CHARACTERISTICS OF NON-LINEAR RESPONSE OF A ROAD BRIDGE WITH FOUNDATION DURING TWO-WAY EARTQUAKE MOTIONS

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Abstract

The research of dynamic soil-structure interactions has a long history of more than 60 years, starting with Housner et al. in 1954. However, seismic design is often performed in a vibration system that ignores both the foundation and ground. There are many unexplained points, and because of the difficulty in modeling and in the phenomenon itself, many problems remain for incorporating the effects of dynamic interaction into design as a standardized method. In addition, large vertical ground motions, including the vertical rigidity of the upper structure compared to the horizontal rigidity, have been observed since the Hyogoken-Nanbu Earthquake, there have been cases of damage to structures that are thought to be due to this event. As such, the influence of the dynamic soil-structure interaction with the ground is likely to be present and therefore needs to be studied.

In general, response calculations for horizontal and vertical input motions are performed by separately performing horizontal motion and vertical motion response calculations and combining them. Dr. Masao proposed the effect of parametric excitation that causes vertical vibrations caused by vertical springs between mass points at the moment when the structure is horizontally displaced by horizontal input motion, and that vertical vibrations excite horizontal vibrations. Additionally, he has proposed the vibration equation incorporated in the above effect, and is conducting a corresponding linear seismic response analysis with a fixed foundation.

In this study, the effect of parametric excitation was modeled on a superstructure by bending shear type beam elements, the foundation-soil system was also supported by a rigid foundation with horizontal, vertical and rotating soil springs. Nonlinear seismic response analysis was performed by considering the dynamic soil-structure interactions of the system. The effect of the vertical motion, parametric excitation and the non-linear characteristics of the three soil springs on the horizontal and vertical vibration characteristics of the superstructure, as well as the response of the foundation caused by soil deformation by the response of the superstructure were investigated.

The results obtained from the numerical calculation examples in this study are as follows.

(1) The influence on the response displacement due to the non-linear characteristics of the soil springs of the foundation are large with the horizontal and vertical displacement of the foundation and small with the displacement of the superstructure regardless of the presence or absence of the vertical motion and parametric excitation.

(2) The effect of parametric excitation appears greatly due to the rotational displacement of the foundation, and the way the linear and non-linear characteristics appear depends on the presence or absence of vertical motion.

(3) There is almost no difference between the horizontal and vertical hysteresis loops of the foundation due to the presence or absence of vertical motion with parametric excitation, however, there is a large difference between the up and down motion of the foundation due to the effect of parametric excitation. Furthermore, a non-linear hysteresis loop is drawn due to the effect of parametric excitation without vertical motion.

Keywords: Horizontal and vertical earthquake motions, Parametric excitation, Dynamic soil-structure interaction

1. Introduction

Research on dynamic soil-structure interactions has a long history of more than 60 years, starting with Housner et al. [1]in 1954. However, seismic design is often performed in a vibration system that ignores both the foundation and ground. Due to difficulty in modeling and of the phenomenon itself, there are several unexplained points and problems in incorporating the effects of dynamic interaction into design as a standardized method [2]. In addition, large vertical ground motions, including the larger vertical rigidity than horizontal rigidity of the upper structure, have been observed since the Hyogoken-Nanbu Earthquake, which resulted in structural damages [3]. Hence, the influence of the dynamic soil-structure interaction on the ground is likely present [4] and therefore, needs to be studied.

Response calculations for horizontal and vertical input motions are generally performed by separately performing horizontal and vertical motion response calculations and combining them. Dr. Masao [5,6] proposed the effect of parametric excitation [7] that causes vertical vibrations from vertical springs between mass points at the moment when the structure is horizontally displaced by horizontal input motion exciting horizontal vibrations. Additionally, he proposed the vibration equation incorporating the above effect and conducted a corresponding linear seismic response analysis with a fixed foundation.

In this study, the effect of parametric excitation was modeled on a superstructure by shear bending of beam elements. The foundation-soil system used was also supported by a rigid foundation with horizontal, vertical, and rotating soil springs. Non-linear seismic response analysis was performed considering the dynamic soil-structure interactions of the system. The effect of vertical motion, parametric excitation, and non-linear characteristics of the three soil springs on the horizontal and vertical vibration characteristics of the superstructure and the foundation caused by soil deformation in response to the superstructure were investigated.

2. Equation of motion considering parametric excitation

A simple spring-mass model of a road bridge is set to examine its dynamic behavior, as shown in Fig. 1. As shown in Fig. 2, the superstructure is a bending shear type with three degrees of freedom (horizontal, vertical, and rotation) at each mass points and at the foundation. Vertical displacement associated is ignored. The global coordinate system is set to $O-XY$ on the upper surface of the earthquake-resistant foundation ground and the input seismic wave horizontally displaced to W_x and vertically displaced to W_y.

The local coordinates of each mass point on the pier are displaced horizontally, vertically, and rotated. However, the mass centers moved to a position below the relevant mass point due to the mass point displacement. The mass points of the piers are numbered in order from the top ($j=1\sim3$) in Fig. 2. The coordinates (x_i, y_i) and (x_{i+1}, y_{i+1}) are determined from the relationship of the elastic deformation of the mass points i and $i+1$ of the superstructure by the local coordinates shown in Fig. 3. The member length ℓ_i is obtained by the Pythagorean theorem. The elongation $\Delta\ell$ of the member between both mass points can be obtained from an approximate expression using the Marcolin series expansion.

