



MEASUREMENT OF NATURAL FREQUENCIES OF THE SHELL PLATE VIBRATION OF A LARGE-SIZED CYLINDRICAL STEEL TANK

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Abstract

We measured the natural frequencies of the fluid-elastic-coupled shell plate vibration (bulging) excited in a large-sized (125,000-m³ capacity, 82-m diameter, and 27.3-m height), flat-bottomed, cylindrical steel tank by observing microtremors. The vibrations excited by ground microtremors were observed at the top and mid-height of the shell plate and on the shoulder of the tank foundation (the tank bottom). The radial-component spectral ratio of the top or mid-height of the shell plate to the tank bottom was examined. Since the tank bottom vibration was measured on the shoulder of the tank foundation, soil-coupled effects should cancel in the spectral ratio. That is, we examined the non-soil-coupled spectral ratio.

Five peaks appear in the observed microtremor spectral ratios. FEM eigenvalue analysis assuming a fixed base was conducted to identify the corresponding bulging mode for each microtremor spectral peak. Comparing the appearance frequencies and the values of the five peaks to the FEM eigenvalue analysis solutions reveals that the five peaks correspond to the bulging modes with a vertical order m of one and a circumferential wavenumber n of one to five, respectively. The appearance frequencies of the five spectral peaks are therefore the non-soil-coupled measured natural frequencies of the corresponding modes. They agree fairly well with the calculated values from the FEM eigenvalue analysis. It should be noted that the natural frequency of the fundamental mode ($m=n=1$) is not the lowest of all the possible theoretical natural frequencies in the examined case.

The measured natural frequency of the fundamental mode is 2.19 or 2.21 Hz, depending on the liquid height when the microtremor observation was conducted. These values agree well with that projected by the simplified equation adopted in the seismic codes established based on the Japanese Fire Service Act (JFSA equation), which was also developed under the assumption of a fixed base. These results suggest that the JFSA equation can project a reliable soil-coupled fundamental-mode natural frequency in the case where a tank is situated on firm ground and the storage-soil-coupled effects are presumed weak.

We also present a simple method to determine the non-soil-coupled fundamental-mode natural frequency. The method uses only the observed microtremor spectral ratio, and does not involve referencing the numerical solutions. The method simply calculates the radial-component spectral ratio of a mid-height to the top of the tank shell plate and infers that the lowest spectral-peak appearance frequency is the fundamental-mode natural frequency. This simple method works extremely well for the examined tank. The method infers that the fundamental-mode natural frequency is 2.22 Hz. This value is almost the same as the value (2.19 Hz), which was inferred by the aforementioned procedure that requires comparing with the numerical solutions of the FEM eigenvalue analysis. This method also has the potential to infer the natural frequencies of the higher modes ($m \geq 2, n=1$) in a very simple way.

Keywords: cylindrical steel tank, shell plate vibration, natural frequency, microtremor



1. Introduction

The natural frequency of the fluid-elastic-coupled shell plate vibration (hereafter referred to as “bulging”) of liquid storage tanks is a vital parameter to evaluate their short-period seismic responses. Theoretical and numerical studies have been intensively conducted to evaluate the natural frequency of bulging for flat-bottomed cylindrical steel tanks, including the FEM eigenvalue analyses (e. g., [1, 2]). Generally in the numerical eigenvalue analyses, the natural frequencies, mode shapes and participation factors are solved for each of possible modes with a vertical order of m ($m \geq 1$) and a circumferential wave number of n ($n \geq 0$), which has an amplitude distribution of $\cos(n\theta)$ for an azimuth θ .

In evaluating the short-period seismic responses of the tank, the fundamental mode ($m=n=1$) is generally conceived important, because higher vertical modes ($m \geq 2$) are presumed harder to be excited than the mode with $m=1$ and higher circumferential modes ($n \geq 2$) cause no resultant horizontal force.

Some of the numerical results on the natural frequencies of bulging have been adopted to seismic codes of flat-bottomed cylindrical steel tanks. In Japan, seismic codes for oil storage tanks are established based on the Japanese Fire Service Act (JFSA) from the point of view of fire prevention, where the fundamental-mode bulging natural frequency f_b of a flat-bottomed cylindrical steel tank can be calculated by the following equations that were proposed by Sakai [3].

$$f_b = \frac{\lambda}{2} \sqrt{\frac{\pi E t_{1/3}}{M}} \cdot \frac{1}{j}, \quad (1)$$

$$\lambda = 0.067(H_L/D)^2 - 0.30(H_L/D) + 0.46, \quad (2)$$

where D , H_L , and M are the diameter, liquid height, and mass of the contained liquid respectively. E is the Young's modulus of the tank material. $t_{1/3}$ is the shell plate thickness at one third of H_L . j is 1.1 if the tank is located on the soft soil and rested on the spread foundation, otherwise 1.0. These equations were developed based on the FEM eigenvalue analyses for the analytical models with a fixed base. We refer to Eq. (1) as “JFSA equation”.

