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BRB SYSTEM DESIGN FOR STABILITY

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Abstract

This paper describes the stability of a BRB as part of a frame. A new, and economical, concept is proposed for the BRB system which allows large deformations without buckling within the system. It involves the use of a specified deformation zone (SDZ) at the brace end. An approach to estimate the deformation capacity for such a system is developed as a function of out-of-plane drift and this is illustrated in an example. Furthermore, to provide the desired performance, brace end boundary elements deformations restricted. The approach developed draws on the perfect-pin concept, but elaborates it further to ensure that the system, rather than just the brace, is likely to perform well.

It is shown that, similar to a beam under lateral loading, three moment-releases (hinges) are required to cause a mechanism. In addition, for the example shown using the new concept developed, monotonic out-of-plane interstorey drifts of 2% may be obtained with a SDZ strain of less than 0.5%, slenderness ratio of 2, and axial force ratio of 0.30. The boundary element restraint is relatively easy to provide when the gusset plate has edge stiffeners and a reinforced concrete floor slab wraps around the column. Finally, it is emphasised that if designers cannot guarantee the performance of a BRB system under bidirectional loading, they should not be recommending this particular system.

Keywords: Buckling; Stability; Buckling Restrained Braces; Out-of-Plane Deformation

Introduction

While Buckling Restrained Braces (BRBs) have become well popular around the world in seismic resisting frames as they add stiffness, and dissipate significant energy under both compression and tension loading. Such braces performed well in some experimental tests. However, concerns exist about the implementation of BRBs into frame systems for a number of reasons (MacRae 2014 [10], MacRae and Clifton 2015 [11], MacRae 2016 [12], Hariri and Tremblay 2019 [7]). In addition, a variety of different failure types have been reported (Palmer et al. 2016 [16] , Sitler et al. 2017 [17]). Two issues relating to the current research are:

(i) Gusset plate design, where Westeneng et al. (2017) [21] showed that current design techniques proposed by AISC may be non-conservative. This was already earlier recognised by Japanese and Taiwanese researchers who have alternative/additional requirements to those of AISC for gusset plate buckling length, and they require gusset plate edge stiffeners. Most Western countries follow AISC design procedures directly and do not use gusset plate edge stiffeners because of their extra cost;

(ii) The performance under earthquake shaking causing out-of-plane deformations. This is because the most common test protocols for evaluating the acceptability of a brace require in-plane testing only. This is in spite of the fact that an earthquake can cause out-of-plane deformation and reduction in BRB performance (e.g. Cui et al., 2019 [3,4]). Both a low, and a high, end stiffness may be problematic in terms of performance. If the BRB end region is stiff, then yielding during out-of-plane displacements may compromise the BRB ability to perform well under subsequent in-plane displacements. If the end region and gusset plate at the end of the BRB is very flexible, then buckling may occur due to in-plane effects.

The issues described above are not due to the BRB itself, but they relate to the performance of the BRB system. As a result, BRB manufacturing companies have not spent much effort to address these issues. Some of these issues have been previously raised (Ozaki et al. 2014 [15], Takeuchi et al. 2014 [18]) and specific design guidance has been developed in Japanese (Takeuchi et al. 2017 [19]). However, the procedures are not simple to apply in practice. Zaboli et al. (2017) [22] have made a proposal where they use a force magnification to design the gusset plate, but this may restrict the use of some systems where yielding is not a problem. Furthermore, none of the current proposals for BRB system design explicitly consider (i) the need for column rotational restraint, which is often necessary for good behaviour, and (ii) the specific zone of permitted yielding. If yielding occurs in an undesirable location it may result in poor subsequent brace behaviour. Furthermore, if it occurs at a strong section of the brace, then large moments enter the BRB itself resulting in reduced cumulative displacement capacity and an increase in compressive strength (Cui et al, 2019) [3].

A smart method to mitigate the out-of-plane deformation effects on a BRB itself is to place a perfect pin/hinge at the end of the BRB (Bruneau and Wei, 2017) [2], but no explicit design considerations are available regarding the performance of a BRB system containing such pins, especially considering boundary element stiffness.

