# STUDY ON FINITE ELEMENT ANALYSIS FOR LARGE-SCALE CONCRETE STRUCTURES USING HIGH-PERFORMANCE COMPUTING

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## Abstract

A lumped mass model is usually used in seismic response analysis of a large-scale reinforced concrete (RC) structure. However, for important facilities such as nuclear power plant buildings, higher reliability is sometimes required. Quality assurance of the numerical simulation, which means revealing the accuracy of the numerical simulation by verification and validation (V&V), is a key issue for higher reliability. In such cases, it is effective to apply a high-fidelity model with solid elements instead of a lumped mass model. When analysing the seismic response of large-scale RC structures using high-fidelity models, high-performance computing (HPC) is indispensable because of the number of degrees of freedom especially in considering soil-structure interaction (SSI). This study shows the development of finite element method (FEM) for seismic response analysis of large-scale reinforced concrete structures utilizing HPC and a trial seismic response analysis of a nuclear power building model. We implemented a previously proposed constitutive relation of concrete to the HPC-FEM program FrontISTR. This constitutive relation employs a tensorial non-linear elasto-plastic and damage model, which expresses the damage to the material in terms of the softening of the stress-strain curve, results in the loss of positive-definiteness of elasto-plastic moduli at large deformation and prevents convergence of the conjugate gradient (CG) method, a solver of HCP-FEM. There are several non-linear analysis techniques or Newton method techniques for dealing with this problem; we adopted a method of calculating the stiffness matrix using elastic tensor of the material, which is a kind of modified Newton method, by comparing its speed of computation with those of other methods. To check the performance, the developed software was validated by a simulated loading test of an RC structure. In this performance check, we also checked the performance of the CG method solver which changed by the non-linear analysis methods. It was shown that the non-linear analysis method using elastic tensor made the performance of the CG method solver higher and more stable. Finally, we constructed a numerical model of a nuclear power plant building with surrounding ground, which was a model of approximately 1.5 million DOF, and carried out a seismic response analysis by using the developed software. The results of the numerical analysis were discussed, focusing the numerical performance of HPC-FEM.

*Keywords: high-performance computing; large-scale reinforced concrete structure; high-fidelity model*

## 1. Introduction

A lumped mass model is usually used in the seismic response analysis of a reinforced concrete (RC) structure. It is easy to validate this model with experiments due to a small number of the model parameter, and its low computational cost is attractive for practical use. A non-linear lumped mass model is used by applying experimental data to the model parameters of the model components, which represent a structural member or a group of structural members.

Meanwhile, it is necessary to examine the applicability (more precisely speaking, the limitation of the applicability) of models for the seismic response analysis, which is usually needed from the viewpoint of quality assurance of the analysis. When a seismic load greater than a design seismic load is considered, the model might fail to be used since a larger portion of the RC structure go to non-linear regime of deformation. A model of larger applicability than a lumped mass model is thus needed. A model of high-fidelity which uses fine solid elements is surely a candidate of such a model. In particular, numerical analysis of a high-fidelity model is required for the evaluation of seismic safety of a nuclear power plant (NPP) building, due to the importance of assuring the highest safety.

A high-fidelity model is also useful to consider soil-structure interaction (SSI) precisely. In a lumped mass model, a few spring elements are used for the SSI even in the seismic response analysis of NPP buildings. These spring elements work well for a certain class of input ground motion, but there is a limitation of applying them to larger ground motion.

There are several researches which applies shell element model to an RC structure [1, 2]. It is difficult for an ordinary finite element method (FEM) program to analyze a solid element base high-fidelity model of an RC together with surrounding ground. This is simply because the model has a large number of the degrees of freedom (DOF) and numerical cost becomes very expensive.

A high-performance computing FEM (HPC-FEM) program is being developed, which implements the conjugate gradient (CG) method as a fast solver algorithm [3, 4]. A key issue of developing HPC-FEM for RC structures is the loss of positive-definiteness for the elasto-plastic moduli of concrete materials when they experience large deformation and softening takes place; strain-stress curve has a negative stiffness, representing local failure of the material, and results in the loss of positive-definiteness. The CG method cannot be applied to a non-positive-definite stiffness matrix, when softening spreads in a sufficiently large portion of the model.

