AN ANALYTICAL FORMULATION TO DESCRIBE THE VIBRATION OF A FLUID-SHELL SYSTEM REPRESENTING A FAST REACTOR

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Abstract

Modern pool-type fast reactors comprise of containers with fluid inside. Besides holding the fluids, the containers are also required to transfer heat from one side to another smoothly. A "thin" container serves the purpose of heat transfer well. However, such containers and associated support systems may be vulnerable during strong earthquake shaking. A possible mitigating strategy is to "isolate" the reactor from horizontal earthquake shaking. Natural period of vibration for a system with horizontal seismic isolation may vary in the range of 1 – 4 seconds, and that corresponding to sloshing of fluids inside the containers may range between 3 – 8 seconds. Possibility of amplification in the sloshing response due to seismic isolation needs to be studied. Such a study can be performed through experiments, or verified and validated analytical models. A third way could be to consider rather simple systems representing a pool-type fast reactor and develop closed-form analytical solutions to describe the vibration of these systems. Such solutions would allow understanding the influence of seismic isolation on the sloshing response for a wide range of geometric and material properties of the simplistic reactor and seismic isolation systems. This paper presents the development of a closed-form analytical solution for a representative pool-type fast reactor. The system comprises of two cylindrical shells and a centrally-placed internal body. The outermost cylindrical shell is supported at the top (through open end). The inner cylinder is supported at the base of the outermost cylinder. These two cylinders are referred to as main vessel and inner vessel, respectively. Fluid is present in the inner vessel and in the annulus between the inner and main vessels. The closed end of the innermost cylinder is at the top, and is attached to the ground. This cylinder is hollow and is partially submerged in the fluid inside the inner vessel. Two broad sets of modes of vibration are observed in such systems: 1) sloshing, and 2) bulging. Analytical formulation presented in this paper considers both. Shells are assumed elastic and their behavior is characterized using Flugee's shell theory. The behavior of the fluid is described by a velocity potential function, which considers coupling between bulging and/or sloshing modes. Natural frequencies and mode shapes of the system are calculated by minimizing the Rayleigh-Ritz quotient. The analytical modeling approach is validated against experiments conducted in the past. A parametric study is carried out to understand the effect of the ratio of radii of main and inner vessels, and density of the fluid on the dynamic properties of the system. Influence of considering coupling between the bulging and sloshing modes on the natural frequencies of the system supported by ground (non-isolated) was found small. Therefore, bulging and sloshing modes can be treated independently for a non-isolated system. Bulging frequency of the system decreases with an increase in the density of fluid or a reduction in the ratio of the radii of main and inner vessels. First mode sloshing frequencies are not considerably affected by the two factors. Second mode sloshing frequency decreases considerably with an increase in the ratio of the radii of main and inner vessels.

Keywords: Bulging; Sloshing; Reactor internals; Velocity potential; Rayleigh-Ritz method

1. Introduction

Modern pool-type fast reactors comprise of a number of vessels containing fluid (e.g., liquid sodium) at a high temperature [1]. Functional needs of such reactors require some vessels to be "thin" so that heat transfer can take place from one chamber to another. These "thin" vessels and associated equipment may be vulnerable to seismic shaking [1]. A way out of this design challenge is to isolate the reactors seismically (e.g., [2]). Two broad classes of modes of vibration are observed for structures containing fluid: 1) sloshing mode associated with the vibration of the free surface of fluid, and 2) bulging mode associated with the vibration of fluid and structure. The period of vibrations associated with the sloshing mode could be in the range of 3 − 8 seconds (e.g., [3-5]), and those corresponding to the vibration of the isolated structures can vary in the range of 1 – 4 seconds (e.g., [6]). The fluid-structure interaction in isolated reactors can be studied through experimental and analytical models. The focus of this paper is to develop an analytical model considering the fluid-structure interaction in a pool-type fast reactor attached to the ground (non-isolated). A summary on the relevant studies is presented below.

