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BEAM-COLUMNS ANALYSIS OF PIPES UNDER AXIAL AND LATERAL FORCE CAUSED BY GROUND DEFORMATION

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Abstract

In order to ensure the safety of underground pipes during earthquakes, it is desirable not only to be able to withstand a certain level of shaking but also to withstand ground deformation caused by earthquakes. The cause of the deformation of ground surface is the subsidence of soft ground, subsidence and lateral flow due to liquefaction of the ground, and displacement due to instability of slope. In addition to these, there is displacement due to faults which affects to pipes significantly.

In the case of a buried pipe with a relatively small diameter, the pipe is often modeled by a beam supported by a ground spring, and an analytical solution of the equation assuming small displacement is applied to designing. When the ground displacement becomes large, the design is being carried out, using the fact that the reaction force by the ground spring can be reduced. If the ground displacement is large to some extent, there is concern about the effect of axial force on bending deformation. It is unlikely that buckling will occur even if an axial force is applied in a normal burial condition. It is necessary to evaluate with. In particular, in the case of fault displacement that is several times the diameter, damage that has been greatly bent due to compressive force has been reported.

In this study, the buried pipe is approximated by an elastic beam, and the problem of the displacement in the axial direction and the direction perpendicular to the axial direction is analyzed using the equation of bean-columns. The beam is a condition that is constrained by surrounded ground, but it is a model in which an equal distribution lord acts in the range where the pipe is deformed by large ground displacement. In addition, the assumption is applied that the axial force is constant, and the analytical solution is obtained within the range of linear equations. The evaluation formula proposed this time is simple calculation formula, but the cross-sectional force to ground displacement can be calculated with the minimum necessary parameters. These are elastic modulus, cross-sectional parameters of pipes and maximum ground reaction force in axial and lateral direction.

Keywords: beam-columns analysis, ground deformation, simple formula of underground pipe

1. Introduction

To ensure safety during an earthquake, buried pipes must be able to resist not only shaking to a certain extent, but also deformation of the ground surface caused by the earthquake. The ground surface deformation may be caused by various factors such as the sinking of soft ground, sinking and flowing due to ground liquefaction, displacements due to destabilization of inclined ground, and fault displacements. Buried pipes, which are subject to these seismic actions, have been constructed in networks over a long time. There is a wide variation in the installation period and pipe types. Old pipes, in particular, are not designed to withstand sufficient external force and suffer from degraded materials. Measures to enhance seismic resistance need to be taken after clarifying the seismic performance of individual pipe designs and network vulnerabilities.

Buried pipes with relatively small diameters are often modeled as beams supported by soil springs, and the analytic solution of the governing equations assuming minute displacement is applied during the design stage. The equations for axial and bending deformations become independent and can be analyzed independently when the minute displacement approximation is valid. Based on this idea, a design system considering soil springs parallel and perpendicular to the axial direction is employed in, for example, water and sewage systems, and utility gas networks. The governing equations of minute displacements are linear differential equations where the sectional force increases in proportion to the ground displacement. The counterforce from soil springs can be reduced to some extent when the ground displacement becomes large. This characteristic is reflected in the design of the buried pipes.

The effect of the axial force on bending deformation becomes a concern when the ground displacement reaches a critical level. Buckling under axial stress is unlikely under standard burial conditions. However, when the aforementioned soil springs yield and the subgrade reaction perpendicular to the pipe axis has an upper limit, the increase in bending from the axial force must be considered appropriately. In particular, significant bending from compressive force has been reported in pipes damaged by fault displacements a few times larger than the pipe diameter.

This study approximates buried pipes as elastic beams and analyzes the translational displacements parallel and perpendicular to the axial direction using the governing equations for finite displacements. The beam is constrained to the ground in the model, and a uniformly distributed force acts where the pipe deforms under a relatively large ground displacement. The analytical solution of the linear equations is obtained under the assumption that the axial force is uniform in the region of bending deformation. This analytical solution can be used to calculate the bending moment, which changes with the axial force.

2. Analysis conditions and the fundamental solution

2.1 Model for analysis

Consider the situation where the soil moves rigidly on one side of an infinitely long pipe buried underground. This model is shown in Fig. 1. The translation displacement with respect to the pipe axis is 2δ at the angle α. Along the boundary of where the translation displacement acts, the axial and bending deformation have line and point symmetry, respectively. At the boundary point, the bending moment is zero and the shear force is the largest. The schematic depicts the diagonal shifting of the soil on the right and left by 2δ, but similar parameters can be set for vertical and horizontal displacements.