The reliability of theoretical and numerical analyses is largely dependent on the various assumptions employed in formulating theoretical and numerical models. Experimental investigations are therefore essential to validate the theoretical and numerical models and methods. So experimental studies have been also intensively conducted to measure the natural frequencies of bulging by means of forced vibration tests or observing the tank vibration excited by seismic ground motions or ground microtremors.

Most of the experimental studies employing the forced vibration test must have examined small-scale tank models because shaking full-scale tanks artificially is very challenging. Sloshing of liquid contained in the tank can be examined well enough even by testing small-scale and rigid tank models, while for study of bulging, testing full-scale tank models or tanks in service would be the only certain way to determine the parameters of major interest including the natural frequency.

From this point of view, Haroun [4] conducted forced vibration tests for two actual cylindrical, cone-roof, steel tanks containing water with a nominal capacity of about 4,000 and 5,600 m³ located in California, USA. He also measured the tank vibration due to ground microtremors at the above two tanks and another smaller water tank in service. As a result, he found that not only the fundamental circumferential mode ($n=1$) of bulging but also significant higher circumferential modes ($n \geq 2$) could be observed in the tank responses



both to the forced vibration and to the ground microtremors. He also found that the observed bulging natural frequencies and mode shapes agreed well with the numerical solutions.

Kawano and Umebayashi [5] and Yamamoto et al. [6] conducted the seismic observation at two cylindrical, cone-roof, steel tanks in service containing water with a nominal capacity of 2,000 and 6,000 m³ located in Japan. They installed seismometers at the top of the tank and on the ground surface directly beneath the top seismometer, and obtained records from three earthquakes. As a result of inferring the transfer function between the two observation points by analyzing the circumferential component, they found that the observed natural frequencies of the fundamental-mode bulging was about half as low as those expected from the numerical models with a fixed base. They furthermore conducted the total system analysis considering the soil-tank-liquid interaction and concluded that it was because of the influence of the soft soil on which the tanks were located that the lower natural frequencies were observed than expected from the fixed-based model.

Ohmachi and Tanida [7] observed microtremors at nine cylindrical, cone-roof, steel tanks in service containing oil with a nominal capacity of 200 to 14,000 m³ located in Japan. They also showed that the observed fundamental-mode bulging natural frequencies were considerably lower than expected from the JFSA equation that was developed based on the fixed-based numerical model.

We got a chance to measure microtremors at a much larger cylindrical, steel tank than the abovementioned ones: the nominal capacity was 125,000 m³. We believe that few studies have examined such a huge tank by observing and analyzing its actual vibration.

In this paper, we report the measured natural frequencies of bulging for the 125,000-m³ tank from the microtremor observation. In inferring the natural frequencies, we needed to identify which bulging mode each of the microtremor spectral peaks correspond to, and we did it by being guided by the solutions of the FEM eigenvalue analysis. We furthermore found a way to figure out the natural frequency of the fundamental-mode bulging without help of the FEM eigenvalue analysis. The method is also presented in this paper.

2. Examined tank and microtremor observation

The examined tank was a flat-bottomed cylindrical, steel tank located in Japan. The tank contained industrial water with a nominal capacity of 125,000-m³, and had a single-deck floating roof. The tank was rested on the mound supported by the spread foundation, and was located on the firm ground where outcrop of sandstone was observed. Table 1 lists the specifications of the tank. The thickness values of the shell plate were not designed ones but measured ones.

We recorded the tank vibration excited by ground microtremors in the manner as shown in Fig. 1 and Table 2. We used palm-sized servo-type velocity meters (VSE-15D-1) manufactured by Tokyo Sokushin Co. Ltd. as sensors. High-gain (1 V/(mm/s)) outputs from the sensors were recorded by 24-bit digital recorders (SAMTAC-802) manufactured by the same company at a sampling rate of 100 Hz after being processed by a high-cut filter with a cut-off frequency of 37.5 Hz.

We discuss two data sets in this paper. One is labeled as Measurement 1 in Table 2, which is mainly discussed. The other is Measurement 2 in Table 2, which is discussed in the discussion part. In Measurement 1, the tank vibration was recorded at three points on or very close to the shell plate: a point on the tank bottom (B in Fig. 1), a point at the top (T) of the shell plate, and a point at a middle height (M) of the shell plate. The three points were aligned vertically at an azimuth of 290°, which was measured clockwise with respect to the plant north (nearly north). Point B was located at the same level as the bottom plate, and Points T and M were at heights of 26.6 and 16.2 m, respectively from the bottom. Three sensors were installed at each point, and three components (radial, circumferential and vertical) of the vibration were recorded. At Point B, the sensors were installed on the shoulder of the tank foundation, and at Points T and M, the sensors were firmly stuck on the vertical wall of the shell plate by pressure-sensitive tapes and the fixtures specially made for this use. Point M was set by making use of the resting-place of the stairway. In Measurement 2, the



tank vibration was recorded at two points: the bottom (B') and the top (T'), which were located at an azimuth of 270°. The way to install the sensors was the same as in Measurement 1. In the both measurements, the tank vibration was recorded for a length of 30 minutes. We tried to infer the natural frequencies of bulging by comparing the Fourier amplitude spectra of the microtremors (the tank vibration) recorded in the above-described way.