Fritz [7] conducted the first analytical study known to the authors, wherein the dynamics of a system of two concentric cylinders with fluid filled between the two cylinders was studied. Cylinders were modelled as beam and the fluid was considered to be potential, i.e., incompressible, inviscid and irrotational. Chung et al. [7] studied a system of a group of cylinders placed inside a larger cylinder. The cylinders were modelled as beam with their longitudinal axis parallel to each other, and the fluid was considered to be potential. Zhu [9] presented the analytical formulation to describe the vibration of a generalized container with fluid. The container was modeled as a shell and the fluid was considered potential. Rayleigh-Ritz approach [9] was adopted to determine the frequencies of vibration associated with the bulging mode. The work was extended by Amabilli [10-11] to incorporate the coupling between the sloshing and bulging modes of vibration. It was concluded that if the frequencies associated with sloshing and bulging were considerably apart, the two modes could be studied independently. Jeong et al. [12] studied a system comprising of two cylinders with their longitudinal axes parallel to each other. Effect of eccentricity on the frequency associated with the bulging mode was studied. Aksari et al. [13] presented a formulation to describe the vibration of concentric cylinders filled with a potential fluid. The inner cylinder was suspended from the top while the outer cylinder was fixed at the bottom. The inner cylinder was modeled as rigid and the outer cylinder was modeled as a flexible shell. The effect of coupling between sloshing and bulging modes was considered. Moshkelgosha et al. [14] considered the system similar to Aksari et al. [13]. However, the inner cylinder was considered flexible and bulging mode of vibration was studied.

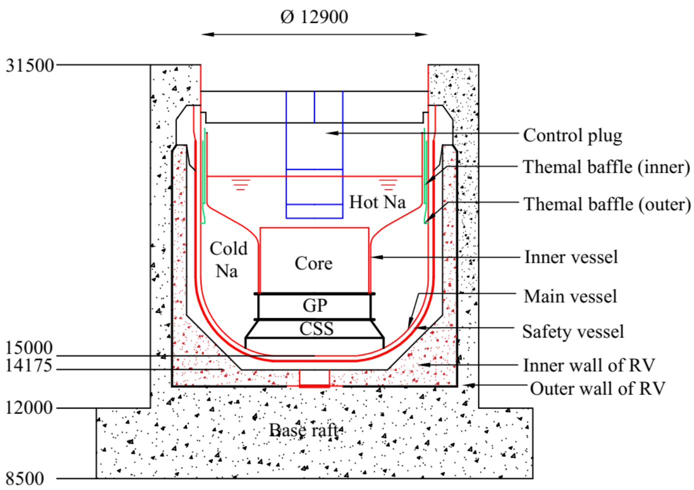
This paper presents an analytical formulation to describe the vibration of a system representing a pool-type fast reactor. The system comprises of three cylinders. All cylinders are modeled as shell and coupling between the bulging and sloshing modes is considered. The considered approach of formulation is validated against experimental studies conducted in the past. A parametric study is carried out to understand the effect of density of the fluid and annular space between the shells on the dynamic properties of the representative system.

2. Development of analytical model

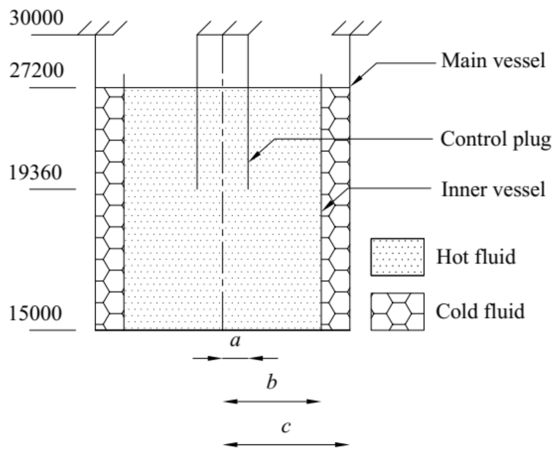
2.1 Idealization of a fast reactor

Fig. 1(a) shows a liquid sodium-based pool-type fast breeder reactor considered by Chellapandi et al. [15]. The diameter and height of the main vessel are 12.9 m and 15.0 m, respectively. The vessel houses control plug, inner vessel, core, grid plate (GP), core support structure (CSS), heat exchanger and pumps. The vessel is supported through the outer wall of the reactor vessel at the top. The inner vessel is supported by the grid plate, which separates the hot and cold sodium pools. The grid plate is supported by the core support structure, which is attached to the base of the main vessel. Fig. 1(b) shows an idealization of the reactor shown in Fig. 1(a). The idealized system comprises of three concentric cylindrical shells. The outermost shell is supported at the top. This cylinder is referred to as main vessel. The inner cylinder (referred to as inner vessel) is supported at the base of the outer cylindrical shell. The inner cylinder contains hot fluid, and cold fluid is filled between the main and inner vessels. The innermost cylinder has its closed end up, which is attached to the ground. This cylinder is partially submerged in the hot fluid. Dimensions of the system are indicated in Fig. 1(b).