The pipe subject to translational displacement also undergoes axial and bending deformation, but the deformation range is limited. The pipe moves along with the soil without any deformation, far away from the translation displacement boundary. Suzuki derived the boundary conditions for the range of beam deformation by investigating the analytic solution for a beam supported by perfectly elastoplastic soil springs with bilinear characteristics, which are used in water and gas pipe design guidelines. After the boundary point, where the deformation converges is determined, calculations can be performed for a beam with a finite length, in which the subgrade reaction after the yielding of the soil springs acts as a uniformly distributed force.

The force F at the translation displacement boundary is the sectional force at this position. This force pushes the pipe down to the left, and the subgrade reaction acts in the opposite direction. On the other hand, another force pushes the pipe up to the right, and the subgrade reaction acts to push the pipe down. The subgrade reaction is generally a function of the relative displacement between the pipe and the soil. Many experimental results have confirmed that the subgrade reaction converges to a fixed value after the relative displacement reaches a certain value. Therefore, bilinear spring models are used. The applicability of these spring models must be carefully determined because the models are empirical, but a constant force can be assumed when the relative displacement is a few tens of centimeters.

The deformation of the beam against the inverting subgrade reaction at the translational displacement boundary is at self-equilibrium. The displacement from the subgrade reaction at the convergence side and the soil displacement are, therefore, the same. The axial force, bending moment, and deflection angle at this point are all zero. Thus, the deformation range can be limited. Considering the symmetry of the deformation, the relative displacement at the deformation convergence boundary with respect to the soil translational displacement boundary is:

$$(\delta \sin \alpha , \delta \cos \alpha) \tag{1}$$

Here, α is positive under tensile deformation and negative under compressive deformation.

The bending equation for finite displacement is used as the governing equation. Considering the symmetry of the deformation, the origin is taken to be the point of translational displacement, and the right-hand side of Fig. 1 is modeled. Fig. 2 shows a schematic of the beam analysis. N_0 and Q_0, which act at the origin in the figure, are counterforces from the decomposition of F in Fig. 1. The subgrade reaction is modeled as a uniformly distributed force based on the maximum soil restraint force under the assumption that the soil springs have yielded. The soil springs parallel and perpendicular to the axis are assumed to yield independently.

The equations of the forces on an infinitesimal length are:

$$N' + p = 0 \tag{2}$$

$$(Nw' + M')' + q = 0 \tag{3}$$

where u is the axial displacement of the beam, w the deflection, N the axial force, M the bending moment, p the uniformly distributed force parallel to the axial direction, and q the uniformly distributed force perpendicular to the axial direction. The axial force and bending deformation can be calculated independently, while the bending moment changes with the axial force. The analytic solution of the differential equation for constant coefficients are

$$N = EAu' \tag{4}$$

$$M = -EIw'' \tag{5}$$

under the assumption that the beam is elastic. Here, E is the Young's modulus, A the cross-section area, and I the geometrical moment of inertia of the cross-section. The differential of axial force $N' \cong 0$ is used under the assumption that the axial force changes gradually. Based on this assumption, the axial force N in equation(3) is constant.

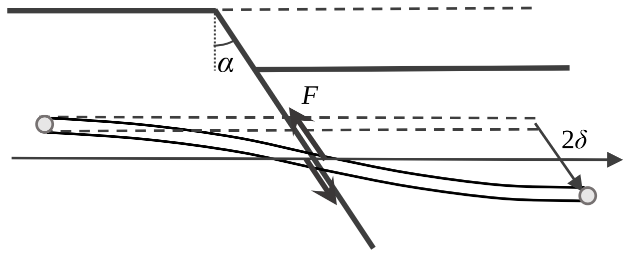

Fig.1–Schematic of deformation of ground and a pipe

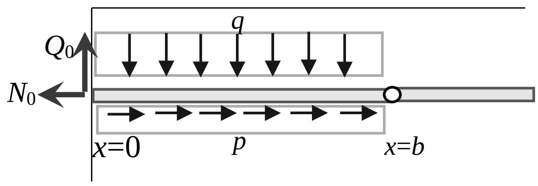

Fig.2–Beam model of half space acting uniformly distributed reaction force

2.2 Bending deformation without any axial force

A beam subject to a displacement at the support and a uniformly distributed force at N = 0 is considered. The governing equation of the beam is:

$$EIw'''' - q = 0 \tag{6}$$

This is a both ends-hinged condition. The solution where the displacement δ at $x = b$ is:

$$w[x] = \delta\frac{x}{b} + \frac{qb^4}{24EI}\left\{\left(\frac{x}{b}\right)^4 - 2\left(\frac{x}{b}\right)^3 + \left(\frac{x}{b}\right)\right\} \tag{7}$$

