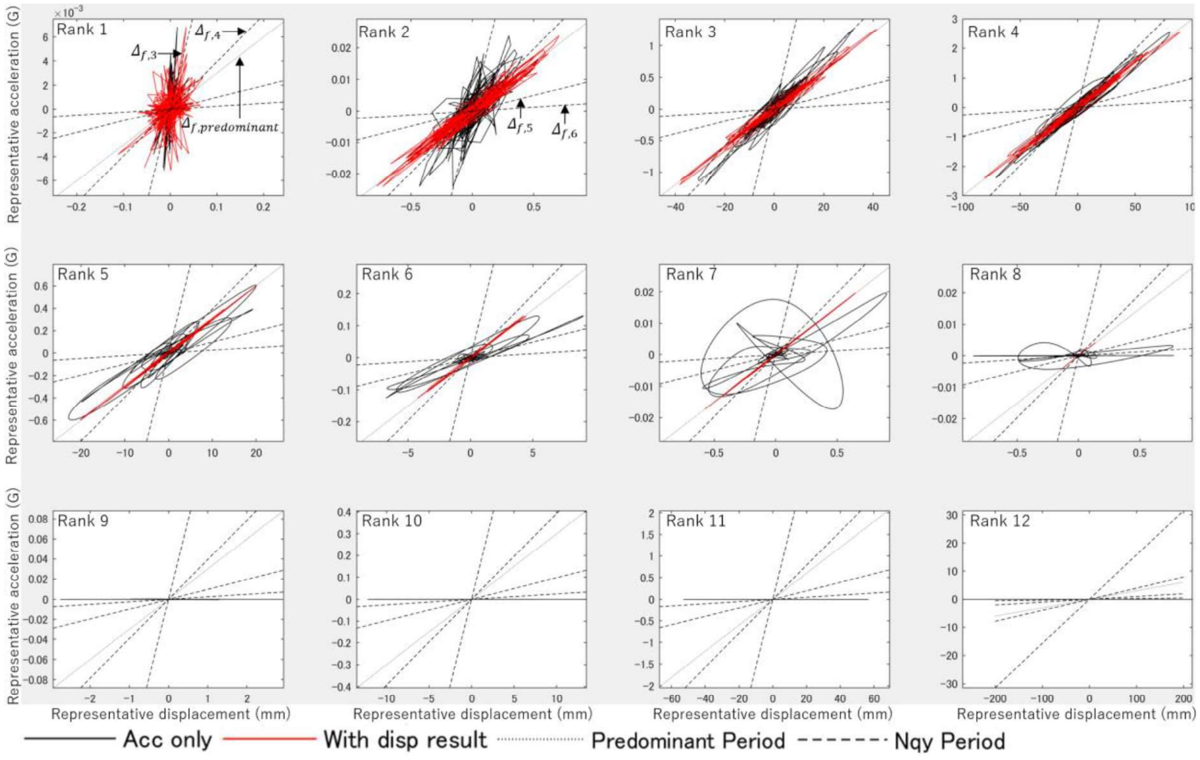

Fig. 6 – Differences between hysteresis curve of each ranks (SDoF linear model)

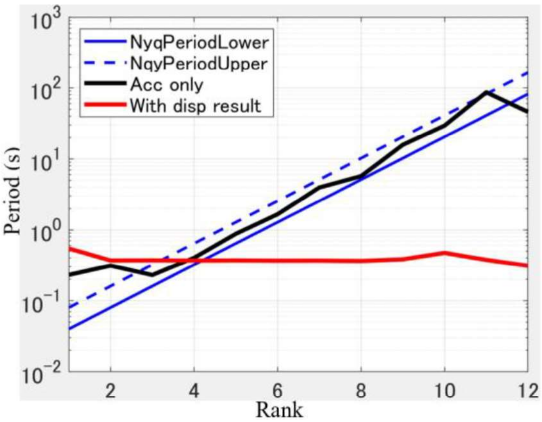

Fig. 7 – The representative linear slope of the ranks (SDoF linear model - in logarithmic scale)

Both Figs. 6 and 7 were used to select the predominant ranks as follows:

(i) Based on the hysteretic response shown in Fig 6, both methods of deriving the rank hysteretic response indicated that the rank with the largest cyclic area was rank 4. As such, rank 4 was selected as the temporary predominant mode.

