IDENTIFYING SEISMIC PERFORMANCE LIMIT STATES OF CIRCULAR CONCRETE BRIDGE PIERS USING MACHINE LEARNING TECHNIQUES

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Abstract

Performance-based earthquake engineering requires the identification of different damage states in structural components. Bridge piers are one of the most critical components in the bridge system that dictates the overall performance of bridges under seismic loading. Identifying and assessing the probable performance is an integral part of performance-based design. The ability to predict different damage states of a bridge pier following an earthquake can be useful in restoring service, or prescribing appropriate remediation. Machine learning techniques are becoming attractive in earthquake engineering for predicting failure modes of building and bridges. The objective of this study is to implement different machine learning techniques to properly capture the different limit states (spalling, core crushing, and bar buckling) of reinforced concrete circular bridge piers under seismic loading. Using experimental data from large scale testing of circular bridge piers, this study aims to compare different machine learning techniques for predicting performance limit states. The efficiency of different methods will be identified using random data sets which are not used for training the models. In addition, this study will identify the influential variables affecting the different performance limit states and provide formulations for predicting different limit states that can be readily used by designers and practitioners.

Keywords: Performance-based design; Machine learning; Bridge pier; Limit states; Seismic performance.

1. Introduction

Recent advances in performance based seismic design focus on improved post-earthquake functionality of structures and accurate prediction of seismic performance over a wide range of earthquake scenarios. Performance-based earthquake engineering requires the identification of different damage states in structural components. Bridge piers are one of the most critical components in the bridge system that dictate the overall performance of bridges under seismic loading. Identifying and assessing the probable performance is an integral part of performance-based design. The ability to predict different damage states of a bridge pier following an earthquake can be useful in restoring service, or prescribing appropriate remediation. Moreover, most of the design guidelines are moving towards performance-based design. AASHTO has already developed performance-based design guidelines for bridges referred as AASHTO SGS (AASHTO 2011). Moreover, the recent edition of the Canadian highway bridge design code (National Research Council of Canada 2019) has also adapted performance-based design and defined some performance levels and performance criteria for different types of bridges.

Due to recent technological progress, Machine Learning (ML) has become a broadly powerful tool. Machine learning techniques are becoming attractive in earthquake engineering for predicting failure modes of building and bridges, predicting concrete strength and elastic modulus [1-2]. A few researchers have applied machine learning methods in the field of structural engineering to classify failure modes in structural components [3-6]. Mangalathu and Jeon [3] applied machine learning techniques to classify failure modes and subsequently predict associated shear strength in beam-column joints. Additional research by Mangalathu and Jeon [4] explores the capabilities of various machine learning classification techniques in identifying bridge column failure modes. Mangalathu et al. [5] applied artificial neural network regression to generate bridge specific fragility curves. Pang et al. [6] additionally applied artificial neural network regression in simulating incremental dynamic analysis (IDA) curves at given intensity levels. Research is mounting in support for classification regression techniques applied to seismic fragility however; a gap exists in the use of machine learning regression techniques when predicting structural limit states. Despite the growth of movements applying ML to problems of structural and earthquake engineering, there remains the need for a concerted effort to identify how these tools may best be applied to performance-based earthquake engineering.

Given the variable nature of bridge pier design process, material and geometric properties, and performance requirements, there exists the need to have predictive tools that enable reasonable estimation for performance limit states of bridge piers. The objective of this study is to implement different machine learning techniques to properly capture the different limit states (spalling, core crushing, and bar buckling) of reinforced concrete circular bridge piers under seismic loading. Unlike previous studies which employ machine learning techniques for failure mode classification [3,4] a comprehensive set of regression tools are employed for the creation a regression model. This study implements several different classes of machine learning techniques such as linear regression, shrinkage methods, nearest neighbors, tree, and ensemble methods to properly capture the different limit states (spalling, core crushing, and bar buckling) of reinforced concrete circular bridge piers under seismic loading. Using experimental data from large scale testing of circular bridge piers, this study aims to compare different machine learning techniques for predicting performance limit states. The efficiency of different methods is identified using random data sets which are not used for training the models.