The common base of the two outer cylindrical shells is assumed rigid. The shells are assumed elastic, and their behavior is characterized using Flugee's shell theory [16]. The fluid is characterized by a velocity potential function, which considers the coupling of bulging and sloshing modes. The coupled vibrational mode of the representative system is determined by minimizing Rayleigh's quotient (equal to the ratio of potential energy to the kinetic energy of the system) [17].



(a) Schematic of reactor vessel adopted from [15]



(b) A representative system

Fig. 1 – A pool-type fast breeder reactor (all dimensions are in mm)

2.2 Analytical formulation

2.2.1 Potential energy of thin cylindrical shell

Potential energy U_s of a thin cylindrical shell is based on Love's hypothesis [16], and is given by:

$$U_s = \frac{E}{2\left(1-\nu^2\right)} \int_0^L \int_{-t/2}^{t/2} \int_0^{2\pi} \left(\varepsilon_x^2 + \varepsilon_\theta^2 + \left(\frac{1-\nu}{2}\right) \varepsilon_{x\theta}^2 + 2\nu \varepsilon_x \varepsilon_\theta \right) (R)\, d\theta\, dz\, dx \tag{1}$$

where, x corresponds to longitudinal axis, θ corresponds to the axis along the periphery, ε_x is normal strain in axial direction, ε_θ is normal strain in tangential direction, $\varepsilon_{x\theta}$ is shear strain in $x-\theta$ plane, z represents the radial distance of any point on the shell from the middle surface of the shell, L, t and R are length, thickness and radius of the cylindrical shell, respectively, and E and ν are elastic modulus and Poisson's ratio of the material of shell. Using Flugee's thin shell theory [16], strains are expressed in terms of displacements u, v and w:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial z^2},\quad \varepsilon_\theta = \left(\frac{1}{R}\right)\frac{\partial v}{\partial \theta} + \frac{w}{R} - z\frac{\partial^2 w}{\partial \theta^2} + \left(\frac{z}{R^2}\right)\frac{\partial v}{\partial \theta},\quad \varepsilon_{x\theta} = \left(\frac{1}{R}\right)\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} - 2z\left(\frac{\partial^2 w}{\partial x \partial \theta}\right) + \frac{z}{R}\frac{\partial v}{\partial x} \tag{2}$$

where, u, v and w are displacements in longitudinal, tangential and radial directions, respectively. For free vibrational analysis of cylindrical shell, the following functional form for displacements is assumed [16]:

$$u=\sum_{n=1}^{N}\sum_{i=1}^{m}\frac{\partial\phi_{ni}}{\partial x}\cos(n\theta)\,e^{i\omega t},\ v=\sum_{n=1}^{N}\sum_{i=1}^{m}\phi_{ni}\sin(n\theta)\,e^{i\omega t},\ w=\sum_{n=1}^{N}\sum_{i=1}^{m}\phi_{ni}\cos(n\theta)\,e^{i\omega t}\quad i=1,2,\ldots,m \tag{3}$$

where n, ω and $\varphi_{ni}(x)$ represent the n^{th} circumferential mode, natural angular frequency and assumed mode shape of the shell, respectively.

2.2.2 Kinetic energy of thin cylindrical shell

Kinetic energy T_s of a thin cylindrical shell is given by [16]:

$$T_s=\frac{1}{2}\rho\int_0^L\int_{-t/2}^{t/2}\int_0^{2\pi}\left(\left(\dot{u}(x,\theta,t)\right)^2+\left(\dot{v}(x,\theta,t)\right)^2+\left(\dot{w}(x,\theta,t)\right)^2\right)(R)\,d\theta\,dz\,dx \tag{4}$$

where ρ is the density of the shell, $\dot{(\ )}$ denotes the derivative with respect to time t, and other parameters were defined previously.

2.2.3 Velocity potential approach for fluid

Behavior of an incompressible, inviscid and irrotational fluid can be described by its velocity potential [9]. Amabilli et al. ([10], [18-19]) extensively studied the vibration of cylindrical shell containing fluid, wherein fluid was assumed to be a potential fluid. In the present study, two coolants, namely, liquid sodium and bismuth, are considered, and both are assumed to be potential fluids. By assuming a simple harmonic motion with radial frequency of ω, the velocity potential of fluid $\tilde{\phi}$ can be expressed in terms of displacement potential ϕ [9]:

$$\tilde{\phi}(r,\theta,x,t)=i\ \omega\ \phi(r,\theta,x)\ e^{i\omega t} \tag{5}$$

where, $\phi(r,\theta,x)$ satisfies the following equation:

$$\left(\frac{\partial^2\phi}{\partial r^2}\right)+\frac{1}{r}\left(\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2\phi}{\partial\theta^2}\right)+\left(\frac{\partial^2\phi}{\partial x^2}\right)=0 \tag{6}$$

Using the superposition principle [11], ϕ can be expressed as follows:

$$\phi=\phi_s+\phi_b \tag{7}$$

where, ϕ_s is the displacement potential associated with sloshing of fluid in a rigid shell, and ϕ_b is the displacement potential associated with the vibration of flexible shell assuming zero pressure at free surface.