In using the HPC-FEM program, we must pay attention to a combination of concrete constitutive relation and an algorithm of non-linear analysis; concrete constitutive relation which allows the loss of positive-definiteness requires a sophisticated algorithm of non-linear analysis since the CG method is implemented in the program. There are several pairs of combinations, and the one that is most computationally efficient must be chosen.

Herein, we seek a suitable combination of the concrete constitutive relation model and the f non-linear analysis algorithm. A numerical experiment is made to examine the performance of the chosen combination, by using a high-fidelity model of a small-scale RC structure. We apply an HPC-FEM program to the seismic response analysis of an NPP building, considering the SSI. The results of the numerical analysis and the computational performance of the HPC-FEM program are discussed.

## 2. Problem in development

In this section, we outline the problem in developing the HPC-FEM program for RC structures. We first explain the constitutive relation of concrete that is used in the program, and then clarifies the problem of a stiffness matrix which loses the positive-definiteness. A need for a sophisticated algorithm of non-linear analysis is explained from the viewpoint of the stability of analysis and the cost of numerical computation.

We employ a model of concrete constitutive proposed by Maekawa et al. [5], the use of which is a de-facto standard in Japan. The model is an elasto-plastic fracture model before cracking, and switches to a cracked concrete mod-el after cracking; see Figure 1. There are two issues in implementing the model in an HPC-FEM program, namely, a high computational cost of the model that is expressed in a general tensorial form and a difficulty in assuring that the stiffness matrix is positive definite.



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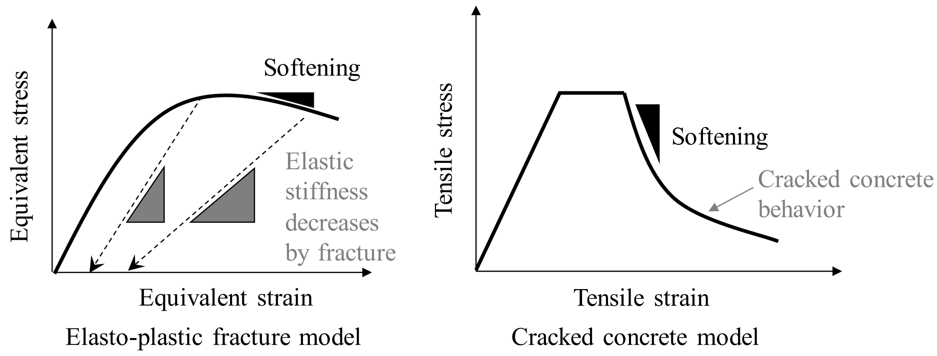


Fig. 1 – Concrete constitutive relation model

Most of numerical computation of HPC-FEM is used in solving a matrix equation, and computing complicated constitutive relation is much cheaper. Still, it is important to save the computational cost of the constitutive relation, together with smaller usage of memories because the number of elements or integration points becomes large for a high fidelity model. The original model of the concrete constitutive relation includes a computation of an inverse of a six-by-six matrix that corresponds to a fourth-order tensor of elasto-plastic tensor. Yamashita et al. [6] reformulated the original model to get rid of computing the inverse matrix. The outline of the reformulation is shown below. Equation 1 presents the method of computing the elasto-plastic tensor $\boldsymbol{c}^{EP}$ in the original model.

$$\boldsymbol{c}^{EP} = (\boldsymbol{c} + \boldsymbol{\nabla c}\colon \boldsymbol{\epsilon}^{E})\colon (\boldsymbol{I} + \boldsymbol{\ell})^{-1}, \tag{1}$$

where $\boldsymbol{c}$ is an elastic tensor which is defined as a function of elastic strain $\boldsymbol{\epsilon}^E$ with $\boldsymbol{\nabla c}$ being a derivative with respect to it, $\boldsymbol{I}$ is the unit tensor, and $\boldsymbol{\ell}$ is a tensor that associates $\boldsymbol{\epsilon}^E$ to an increment of plastic strain $\boldsymbol{\epsilon}^P$ playing a unique role of the model. Yamashita derived the following equation from Equation 1, considering the relation between the total strain, $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^E + \boldsymbol{\epsilon}^P$, and $\boldsymbol{\epsilon}^P$:

$$\boldsymbol{c}^{EP} = (\boldsymbol{c} + \boldsymbol{\nabla c}\colon \boldsymbol{\epsilon}^{E})\colon (\boldsymbol{I} - \boldsymbol{L}), \tag{2}$$

where $\boldsymbol{L}$ is a tensor that associates $\boldsymbol{\epsilon}$ to the increment of $\boldsymbol{\epsilon}^P$. As is seen, Equation 2 avoids computing an inverse of a fourth-order tensor. A technique of implementing this equation into a program is shown in the authors' work.

As for the second issue of assuring the positive-definite of the stiffness matrix, we first used an elasto-plastic tensor of Equation 2, to examine the effects of the loss of positive-definiteness of local material upon an overall stiffness matrix [7]. It was shown that the stiffness matrix was not assured as positive definite when a sufficiently large portion of the model enters large plastic strain regime. In the seismic response analysis, however, the mass matrix (and the damping matrix) was added to the stiffness matrix; the matrix that was used in the seismic response analysis is expressed in terms of the stiffness matrix $\boldsymbol{K}$ and the mass matrix $\boldsymbol{M}$ as $\boldsymbol{A} \approx (1/\mathrm{d}t^2)\,\boldsymbol{M} + \boldsymbol{K}$ with a time increment $\mathrm{d}t$. A smaller $\mathrm{d}t$ would improve the condition of $\boldsymbol{A}$ even if $\boldsymbol{K}$ lost the positive-definiteness, since $\boldsymbol{M}$ is positive definite. It is thus rational to make a trial of acquiring numerical stability by setting a smaller value on $\mathrm{d}t$.

This trial was not successful, as the numerical stability and the elapsed time changed depending on target structures. In general, the elapsed time increased as the numerical stability was maintained. This was just because we needed a smaller $\mathrm{d}t$ and a larger number of time steps.

We need to develop a suitable method in which the numerical stability does not depend on $\mathrm{d}t$. We focus on improving the algorithm of non-linear analysis; a Newton method is implemented in the HPC-FEM program. A new algorithm uses an elastic tensor, rather than, an elasto-plastic tensor, to calculate the stiffness matrix. In this method, the numerical stability is assured even with a larger $\mathrm{d}t$ because the stiffness matrix is always positive definite. On the contrary, the estimation of the increment of displacement includes a larger error,



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which introduces the increase in the iteration of computing the non-linear constitutive relation. It is thus necessary to choose a suitable non-linear analysis for the HPC-FEM program for RC structures, by comparing the computing performances.

## 3. Non-linear analysis algorithm for RC structures

### 3.1 Proposed method

This section explains a trial algorithm of the non-linear analysis. As for convenience, we call this algorithm as a proposed method.

Unlike the existing algorithm (or the existing method) explained in Section 2, the elastic tensor is used in the proposed method for calculating the stiffness matrix. This method can directly avoid dealing with the softening behavior that leads to a negative stiffness; hence, the positive definiteness of the stiffness matrix is assured. In the implementation, we need to ensure that the elastic stiffness is changed by the strain increase. This method is regarded as a kind of a modified Newton method which ignores the change in the elastic stiffness. Figure 2 depicts the method outline.