Then, coordinates (x_i, y_i) and (x_{i+1}, y_{i+1}) are returned to the coordinate system of the mass point of the superstructure shown in Fig. 2. The kinetic and potential energy of the entire system (base spring, pier spring, and spring due to shaft expansion and contraction of the pier) are then calculated. The equation of motion by the Lagrangian equation with omitted terms is given by Eq. (1). $\{m\}$ and $\{R\}$ are seismic external force and corrected external force, respectively. The vertical and horizontal displacement response are related from the latter $\{R\}$ element, which expresses the effect of parametric excitation.
where,

$$[M]\{\ddot{x}\}+[K]\{x\}=-\{m\}-\{R\} \tag{1}$$

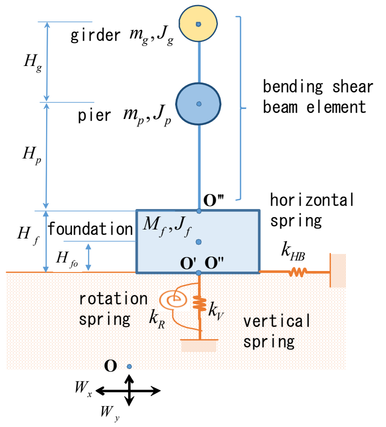


Fig. 1 Spring-mass system model

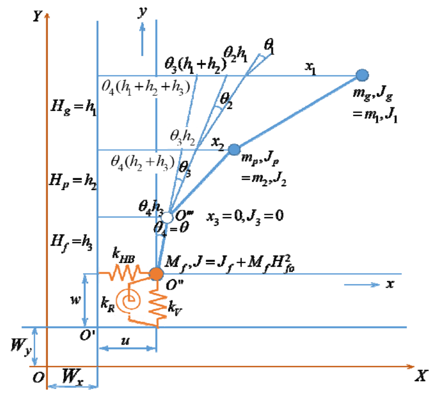
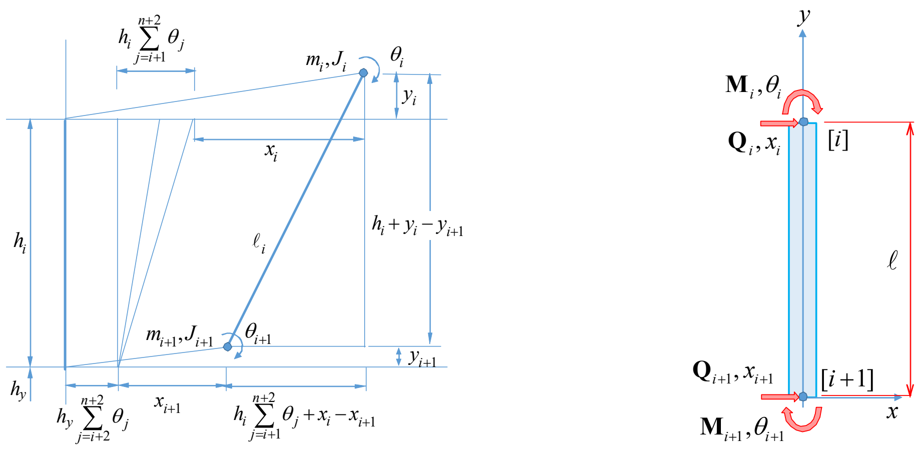


Fig. 2 Global coordinate system



(a) Coordinate relationship between nodes (b) Bending shear beam element

Fig. 3 Coordinate relationship between nodes and bending shear beam

3. Analysis method and model

The earthquake response analysis of Eq. (1) is calculated using Newmark's β method to obtain the sum of the absolute values of displacement $\{x\}$ and the relative error (difference between the sum of time t and the sum of time $t-1$ in a specific time section). If the value is smaller than the set value, the method proceeded to the next step. The analysis model used the parameters for road bridges shown in Table 1, where three types are set: Case A (level 1, linear), Case B (level 2, linear), and Case C (level 2, non-linear). Level 1[8] indicates that the NS component of the JMA Kobe conforms to the standard acceleration response spectrum (Class I ground) of the specifications for highway bridges and the maximum UD and NS component of the observed wave in the NS component amplitude. This is the input waveform of the UD component created by multiplying the ratio. Fig. 4 shows the input waveform of the NS and UD components of Level 1 and their acceleration response spectrum. The spectrum also shows each Level 2 component (Kobe Marine Meteorological Observatory) for comparison. The non-linear seismic response analysis for Case C is performed using the bilinear type for restoring force characteristics of the ground spring with linear superstructure. At that time, the yield displacement [9] is set from the maximum displacement of response value of Case A (in Table 2 described in Section 4.1) and the plastic secondary gradient is 10% of the initial stiffness. In addition, numerical integration method is used for the linear acceleration method (β = 1/6) using the Rayleigh attenuation term determined from the first and third modes of eigenvalue analysis with a minute time interval set to 0.001 s.

Table 1 Parameters for road bridge

superstructure and foundation	mass	m_g	m_p	M_f
	m (kN・s^2/m)	407.89	169.88	899.39
	elastic modulus	E_g	E_p	
	E (kN/m^2)	2250000	8799738	
	second moment of area	I_g	I_p	
	I (m^4)	0.0043279	9	
	shear modulus	G_g	G_p	
	G (kN/m^2)	900000	3519895	
	cross section	A_g	A_p	
	A (m^2)	1.44	12	
	member length	H_g	H_p	H_f
	H (m)	1.6	11	2.5
	moment of inertia	J_g	J_p	J_f
	J (kN・m・s^2)	54472	177.55	12666
ground	spring	k_{HB}	k_V	k_R
	k (kN/m) or (kN・m)	1459955	1713212	54225414

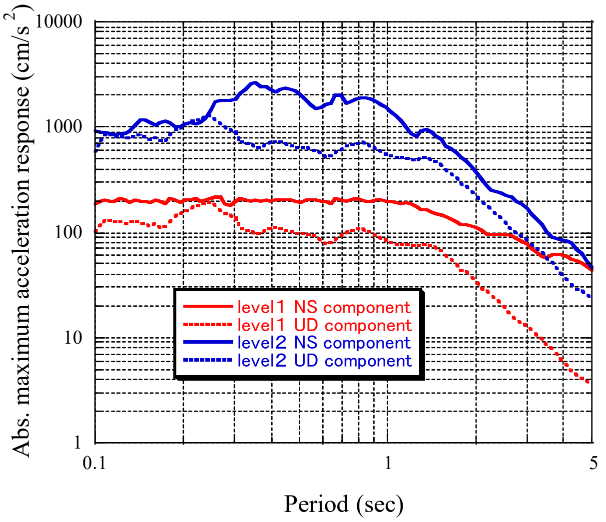


Fig. 4 Level 1 NS and UD input motions and their acceleration response spectra

4. Examples of numerical calculations

4.1 Linear analysis

Tables 2 and 3 show the maximum displacement response values of Case A and B with and without vertical motion. The horizontal axis shows the horizontal (m), vertical (m), and rotational (rad.) motion. Figs. 5 to 6 show the values in Table 2 and 3 as bar graphs, respectively. Due to the difference in units, the rotation is relatively smaller than the horizontal and vertical motions. Subscript p of Upmax represents the parametric excitation of the superstructure.