2. Performance Limit States

For performance based seismic design and assessment of bridges, it is critical to predict different damage states or performance levels under different levels of ground motions [7]. Damage experienced by a bridge pier, during an earthquake, is a function of ductility, deformation, energy dissipation, as well as strength and stiffness degradation. Damage indices are often used to quantify the damage sustained by a concrete bridge pier. Since damage is a complex function of different structural and response parameters, different cumulative and noncumulative damage indices are available in the literature. The Canadian Highway Bridge

Design Code, CAN/CSA S6-19 [8], has implemented Performance Based Design (PBD) as a requirement for seismic design of more important or irregular bridges. This means explicit performance objectives are defined and have to be demonstrated, in contrast to traditional codes where performance objectives are implicit. In S6-19, the damage-level criteria are defined as quantitative limits on concrete and reinforcement strains as well as qualitative limits on displacements or damage to bridge components. In this study, the focus is on the drift limits that apply to concrete bridge piers since they are the most common lateral force resisting element in typical highway bridges. The performance limits considered in this study are the spalling of cover concrete, crushing of core concrete, and buckling of longitudinal rebar (Fig. 1). Although, yielding of longitudinal rebar is the first significant damage that a reinforced concrete pier will encounter, it is not considered here since it can be easily predicted when the rebar strains exceed the yield strain for the first time. The yield strain of the rebar can be obtained by dividing the design yield strength of longitudinal rebar with the elastic modulus of steel (E=200 GPa).

On the other hand, for RC columns and bridge piers, determining the exact occurrence of cover spalling/significant spalling from testing is challenging because damage is typically evaluated and reported at the loading cycle peaks. Also predicting the occurrence of spalling is not straightforward. Similarly, the crushing of core concrete depends on the compressive strength of concrete and the detailing of longitudinal and transverse reinforcement. Likewise, predicting the drift corresponding to core crushing is not straightforward and different empirical expressions exist in the literature [10,15]. Buckling of longitudinal bars is a common form of damage in reinforced concrete (RC) structures subjected to earthquakes. However, it is difficult to numerically simulate an RC structural member including the inelastic buckling of longitudinal reinforcing bars. Modeling localized nonlinear behavior and the complicated boundary conditions as well as their interactions with the reinforcing bar requires extensive computational cost, because failure of convergence often occurs in analysis. To resolve these limitations, this paper explores the application of machine learning techniques to identify the different performance limit states circular RC bridge piers.