Table 1 – Specifications of the examined tank

Diameter (m)		82.0	
Tank Height (m)		27.3	
Shell Plate	Course from the bottom	Thickness (mm)	Width (m)
	1	40.7	3.9
	2	30.5	3.9
	3	26.1	3.9
	4	21.3	3.9
	5	16.2	3.9
	6	13.7	3.9
	7	13.7	3.8
Top Angle		13.7	0.1

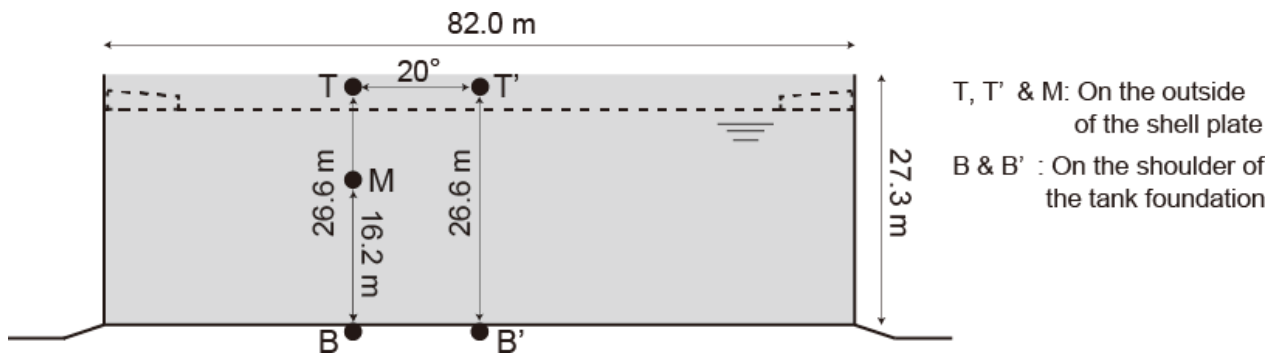


Fig. 1 – Schematic cross section of the tank and deployment of microtremor sensors

Table 2 – Specifications of the microtremor measurements

Measurement No.	1	2
Measurement Period (JST)	November 15, 2012 14:15 – 14:45	November 14, 2012 15:50 – 16:20
Measurement Points	B, M and T	B' and T'
Liquid Height (m)	22.982	23.542
f_b (Hz) by the JFSA equation	2.43	2.21
$t_{1/3}$ (mm) in the JFSA equation	26.1	30.5



3. Theoretical natural frequency and mode shape

To facilitate the analysis and interpretation of the spectral characteristics of the measured microtremors, we obtain the numerical natural frequencies and mode shapes of bulging for the examined tank by the FEM eigenvalue analysis. The FEM analysis was performed by using the codes developed by Yoshida and Miyoshi [8, 9] and Yoshida et al. [10]. In the analysis, a fixed base was assumed, and initial stress occurring in the shell plate was considered.

Figs. 2 and 3 show the numerical natural frequencies and mode shapes. The liquid height was assumed to be 22.982 m, which was observed when Measurement 1 was conducted. In Fig. 2, the circles, squares and triangles denote the natural frequency of the possible modes with $m=1, 2,$ and $3,$ respectively. The mode of $n=0$ can be excited by vertical ground motions. The fundamental-mode natural frequency was solved to be 2.15 Hz. The natural frequency decreases with a circumferential wave number when it is small (e. g., $n \leq 7$ for $m=1$), while for larger circumferential wave numbers the natural frequency increases. We should note that the natural frequency of the fundamental mode is not the lowest of all the possible natural frequencies, according to theory. The natural frequency of the fundamental mode obtained using the JFSA equation was 2.43 Hz (horizontal black line in Fig. 2). The JFSA equation gave a slightly higher natural frequency than the FEM analysis. In Fig. 3, the vertical amplitude distributions (mode shapes) of the radial-component displacement for the modes with $m=1$ and $n=0-17$ are shown. For a small circumferential wave number with $n \leq 3$, the maximum amplitude appears not at the top but at a middle height (16–20 m) of the shell plate, while for larger circumferential wave numbers the amplitude increases monotonically with a height and the maximum amplitude is expected at the top.