The condition where, additionally, the deflection angle is 0 at $x = b$ can be written as:

$$w'[b] = \frac{\delta}{b} - \frac{qb^3}{24EI} = 0 \tag{8}$$

The pipe can smoothly connect to the undeformed region if the deflection angle due to the elastic deformation of the pipe matches the displacement angle at the support. The length b that matches this condition is :

$$b = \sqrt[4]{\frac{24EI\delta}{q}} \tag{9}$$

The maximum bending moment and shear force can be calculated once the length b is determined:

$$M_{max} = \frac{qb^2}{8} = \frac{1}{4}\sqrt{6EIq\delta} \tag{10}$$

$$Q_0 = \frac{qb}{2} \tag{11}$$

When the soil displacement δ increases, the maximum bending moment increases proportionally to the square root of δ. This equation for the bending deformation in the absence of axial force is utilized in the liquefaction design of gas pipes in Japan.

3. Analysis of an axial force

3. 1 Compressive force

The governing equation of a beam subject to a constant compressive force P and a uniformly distributed subgrade reaction q is:

$$EIw'''' + Pw'' - q = 0 \tag{12}$$

This is a linear equation with constant coefficients, and the analytical solution under given boundary conditions can be obtained. The solution for the both ends-hinged condition, where $w = 0$ at $x = 0$ and $w = \delta$ at $x = b$, is:

$$w[x] = \frac{x\delta}{b} + \frac{q\left(-2EI + Px(-b+x)\right)}{2P^2} + \frac{EIq\cos\left[\frac{\sqrt{P}(b-2x)}{2\sqrt{EI}}\right]\sec\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right]}{P^2} \tag{13}$$

The deflection angle is 0 at the boundary of the region where the beam moves together with the soil without deformation. The expression for this condition is:

$$w'[b] = \frac{\delta}{b} + \frac{bq}{2P} - \frac{\sqrt{EI}q\tan\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right]}{P^{3/2}} = 0 \tag{14}$$

This is a non-linear equation in the variable b, but b can be numerically obtained using Newton's method. The deflection and sectional force can be calculated once b is determined. For example, the maximum bending moment is:

$$M_{max} = \frac{EIq}{P}\left(\sec\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right] - 1\right) \tag{15}$$

The Euler's buckling load under the both ends-hinged condition, P_E, is defined as:

$$P_E = \frac{\pi^2 EI}{b^2} \tag{16}$$

The equation for M_{max} can be rewritten using P_E as:

$$M_{max} = \frac{qb^2 P_E}{\pi^2 P}\left(\sec\left[\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right] - 1\right) \tag{17}$$

The sectional force diverges as $P \rightarrow P_E$.

3.2 Tensile force

The situation when a tensile force acts is considered next. The governing equation under a tensile force P is:

$$EIw'''' - Pw'' - q = 0 \tag{18}$$

A solution under the boundary conditions can be similarly obtained in the compressive force case:

$$w[x] = \frac{x\delta}{b} + \frac{q\left(-2EI + Px(b-x)\right)}{2P^2} + \frac{EIq\cosh\left[\frac{\sqrt{P}(b-2x)}{2\sqrt{EI}}\right]\operatorname{sech}\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right]}{P^2} \tag{19}$$

The cos and sec functions in Eq. (13), which is the solution for a compressive force, are replaced by cosh and sech, respectively, which are exponential functions. The condition where the deflection angle is 0 at $x = b$ can be written as:

$$\delta = \frac{bq\left(b\sqrt{P} - 2\sqrt{EI}\tanh\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right]\right)}{2P^{3/2}} \tag{20}$$

Although b cannot be given explicitly, b can be numerically obtained using Newton's method. The deflection and sectional force can be derived once b is determined. The maximum bending moment is:

$$M_{max} = \frac{EIq}{P}\left(-\mathrm{sech}\left[\frac{b\sqrt{P}}{2\sqrt{EI}}\right] + 1\right) \tag{21}$$