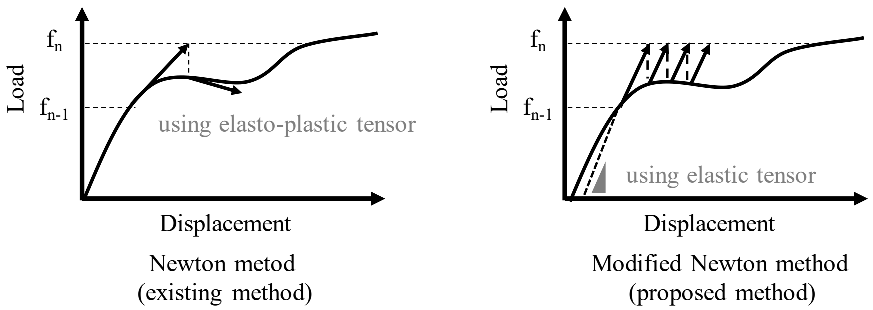


Fig. 2 – Outline of the nonlinear method

### 3.2 Model for performance check

A cyclic loading test of a simple RC structure made by Habasaki et al. [8] was used as a target problem of examining the computational performance of the proposed method. We constructed an analysis model which consists of solid elements; the target structure was a box structure, constructing of four RC walls, a ceiling slab and a base slab. The thickness, width, and height of each wall was 75, 1500, and 1000 mm, respectively. The reinforced bars were 6 mm in diameter (D6), equivalent to SD345. The steel ratio in the wall was 1.2%. The material parameter used in the model was set from the literature of the experiment.

Figure 3 shows an overview of the analysis model. The ceiling slab and the base slab are modeled as a rigid body. Figure 3 also shows the cross-section of the wall. A shell element is used to a group of reinforced bars. The bottom face of the base slab is fixed, and the loading is applied to the center of loading slab. The loading condition is displacement control, i.e., increasing displacement is added at suitable nodes as boundary conditions.

Figure 4 depicts the comparison of the experiment and the numerical analysis that uses the proposed method; the main experimental data are traced by the literature. The load condition is simplified to reduce time steps; numerical analysis is single cyclic loading compared to the experiment of double cyclic loading. The numerical analysis succeeds to reproduce the experimental data fairly well considering that this result does not include parameters fitting of strength. It is certainly true that further studies are needed to improve the accuracy of computing hysteresis curve. However, this model can estimate the skeleton curve with enough accuracy.



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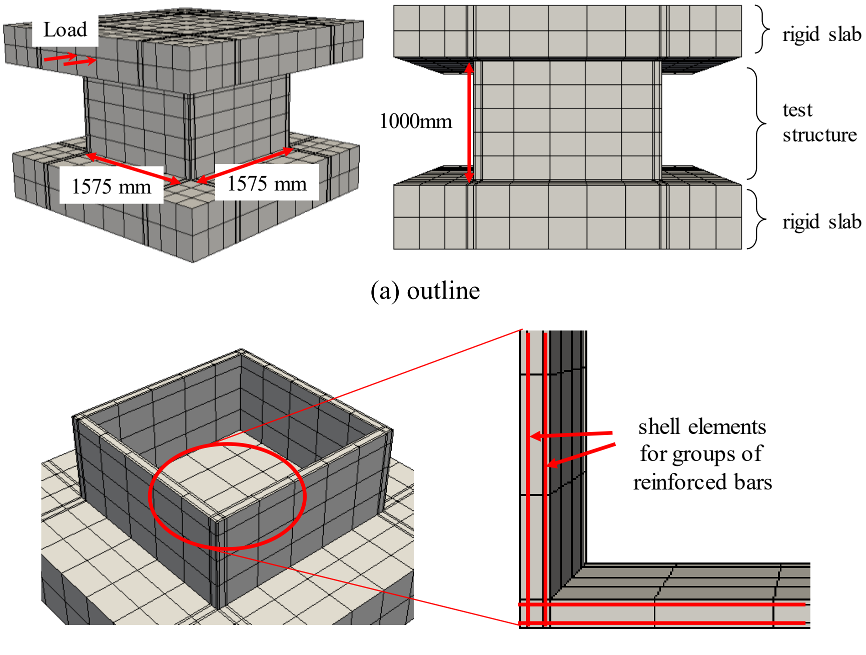