The lack of agreement on the best approach to design BRBs in NZ has contributed to the cancellation of the development of national design recommendations for BRB systems in New Zealand, and also to a reduction in the use of BRBs in NZ structures.

It may be seen from the discussion above that for BRBs to behave well under both in-plane and out-of-plane loading, appropriate design procedures are needed. This paper seeks to address this need by seeking answers to the following questions:

a) What is required for a BRB frame system under in-plane loading to undergo large out-of-plane drifts without out-of-plane drift damage or a buckling failure?

b) Can a new concept be developed to allow the system to carry large bi-directional horizontal drifts?

c) Can design methods be developed?

d) What is the out-of-plane deformation capacity of such a system?

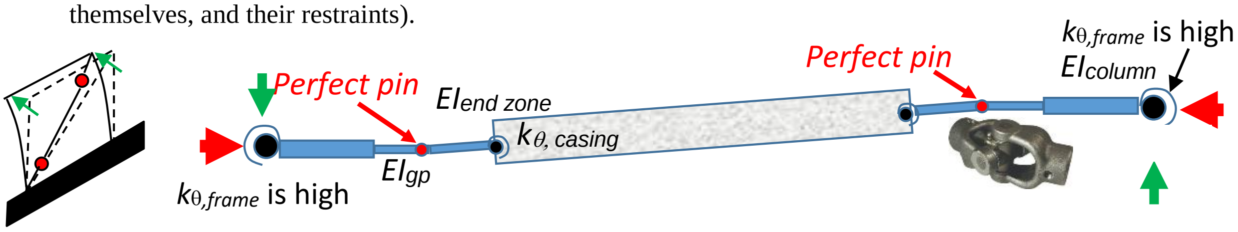
BRB deformation and buckling

As a BRB frame undergoes out-of-plane deformation, and one or both ends have some restraint against rotation, then the BRB with its end zones, and gusset plate as well as the column are subject to bending. Such bending can cause yielding somewhere within the system, and the possibility of poor behaviour during in-plane and out-of-plane cyclic loading.

By placing perfect pins at the end of the BRB itself, such as suggested by Bruneau and Wei (2017) [3], the BRB can remain straight during out-of-plane deformation as shown in Figure 1. However, this behaviour is only possible if the boundary elements are relatively stiff and do not buckle under the compressive axial force applied to it by the BRB. In addition, the end elements must be strong enough to resist the bending caused by the compressive axial force in the BRB. Analysis of such a system involves assessing the stability of:

a) The region between the pins (i.e. the BRB with its casing and end regions) using a methodology similar to that developed by Alizadeh (2018) [1], and

b) The region at each end of the system outside the hinges (containing the gusset plates, beam/column themselves, and their restraints).



(a) Overall Frame (b) Plan of BRB in Frame

Figure 1. Out-of-Plane Frame Deformation of Idealised BRB with End Pins and Rigid Boundaries

It may be noted that if there are no perfect pins, but the boundary elements (gusset plate, beam or column) are flexible (i.e. $k_{\theta,frame} = 0$) as shown in Figure 2, out-of-plane deformation may occur without compromising the ability of the BRB to carry axial force. However, flexible boundary elements are not generally desirable in practice, because other modes of failure, such as column twisting, may occur.

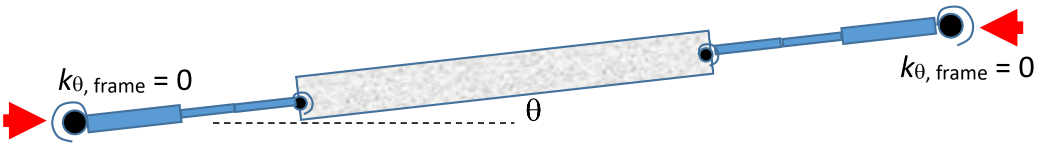


Figure 2. Out-of-Plane Frame Deformation of Idealised BRB with Flexible Boundaries

If out-of-plane deformation causes either of the sway modes shown in Figure 1 or 2, then the axial strength of the BRB remains the same as the in-plane case. However, the frame in-plane horizontal force capacity, due to the BRB resistance only, decreases. The frame in-plane horizontal force capacity is cosine θ times that of the brace if it remained in-plane, where θ is the angle shown in Figure 2. Since out-of-plane drifts are normally small, cosine θ is close to unity, and there is little strength reduction.