2.2.4 Displacement potential associated with sloshing of fluid ϕ_s

Fig. 2 shows the schematic considered for calculating sloshing displacement potential ϕ_s for the representative system. The radial, tangential and axial coordinates are denoted by r, θ and X, respectively. The radius of control plug, inner vessel, main vessel are denoted by a, b, and c, respectively, and the height of control plug, inner vessel, main vessel and fluid are denoted by L_c, L_i, L_m and H, respectively. The computation of sloshing displacement potential ϕ_s is simplified through the following transformation: $X=x-(L_m-L_c)$ (see [20] for further details). The fluid domain is divided into four regions to calculate sloshing displacement potential ϕ_s (e.g., [13], [21]). These regions are identified below.

$$R_1^s = \{(r,x,\theta): 0 \le x \le L_m - L_c, r < b\},\ R_2^s = \{(r,x,\theta): L_m - L_c \le x \le H, r < a\},$$
$$R_3^s = \{(r,x,\theta): L_m - L_c \le x \le H, a < r \le b\},\ R_4^s = \{(r,x,\theta): 0 \le x \le H, b < r \le c\} \quad (8)$$

By solving Eq. (8) using method of separation of variables and applying the rigid boundary condition at $r=b$, the sloshing displacement potential ϕ_s for R_1^s can be given as:

$$\phi_s^1 = A_{mns} J_n\left(\frac{\lambda r}{b}\right)\left[\cosh\left(\left(\frac{\lambda}{b}\right)\left(x-(L_m-L_c)\right)\right)+\tanh\left(\left(\frac{\lambda}{b}\right)\left(x-(L_m-L_c)\right)\right)\sinh\left(\left(\frac{\lambda}{b}\right)\left(x-(L_m-L_c)\right)\right)\right] \quad (9)$$
$$J_n^{'}(\lambda)_{r=b} = 0$$

where, J_n and Y_n represent Bessel's function of first and second kind, respectively, while the constant A_{mns} can be obtained using compatibility and boundary conditions (see [20] for further details).

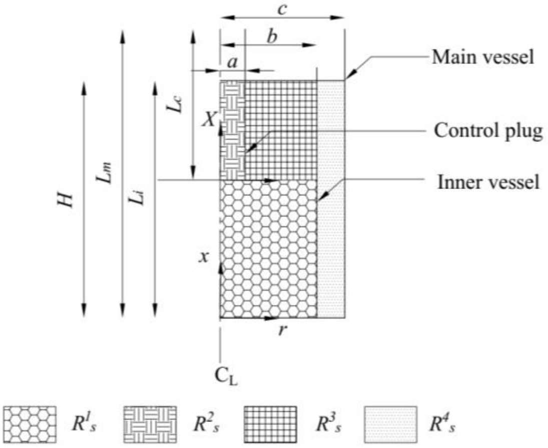


Fig. 2 – Segments of fluid considered for calculating sloshing displacement potential ϕ_s

2.2.5 Displacement potential associated with vibration of flexible shell ϕ_b

Fig. 3 shows the segments considered for calculating displacement potential ϕ_b of the representative system. Radial, tangential and axial coordinates are denoted by r, θ and x, respectively. The fluid domain is divided into three regions to calculate bulging displacement potential ϕ_b. These regions are identified below.

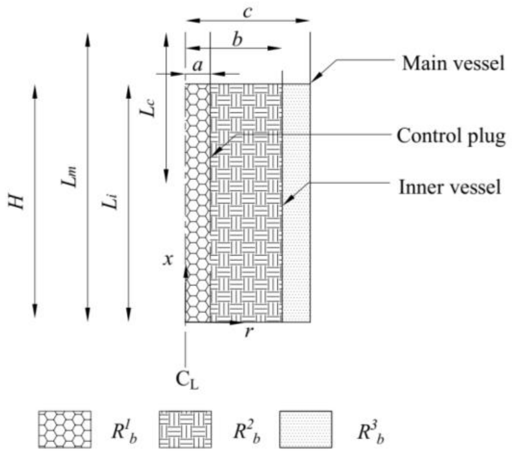


Fig. 3 – Segments of fluid considered for calculating bulging displacement potential ϕ_b

$$R_1^b = \{(r,x,\theta): x<H, r<a\},\ R_2^b = \{(r,x,\theta): x<H, a<r\le b\},\ R_3^b = \{(r,x,\theta): x<H, b<r\le c\} \quad (10)$$