First, the presence or absence of vertical motion (up and down in Tables 2 to 3) is considered. In Case A, the displacement response increased with up and down motions in three displacements (y_1, y_2, w) without parametric excitation and seven displacements ($x_1, y_1, y_2, \theta_1, u, w, \theta$) with parametric excitation.

Next, the presence or absence of parametric excitation is considered. In Case A, the displacement response increased with parametric excitation in four displacements (θ_1, u, w, θ) with vertical motion and four displacements (y_1, y_2, w, θ) without vertical motion. In Case B, the displacement response increased with parametric excitation in two displacements (θ_1, θ) with vertical motion and six displacements ($y_1, y_2, \theta_1, u, w, \theta$) without vertical motion.

However, in a linear Case, the engineering effect on the response displacement is minimal regardless of the vertical motion and parametric excitation.

4.2 Non-linear analysis

a) Effect of parametric excitation with and without vertical motion

Tables 4 to 6 (Figs. 7 to 9) show the plastic secondary gradient of Case C as 0, 5 and 10% of the initial stiffness and similar display with the linear analysis.

First, focusing on the presence or absence of vertical motion (up and down in Table 4), there is almost no difference in the response displacements with the exception of the three vertical displacements (y_1, y_2, w) same as linearity.

Table 2 Maximum displacement response value with or without vertical motion in Case A (level 1, linear)

CaseA(eV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	1.4711E-02	1.3637E-02	4.2498E-04	9.1930E-05	2.6256E-05	1.2356E-03	1.2372E-03	1.1388E-03	1.0997E-03	6.4497E-07
Upmax	1.4708E-02	1.3614E-02	4.2391E-04	9.0754E-05	2.6385E-05	1.2338E-03	1.2354E-03	1.1392E-03	1.0998E-03	8.2153E-07
CaseA(nV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	1.4711E-02	1.3637E-02	0.0000E+00	0.0000E+00	2.6256E-05	1.2356E-03	1.2372E-03	1.1388E-03	0.0000E+00	6.4497E-07
Upmax	1.4706E-02	1.3618E-02	4.1582E-05	3.3897E-05	2.6237E-05	1.2341E-03	1.2357E-03	1.1388E-03	1.8337E-06	6.4782E-07

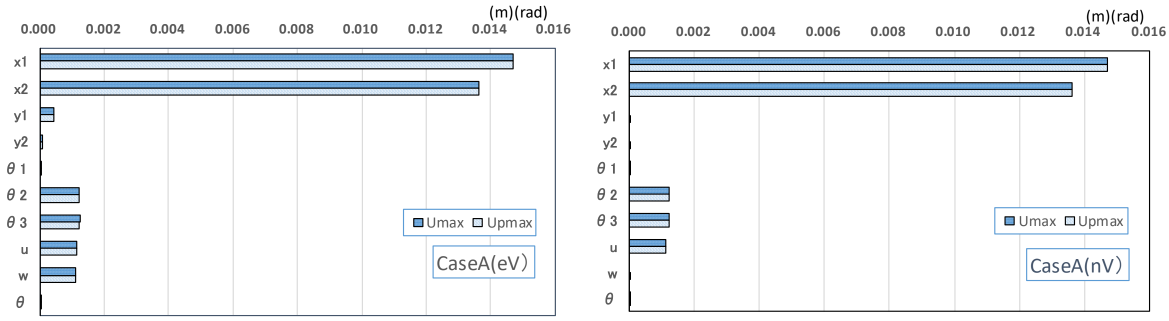


Fig. 5 Effect of parametric excitation with or without vertical motion in Case A (level 1, linear)

Table 3 Maximum displacement response value with or without vertical motion in Case B (level 2, linear)

CaseB(eV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.1183E-02	2.9723E-02	2.8122E-03	6.0832E-04	3.0812E-05	2.6858E-03	2.6896E-03	6.2398E-03	7.2767E-03	4.6966E-06
Upmax	3.1157E-02	2.9709E-02	2.6417E-03	5.7528E-04	3.2197E-05	2.6843E-03	2.6881E-03	6.2029E-03	7.2528E-03	9.6702E-06
CaseB(nV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.1183E-02	2.9723E-02	0.0000E+00	0.0000E+00	3.0812E-05	2.6858E-03	2.6896E-03	6.2398E-03	0.0000E+00	4.6966E-06
Upmax	3.1181E-02	2.9633E-02	2.0136E-04	1.6246E-04	3.2707E-05	2.6782E-03	2.6818E-03	6.2425E-03	3.4691E-05	4.8326E-06