However, if the boundary elements are not rigid (e.g. $k_{\theta,frame} = 0$) and pins exist, then zero force can be carried by the system as it buckles. A minimum of three hinges are required for a flexural mechanism as per normal beams.

Takeuchi (2017) [19], Westeneng et al. (2017) [21] and Zaboli et al. (2018) [22] have developed, or partly developed design/assessment procedures that consider brace system stability. Unfortunately, the solutions obtained from such methods are not simple, and designers do not like to use a black box as part of their design.

Proposed BRB System Design Concept

The proposed concept is based on Bruneau and Wei (2017) [2]. Here perfect pins are at the BRB ends as shown in Figure 1. This allows lateral movement, but the system is stable. It also puts no moment into the BRB itself. However, simply using perfect pins may not be suitable for the following reasons:

1) A lateral stability failure may occur if the boundary elements (gusset plate, column/beam. and their restraints) have low stiffness, as shown in Figure 3.

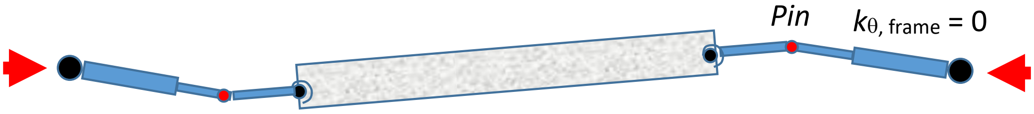


Figure 3. Out-of-Plane Frame Deformation of Idealised BRB with End Pins and Flexible Boundaries

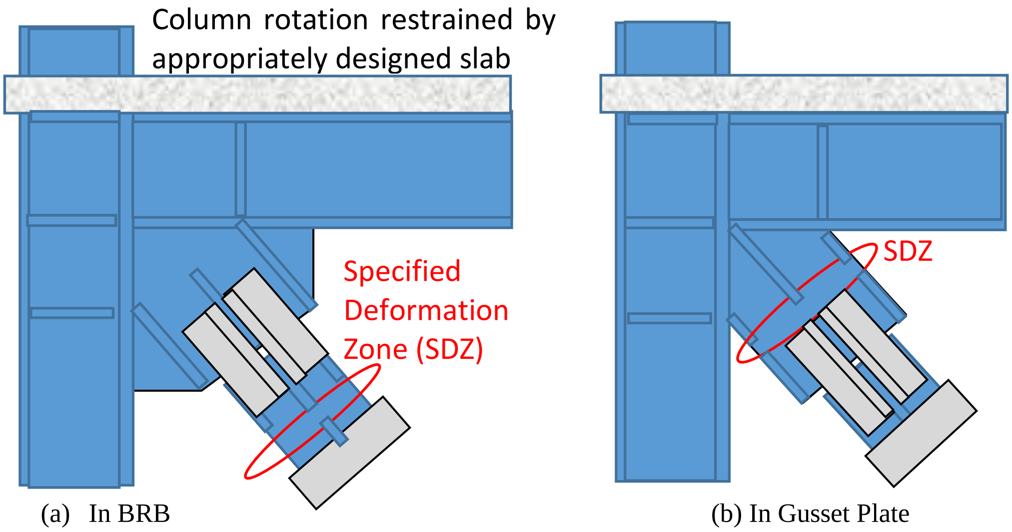
2) Perfect pins are expensive.

If perfect pins are used, both (i) boundary element out-of-plane deformations under the proposed loading, and (ii) BRB buckling between pin locations, must be considered.