Using the superposition principle [18], the bulging displacement potential ϕ_b can be written as follows:

$$\phi_b = \phi_b^1 + \phi_b^2 + \phi_b^3 \quad (11)$$

where, ϕ_b^1, ϕ_b^2 and ϕ_b^3 represent the displacement potential in the fluid due to vibration of control plug, inner vessel and main vessel, respectively. Solving Eq. (8) using method of separation of variables and assuming zero pressure at the free surface, the bulging displacement potential ϕ_b for R_I^b due to vibration of control plug can be given as:

$$\left(\phi_{bI}^1\right) = A_{mns1} I_n(\gamma_m r)\cos(\gamma_m x),\ \gamma_m = \left(\frac{(2m-1)\pi}{2}H\right) \quad (12)$$

where, I_n and K_n represent modified Bessel's function of first and second kind, respectively, while the constant A_{mns1} can be obtained through compatibility and boundary conditions (see [20] for further details).

2.2.6 Compatibility and sloshing boundary conditions

Compatibility conditions are defined as follows: 1) displacement potential and its derivative calculated at either side of an interface should be equal, and 2) velocity of the fluid at a fluid-structure interface is equal to the velocity of the shell at the interface. The two conditions ensure the continuity of the displacement potential at the interfaces identified in Figs. 2 and 3.

Sloshing boundary condition (e.g., [11], [13]) for R_4^s in Fig. 2 (see [20] for further details) at the free surface considering both the bulging and sloshing potentials is given as follows:

$$\left(\frac{\partial \phi_s^4}{\partial x}\right)_{x=H} + \left(\frac{\partial \phi_{bI}^1}{\partial x}\right)_{x=H} = \frac{\omega^2}{g}\left(\phi_s^4\right)_{x=H} \quad (13)$$

where, ϕ_{bI}^1 represent the bulging displacement potential in R_b^1 due to vibration of control plug, g is acceleration due to gravity, and other parameters were defined previously. Sloshing boundary conditions for other regions identified in Fig. 2 are presented in Saboo [20].

2.2.7 Natural frequencies of the system considering only bulging mode

As noted previously, bulging and sloshing modes in a structure containing fluid can be well separated, and the two modes can be considered separately in such a scenario (e.g., [11]). Natural frequency associated with the bulging mode for the representative system (ignoring sloshing in the system) shown in Fig. 1(b) can be calculated by minimizing the Rayleigh's quotient [17], wherein the problem is reduced to an eigenvalue problem (see [20] for further details):

$$\omega^2 = \frac{U_s^1 + U_s^2 + U_s^3}{T_s^1 + T_s^2 + T_s^3 + T_L^*},\ \ T_L^* = \frac{\rho_f}{2}\int_S (\phi_b)\left(\frac{\partial \phi_b}{\partial r}\right) dS \quad (14)$$

$$[K]\{q\} - \omega^2 [M]\{q\} = 0 \quad (15)$$

where, ω, U_s^1, U_s^2, U_s^3, T_s^1, T_s^2, T_s^3, T_L^*, ρ_f, S, $[K]$, and $[M]$ represent natural frequency associated with bulging mode, potential energy associated with control plug, inner vessel and main vessel, kinetic energy associated with control plug, inner vessel and main vessel, reference kinetic energy of the fluid, density of fluid, interface area between the shell and fluid, and bulging stiffness and mass matrices of the system, respectively.

2.2.8 Natural frequencies of the system considering only sloshing mode

Natural frequency associated with the sloshing mode for the representative system (ignoring bulging in the system) shown in Fig. 1(b) can be calculated by eliminating the bulging displacement potential from the sloshing boundary condition. The corresponding sloshing boundary condition is given by:

$$\left(\frac{\partial \phi_s}{\partial x}\right)_{x=H} = \frac{\omega_1^2}{g}(\phi_s)_{x=H} \tag{16}$$

where, ω_1 represents natural frequency associated with the sloshing mode, and other parameters were defined previously.