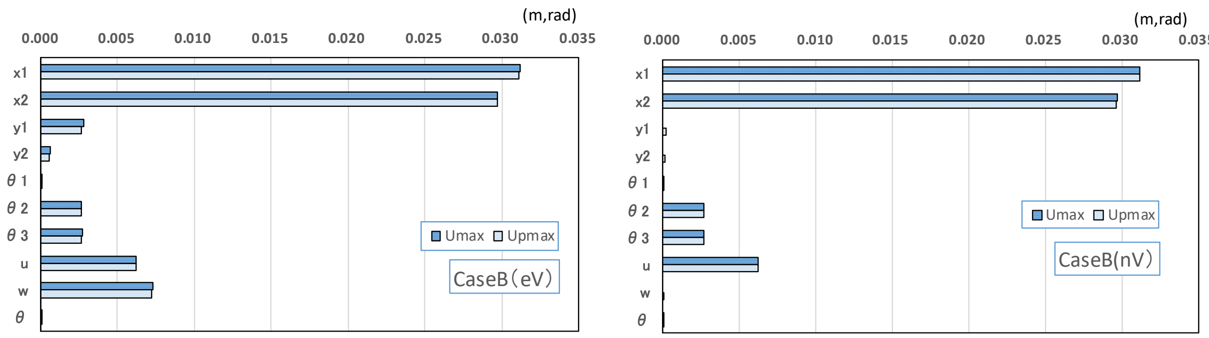


Fig. 6 Effect of parametric excitation with or without vertical motion in Case B (level 2, linear)

Next, the presence or absence of parametric excitation is considered. Five displacements ($x_1, x_2, \theta_2, \theta_3, u$) are reduced by the effect of parametric excitation regardless of the presence or absence of vertical motion. In addition, the foundation horizontal displacement (u) is relatively large due to the non-linear characteristics of the ground spring regardless of the presence or absence of vertical motion or parametric excitation.

A plastic secondary gradient of 5% (up and down in Table 5) and 10% are compared with or without vertical movement or parametric excitation. A similar trend is observed in both situations, except when the displacement (u) is large. The 0% secondary slope of the plastic (up and down in Table 6) is greater than the 5 and 10% slopes at eight displacements ($x_1, x_2, y_1, y_2, \theta_2, \theta_3, w, \theta$), with or without vertical movement. The horizontal displacement (x_1) of the superstructure is nearly 1.2 times larger of the vertical motion displacement. The horizontal and rotational displacements of the foundation(u, θ) are large regardless of vertical motion and parametric excitation.

Therefore, when the plastic secondary gradient is reduced, the response displacement is greatly affected by parametric excitation regardless of the vertical motion. Furthermore, the horizontal and rotational displacement of the foundation increases.

Fig. 10 shows the horizontal and vertical hysteresis loops of Case C (plastic secondary gradient 0, 5, and 10%). The horizontal movement of the foundation is almost constant regardless of the vertical movement. The lower the secondary gradient of the plastic, the greater the displacement. The vertical motion of the foundation largely differed,With vertical movements, the displacement increased with a plastic secondary gradient of 10 to 5% followed by a decrease from 5 to 0%. Without vertical motion, the displacement is smaller due to the smaller plastic secondary gradient and a hysteresis loop is drawn instead of a linear relationship. This indicates that the vertical and horizontal displacement response is related by the modified external force element.

Table 7 shows the horizontal displacement from the top of the foundation to the mass position with or without parametric excitation of a secondary gradient of 0, 5, or 10% plastic. Parametric excitation did not affect the plastic secondary gradient. With 10 to 5% and 5 to 0% plastic secondary gradient, the horizontal displacement is about 0.85 times smaller and 2 times larger, respectively. Hence, the effect on the horizontal displacement of the superstructure is smaller regardless of the vertical motion and parametric excitation and is larger with changes in the plastic secondary gradient.

Therefore, the effect on non-linear characteristics and parametric excitation differs with the plastic secondary gradient. Below is a detailed example of the numerical calculations.

b) Effect of parametric excitation by plastic secondary gradient

In Fig. 11, the vertical axis represents the horizontal displacement at the mass point position from the top of the foundation to the girder, the horizontal axis represents the plastic quadratic gradient (k2), and eV or nV signifies with and without the vertical movement. eP or nP represents the presence or absence of parametric excitation. Regardless of vertical motion or parametric excitation, the displacement is maximum at 0% plastic secondary gradient and minimum at 3%. With this, the property of 0, 3 and 10% plastic secondary gradient investigate in detail.

The upper parts of Figs. 12 to 14 show the ratio of parametric excitation (with/without) on the vertical axis expressed as the maximum displacement response value. When this ratio exceeds 1, the maximum

Table 4 Maximum displacement response value with or without vertical motion in Case C (level 2, non-linear, 10% plastic secondary gradient)

CaseC(eV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.6065E-02	3.2072E-02	1.0381E-03	2.0042E-04	3.8632E-05	2.9424E-03	2.9320E-03	4.5446E-02	1.1602E-02	5.1250E-05
Upmax	3.5889E-02	3.1811E-02	8.7385E-04	1.9841E-04	4.1553E-05	2.9204E-03	2.9093E-03	4.5347E-02	1.1589E-02	5.4060E-05
CaseC(nV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.6065E-02	3.2072E-02	0.0000E+00	0.0000E+00	3.8632E-05	2.9424E-03	2.9320E-03	4.5446E-02	0.0000E+00	5.1250E-05
Upmax	3.5931E-02	3.1815E-02	2.4942E-04	1.8919E-04	4.1171E-05	2.9208E-03	2.9097E-03	4.5377E-02	8.1971E-05	5.6744E-05

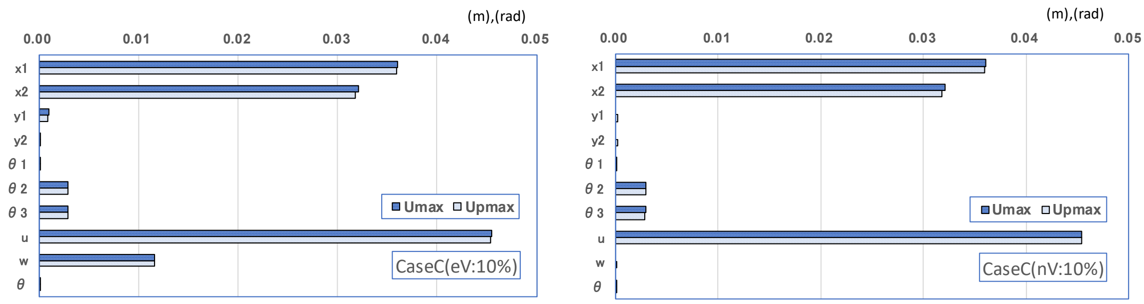


Fig. 7 Effect of parametric excitation with or without vertical motion in Case C (level 2, non-linear, 10% plastic secondary gradient)