(a) outline

(b) model of reinforced bars

Fig. 3 – Model of the experiment structure

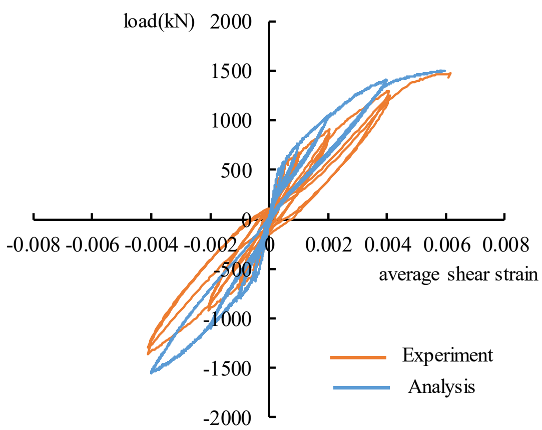


Fig. 4 – Example of the simulation result

### 3.3 Performance check

The computation performance was checked for the non-linear analysis for the RC box structure; the number of nonlinear analysis iteration and the elapsed time were compared for the existing method that used the elasto-plastic tensor and the proposed method that used the elastic tensor. A target problem was monotonic loading up to 2 mm displacement, in terms of strain, up to 0.002 deformation with a simplified model. A dynamic analysis was used, with a loading time of 10 s. As for the existing method, we used three values of $dt$, namely, $dt$ = 0.1, 0.01, 0.001 s. The stiffness matrix was fixed at each time step (the model was linearized), indicating no repeated calculation for convergence. The proposed method used $dt$ =0.01 s only, and the maximum iteration number of computing non-linear constitutive relation to reach the numerical convergence was set as five times; the non-equilibrating nodal forces were carried over to the next time step if the iteration number exceeded the maximum number.



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Figure 5 shows the results which are achieved from the performance check simulation. It is seen that although a smaller time increment improves the numerical stability, the existing method cannot acquire the convergence of the CG method even in the case of $dt$ =0.001 s. On the contrary, the proposed method is stable for this target case.

Table 1 lists the number of nonlinear analysis iteration and the averaged value of the elapsed time per each time step. The number of the non-linear analysis iteration means the total number of iterative computations for the non-linear analysis, which is equal to the number of solver functions called. We also show the number of calculation steps for the CG method to have a converged solution. The existing method failed to produce a converged solution, and the number of calculation steps for the CG method stands for the number at which the CG method is ceased.

Using the proposed method, the number of non-linear analysis iteration is decreased by approximately one third, compared with the existing method that uses $dt$ =0.001 s to have numerically stable and converged solutions. Even though the elapsed time per each time step is increased, the computational performance of the proposed method is high. Indeed, if the existing method uses a smaller value for $dt$ depending on the target problem, the difference in computational performance becomes larger than that shown in Table 1.

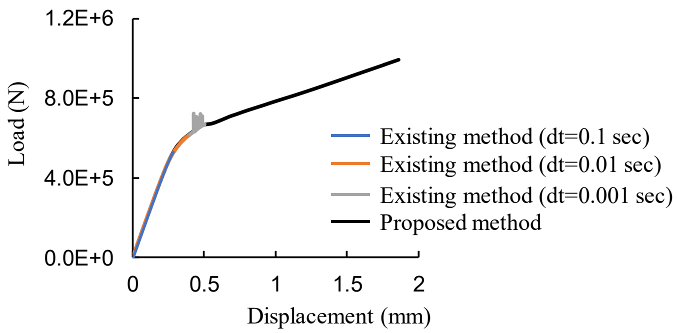


Fig. 5 – Result of the performance check simulation

Table 1 – Comparison of the performances in each case

| | Existing ($dt$=0.1) | Existing ($dt$=0.01) | Existing ($dt$=0.001) | Proposed ($dt$=0.01) |
|---|---|---|---|---|
| Number of analytical steps | 100 (25) | 1,000 (304) | 10,000 (3278) | 3,485 |
| Elapsed time of solver per one step | 0.328 s | 0.332 s | 0.300 s | 0.322 s |
| Number of CG steps per one step | 898.5 | 894.5 | 786.4 | 800.9 |

We compared the change in the number for the CG method to have a converged solution, depending on the progress of the loading step (or the growth of nonlinear elasto-plastic regime). Figure 6 shows the summary of the comparison. The x- and y-axis denote the loading displacement and the number of the CG steps at each displacement. As for the existing method, the number of the CG steps increases at a larger displacement. In contrast, as for the proposed method, the change in the number of the CG steps is smaller, because the change in the stiffness matrix of the proposed method is smaller than that of the existing method. From the view of the numerical stability and the computational performance, this result demonstrates the superiority of the proposed method compared with the existing method, in numerically analyzing the RC structures, using an HPC-FEM program to which the CG method is implemented.