(ii) The representative linear slope of each rank has been examined and visualized in Fig.7. Then, the ranks that have a similar linear slope to the temporary predominant rank has been included in the predominant rank.

In conclusion, the predominant rank selected in final is shown in Table 2. and the hysteresis curve of the combination of the selected ranks were shown in Fig.8. In this case, the original hysteresis curve has been overlapped with that of the selected ranks combination of using displacement data.

Table 2 – Selected predominant ranks based on rank hysteretic response derivation methods. (SDoF linear model)

	Using only acceleration data	**With displacement data**
Selected Predominant Ranks	Rank 2 - 4	Rank 2 - 12

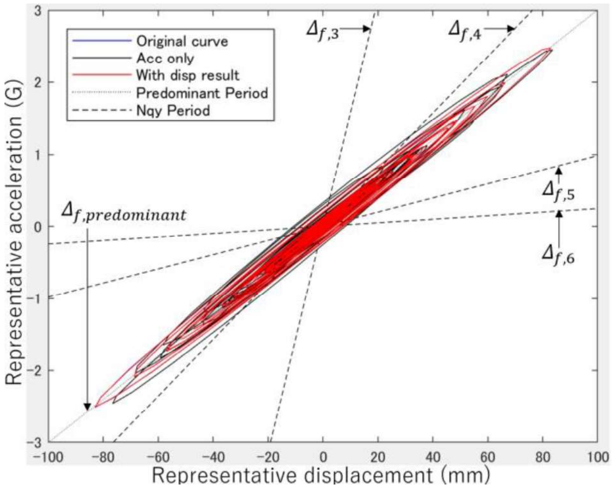

Fig. 8 – Hysteresis curve of the selected predominant ranks (SDoF linear model)

4. Case study on obtaining the capacity curve of an in elastic SDoF structure

4.1 Case study details

In this section, comparison between the different approaches to derive the hysteretic response of individual ranks similar to that examined in Section 3 was again performed. However, the Takeda model was adopted instead of a linear model. The properties of Takeda model and the hysteresis curve are as shown in Fig.9.