Table 5 Maximum displacement response value with or without vertical motion in Case C (level 2, non-linear, 5% plastic secondary gradient)

CaseC(eV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.0524E-02	2.7139E-02	8.7476E-04	1.7442E-04	3.2874E-05	2.4849E-03	2.4780E-03	5.5731E-02	1.1783E-02	6.3508E-05
Upmax	3.0350E-02	2.6935E-02	8.8643E-04	1.8739E-04	3.5038E-05	2.4665E-03	2.4597E-03	5.5629E-02	1.1775E-02	5.3668E-05
CaseC(nV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	3.0524E-02	2.7139E-02	0.0000E+00	0.0000E+00	3.2874E-05	2.4849E-03	2.4780E-03	5.5731E-02	0.0000E+00	6.3508E-05
Upmax	3.0429E-02	2.6966E-02	1.7753E-04	1.3519E-04	3.4681E-05	2.4702E-03	2.4629E-03	5.5674E-02	4.5886E-05	6.3825E-05

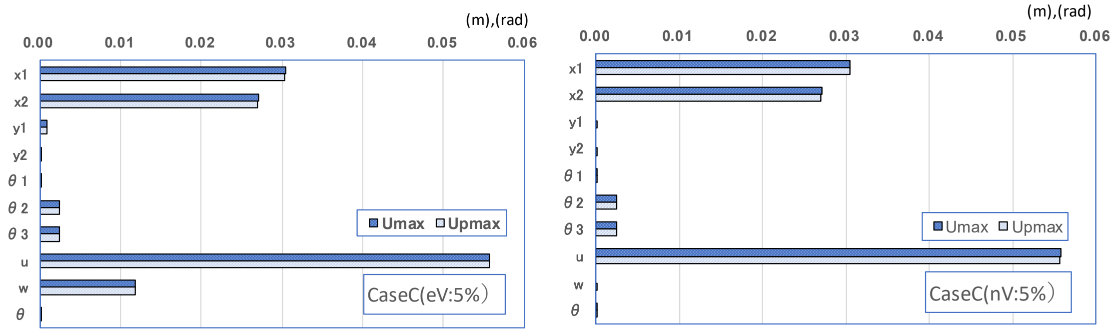


Fig. 8 Effect of parametric excitation with or without vertical motion in Case C (level 2, non-linear, 5% plastic secondary gradient)

Table 6 Maximum displacement response value with or without vertical motion in Case C (level 2, non-linear, 0% plastic secondary gradient)

CaseC(eV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	5.1949E-02	4.8602E-02	9.4506E-04	1.8075E-04	5.7291E-04	4.4061E-03	4.4112E-03	1.1028E-01	1.2626E-02	2.1506E-04
Upmax	6.2021E-02	5.7726E-02	1.1402E-03	5.2402E-04	3.2626E-04	5.2362E-03	5.2418E-03	1.0905E-01	1.2633E-02	1.1707E-02
CaseC(nV)	x1	x2	y1	y2	θ1	θ2	θ3	u	w	θ
Umax	5.1949E-02	4.8602E-02	0.0000E+00	0.0000E+00	5.7291E-04	4.4061E-03	4.4112E-03	1.1028E-01	0.0000E+00	2.1506E-04
Upmax	6.8806E-02	6.3972E-02	5.2362E-04	4.2942E-04	3.8022E-04	5.8038E-03	5.8097E-03	1.0915E-01	4.3199E-05	1.2732E-02

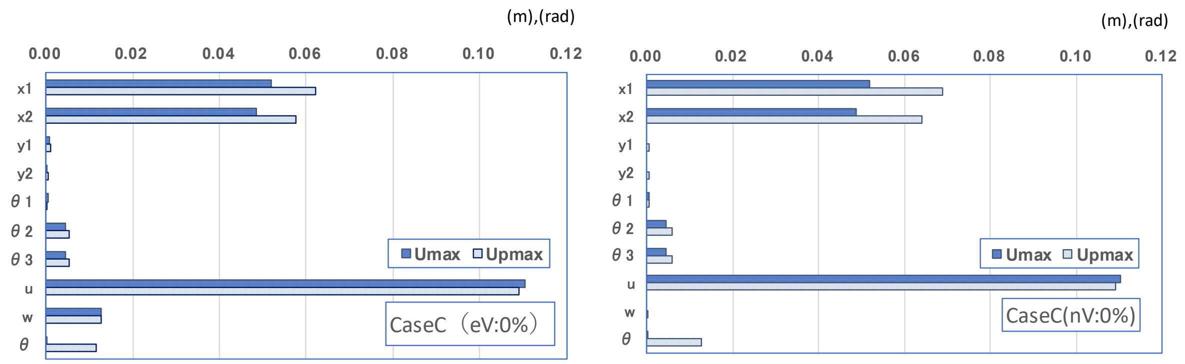


Fig. 9 Effect of parametric excitation with or without vertical motion in Case C (level 2, non-linear, 0% plastic secondary gradient)