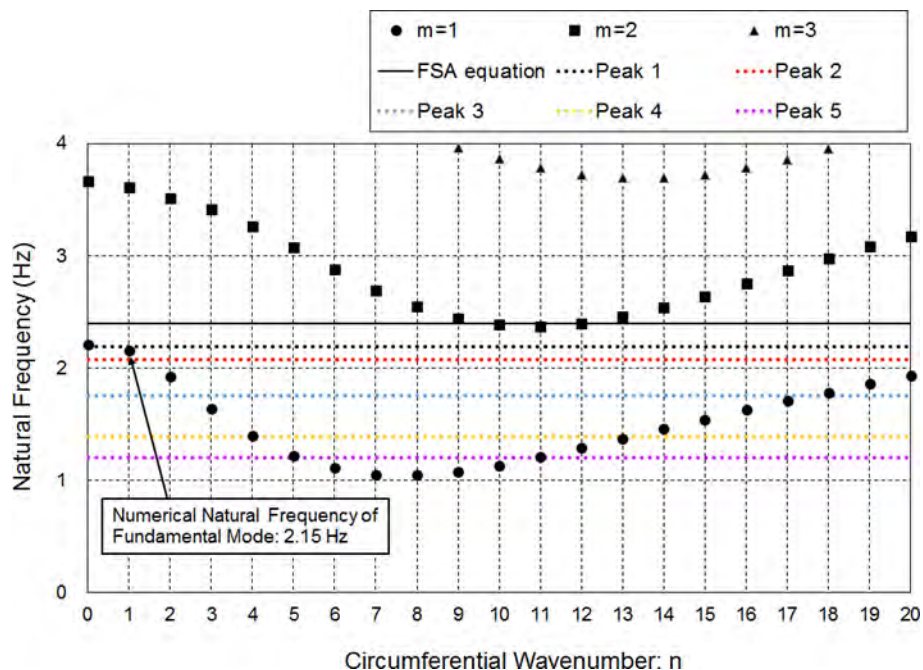


Fig. 2 – Numerical natural frequencies of the tank using the liquid height encountered for Measurement 1 data

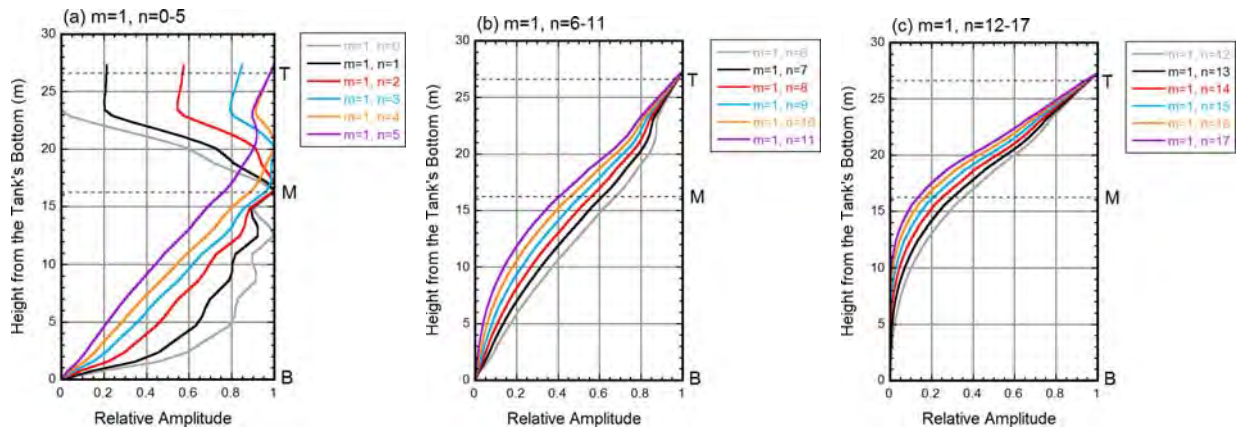


Fig. 3 – Numerical mode shapes (radial component) of the tank using the liquid height encountered for Measurement 1 data

4. Spectral analysis of microtremors

We expect that the natural frequency of bulging could be observed as the frequency at which the peak of the microtremor spectral ratio (hereafter referred to as “peak appearance frequency”) of the top or the middle height of the shell plate to the bottom appear. The spectral ratio was calculated from Fourier amplitude spectra of the time-history velocity records of the measured microtremors. The 30-minute-long velocity records were divided into 10 analysis time sections with a length of 163.84 s and an appropriate tapering, and the Fourier amplitude spectra were calculated for each of the time sections. Then the spectral ratio was calculated for each of the time sections after smoothing the spectra by a frequency window with a width of 0.05 Hz. Finally the spectral ratio from the ten time sections were averaged.

We here discuss the results from Measurement 1, and we focus on the radial component. Fig. 4 (a) and (b) show the Fourier spectral ratio of Point M to B and Point T to B, respectively. In the M-to-B spectral ratio (Fig. 4(a)), we find five significant peaks at frequencies near or below the numerical fundamental-mode natural frequency (2.15 Hz). In the T-to-B spectral ratio (Fig. 4(b)), we also find five peaks at almost the same frequencies as in the M-to-B spectral ratio, although the peak at the highest peak appearance frequency is less obvious. We name these five pairs of the spectral peaks Peaks 1–5 in a descending order of their appearance frequencies (Table 3). For each pair of the spectral peaks, however, the peak appearance frequencies are slightly different. We so averaged the two peak appearance frequencies and regard the average as the peak appearance frequency of the spectral-peak pair (the values in row (6) of Table 3).