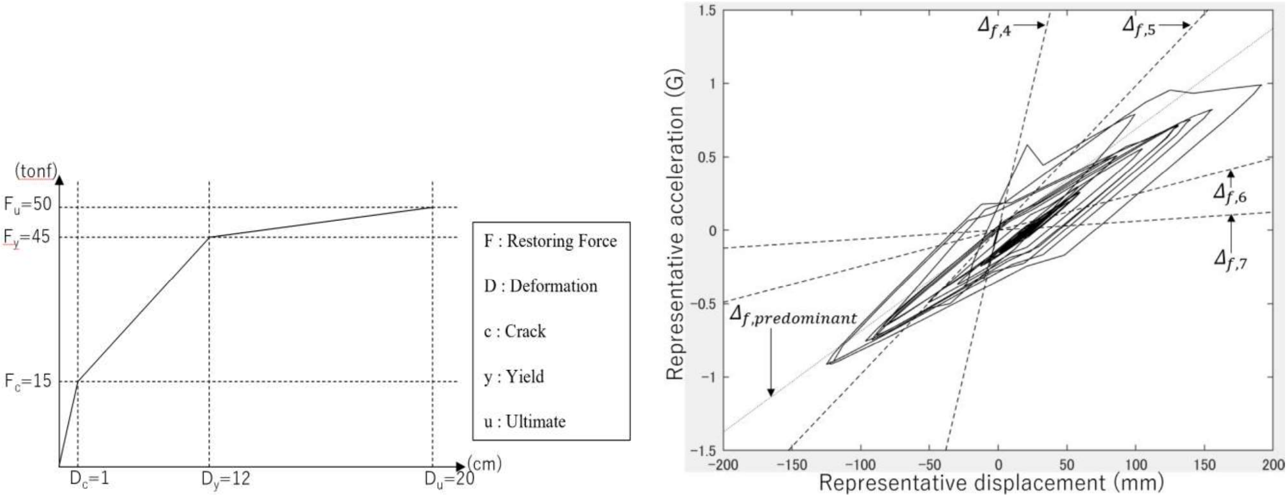

Fig. 9 – Properties and the hysteresis curve of the used Takeda Model

The transfer function obtained from analysis results for the SDoF Takeda model is shown in Fig.10. The Nyquist period of each rank and predominant period of the model are indicated using dashed lines.

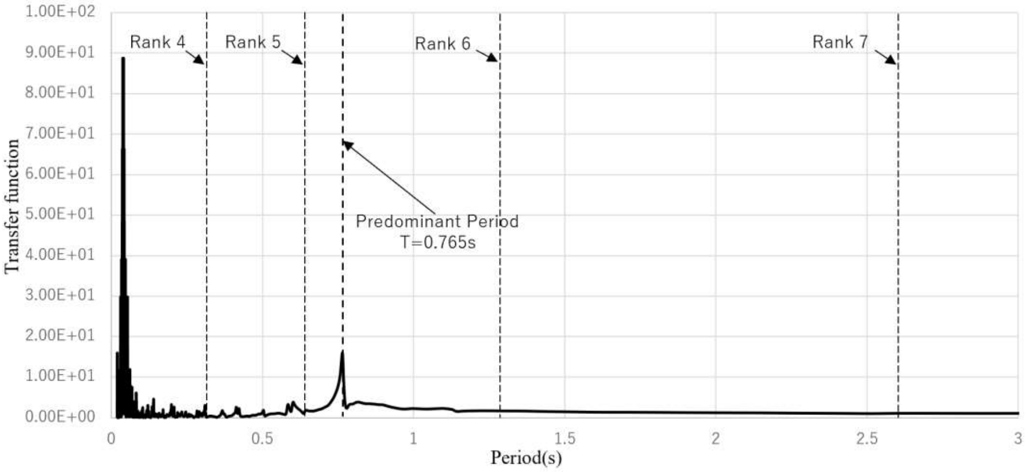

Fig. 10 – Transfer function of SDoF Takeda model

4.2 Predominant rank selection

In this section, the hysteretic response for each rank and its predominant period using results from the Takeda model is derived and compared by the same way described in Chapter 3. Fig.11 shows the results of finding the capacity curve by each method, while Fig. 12 shows the main period of each rank.