In addition, to reduce construction cost, instead of using perfect pins/hinges, *specified deformation zones*, SDZ, may involve potential plastic hinge zones located in steel plate at both ends of the brace. SDZs should:

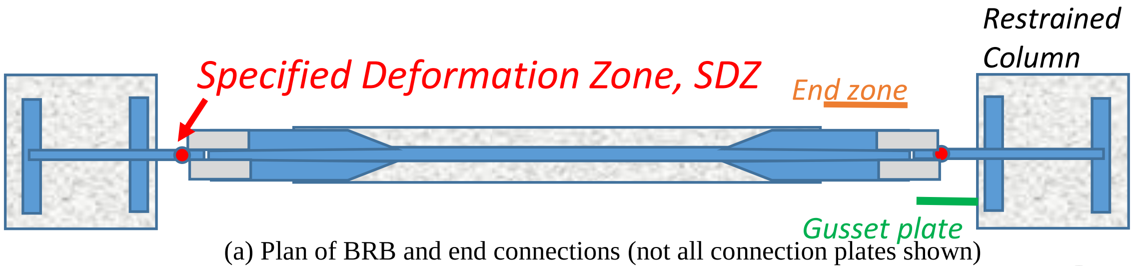
1) easily carry the direct axial force from in-plane loading to the brace,
2) be short enough to not buckle over their length,
3) be long enough to avoid excessive strain demands during expected out-of-plane deformations, and
4) be weak in flexure to limit moments which affect the BRB and the brace-column joint.

Simple details are given in Figure 4 with the SDZ placed outside the BRB end zone. It may be within the brace itself as shown in Figure 4a, or within the gusset plate, as shown in Figure 4b. Variations on this are also possible, for example a perpendicular plate may be welded on the BRB side of the SDZ. Having the SDZ in the gusset plate may provide the most versatility to (i) control the SDZ length due to more stiffener placement options, and (ii) to change the gusset plate thickness and width. However, the option that most economically delivers the desired performance is best. For simplicity, further discussion in this paper concentrates on the gusset plate SDZ shown in Figure 4b. A plan of such a member is given in Figure 5.

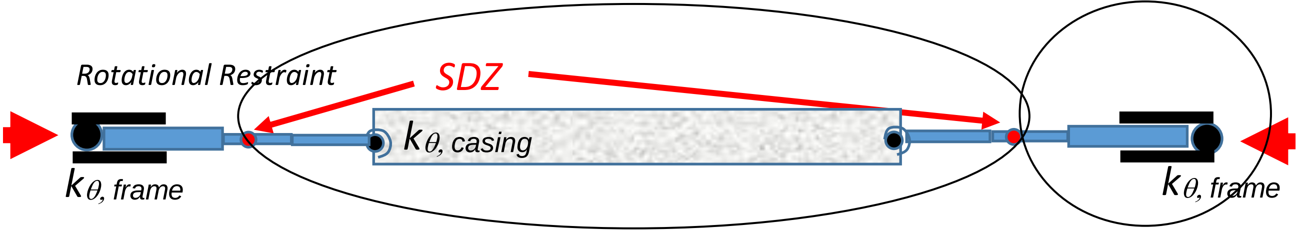


(a) In BRB (b) In Gusset Plate

Figure 4. Specified Deformation Zones (SDZs) for Out-of-Plane Bending



(a) Plan of BRB and end connections (not all connection plates shown)



(b) Simple Model

Figure 5. Simple model with specified deformation zones and k_θ frame → ∞ (not all plates shown)

Design Considerations

a) Specified Deformation Zone (SDZ) strain considerations:

Strains in the SDZ result from axial force, flexural, and cyclic effects. The strain due to axial force, ε_{axial}, needs to be limited to a fraction, say α, of the yield strain, ε_y. The monotonic flexural strain, $\varepsilon_{flexural}$, may be computed from the curvature over the SDZ, ϕ, which is related to the maximum drift causing strains in the SDZ, θ_{SDZoop}. This drift is less than the total out-of-plane drift, θ_{oop}, due to elastic deformation in other elements of the frame. The strain associated with this is $\varepsilon_{flexural} = \phi \, . \, (t/2) = \theta_{SDZoop} \, / \, l_{SDZ} \, . \, (t/2)$, where l_{SDZ} is the length of the SDZ, and t is the thickness of the SDZ plate. The cyclic axial strain increases rapidly for a member subject to repeated yielding flexural strains in both directions of loading for an element in constant tension or compression (MacRae et al. 2009) [9]. For a typical frame, the out-of-plane bending is unlikely to be simply correlated with brace tension or compression loading. Because of this, very large increases in strain may not always be expected. However, it is still prudent to limit the total SDZ strain, $\varepsilon_{SDC,monotonic} = \varepsilon_{axial} + \varepsilon_{flexural}$, to a small level. This can be achieved by providing a large l_{SDZ}, as long as l_{SDZ} is not so large that buckling occurs. A maximum monotonic strain capacity of 1.5% is tentatively suggested as a reasonable limit until better information becomes available.

For example, if θ_{oop} = 2%, θ_{SDZoop} = 1.5%, $\varepsilon_y = 0.0015$, $\alpha = 0.30$ say, and aspect ratio, l_{SDZ}/t, is 2 say, then

$$\begin{array}{lll} \varepsilon_{SDC,monotonic} & = \alpha\varepsilon_y & + \theta_{SDZoop}.\,(t/2)/l_{SDZ} \\ & = 0.30 \text{ x } 0.0015 & + 0.015 \text{ x } (1/2)\,/2 \\ & = 0.00045 & + 0.00375 \\ & = 0.00420 & = 2.8\varepsilon_y \end{array}$$

Here, since $\varepsilon_{SDC,monotonic}$ = 0.420% is less than 1.5%, so this is satisfactory.

b) Specified Deformation Zone (SDZ) local buckling

Local buckling may occur within the SDZ if its aspect ratio is too high. Therefore, l_{SDZ}/t should not be too large. Values of up to 5 have been recommended for CBFs with buckling braces. Until further information is available, it is suggested that this limit also not be exceeded for BRBs.

c) Effect of Moment on Brace Performance

Out-of-plane moment generally reduces BRB cumulative ductility capacity under cyclic loading and increases the compressive force (Cui et al, 2019) [3]. The maximum out-of-plane moment applied to the BRB is the SDZ flexural capacity under no axial force. For BRB systems with no specified SDZ it may be difficult to estimate the maximum out-of-plane moment demand on the BRB. Using a perfect pin, or a plate SDZ, keeps the BRB moments small mitigating undesirable behaviour. Engineers should consider the sensitivity of BRB axial strength and deformation capacity to out-of-plane moments as part of a responsible design.

Capacity design principles should be used to ensure that the brace and gusset plate flexural strengths away from the SDZ are significantly high that large/inelastic deformations only occur in the SDZ.

d) Brace Stability

The brace between the SDZs shown in Figure 5 must be designed against buckling. Until a more definitive approach is available, it is suggested that the maximum axial force be less than $F_e/3.5$ according to the NZ Steel Standard (NZS3404:2009), where F_e is the lowest elastic buckling force, to account for the many uncertainties associated with the residual stresses, out-of-straightness, member/material overstrength, *M-P* effects on the axial strength, and dynamic effects.

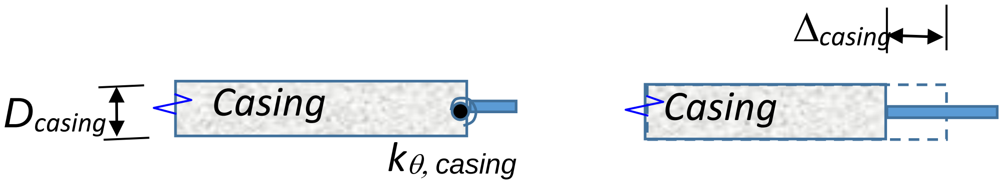
In order to conduct the buckling analysis, it is necessary to know the flexural stiffness of the:

(i) casing,
(ii) end of casing, and
(iii) end zone.