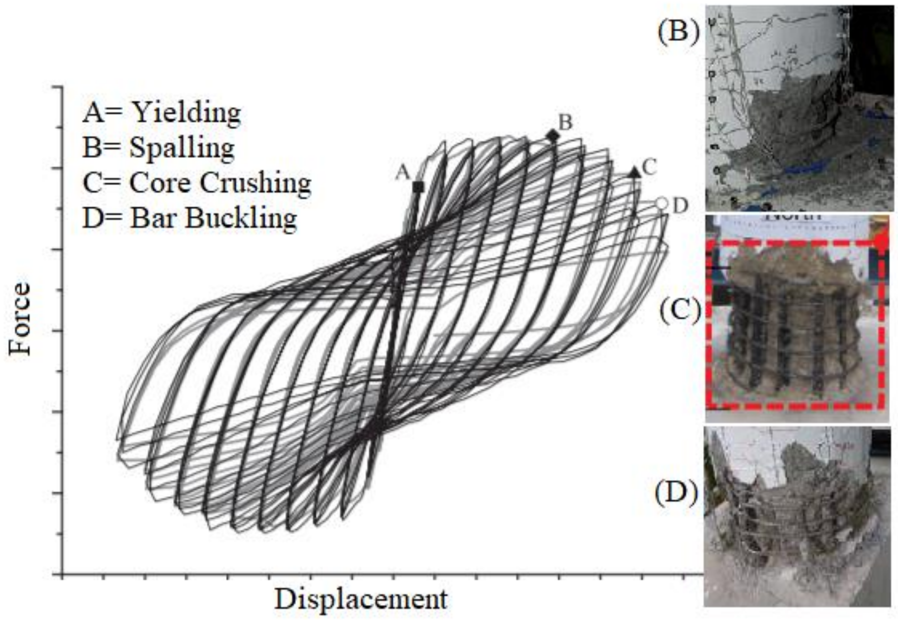

Fig. 1 – Typical definition of circular column performance limit states

3. Column Database

A comprehensive database of 143 experimental circular column results has been assembled for this study. The database consists of experimental column results provided by the PEER structural performance database

(SPD) [9], along with newly collected test data from additional experimental studies [10-13]. Results from the original PEER SPD consist of 163 experimental results. However, data which did not have limit state drifts recorded was excluded for this study, and experimental data for columns with unusual material properties (i.e. high yield strength rebar, high concrete compressive strength, and tensile column tests) were also excluded. Furthermore, the preliminary database was filtered for columns which experienced damage states outside of expected drift ratios as shown in Fig. 2.

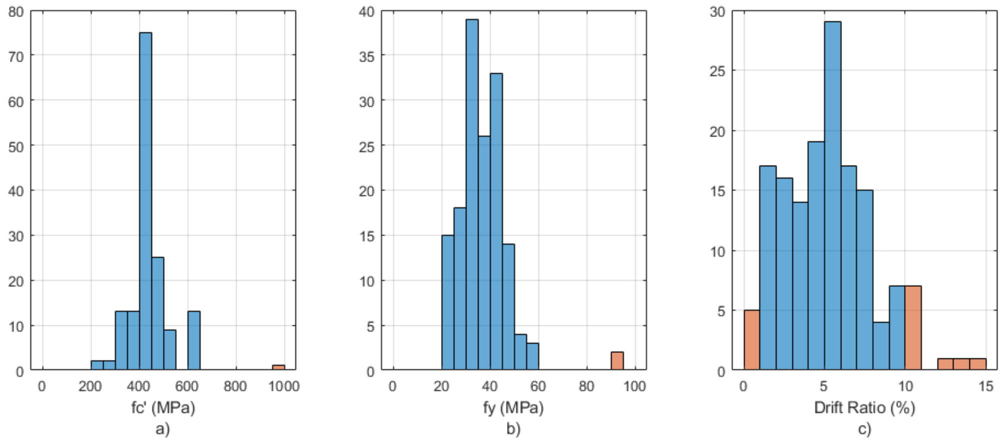

Fig. 2 – Filtered column input parameters

The resulting database of 141 experimental circular columns has design parameters that fall within the following ranges:
Axial load ratio: $0.003 \leq P/fc'A_g \leq 0.700$
Concrete compressive strength: 22.40 MPa $\leq fc' \leq$ 57.00 MPa
Aspect ratio: $1.5 \leq l/d \leq 10.0$
Yield strength of longitudinal reinforcement: 294 MPa $\leq f_{yl} \leq$ 565 MPa
Longitudinal reinforcement ratio: 0.46% $\leq \rho_l \leq$ 5.21%
Yield strength of transverse reinforcement: 207 MPa $\leq f_{yt} \leq$ 607 MPa
Transverse reinforcement ratio: 0.13% $\leq \rho_s \leq$ 2.84%

Where $P/fc'A_g$= axial load ratio, defined as the axial compressive load (P) divided by the concrete compressive strength (fc') and gross cross sectional area (A_g); l/d = aspect ratio, defined as the column length (l) divided by the column diameter (d); f_{yl} = yield strength of longitudinal reinforcement; ρ_l = longitudinal reinforcement ratio; f_{yt} = yield strength of transverse reinforcement; ρ_s = transverse (spiral or hoop) reinforcement ratio.