2.2.9 Natural frequencies of the system considering both bulging and sloshing modes

Natural frequencies of the system shown in Fig. 1(b) considering coupling between the bulging and sloshing modes can be determined in a manner similar to Section 2.2.7 with the displacement potential defined using Eq. (7), and the sloshing boundary condition defined by Eq. (13):

$$\omega_2{}^2 = \frac{U_s^1 + U_s^2 + U_s^3}{T_s^1 + T_s^2 + T_s^3 + T_L^*}, \quad T_L^* = \frac{\rho_f}{2}\int_S (\phi)\left(\frac{\partial \phi}{\partial r}\right) dS, \tag{17}$$

$$[K_2]\{q_1\} - \omega_1^2 [M_2]\{q_1\} = 0 \tag{18}$$

where, ω_2, $[K_2]$, and $[M_2]$ are natural frequency, and stiffness and mass matrices of the system, respectively, and other parameters were defined previously.

3. Validation

The approach to analytical formulation described in Section 2 is validated against experimental studies conducted in the past. Table 1 describes the experimental data of Tani et al. [22] for two concentric cylinders (clamped at the bottom) filled with water in the annulus. Parameters R_i, R_o, t, L, E, ν, ρ_s, ρ_f and H represent the radius of inner cylinder, radius of outer cylinder, thickness of cylinders, length of cylinders, Young's modulus, Poisson's ratio, density of cylinders, density of fluid and height of fluid, respectively. Table 2 presents experimental and analytically obtained natural frequencies for different bulging modes. Parameters n and m represent the wavenumber in the tangential direction and the number of nodal lines in the axial direction [23], respectively. These analytical results are generated considering only the bulging mode. The analytical frequencies compare well with the experimental observations.

Table 1 – Material and geometrical properties of the system considered by Tani et al. [22]

Case	R_i (mm)	R_o (mm)	t (mm)	L (mm)	E (GPa)	ν	ρ_s (kg/m³)	ρ_f (kg/m³)	H (mm)
1	100	120	0.25	197.5	5.56	0.3	1410	1000	49.37
2	100	120	0.25	197.5	5.56	0.3	1410	1000	98.75
3	100	107	0.25	197.5	5.56	0.3	1410	1000	98.75
4	100	150	0.25	197.5	5.56	0.3	1410	1000	98.75

Table 2 – Comparison of bulging frequency in the system considered by Tani et al. [22]

Case	**Mode (*n m*)**	**Experimental (Hz)**	**Analytical (Hz)**	**Error (%)**
1	(6 1) [1]	99.0	102.2	-3.2
1	(6 1) [2]	110.3	109.8	0.5
1	(8 2) [1]	150.0	166.8	-11.2
2	(9 1) [1]	29.3	32.4	-10.6
2	(9 2) [1]	73.7	74.5	-1.09
2	(8 1) [2]	39.3	44.5	-13.2
3	(10 1) [1]	26.2	29.9	-14.1
3	(10 2) [1]	49.3	56.5	-14.6
3	(8 1) [2]	65.7	67.4	-2.6
3	(11 2) [2]	146.5	148.2	-1.2
4	(10 1) [1]	32.5	28.7	11.7
4	(10 2) [1]	85.5	95.9	-12.2
4	(8 1) [2]	31.2	40.5	-29.8

[1] This mode is characterized by the out of phase vibration of inner and outer cylinder

[2] This mode is characterized by the in-phase vibration of inner and outer cylinder

Table 3 presents the experimental data reported by Chiba et al. [24] for two concentric cylinders clamped at the bottom with water filled in the annulus. Parameters t_i and t_o represent the thickness of inner and outer cylinders, respectively, and other parameters were defined previously. Table 4 presents a comparison of the experimental and analytical frequencies for first circumferential (bulging) mode of the system. The analytical results are generated considering only the bulging mode.

Table 3 – Material and geometrical properties considered by Chiba et al. [24]

Case	R_i **(mm)**	R_o **(mm)**	t_i **(mm)**	t_o **(mm)**	L **(mm)**	E **(GPa)**	ν	ρ_s **(kg/m³)**	ρ_f **(kg/m³)**	H **(mm)**
1	150	400	10	2	850	3.32	0.30	1187	1000	400
2	150	400	10	2	850	3.32	0.30	1187	1000	600
3	150	400	10	2	850	3.32	0.30	1187	1000	800

Table 4 – Natural frequencies for first circumferential mode [24]

Case	**Experimental (Hz)**	**Analytical (Hz)**	**Error (%)**
1	54.40	50.00	8.10
2	35.30	33.20	5.90
3	24.30	23.80	2.10

Table 5 presents the experimental data reported by Fujita et al. [25] for concentric cylinders (clamped at the bottom) filled with water in the annulus. It also presents analytical results generated considering only the sloshing mode. Experimental and analytically obtained sloshing frequencies corresponding to the first circumferential mode of the system compare well.