The casing end minimum rotational stiffness, $k_{\theta,\, casing}$ in Figure 6a, resulting in the lowest buckling load, should be obtained experimentally. This may be obtained by a three-point loading test, where a BRB is simply supported (providing support transverse to the direction of the member axis) at one end of the casing as well as near the far connection of the BRB. Transverse loading is applied to the BRB near the casing end zone close to the supported connection, for example. The displacement obtained at the loading position may be compared to a model with the connection end properties, and the cased section properties, as well as a rotational spring at the casing end, $k_{\theta,\, casing}$. Since the connection end properties and cased section properties may be estimated, the value $k_{\theta,\, casing}$ to obtain the same displacement may be estimated. This testing should all be conducted with the brace is in its most extended position to obtain the most critical (i.e. lowest) $k_{\theta,\, casing}$ values. The stability analysis may follow Westeneng et al. (2017) [21]. For the stability analysis also, the brace should be considered at its maximum extension because longer members have lower buckling strengths.

A simplification to this may be obtained by replacing the rotational stiffness at the end of the casing, $k_{\theta,\, casing}$, by a shorter length of casing as shown in Figure 6b. This is discussed by Cui (2020) [4]. In this case, only two stiffnesses and two lengths need to be considered, and these are those of the BRB casing and the end zone. This considers lateral, as well as rotational, flexibility at each end of the casing. As a first estimate, the casing diameter, D_{casing},, is considered to be a reasonable estimate of the casing shortening on each end until better information is available. That is Δ_{casing} =

D_{casing} in Figure 6. The lowest buckling mode may be found using stability functions, and then simplified for design, with techniques similar to that of Alizadeh et al. (2018) [1].



(a) Rotational spring (b) Reduced casing length

Figure 6. Modelling Casing End Connection

The brace between the SDZs in Figure 5, under axial force can deform in both a (1) symmetric, and an 2) anti-symmetric, buckling modes, as shown in Figure 7. Based on simple column buckling theory and stability function concepts, the anti-symmetric buckling mode results in higher critical axial loads (Alizadeh, 2019) [1]. Therefore, only the symmetric buckling mode is further considered. Equation 1 estimates the first mode elastic buckling load, P_{cr1}, as there is no closed-form solution. Here EI_{casing} is the effective flexural stiffness of the cased zone (often estimated as that of the casing alone) for the material considered; $EI_{endzone}$ is the effective flexural stiffness of the endzone; L_{casing} is the effective length of casing considering the effect of the reduced casing length as shown in Figure 6; $L_{endzone}$ is the effective length of endzone where the brace is considered to be at its maximum extension; $L = L_{casing} + 2{*}L_{endzone}$ is the total member length. The empirical equation for P_{cr1} has been developed over the practical range of parameters when L_{casing}/L is in the range 0 to 1, and $EI_{endzone}/EI_{casing}$ is in the range 0 to 1. It is accurate and generally conservative. It may also be noted that as L_{casing} tends to:

- zero (i.e. $L_{casing} \rightarrow 0$), then $\alpha \rightarrow r = EI_{endzone}/EI_{casing}$, and P_{cr1} tends to the Euler value for $EI_{endzone}$ over the total member length L, and
- the full member length (i.e. $L_{endzone} \rightarrow 0$), then $\alpha \rightarrow 1$ and P_{cr1} tends to the Euler value for EI_{casing} over the total member length (which is L_{casing} in this case).

It should be noted that when $EI_{endzone} = EI_{casing}$, i.e. $r = 1$, then the buckling axial force differs slightly from the expected condition of the Euler buckling load for a uniform member. However, the error in P_{cr1} ranges between -3.7% and 1.3% within the range of s, which is considered negligible.

$$P_{cr1} = \alpha * \pi^2 EI_{casing}/L^2 \tag{1a}$$
$$\alpha = \alpha_0 * f \tag{1b}$$

$$\alpha_0 = r/(r{*}s^2 + (1-s)^2 + 2.4{*}r{*}s{*}(1-s)) \tag{1c}$$
$$f = 2/(1+\sqrt{(1-\theta)}) \tag{1d}$$

$$\theta = 2.3{*}r{*}s{*}(1-s){*}(1-s^2)/(r{*}s^2+(1-s)^2+3{*}r{*}s{*}(1-s))^2 \tag{1e}$$
$$r = EI_{endzone}/EI_{casing} \tag{1f}$$
$$s = L_{casing}/L \tag{1g}$$