In Fig. 2, the observed peak appearance frequencies obtained by the above-described way are compared with the numerical natural frequencies. Noting that our spectral ratios are the ones with respect to the records measured on the shoulder of the tank foundation, the interaction between the soil and the tank-liquid system should be canceled out in our spectral ratios. So we may well expect some degree of agreements between the numerical natural frequencies solved under assumption of the fixed base and our observed peak appearance frequencies, if the observed spectral-ratio peak corresponds to one of the possible modes. With this in mind, we tentatively identify from Fig. 2 the following possibilities: Peak 1 may correspond to modes $(m, n)=(1, 0 \text{ or } 1)$; Peak 2, $(1, 1 \text{ or } 2)$ and/or $(1, 20 \text{ or more})$; Peak 3, $(1, 2 \text{ or } 3)$ and/or $(1, 17-19)$; Peak 4, $(1, 3-5)$ and/or $(1, 12-14)$; Peak 5, $(1, 4-6)$ and/or $(1, 10-12)$.

To determine which mode the observed spectral-ratio peaks can correspond to, we compared the observed T-to-M spectral ratio values at the five peak appearance frequencies with those predicted from the numerical mode shape (Fig. 5). In Fig. 5, the numerical mode shapes shown in Fig. 3 are normalized by the amplitudes at the height of Point M (16.2 m), and the squares show the observed T-to-M spectral ratio (the values in row (7) of Table 3). We find that Peaks 3–5 cannot correspond to modes with large circumferential



wave numbers ($n \geq 10$) because the theory predicts much higher amplitude ratios than those observed. We can instead consider that Peaks 1–5 can correspond to the modes with $m=1$ and $n=1-5$, respectively, because the observed ratios increase in order with the circumferential wave number as predicted by the theory and the values of the ratio are not very different from the numerical values.

We finally consider that the five peak appearance frequencies of Peaks 1–5 are the observation of the natural frequencies of the bulging modes with $m=1$ and $n=1-5$, respectively, and the observed and the numerical values are compared in Table 3 (the values in rows (6) and (9)), where we find fairly good agreements.

From the above discussions, we concluded that the natural frequency of the fundamental mode of bulging was inferred to be 2.19 Hz from the microtremor measurements for the examined tank and the liquid height when the Measurement 1 was conducted. The JFSA equation projects the natural frequency of 2.43 Hz for the same situation and mode, also giving a good estimation as well as the FEM eigenvalue analysis, although the JFSA equation projection was about 10 % higher than the observed value.

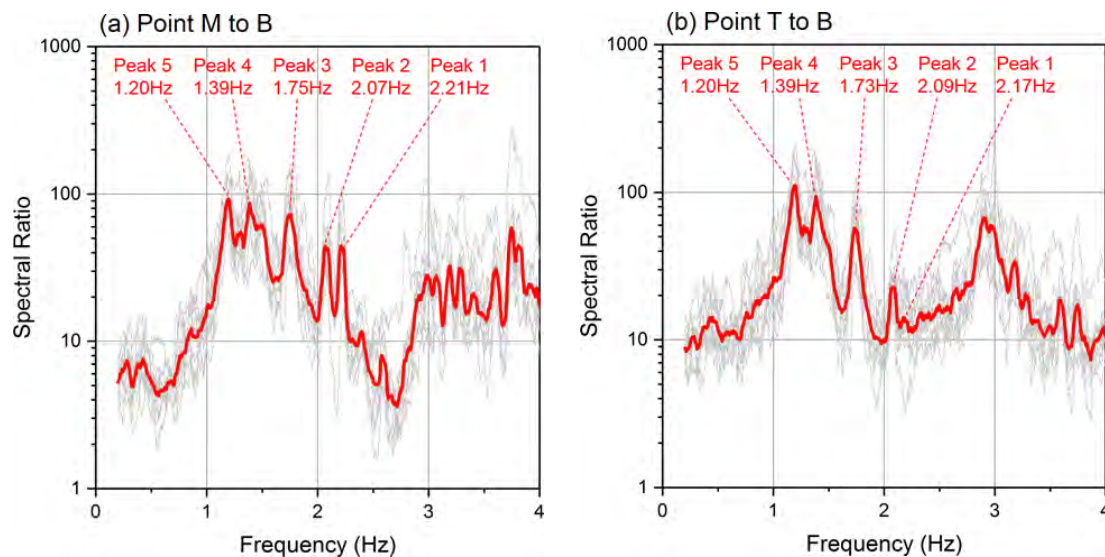


Fig. 4 – Fourier amplitude spectral ratio (radial component) of the microtremors from Measurement 1

Table 3 – Peaks of the Fourier amplitude spectral ratio (radial component) of the microtremors from Measurement 1 data

(1) Peak No.	1	2	3	4	5
(2) Peak appearance frequency in the M-to-B spectral ratio (Hz)	2.21	2.07	1.75	1.39	1.20
(3) Peak value at the peak appearance frequency in the M-to-B spectral ratio	44.7	44.0	72.7	86.7	92.3
(4) Peak appearance frequency in the T-to-B spectral ratio (Hz)	2.17	2.09	1.73	1.39	1.20
(5) Peak value at the peak appearance frequency in the T-to-B spectral ratio	14.2	22.8	57.4	94.2	112.0
(6) Peak appearance frequency (Hz) [Average of (2) and (4)]	2.19	2.08	1.74	1.39	1.20
(7) T-to-M spectral ratio at the peak appearance frequency [(5) over (3)]	0.32	0.52	0.79	1.09	1.21
(8) Bulging mode (m, n) corresponding to the spectral-ratio's peak	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
(9) Numerical natural frequency of the corresponding bulging mode	2.15	1.92	1.64	1.40	1.22