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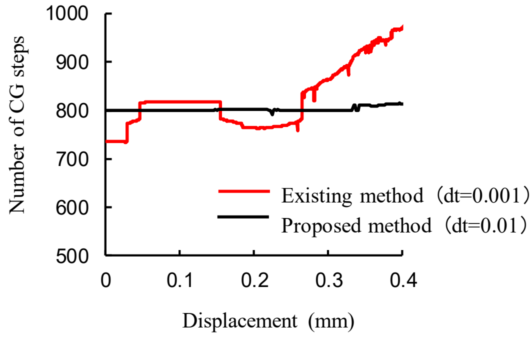


Fig. 6 – Number of CG steps at each step

## 4. Seismic response analysis of an NPP building

### 4.1 Target building

Using the developed HPC-FEM program, we carried out a seismic response analysis of a high fidelity model for an NPP building with surrounding ground. Figure 7 illustrates the model outline. The surrounding ground size is 600 m × 800 m horizontally and 200 m vertically. Table 2 lists the computation scale.

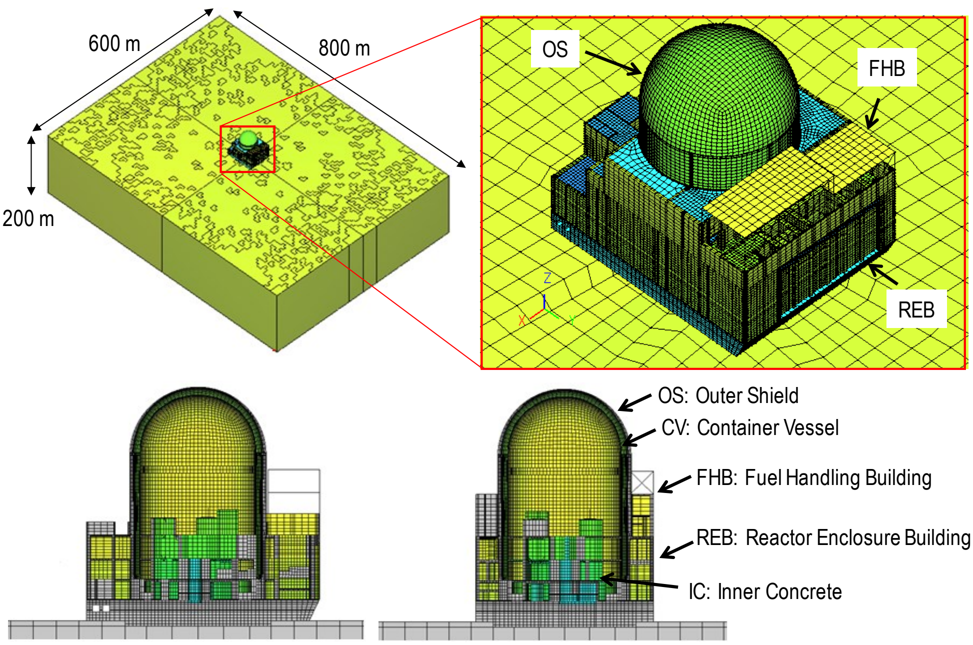


Fig. 7 – Outline of the model of the NPP buildings

Table 2 – Computation scale

| Node | Element | DOF |
|---|---|---|
| 405,326 | 444,060 | 1,472,529 |



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The input seismic wave was synthesized, to be compatible to the spectrum proposed by Kato et al. [9]. The duration time of the input seismic wave was 22.08 s. The displacement boundary conditions that were compatible to the input seismic wave was posed to the bottom as well as on each side of the surrounding ground; the one-dimensional wave propagation in the ground was computed and the computed displacement was used for the sides. The time increment was set as $dt$ =0.005 s, and the total number of time steps was 4,416.

### 4.2 Results of the seismic response analysis

First, we examine the computational performance. The elapsed time of this analysis was approximately 10 h, when we used a parallel computer of 64 cores. This elapsed time is short enough for the practical use.