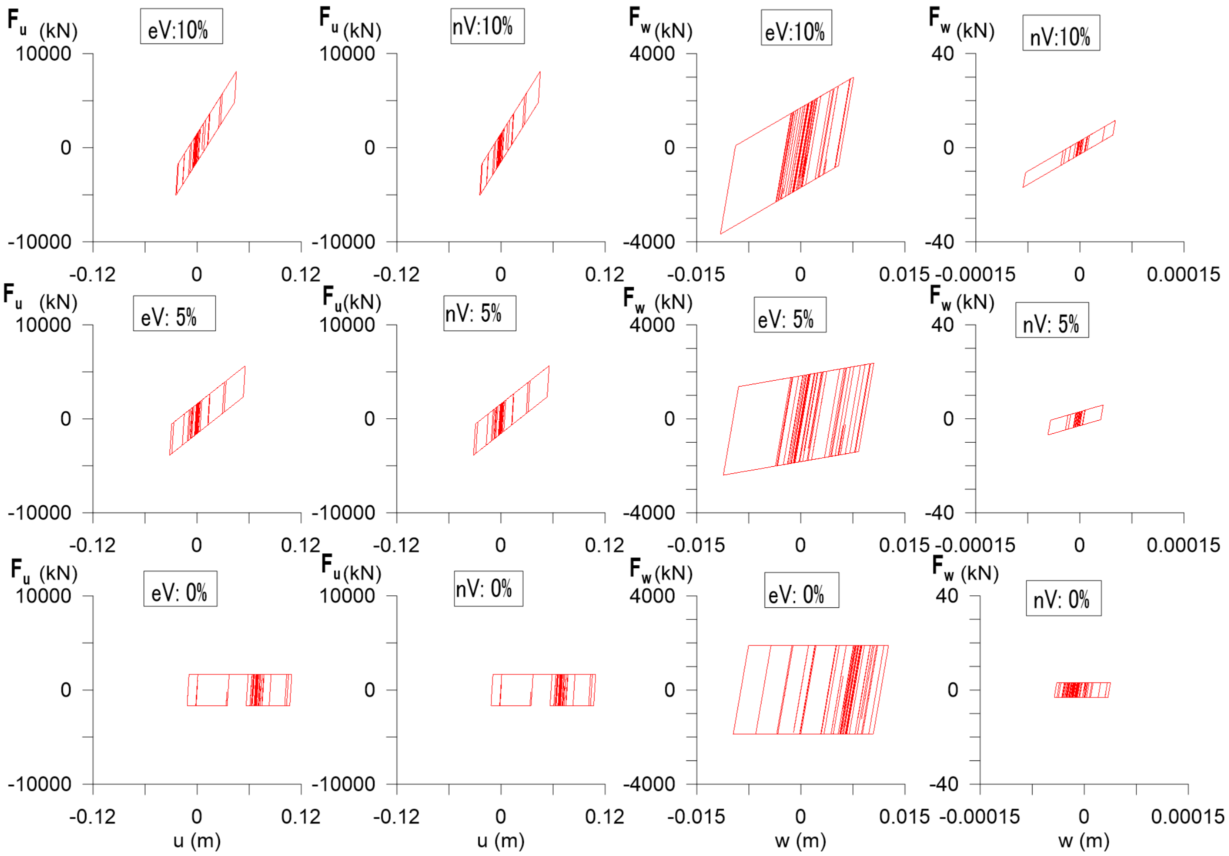


Fig. 10 Horizontal and vertical hysteresis loop of foundation with and without vertical motion (with parametric excitation; 0, 5, and 10% plastic secondary gradient)

Table 7 Horizontal displacement x_T (m) from the top edge of the foundation to the mass point position of the girder

plastic secondary gradient k2 (%)	eV		nV	
	eP	nP	eP	nP
10	0.7713E-01	0.7763E-01	0.7741E-01	0.7759E-01
5	0.6508E-01	0.6553E-01	0.6563E-01	0.6579E-01
0	0.1362E+00	0.1143E+00	0.1124E+00	0.1156E+00

displacement increased due to the effect of parametric excitation. The horizontal axis shows the number of stages in which the yield displacement is gradually increased from Case A, which is the difference between the maximum displacement response values of Case B and Case A divided into 20.

The lower parts of Figs. 12 and 14 show the vertical hysteresis loop of the foundation corresponding to the maximum and minimum values of the ratio of the rotational displacement θ of the foundation where the change in the upper part is largest. The 3% plastic secondary gradient in Fig. 14 is compared to the 10% as follows, the vertical displacement y_2 also increased from 1 to 18 stages due to the effect of parametric excitation when there is up and down motion in the left figure. That is, there is almost no change. Also, the rotational displacement θ of the foundation increased from 6 to 7 stages due to the effect of parametric excitation. However, as well as for a 10% plastic secondary gradient, the minimum (1) (0.50) is on the non-linear side and the maximum (2) (2.07) is on the linear side.

In the Case of no vertical motion as shown in the right figure, the maximum value (3)(248.39) appeared on the non-linear side of the foundation rotational displacement θ due to the effect of parametric excitation as in the case of vertical motion , it is about 250 times greater than 3 and 10% of the secondary gradient of plastic. The minimum value (4) (0.92) appeared in 16 stages with large difference from the maximum value (3). The hysteresis loop is drawn on both the nonlinear and linear sides. Therefore, the effect of parametric excitation differs depending on the difference in the plastic secondary gradient, non-linear characteristics, and vertical motion.

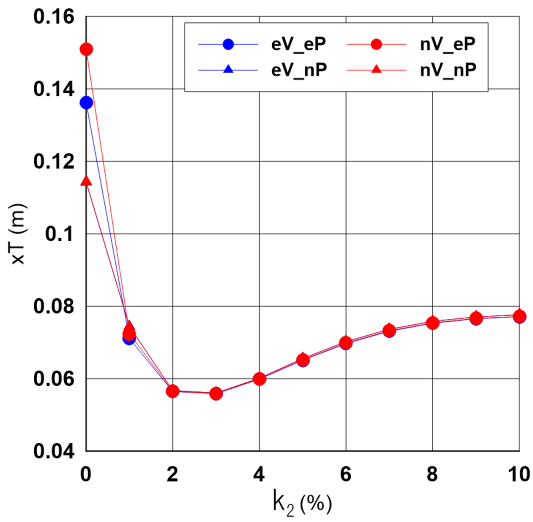


Fig. 11 Relation between displacement and plastic secondary gradient with and without vertical motion and parametric excitation