Performance-based earthquake engineering in reinforced concrete structures requires the prediction of deformations at the onset of different damage states in structural components Typically, damage states are defined in terms of drift or displacement where they are defined as discrete observable damage states such as rebar buckling, concrete spalling, and core crushing [7,14]. The current edition of the Canadian highway bridge design code [8] defines performance criteria for four different performance levels such as immediate, limited, service disruption, and life safety. Previous work by Berry and Eberhard [15] identifies key damage states for bar buckling and concrete spalling as a function of deformation which will be used in classifying limit states performance.

The table below (Table 1) summarizes relevant drift limit states prescribed in the current edition of the Canadian highway bridge design code [8]. It should be noted that due to an inadequate amount of cracking experimental results, the limit states model for this damage state has been excluded in this research.

Table 1 – Bridge component damage states

Damage parameter	Functionality level	Damage Level	Description
Cracking*	Immediate	Minimal	Onset of hairline cracking
Spalling	Limited	Repairable	Onset of concrete spalling
Core Crushing	Service Disruption	Extensive	Crushing of core concrete
Buckling	Life Safety	Probable Replacement	Theoretical first buckling of longitudinal rebar

4. Machine Learning Regression Methods

The objective of this research is to develop a regression model for bar buckling, concrete spalling, and core crushing limit states of circular reinforced concrete bridge columns as a function of their selected input parameters. As such, various regression methods such as decision trees (DT), K-nearest neighbors (KNN), least angles regression lasso (LARS Lasso), and artificial neural networks (ANN) are explored. An overview of the regression algorithm is provided in the following section, along with the author's reasoning behind input parameter selection. As this section is not intended to be a comprehensive overview of the selected regression methods, readers are directed to either additional sources [16-18] for further insight.