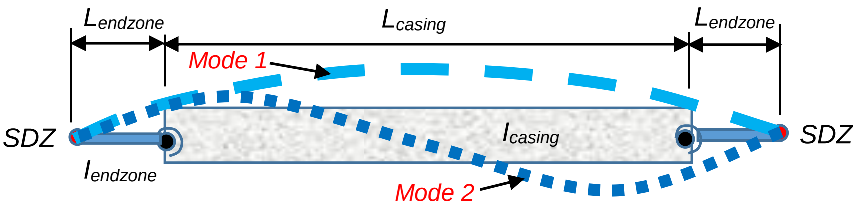


Figure 7. Buckling modes of the brace between the SDZs

e) Boundary Element Stability

The boundary element (connection and beam/column area) should be stiff and not buckle when the SDZ is subject to high axial force. Special methods may be provided to do this as shown by MacRae et al. (2020a) [13] but these are beyond the scope of this paper.

The proposal described above uses the deformation zone general concept developed previously for concentrically braced frame (CBF) gusset plates. However there are differences in the BRB frames and CBFs both behaviour and detailing.

(i) Behaviour: Flexural yielding in gusset plates of BRB frames occurs due to out-of-plane deformations. For CBFs yielding may also occur at the centre of the braces and in the gusset plates as the braces buckle out-of-plane under large compressive deformations.

(ii) Detailing of deformation zone:

The detailing is proposed in this paper for BRB gusset plates is more explicit (including consideration of the SDZ buckling, SDZ inelastic demands, and location of the SDZ) than it is for BRBs.

There are also some requirement similarities for both BRB frames and for CBFs. Both require:

(i) the strength hierarchy of brace elements must be sufficient to prevent yielding outside the SDZ, and

(ii) sufficient boundary element stiffness to prevent an overall buckling mechanism.

However, explicit checks are seldom undertaken to prevent undesirable behaviour related to both these issues even in standard CBF design. The concepts described in this paper therefore provide a better understanding of behaviour from which CBF, as well as BRB frame, construction can benefit.

Conclusions

This paper describes the behaviour of Buckling Restrained Braces (BRBs) within frames considering out-of-plane deformations and stability. It is shown that:

1) For a BRB frame with significant boundary element (gusset plate/beam/column) out-of-plane restraint, a rotational hinge is required at each end of the BRB member to accommodate out-of-plane lateral deflection without causing stress in the member. Three rotational hinges within the member cause a mechanism.

2) A new concept is proposed for the design of BRBs. The concept, which builds on similar ideas for CBFs, allows large frame out-of-plane deformations without a buckling failure at the BRB member end. A rotational hinge is provided at each end of the BRB member (a perfect hinge, or a weak plastic hinge referred to as a specified deformation zone (SDZ)) outside the BRB end region. The moment capacity of the SDZ is small, so moments applied to the brace are small. This means that the brace’s cyclic deformation capacity should not decrease nor axial compression strength increase as a result of out-of-plane deformations. Capacity design can ensure that the BRB end zone remains elastic so that the BRB has repeatable in-plane performance. Boundary elements (beams-columns and gusset plates) are designed to provide sufficient lateral stiffness at the locations of the pins/hinges to prevent instability.

3) Details for a SDZ outside the BRB end-region are provided. Checks to ensure that buckling does not occur (i) within the brace and end regions between the locations of pins/hinges, or (ii) at the boundary elements, are provided. Specific guidance is given for the case where lateral restraint of the beam top flange and column is provided by a slab.

4) An approximate relationship between BRB frame system out-of-plane deformation, and strain in a rectangular plate plastic hinge SDZ at the end of a brace, is provided. Brace stability checks are critical when the in-plane deformation is maximum. In addition, the moment affecting the BRB is maximum when the in-plane forces are zero.

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