4. Approximate calculations using the buckling amplification function

4.1 Approximate equation of deflection and sectional force

The analytical solutions were calculated separately for compressive and tensile forces in section 3. The effect of the axial force is evaluated with a simple equation in a fractional form in both cases. The amplification function β is defined using N_0 in the definition of the axial force:

$$\beta = \frac{1}{1 + N_0/P_E} \tag{22}$$

The equation for deflection without any axial force is given in Eq. (7). The appropriate expression for deflection is β times the distributed force term in the equation plus the support displacement:

$$w[x] = \delta\frac{x}{b} + \beta\frac{qb^4}{24EI}\left\{\left(\frac{x}{b}\right)^4 - 2\left(\frac{x}{b}\right)^3 + \left(\frac{x}{b}\right)\right\} \tag{23}$$

The Euler's buckling load P_E is a function of b^2. The compatibility condition of the deflection angle is a thus quadratic equation in b^2 and can be solved analytically:

$$w'[b] = \frac{\delta}{b} - \beta\frac{qb^3}{24EI} = \frac{\delta}{b} - \frac{\pi^2 qb^3}{24(\pi^2 EI + N_0 b^2)} = 0 \tag{24}$$

$$b^2 = \frac{12N_0\delta}{\pi^2 q} + 2\sqrt{\frac{6EI\delta}{q} + \left(\frac{6N_0\delta}{\pi^2}\right)^2} \tag{25}$$

The range that the soil yields, b, obtained from Eq. (25) is good approximation of the result of Eq. (14) or Eq. (20), respectively confirmed by numerical calculation. The range b becomes longer and shorter compared to when there is no axial force at $N_0 > 0$ and $N_0 < 0$, respectively. The approximate equation of the maximum bending moment is:

$$M_{max} = \frac{1}{4}\sqrt{6EIq\delta + \left(\frac{6N_0\delta}{\pi^2}\right)^2} - \frac{3N_0\delta}{2\pi^2} \tag{26}$$

This function increases and decreases in the presence of the axial force compared to when the axial force is absent for $N_0 > 0$ and $N_0 < 0$, respectively. The boundary of the region that moves with the surrounding soil without displacement becomes closer and farther to the origin when compressive and tensile forces are acting, respectively. The displacement at the support δ behaves in the same manner. Thus, the deflection increases when the distance decreases.

4.2 Example calculations

Calculations when both axial and bending deformation are present, as in Fig. 1, are performed. The range of deformation is determined by the yielding of the pipe material. The relative amount and direction of soil movement are considered and the sectional forces are superimposed. The sign of the axial force changes the equation, thus:

$$N_0 = \text{sign}[\alpha]\sqrt{2EAp\delta\sin|\alpha|} \tag{27}$$

is used. Here, sign is the sign function and the direction in Fig. 1 is taken as the positive direction. The sine function in the square root is calculated using the absolute value. The δ in this equation is the displacement on one side, and the displacement of the soil is 2δ.

The range that the soil yields, b, and the maximum bending moment are calculated using below:

$$b^2 = \frac{12N_0\delta\cos\alpha}{\pi^2 q} + 2\sqrt{\frac{6EI\delta\cos\alpha}{q} + \left(\frac{6N_0\delta\cos\alpha}{\pi^2 q}\right)^2} \tag{28}$$

$$M_{max} = \frac{1}{4}\sqrt{6EIq\delta\cos\alpha + \left(\frac{6N_0\delta\cos\alpha}{\pi^2}\right)^2} - \frac{3N_0\delta\cos\alpha}{2\pi^2} \tag{29}$$

As the pipe material is elastic, the superposition of the sectional forces is obtained by a simple sum. The evaluation is performed for compressive and tensile stress when the deformation is compressive and tensile, respectively.

The calculation results under the conditions in Table 1 are shown below. The axial and bending stresses are calculated for increasing soil displacement δ and constant translational displacement angle α. Fig. 3(a) shows the results for a tensile force and $\alpha=\pi/4$. One side is modeled for δ = 0.1 m, which corresponds to a translational displacement of 0.2 m. The axial and bending stresses are about the same in this case. In Fig. 3(b), a compressive force is present and $\alpha=-\pi/4$. The axial stress is the same as in Fig. 3(a), but the change in bending stress is increased, indicating that a compressive force increases the bending stress.

The results when the angle α is varied and the soil displacement kept constant are shown in Fig. 4. δ=0.03 m, and positive and negative α indicate tension and compression, respectively. Bending dominates when the angle is small, but the axial force becomes larger when the angle is increased. The bending stress becomes larger on the compression side and peaks at α=-0.5 –1 rad. These examples are introduced with the elastic range of steel pipes in mind, but the response to soil displacement can be easily derived using the approximate solution and the bare minimum of parameters.