4.1 Artificial Neural Network

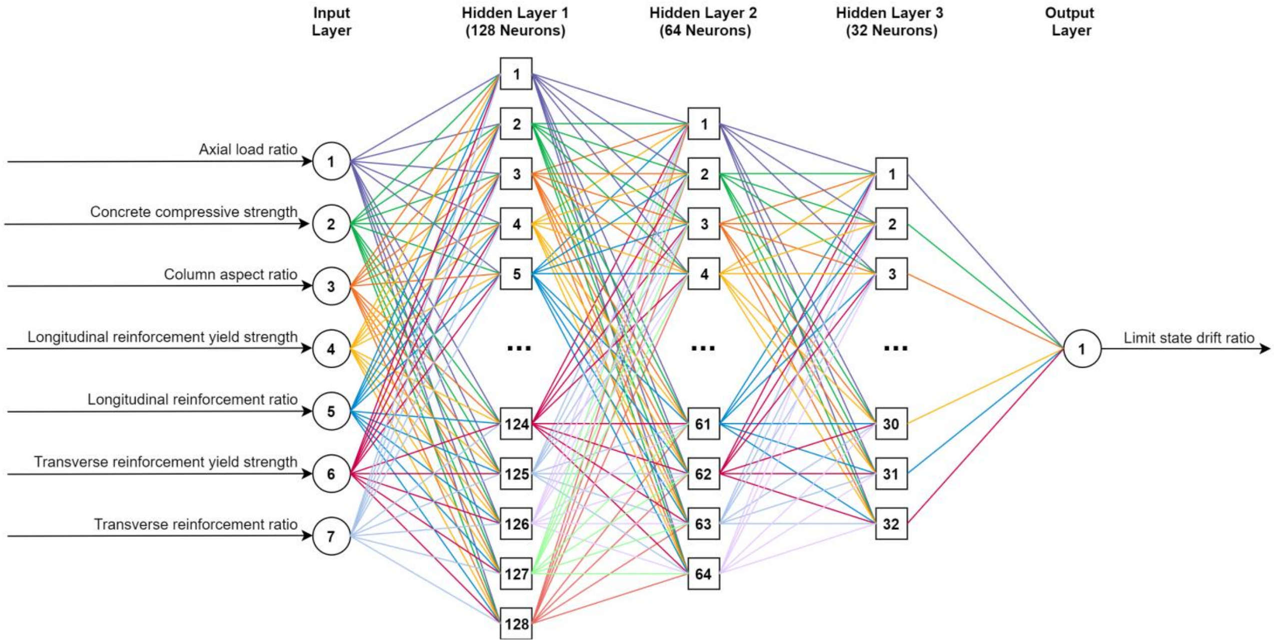

Fig. 3 – Artificial neural network layout for limit state drift ratio regression

Artificial neural networks are a type of nonlinear regression model which operate by emulating the function of neurons found in biology. The neural network arrangement used in this study as shown in Fig. 3 is defined as the feedforward multilayer perceptron (MLP)[19,20]. Typical MLP neural networks are composed of an input layer, several hidden layers, and output layer. Hidden layers used in this study are comprised of varying amounts of neurons which feed forward to the next hidden layer using a weighting function. Each neuron in a hidden has the following output signal:

$$y_k = \varphi\left[\sum_{j=1}^{m} w_{kj}x_j + b_k\right] \quad (1)$$

In which φ is the neuron's activation function; m is the number of neurons in the hidden layer; w_{kj} are the synaptic weights; x_j are the input signals; and $b_k$is the hidden neuron bias. The particular activation function used in this study is the rectified linear unit (ReLU) as it lends itself particularly well to better performance of deep networks [21]:

$$\varphi(v) = max(0, v) \quad (2)$$

The samples from the database are first permutated to randomize their order, and 25% of the samples are reserved for model testing. The remaining 75% of sample data is used to train the network model and develop a regression function. A loss function of the neural network model is tested against the reserved data at each epoch where the model weighting factors are refined.

4.2 Linear Regression

Linear regression (LR) is the simplest and most commonly applied regression technique that generates a line of best fit through a specified set of points, in the form of:

$$y = \beta_o + \sum_{j=1}^{p} x_j\beta_j \quad (3)$$

Where p number of unknown β_j coefficients exist for each input parameter. Coefficients $\beta_j, \beta_k, \beta_l \cdots \beta_p$ are typically estimated using the least squares method, in which the coefficients are selected to minimize the residual sum of squares in equation 4.

$$RSS(\beta) = \sum_{i=1}^{N}\left(y_i - \beta_o - \sum_{j=1}^{p} x_{ij}\beta_j\right)^2 \quad (4)$$

4.3 Ridge Regression

Ridge regression (RR) is a shrinkage method that implements a size penalty on regression coefficients. Shrinkage methods retain a subset of predictors to produce a model which tends to have lower variance as compared to the least squares estimator as discussed previously. Ridge regression penalizes the residual sum of squares as

$$\hat{\beta}^{ridge} = \underset{\beta}{argmin}\left\{\sum_{i=1}^{N}\left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_{ij}\right)^2 + \alpha\sum_{j=1}^{p}\beta_j^2\right\} \quad (5)$$

Where α is the nonzero complexity parameter that controls the amount of shrinkage. A larger α corresponds to greater amount of model shrinkage and thus the models become more resistant to collinearity. A value of $\alpha = 0.5$ has been used in the ridge regression model in this research. Typically a size constraint t for inputs is also implemented in ridge regression, and coefficients are scaled respectively such that

$$\sum_{j=1}^{p} \beta_j^2 \leq t \tag{6}$$

Imposing a size constraint alleviates a problem of high variance in the correlation variables, however the inputs need also be standardized to ensure solutions are equivariant.

4.4 Least Angle Regression Lasso

Least angle regression is a newer [22] shrinkage method similar to forward stepwise regression. Much like ridge regression, least angles regression tends to produce a model with lower variance as compared to least squares regression.