Table 5 – Comparison of sloshing frequency for first circumferential mode in the system considered by Fujita et al. [25]

Case	R_i (mm)	R_o (mm)	ρ_f (kg/m^3)	H (mm)	Experimental			Analytical		
					Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
1	825	880	1000	745	0.43	-	-	0.45	3.77	5.33
2	230	880	1000	1125	0.66	1.17	1.59	0.67	1.19	1.59

4. Parametric study

4.1 Parameters of the system

A parametric study on the representative system shown in Fig. 1(b) is carried out to understand the influence of density of fluid and annular gap ratio (ratio of the radii of main and inner vessels) on the dynamic properties of the system. Six cases are considered for the study: two values of density of fluid combined with three values of annular gap ratio. The radii of the control plug and inner vessel are set equal to 1.29 m and 5.00 m, respectively, and the heights of the control plug, inner vessel, main vessel and fluid are set equal to 7.84 m, 13.00 m, 15.00 m and 12.32 m, respectively. The thicknesses of the control plug, inner vessel and main vessel are set equal to 0.015 m, 0.030 m and 0.035 m, respectively. These ratios are realized by varying the radius of main vessel. Two fluids are considered as coolant: liquid sodium and liquid bismuth. Densities of the two fluids are 900 kg/m^3 (e.g., [26]) and 9,854 kg/m^3 (e.g., [27]), respectively. Properties of fluids are assumed independent of temperature. Cases 1, 2 and 3 of parametric study correspond to liquid sodium and annular gap ratios of 1.3, 1.5 and 1.7, respectively. Cases 4, 5 and 6 of the study correspond to the three values of annular gap ratios, respectively, and liquid bismuth.

4.2 Natural frequency of the system

Table 6 presents the bulging and sloshing frequencies for Case 1 (annular gap ratio of 1.3, liquid sodium). Bulging frequencies obtained by considering only the bulging modes (A) are marginally lower than those obtained by considering the coupling between bulging and sloshing modes (B). Sloshing frequencies obtained by considering only the sloshing modes (C) are identical to those obtained by considering the coupling between bulging and sloshing modes (B). A similar trend is observed for other cases 2 through 6.

4.3 Results

Fig. 4(a) presents the bulging frequency (considering bulging and sloshing mode) for an annular gap ratio of 1.3 (Case 1). The frequency is smaller for a denser fluid and/or a smaller annular gap ratio. This observation is true for other annular gap ratios (see Figs. 4(b) and 4(c)) as well. Similar observations were made by Tani et al. [22] and Chiba et al. [24] for a system of two concentric cylinders.

Panels (a), (b) and (c) of Fig. 5 present sloshing frequencies of the system. The first mode sloshing frequency associated with the outer annulus decreases marginally with an increase in annular gap ratio. However, the frequency corresponding to the second sloshing mode reduces considerably with an increase in the ratio. A similar trend was observed by Fujita et al. [25] for coaxial cylinders filled with fluid in the annulus.

5. Summary and conclusions

This paper presents an analytical formulation to study the vibration characteristics of a fluid-shell system representing a pool-type fast reactor. The reactor is idealized as three concentric cylinders separated by fluids. The cylinders are modeled as shells, while the fluid is modeled as potential fluid. The coupling between bulging and sloshing modes is considered. The approach to analytical formulation was validated against available experimental results. A parametric study was carried out to understand the effect of the density of the fluid and the ratio of the radii of outermost and inner cylinders (main and inner vessels, respectively) on the dynamic properties of the system. The influence of considering coupling between the bulging and sloshing modes on the determination of bulging and sloshing frequencies is small. Therefore, bulging and sloshing modes can be treated independently. The bulging frequency decreases with an increase in density of fluid or a reduction in the ratio of the radii of outermost and inner cylinders. First mode sloshing frequency is not considerably affected by the parameters considered. However, the second mode sloshing frequency was found considerably smaller for a greater ratio of radii of main and inner vessels.