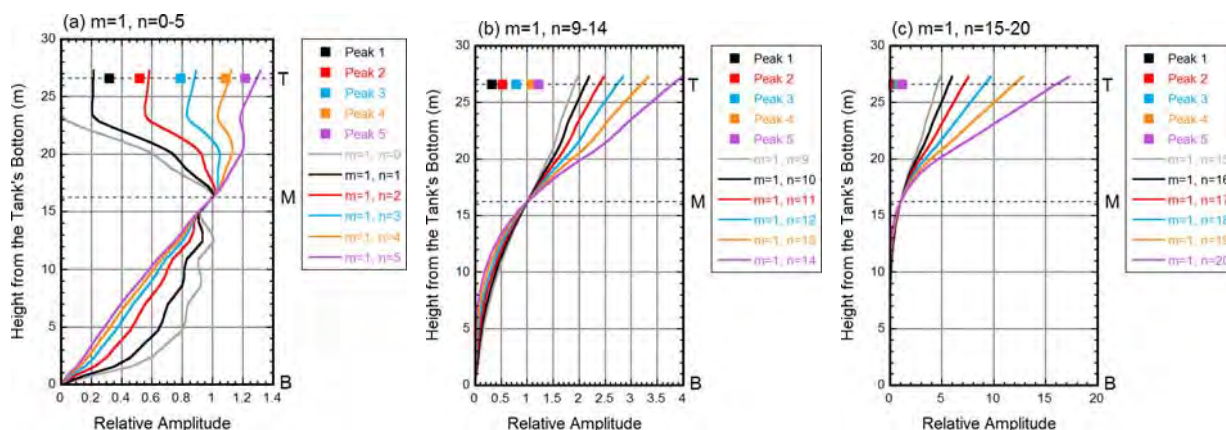


Fig. 5 – Numerical mode shapes (radial component) of the tank using the liquid height encountered for Measurement 1 data, normalized by the amplitude at the height of Point M

5. A simple way to determine the fundamental-mode bulging natural frequency from microtremor spectral ratio

One of the challenging aspects in inferring the natural frequency of the fundamental mode of bulging from the spectral ratio of measured vibration of tanks is that the spectral peak corresponding the fundamental mode does not necessarily appear at the lowest peak appearance frequency, as seen in the previous sections. Therefore, numerical analyses, including the FEM eigenvalue analysis, seem indispensable in identifying which spectral-ratio peak corresponds to the fundamental mode.

Through the above-described examinations, however, we came up with a simple way to determine the fundamental-mode natural frequency solely from the observed microtremor spectral ratio without help of numerical solutions. From Figs. 3 and 5, we notice that the M-to-T amplitude ratios will have a much larger value than one for only the fundamental mode ($m=n=1$), according to theory. So when we calculate the M-to-T spectral ratio for the measured microtremor records, we should be able to get a single peak corresponding to the fundamental mode.

With this in mind, we plotted the M-to-T spectral ratio of the measured microtremors from the Measurement 1 data (Fig. 6). As expected, a single clear peak (Peak A) appears in the frequency range where the fundamental-mode natural frequency is anticipated by theory. This peak appearance frequency is 2.22 Hz, agreeing with the numerical fundamental-mode natural frequency and also with that inferred from the observation in the previous section. Note that we did not use the records at Point B in calculating this spectral ratio. This means that measurements on the shoulder of the tank foundation are not necessarily required when only the fundamental-mode natural frequency wants to be known.

One thing that we should be careful about would be the height where the middle-height sensor is installed. In the case of Measurement 1, Point M was incidentally located around the antinode of the fundamental mode. The M-to-T spectral ratio would work well in such a case to detect the fundamental mode. The heights where the antinode of the fundamental mode appears should be dependent on the liquid height. So we might need to predict the antinode heights for the liquid height when the microtremor measurement is conducted before installing the sensors. It follows that the necessity of numerical solutions might not be entirely eliminated even when measuring only the fundamental-mode natural frequency is aimed at.

In Fig. 6, we also find other clear peaks (Fig. 6, Peaks B to D) at frequencies above the fundamental-mode natural frequency. In the figure, the numerical M-to-T amplitude ratio predicted by the mode shapes from the FEM eigenvalue analysis were plotted by the circles (the modes with $m=1$ and $n=1-5$), the squares



($m=2$ and $n=1-5$), the triangles ($m=3$ and $n=1-5$), and the reverse triangles ($m=4$ and $n=1-5$). The theory predicts the largest M-to-T amplitude ratio when $n=1$ even for the higher vertical modes ($m \geq 2$). And the peak appearance frequencies of Peaks B to D are close to the numerical natural frequency of the modes with $m=2-4$ and $n=1$. Peaks B to D are therefore fairly likely to correspond to these higher modes. If so, the observed M-to-T microtremor spectral ratio suggests the natural frequency of the modes with $m=2-4$ and $n=1$ to be 3.83, 4.98, 5.76 Hz, respectively, which are slightly higher than the numerical values as well as the fundamental-mode case.