The least angle regression algorithm first standardized the predictors to have mean zero and unit norm. Initial residual response is defined as:

$$r = y - \bar{y} \tag{7}$$

The algorithm selects the input variable x_j most correlated with the residual response, r. The value of the corresponding β_j coefficient is modified from 0 towards its least-squares coefficient (x_j, r) by the algorithm until a second predictor x_k has equal correlation. Coefficients β_j, β_k are modified together in the direction towards their joint least-squares coefficient (x_j, x_k) until another predictor x_l has equal correlation with the combined current residual. The process of evaluating joint correlation and progressively including the remainder of predictors is repeated until all p coefficients have been inputted. At any point where a joint least squares coefficient is evaluated, the lasso modification to the least angle regression algorithm monitors the existing pool of nonzero $(\beta_j, \beta_k, \beta_l \cdots \beta_p)$ coefficients. If any of the coefficients reach zero, the LARS lasso algorithm removes them from the active set and the joint least squares direction is recalculated. As such, least angle regression always takes p steps to reach the full least squares estimates, whereas the lasso modification of LARS can have more than p steps.

4.5 K-Nearest Neighbors Regression

K-nearest neighbors regression (KNN) is a nonparametric method which predicts a numerical target based on a distance function to its K-nearest neighbors. In this study's defined search space, the KNN makes use of an automatic search algorithm which tests both k-d tree and ball tree search within the multidimensional search space. For the predicted data x, the KNN identifies K number of neighbors from its training data which are closest to x. Using the following Euclidian distance function a weighting factor is generated to produce the prediction:

$$D = \sqrt{\sum_{i=1}^{k} (x_i - y_i)^2} \tag{7}$$

The choice of K has great influence on the performance of the KNN model. Lower K values limit model bias and allow for higher variance, whereas high K values limit model variance and allow for higher bias. In this study K values between 1 and 10 were tested, the mean absolute error (MAE) values for the models are shown in table 2. The MAE values are normalized and K=3 is shown to have the best overall performance for the three limit states models.

Table 2 – Influence of K parameters

	Normalized Buckling MAE	Normalized Spalling MAE	Normalized Crushing MAE	Average Normalized MAE
K = 1	1.000	1.000	0.878	0.959
K = 2	0.832	0.885	0.974	0.897
K = 3	0.799	0.780	0.937	**0.839**
K = 4	0.785	0.828	0.973	0.862
K = 5	0.848	0.838	0.959	0.882
K = 6	0.921	0.882	1.000	0.934
K = 7	0.930	0.913	0.996	0.947
K = 8	0.918	0.860	0.977	0.919
K = 9	0.912	0.787	0.960	0.886
K = 10	0.947	0.779	0.991	0.905

4.6 Decision Tree Regression

Decision trees are another nonparametric regression method that operate by forming a tree-like decision network by learning simple decision rules inferred from the data features. The decision tree algorithm uses recursive partitioning to divide the data into unique regions with distinct boundaries. Decision trees are initiated with a root node, and using binary splits henceforth, interior and terminal nodes are formed where each internal node has only one parent and two children, and each terminal node has one parent only. Fig. 4 shows the layout of the decision tree as an example for the regression of core crushing drift limit.

In this study, a nonlinear relationship between predictors and response warrants the use of the classification and regression trees (CART) algorithm by the decision tree regressor. CART is a similar implement to C4.5, with the exception of support for regression, and exclusion of computed rulesets.

4.7 Random Forest Regression

Random forest (RF) is an ensemble method that creates multiple parallel tree structures consisting of decision trees [23]. RF implements bootstrap aggregation (bagging) and random feature selection which generates each tree using the bootstrap sampled versions of the training data. The regression model generated is the averaged regressor of each decision tree subassembly. Random forests make for a powerful regression model since the relatively low bias of Decision trees is maintained, while a common problem in decision trees, noise is reduced by averaging.

A generalized random forests algorithm consists of the following procedure:

For $b = 1 \ to \ B$

1. Draw a bootstrap sample Z from the available training data
2. Grow a decision tree T_b from each bootstrap sample data, by randomly selecting variables from those available and selecting the best split among the data until the minimum node size is reached.