Figure 8 presents the examples of the deformation of the NPP building. Shown is the distribution of cracking in concrete on the outer shield and the distribution of the maximum shear strain; the same time step was chosen at the maximum shear strain takes the maximum value during the analysis. These results indicate that a numerical analysis of a high-fidelity model, which is enabled by using the HPC-FEM program, visualizes the local deformation and damage which are caused by the seismic resolution with high spatial resolution.

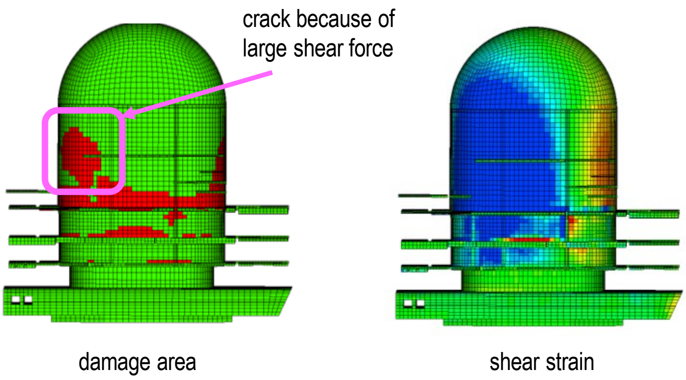


Fig. 8 – Distribution of the crack and shear strain on RC walls of OS

Figure 9 exhibits the acceleration response and the acceleration response spectrum. These are measured for the two target points at the same level of the reactor enclosure building. The response and spectrum are quite different even though they are on the same level, and such difference cannot be computed by using an ordinary lumped mass model in which responses at the same level are identical. It confirms a need for the analysis of a high-fidelity model for accurate estimation of local seismic responses which are used as input for equipment and facilities located in the NPP building. The results of linear analysis are shown in this figure for the comparison of the nonlinear analysis. The acceleration response spectrum is quite different in a certain range of frequency, which confirms the need for the nonlinear analysis. Precise treatment of material nonlinearity is needed for to make the device design more rational.

## 5. Conclusions

We developed the non-linear analysis method for an HPC-FEM program when seismic response analysis is made for a high-fidelity model of an RC structure. Good computational performance of the proposed method of non-linear analysis inherent to the concrete constitutive relation employed was confirmed by carrying out the loading test of a small RC structure. We carried out the seismic response analysis of an NPP building with surrounding soil, applying the proposed method to its high-fidelity model. Satisfactory computational performance was confirmed, as the program was able to finish the numerical analysis shorter than half a day when a suitable computer environment was used. Local deformation and damage, which was induced by



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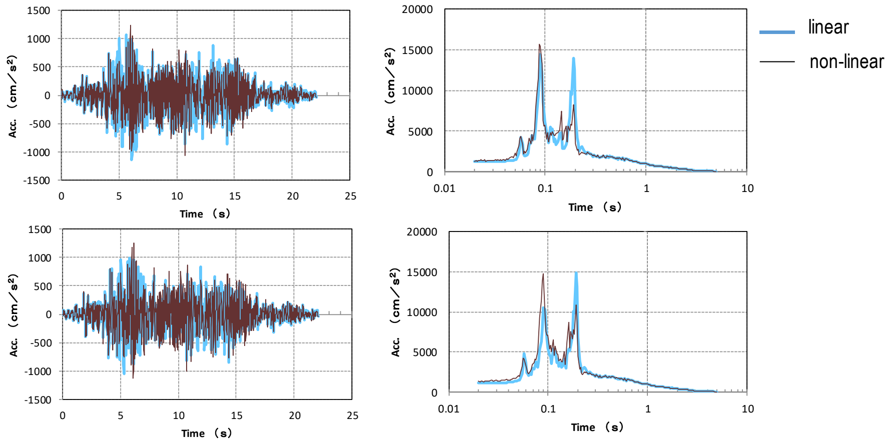


Fig. 9 – Acceleration and acceleration response spectrum

concrete non-linearity, were computed when large ground motion was input. High spatial resolution of the numerical analysis of the high fidelity model was practically important.

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