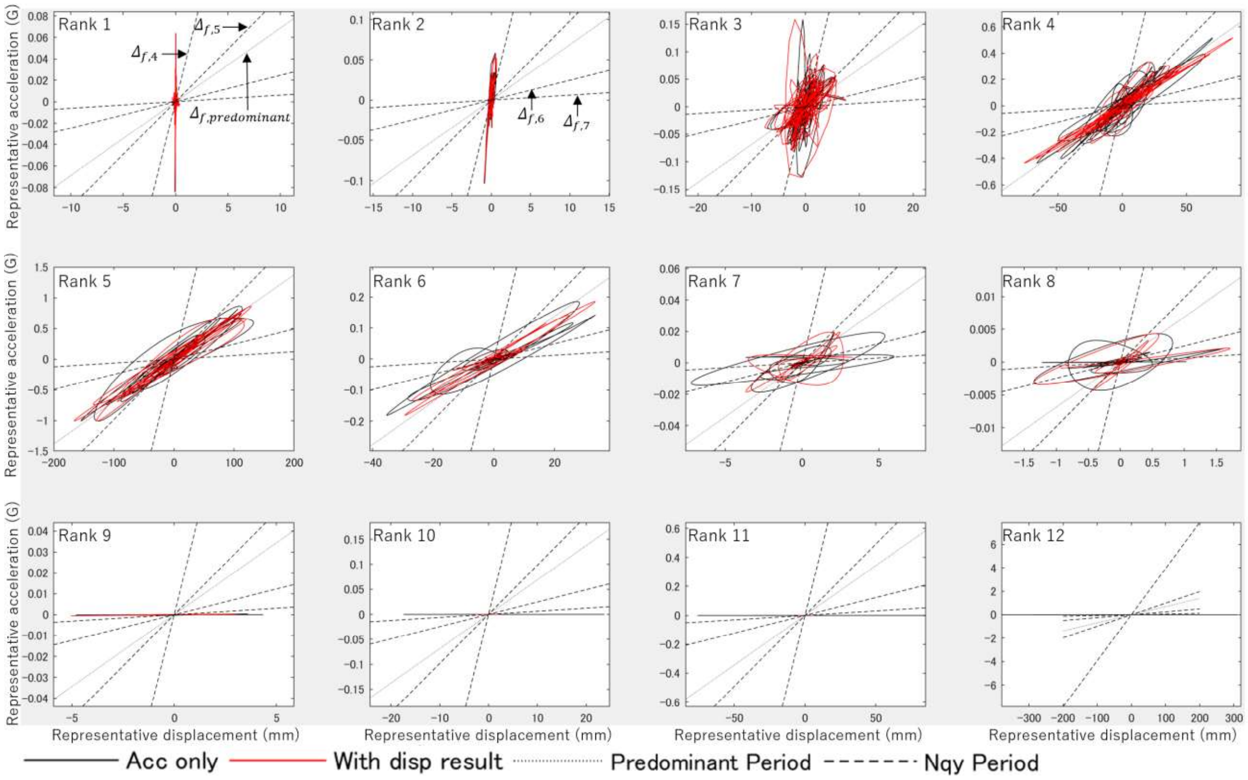

Fig. 11 – Differences between hysteresis curve of each ranks (SDoF Takeda model)

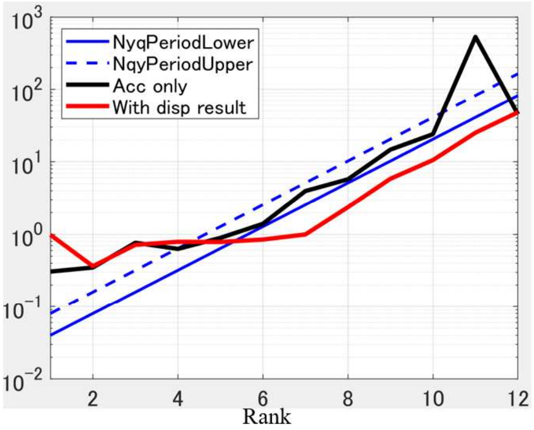

Fig. 12 – The representative linear slope of the ranks (SDoF Takeda model – in logarithmic scale)

As with previously, both Fig. 11 and 12 were used to select the predominant rank as follows:

(i) The rank with the largest cyclic area is rank 5 regardless of the approach adopted to obtain the displacement response, which means rank 5 is selected as the temporary predominant mode.

(ii) The representative linear slope of each rank has been examined and visualized as Fig.12. Then, the ranks that have a similar linear slope to the temporary predominant rank has been included in the predominant rank, In conclusion, the last selected predominant rank is shown in Table 3. and the hysteresis curve of the combination of the selected ranks were shown in Fig.13. In this case, the original hysteresis curve has not been matched for both combination of selected ranks because of the residual deformation of the building.