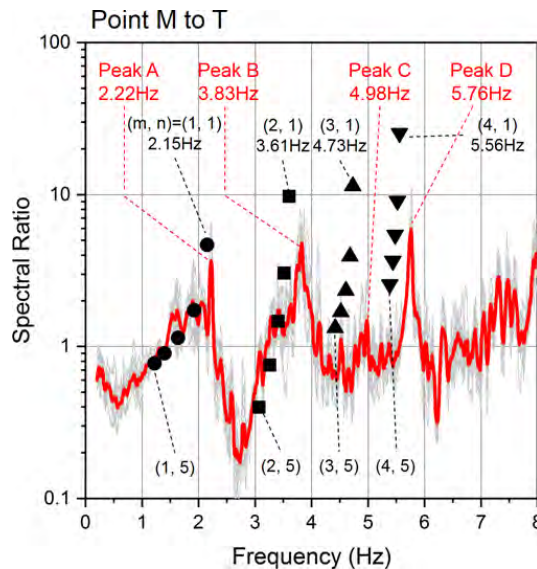


Fig. 6 – Point M to T Fourier amplitude spectral ratio (radial component) of the microtremors from Measurement 1

6. Discussions

What we showed in the previous sections is that as long as analyzing a 30-minute-long microtremor data set from Measurement 1, five peaks were observed in the spectral ratio and they could correspond to the bulging modes of with $m=1$ and $n=1-5$. The problem is whether the five peaks can be always observed. So we at first discuss the data from Measurement 2 (Table 2), which was conducted on the previous day of Measurement 1.

Again, we first computed the numerical natural frequencies by the FEM eigenvalue analysis that was performed for the liquid height 23.542 m, which was observed when the measurement was conducted. The liquid height was slightly different from Measurement 1, and the results are shown in Fig. 7. The FEM analysis predicted the fundamental-mode natural frequency to be 2.11 Hz, and the JFSA equation did 2.21 Hz (Table 2). Fig. 8 shows the Point T'-to-B' spectral ratio obtained in the same manner as before. We again find five peaks (Peaks 1'-5') at frequencies near or below the numerical fundamental-mode natural frequency even in the data measured at a different time and at different points of the tank, although the spatial difference was slight. The peak appearance frequencies are compared with the numerical natural frequencies in Fig. 7. Again, we find good agreements between the five peak appearance frequencies and the numerical natural frequencies of the modes with $m=1$ and $n=1-5$. From the above discussions, we may well assume the temporal and spatial stability of the microtremor spectral-ratio characteristics in regard to inferring the natural frequency of the fundamental mode and the modes with $m=1$ and a small circumferential wave number.

We next discuss the conclusion of Ohmachi and Tanida [7] that the JFSA equation tends to give a considerably higher fundamental-mode natural frequency than it is. What Ohmachi and Tanida [6] examined



were (i) the spectral ratio of the microtremors recorded at the tank top to those recorded on the tank foundation, and (ii) the spectral amplitude of the microtremors recorded at the tank top (not the spectral ratio). They thought that from the examination (i), the natural frequency of the non-soil-coupled system could be inferred, and from (ii) the natural frequency of the soil-coupled system could be inferred. As a result of their examinations, they concluded that non-soil-coupled system fundamental-mode natural frequencies were considerably lower than the natural frequencies projected by the JFSA equation, and that the soil-coupled system values were inferred to be even lower than the non-soil-coupled system values. The examination (i) was basically the same as we did in this paper, and the soil-coupled effects could have been canceled out in the spectral ratio that they examined. If so, the fundamental-mode natural frequency inferred from the spectral ratio could not be very different from the JFSA equation prediction that is based on the assumption of the fixed base. In their examinations, they seem to have read the lowest spectral-peak appearance frequency as the fundamental-mode natural frequency. But as seen in the previous sections, the fundamental mode does not necessarily appear at the lowest peak appearance frequency in the microtremor spectral ratio. This may be the reason for their conclusion.