Output the forest ensemble $\{T_b\}_1^B$

$$\hat{f}_{rf}^{B}(x) = \frac{1}{B}\sum_{b=1}^{B} T_b(x) \tag{8}$$

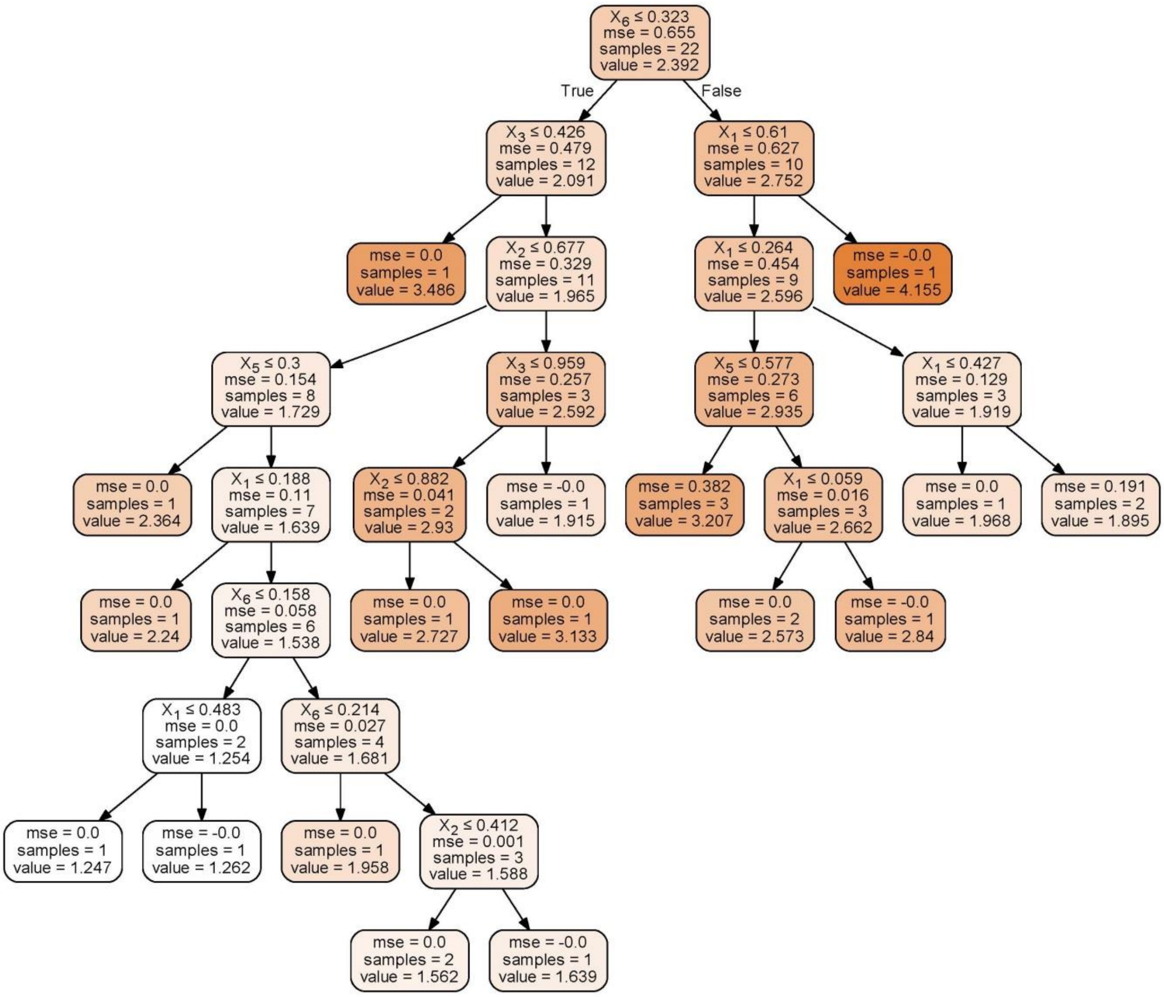

Fig. 4 – Decision Tree layout for concrete core crushing limit state drift ratio regression