Table 1 –Data list of example calculation

Items	Values
Diameter *D*	0.4[m]
Thickness *t*	0.006[m]
Young's Modulus *E*	200[GPa]
Yield stress of ground in axial direction, τ_g	15[kPa]
Yield stress of ground in lateral direction, σ_g	170[kPa]
Area of cross-section, *A*	πDt
Geometrical moment of inertia, *I*	$(\pi D^3\ t)/8$
Section modulus, *W*	$(\pi D^2\ t)/4$
Uniformly distributed load in axial direction, *p*	$\pi D\tau_g$
Uniformly distributed load in lateral direction, *q*	$D\sigma_g$

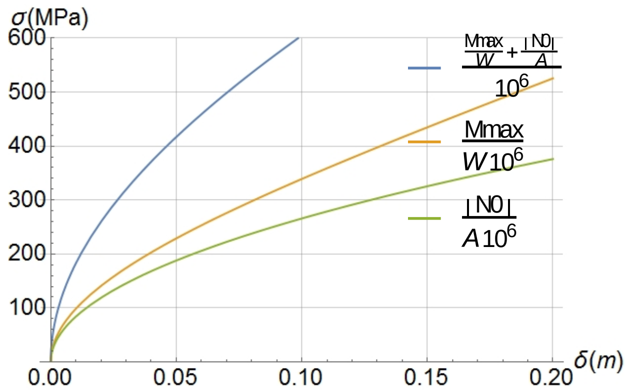

Fig.3(a)–Relationship between ground displacement and compressive stress (α=–π/4)

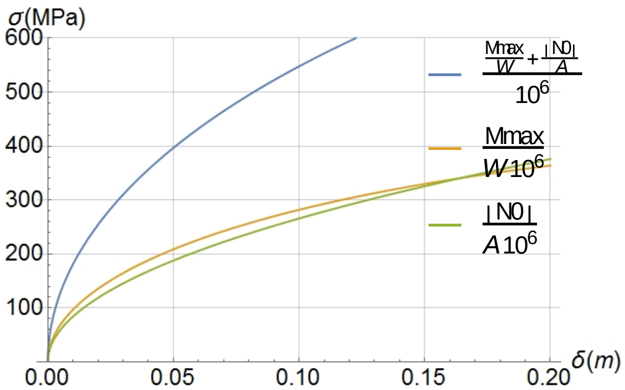

Fig.3(b) – Relationship between ground displacement and tensile stress (α=π/4)

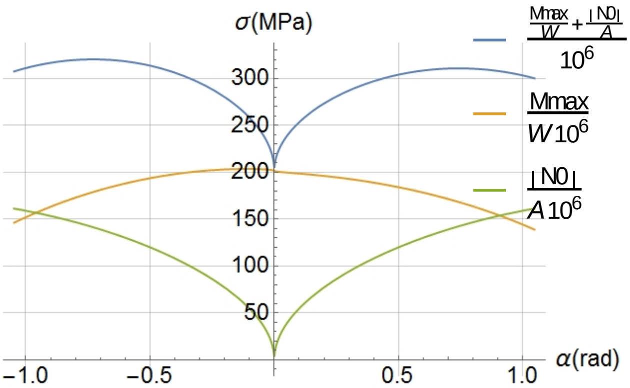

Fig.4 – Relationship between ground shifting angle and stress (δ=0.03 m)

5. Conclusions

This study calculated the deflection when a translational displacement acts at a point on a pipe buried near the ground surface. It was assumed that the pipe deforms elastically, and that the pipe is supported by soil springs that completely and independently yield parallel and perpendicular to the axial direction. The elastic range of the soil springs is smaller when they yield perpendicular to the axial direction than that when they yield parallel to the axial direction. A uniform axial force approximation is thus possible in the range considered in the deflection calculations. The fundamental equation considers the action of a uniformly distributed force on a beam-column under a constant axial force. Translation of the soil springs support results in the deflection of the pipe limited to the near vicinity of the translational displacement. The pipe simply follows the soil outside this region. The procedure to calculate the range and shape of the deflection as well as the maximum sectional force was outlined. The relations between the bending moment and the compressive or tensile force, which vary with respect to the translational displacement angle, were formularized. The bending moment increases and decreases under a compressive and tensile force, respectively.

6. References

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