Table 6 – Bulging and sloshing frequencies (Hz) for Case 1

Mode (n m)	***Bulging mode***		***Mode (n m)***	***Sloshing mode (Inner vessel)***		***Sloshing mode (Outer annulus)***	
	A[3]	***B***[3]		***C***[3]	***B***[3]	***C***[3]	***B***[3]
(1 1)[1]	4.905	5.082	(1 1)	0.292	0.290	0.206	0.206
(1 1)[2]	10.454	10.680	(1 2)	0.499	0.499	0.736	0.736
(2 1)[1]	3.219	3.302	(2 1)	0.386	0.386	0.295	0.295
(2 1)[2]	4.824	4.887	(2 2)	0.561	0.561	0.740	0.740
(3 1)[1]	2.295	2.343	(3 1)	0.456	0.456	0.361	0.361
(3 1)[2]	3.070	3.119	(3 2)	0.624	0.624	0.745	0.745
(4 1)[1]	1.998	2.025	(4 1)	0.514	0.514	0.417	0.417
(4 1)[2]	2.545	2.571	(4 2)	0.677	0.677	0.754	0.753

[1] Out-of-phase vibration of control plug and inner vessel, and out-of-phase vibration of inner and main vessels
[2] Out-of-phase vibration of control plug and inner vessel, and the in-phase vibration of inner and main vessels
[3] *A*: Only bulging mode considered; *B*: Bulging and sloshing considered; *C*: Only sloshing mode considered

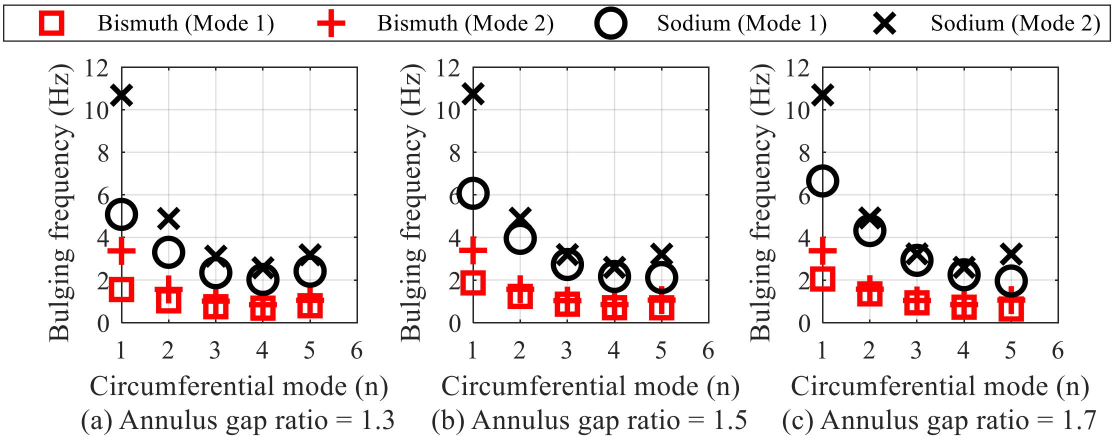


Fig. 4 – Effect of density and annular gap ratio on the bulging frequencies of the system (modes 1 and 2 are defined in Table 6)

6. Acknowledgements

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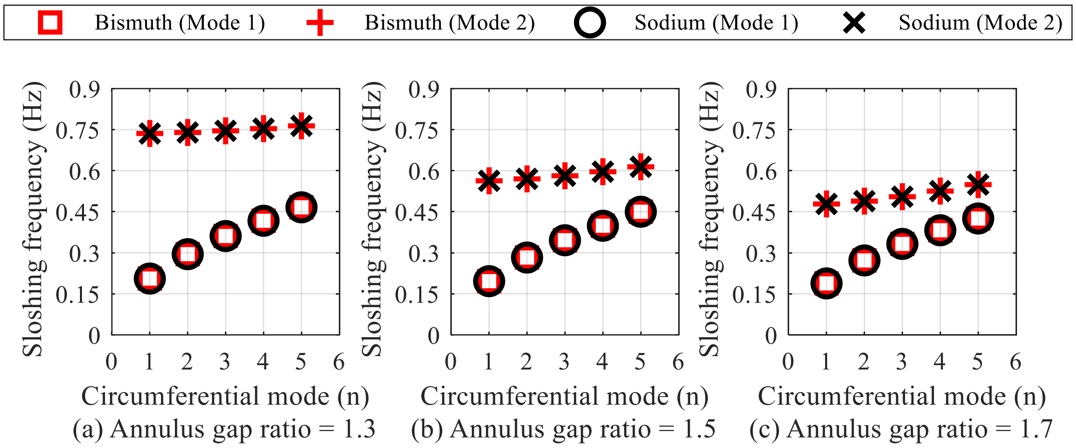

Fig. 5 – Effect of density and annular gap ratio on the sloshing frequencies of the system

7. Copyrights

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