5. Performance of Different Regression Methods

To evaluate the performance of the proposed regression models in predicting future data, the database is divided into training and testing set with a 75%-25% ratio respectively. Training and testing sets assignment is randomized by permutating the database using a predefined seed for repeatability. Training data makes up the majority of the database and is used to train all regression models, whereas the testing data is used to estimate fitting accuracy of the proposed models and to avoid overfitting. The regression model predictions are compared against the test set actual values to determine error. In this study, Mean absolute error (MAE) and mean square error (MSE) criteria are evaluated and compared amongst model performance for each of the three drift limit states. MAE and MSE can be estimated as

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|Y'_i - Y_i| \tag{9}$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y'_i - Y_i)^2 \quad (10)$$

Where Y' is the actual testing set result and Y is the corresponding regression model prediction. The results of the three limit states' regression models have been summarized in the tables 3 and 4 as follows.

Table 3 – Limit state regression mean absolute error

MAE	ANN regression	Linear regression	Ridge regression	LARS lasso regression	KNN regression (K=3)	Decision tree regression	Random forest regression
Bar buckling	1.310	1.407	1.342	1.381	**0.789**	0.921	0.987
Concrete spalling	0.762	0.744	0.552	0.616	**0.461**	0.822	0.518
Core crushing	1.127	1.421	1.168	1.039	**0.907**	1.412	1.071

Table 4 – Limit state regression mean square error

MSE	ANN regression	Linear regression	Ridge regression	LARS lasso regression	KNN regression (K=3)	Decision tree regression	Random forest regression
Bar buckling	2.485	2.709	2.324	3.258	**0.913**	1.508	1.234
Concrete spalling	0.776	0.834	0.499	0.610	**0.438**	1.027	0.543
Core crushing	1.666	3.126	1.821	1.355	**1.433**	2.851	1.578

K-nearest neighbors regression outperforms neural network, linear, and tree based regression methods by an average margin of 51.9%, 34.8%, and 15.7% respectively. Therefore, the K-nearest neighbors model with K=3 is the suggested regression model for limit states identification of all three studied states. The model provides a reasonable accuracy in identifying a drift limit state for a provided bridge column suitable for performance based design.

Note that the regression results are relevant to circular reinforced concrete bridge columns with material properties falling into the ranges of those discussed in section 3. Further research is recommended in populating a larger experimental database, and exploration of influence of additional column parameters.

6. Conclusions

The ability to predict different damage states of a bridge pier following an earthquake can be useful in restoring service, or prescribing appropriate remediation. This study explores the feasibility of applying machine learning regression techniques in predicting circular bridge column limit states. This is achieved by compiling an extensive experimental database of bridge columns with varying material, geometrical, and loading properties. To minimize outliers for the developed regression models, abnormal experimental configurations are excluded from the database as the interaction between inputs is nonlinear and complex.

The experimental database is permutated and divided into 75% training and 25% testing data set. Various machine learning regression methods such as artificial neural network (ANN), linear regression (LR), least angle regression lasso (LARS Lasso), ridge regression (RR), K-nearest neighbors (KNN), decision tree (DT), and random forests (RF) are established using the training data set. The remaining test dataset is used to categorize the efficiency of the proposed regression models using mean absolute error (MAE) as the performance criteria, where it is consistently shown that KNN consistently achieves the most accurate prediction.

In this study, the K-nearest neighbors model with K=3 is the suggested regression model for limit states identification with reasonable accuracy. It should however be noted, that the underperformance of the ANN model suggests that a larger experimental database would yield more accurate results. Nevertheless, the comparison of various machine learning regression models highlights the complex nonlinear relationship in predicting seismic performance limit states.

7. Acknowledgment

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