Table 3 – Selected predominant ranks based on rank hysteretic response derivation methods. (SDoF Takeda model)

	Using only acceleration data	**With displacement data**
Selected Predominant Ranks	Rank 3 - 6	Rank 3 - 7

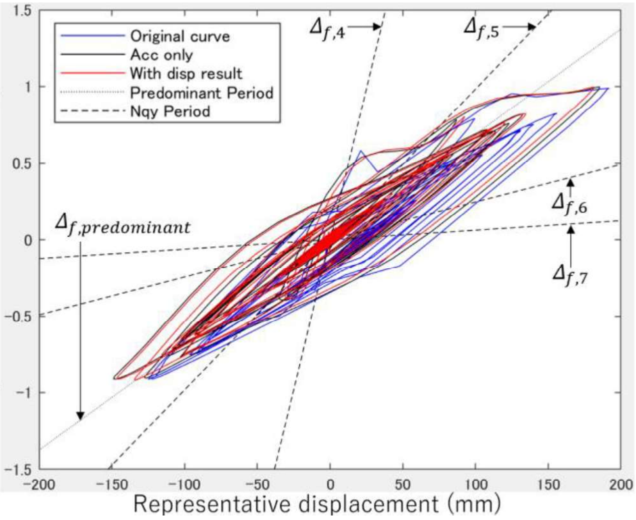

Fig. 13 – Hysteresis curve of the selected predominant ranks (SDoF Takeda model)

5. Conclusion

In this study, simple elastic and inelastic single-degree-of-freedom (SDoF) model were used to investigate how measured displacement data affects wavelet transform-based capacity curve evaluation.

Based on the comparison results for the elastic SDoF linear model case, using building displacement response data explicitly causes the representative total acceleration versus relative displacement relationship at each decomposition level to have almost identical slope. In contrast, using double-integrated acceleration data as displacement resulted in the period of each rank matching the Nyquist period of each rank. Similar issues were also observed for the inelastic SDoF case. This is likely because when double integration is used to obtain displacements, the acceleration data was already decomposed, and thus the period of acceleration response was more representative of the rank Nyquist period than the structure period itself. Therefore, using the displacements obtained directly from structural analyses make the representative linear slope of the ranks more dependent on structural properties than the Nyquist period. While the slope matches the predominant period of the structure, this resulted in there being less indicators for selecting the ranks which should be removed.

The result from the nonlinear case, using the displacement data explicitly was not an answer for capturing the residual displacement of the building. However, the results of this paper indicate that the decomposed non-predominant ranks, adjacent to the predominant ranks, could implicit the predominant mode information of the structure but discarded as a noise and error.

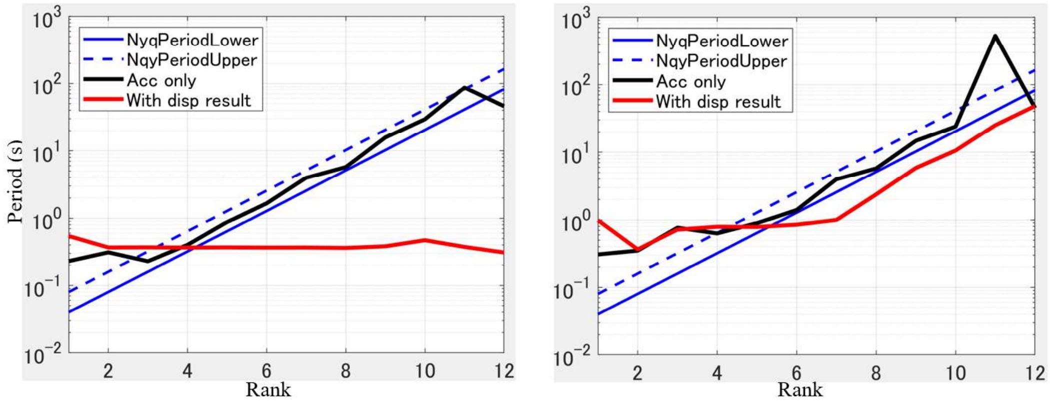

Fig. 14 – The representative linear slope of the ranks (both SDoF models – in logarithmic scale)

Unfortunately, capturing the displacement of the building in real-time using displacement sensor is not as simple as using dynamic analysis results due to several factors such as low performance of displacement capturing sensors or noises coming from other source (e.g. wind, movement of occupants). Therefore, the selection of predominant ranks using acceleration data should be improved in the direction of the way to capture the residual displacement of the building along with to capture the predominant mode information hidden in neglected ranks.

6. References

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