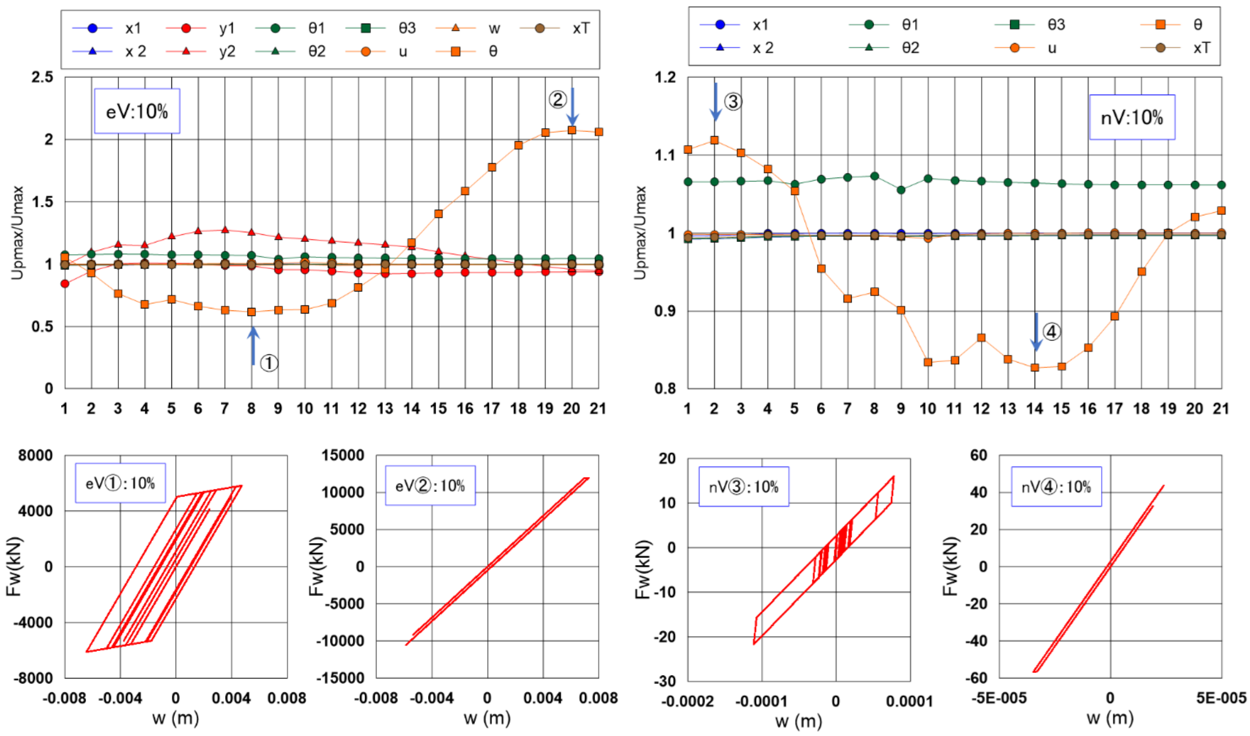


Fig. 12 Effect of parametric excitation due to vertical motion between Case B and Case C (10% plastic secondary gradient)

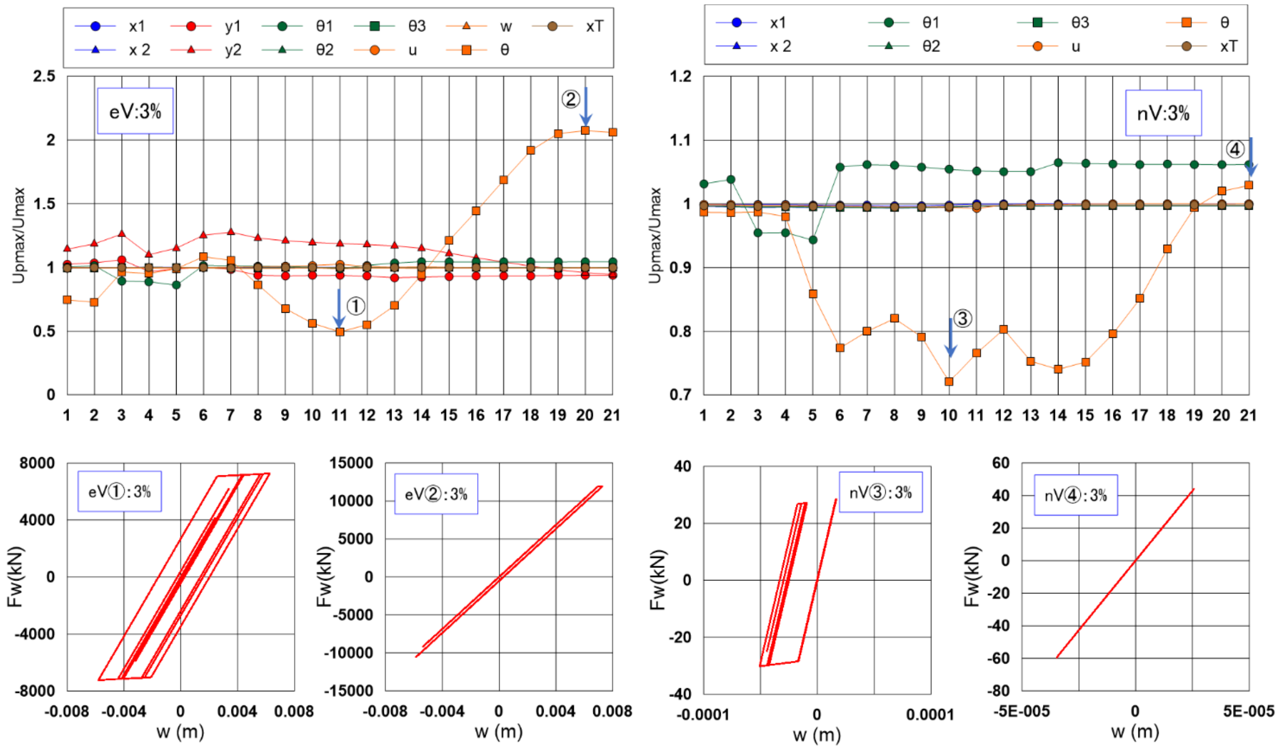


Fig. 13 Effect of parametric excitation due to vertical motion between Case B and Case C (3% plastic secondary gradient)