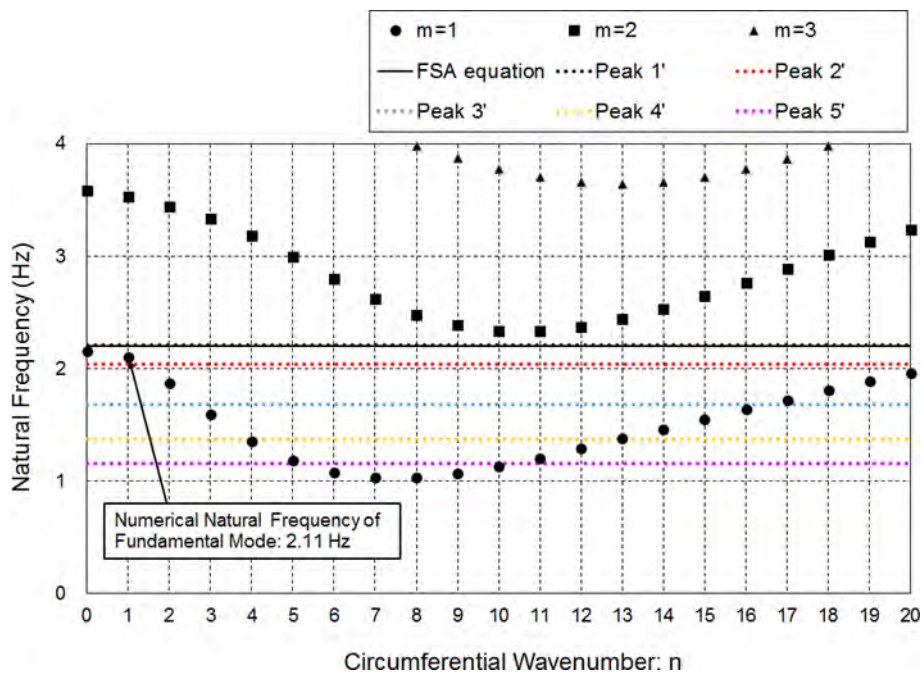


Fig. 7 – Theoretical natural frequencies of the tank using the liquid height encountered for Measurement 2 data

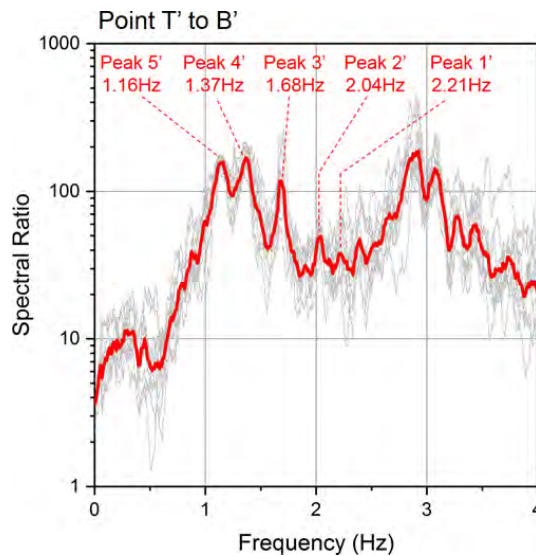


Fig. 8 – Fourier amplitude spectral ratio (radial component) of the microtremors from Measurement 2

6. Conclusions

To measure the natural frequencies of the fluid-elastic-coupled shell plate vibration (bulging) modes of a large-sized (125,000 m³ in capacity, 82 m in diameter, and 27.3 m in height), flat-bottomed, cylindrical steel tank that contained industrial water and was located in Japan, we observed the shell plate vibration of the tank excited by ground microtremors and examined the radial-component spectral ratio of the top or a middle height of the shell plate to the tank bottom. Since the vibration at the tank bottom was measured on the shoulder of the tank foundation, soil-coupled effects should have been canceled out in the spectral ratio, that is, the non-soil-coupled spectral ratio was examined.

By comparing the appearance frequencies and values of the five peaks seen in the observed microtremor spectral ratio with the numerical natural frequencies and the radial-component mode shapes obtained by the FEM eigenvalue analysis that was conducted under assumption of a fixed base, we could identify the five peaks as the modes with a vertical order (m) of one and a circumferential wave number (n) of one to five, respectively and succeeded in inferring the natural frequency of each of the modes from the microtremor observation. The non-soil-coupled measured natural frequencies inferred from the spectral ratio agreed fairly well with the numerical values obtained from the FEM eigenvalue analysis, which we considered reasonable, because the fixed-base-assumed FEM eigenvalue analysis should have given non-soil-coupled natural frequencies.

The measured fundamental-mode ($m=n=1$) frequency was 2.19 or 2.21 Hz, dependent on the liquid height when the microtremor observation was conducted. These values were in good agreement with the values projected by the simplified equation (JFSA equation) adopted in the seismic codes of the Japanese Fire Service Act, which was also developed under assumption of a fixed base. It follows that the JFSA equation can be expected to project a reliable soil-coupled fundamental-mode natural frequency in the case of the tank rested on the firm ground where the storage-soil-coupled effects are presumed to be little.

We finally presented a simple way to determine the non-soil-coupled fundamental-mode natural frequency from the observed microtremor spectral ratio without help of numerical solutions for which involved computations such as the FEM eigenvalue analysis are needed. The way is simply to calculate the radial-component spectral ratio of a middle height to the top of the shell plate of the tank, and to infer the lowest spectral-peak appearance frequency as the fundamental-mode natural frequency. In this method, microtremor observation at the tank bottom is not necessary. This simplified method worked very well for



the examined tank: the method inferred the fundamental-mode natural frequency of 2.22 Hz, which is almost the same as the value (2.19 Hz) inferred by the above-mentioned involved procedure that needed comparison with the numerical solutions of the FEM eigenvalue analysis. This method also has a possibility to infer the natural frequencies of the higher modes ($m \geq 2, n=1$) in a very simplified way.

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