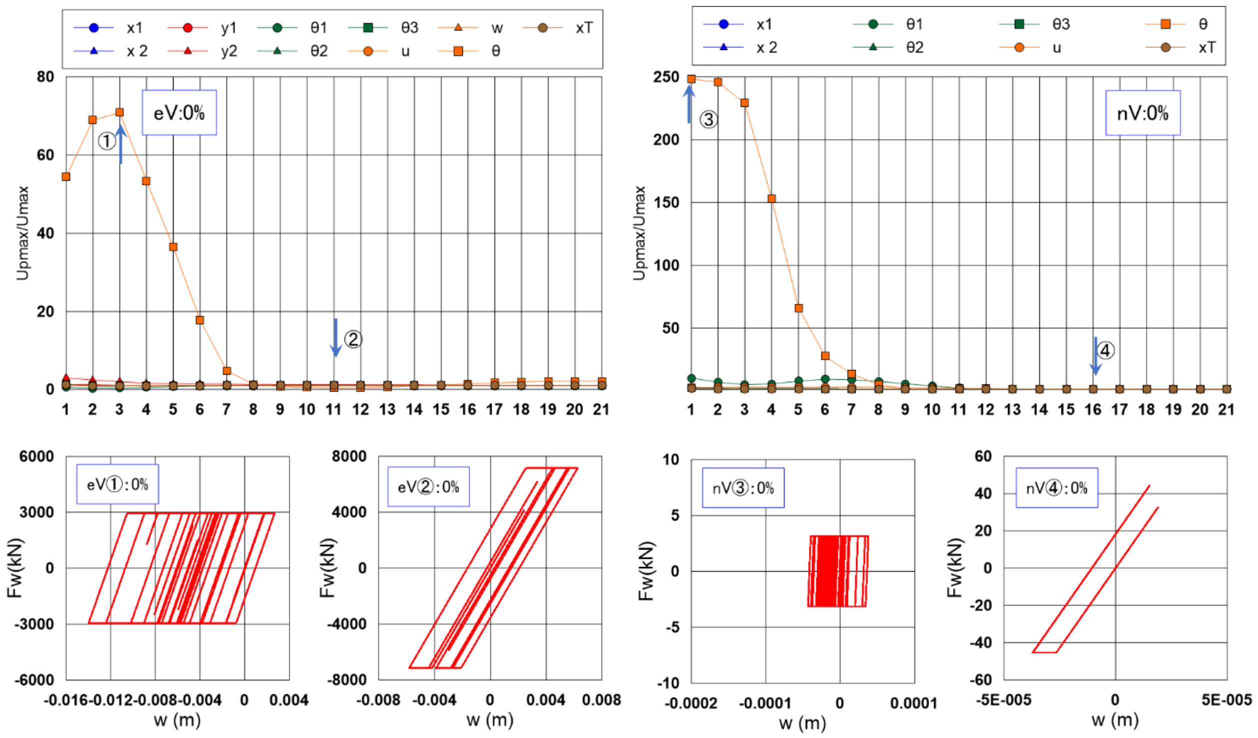


Fig. 14 Effect of parametric excitation due to vertical motion between Case B and Case C (0% plastic secondary gradient)

5. Conclusion

(1) The influence on the response displacement due to the non-linear characteristics of the soil springs of the foundation are large with horizontal and vertical displacements of the foundation and small with the displacement of the superstructure regardless of the presence or absence of vertical motion and parametric excitation.

(2) The effect of parametric excitation is significant due to the rotational displacement of the foundation. The linear and non-linear characteristics depend on the presence or absence of vertical motion.

(3) There is almost no difference between the horizontal and vertical hysteresis loops of the foundation due to the effect of vertical motion with parametric excitation. However, due to parametric excitation, there is a large difference between the up and down motions of the foundation and a non-linear hysteresis loop is drawn without vertical motion.

6. Acknowledgments

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7. References

[1] R.G.Merritt, G.W.Housner (1954): Effect of foundation compliance on earthquake stresses in Multistory Buildings, Bulletin of the Seismological Society of America, 44 (4), 551-569.

[2] Architectural Institute of Japan Structural Committee (2017): Workshop of phenomenon elucidation and seismic design of dynamic soil struture interaction of ground and building (in Japanese).

[3] Japan Society of Civil Engineers Kansai Branch of Research Committee (2019): Research Committee report on the vertical direction of the seismic isolation structure of the city directly under earthquake (in Japanese).

[4] T., Kinoshita, T., Sato, M., Nagano, H., Kitamura (2015):Validation of vertical response model of seismically isolated structure based on strong motion observation and simple evaluation method of dynamic soil-structure-interaction, Journal of structural and construction engineering (Transaction of AIJ) 80 (709), 391-400 (in Japanese).

[5] T., Masao (1998):Response characteristics of structure during horizontal and vertical earthquake motions -Earthquake response with parametric excitation-, Journal of structural engineering, 44 (B), 349-357 (in Japanese).

[6] T., Masao (1998):Characteristics of structure during horizontal and vertical earthquake motions, Proceedings of the 10th earthquake engineering symposium, G3(13), 2585-2590 (in Japanese).

[7] T., Kodera (2010):Parametric excitation, Morikita Publishing Co., Ltd (in Japanese).

[8] Japan road association (1990):Specifications for highway bridges, V Seismic design (in Japanese).

[9] K., Nagata, E., Watanabe, K., Sugiura, T., Utsunomiya (1995): Study on non-linear dynamic soil structure interaction of foundation-structure system, Proceedings of the 23rd JSCE earthquake engineering symposium, 473-476 (in Japanese).

