OPTIMUM PARAMETERS AND LOCATION OF MULTIPLE TUNED MASS DAMPERS UNDER SEISMIC EXCITATION

H. Cetin[(1)], E. Aydin[(2)], B. Ozturk[(3)]

[(1)] *Ph.D., Nigde Omer Halisdemir University, Department of Civil Engineering, henainsaat@gmail.com*
[(2)] *Associate Professor, Nigde Omer Halisdemir University, Department of Civil Engineering, eaydin@ohu.edu.tr*
[(3)] *Associate Professor, Hacettepe University, Department of Civil Engineering, bakiozturk@hacettepe.edu.tr*

Abstract

In this study, a proposed method considering the parameters of multiple tuned mass dampers (MTMDs) and location of MTMDs on a one-story single degree of freedom (SDOF) structure is presented. The equation of motion evaluated in the frequency domain and the seismic input for design is considered as white noise excitation in a particular critical frequency range interval. In the first step, tuned mass dampers (TMDs) are located on the structure. By using random vibration theory, the mean square displacement is minimized via differential evolution (DE) algorithm in this range under the specified constraints. If the mass or stiffness of the specified TMD is almost zero, it means that this TMD is eliminated so that the optimum location of MTMDs is executed. Thanks to the mass quantity optimization, the quantity of TMD on the floor is attained. After all, designed and located MTMDs performance is tested in time domain under seismic excitation. The results have shown that the proposed method is successful in reducing the response of structure under seismic excitation.

Keywords: Multiple tuned mass dampers, tuned mass damper location, frequency domain, differential evolution

1. Introduction

Classical design in structures under dynamic effects such as earthquakes and wind loads try to absorb the energy with internal force-deformation behavior. The energy is damped by nonlinear behavior and local damage, after the building material reaches its yielding limit under earthquake excitation or the other dynamic external influences. In the classical and modern periods of earthquake-resistant structural design, the structure consumes energy by making deformations against big external effects. However; in the modern period active, passive, semi-active and mixed control systems are added to the structure and they are energy absorbing technological elements. It is known that correctly designed tuned mass damper (TMD), which is one of the passive control elements, reduces structural vibrations under dynamic external forces such as earthquakes and wind. These dampers have been extremely effective in preventing damage or collapse of vibration-affected structures. Passive TMDs are usually adjusted to a specific mode which is especially the first mode of structure [1-4]. TMDs are usually more effective, when they are located on the top floor of the structure [5-7]. Adjustable mass dampers reduce the amount of hysterically damped energy of the system. The amount of this damped energy in the system is directly related to the damage of the structure. Therefore, TMDs will be effective in protecting nonlinear structures under dynamic effects [8-11]. Furthermore, if TMDs are designed correctly, they can reduce both structural and non-structural earthquake damage [12, 13] so that they can be effectively used to reduce both earthquake and wind effects. Several researchers have investigated both the TMDs and multiple tuned mass dampers (MTMDs) considering their effects on the structures [14-19].

The optimum parameters of MTMDs and their placement on a one-story single degree of freedom (SDOF) structure are investigated in this study. It is suggested that two pieces of single TMDs are placed on the one-story SDOF structure for this purpose. The seismic input is defined as a band-limited white noise excitation in the frequency domain of the structure. Taking into consideration the boundary condition, Differential Evolution (DE) algorithm is executed to minimize the objective function which is the mean square of displacement (σ_D^2). If the results of TMD mass or stiffness quantities are almost equal to zero on the floor, the TMD is cancelled from the floor since it becomes unable to reduce the objective function. After this optimization, the effectiveness of designed MTMD is compared with the structure without structural control. It is observed that MTMD is effective in decreasing the displacement and acceleration transfer functions of the structure. Additionally, MTMD is also effective in reducing the displacement and acceleration responses under seismic excitation in the time domain.

2. Building Model with TMDs

MTMD system placed on a shear frame is shown in Figure 1. k_{d1j} , m_{d1j}, and c_{d1j} represent stiffness, mass and damping coefficients of j^th TMD placed on the SDOF system, respectively. The power spectral density (PSD) of the seismic input acceleration $\ddot{x}_g$ is evaluated as a white-noise excitation. $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are the displacement, velocity and acceleration vectors of the shear building with MTMD, respectively. The equation of motion of the structure-MTMD system can be expressed as,

$$(\boldsymbol{M}_s + \boldsymbol{M}_{TMD})\, \ddot{\boldsymbol{x}}(t) + (\boldsymbol{C}_s + \boldsymbol{C}_{TMD})\, \dot{\boldsymbol{x}}(t) + (\boldsymbol{K}_s + \boldsymbol{K}_{TMD})\, \boldsymbol{x}(t) = -(\boldsymbol{M}_s + \boldsymbol{M}_{TMD})\boldsymbol{r}\ddot{x}_g(t) \quad (1)$$

in which $\boldsymbol{C}_s$, $\boldsymbol{M}_s$ and $\boldsymbol{K}_s$ denote the damping, mass and stiffness matrices of the structure without MTMD and $\boldsymbol{r} = \{1,1,1,\dots\}^T$ is the influence vector. $\boldsymbol{C}_{TMD}$, $\boldsymbol{K}_{TMD}$ and $\boldsymbol{M}_{TMD}$ are the damping, stiffness and mass matrices of MTMD which can be written as given below.

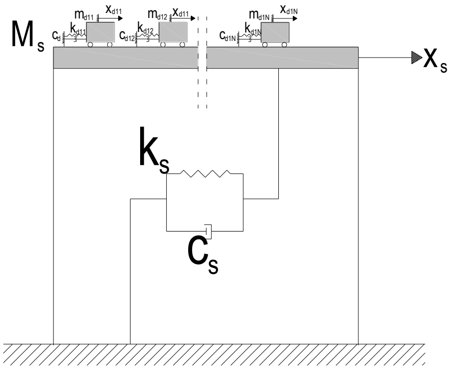

Fig. 1 - Multiple tuned mass dampers placed on a shear frame

Fourier Transform of Equation 1 given above can be written as follows.

$$(\boldsymbol{K} + \mathrm{i}\omega\boldsymbol{C} - \omega^2\boldsymbol{M})\boldsymbol{X}(\omega) = -\boldsymbol{M}\boldsymbol{r}\ddot{X}_g(\omega) \tag{2}$$

In this equation, $\boldsymbol{M}, \boldsymbol{K}$ and $\boldsymbol{C}$ are the mass, stiffness and damping matrices of the structure-MTMD system. $\boldsymbol{X}(\omega)$ is the Fourier Transform of displacement vector and $\ddot{\boldsymbol{X}}_{\boldsymbol{g}}(\omega)$ is the Fourier Transform of ground acceleration. $\boldsymbol{X}(\omega)$ can be expressed as,

$$\boldsymbol{X}(\omega) = -(\boldsymbol{K} + \mathrm{i}\omega\boldsymbol{C} - \omega^2\boldsymbol{M})^{-1}\boldsymbol{M}\boldsymbol{r}\ddot{\boldsymbol{X}}_{\boldsymbol{g}}(\omega) \tag{3}$$

Let $\boldsymbol{H}_D(\omega)$ be a displacement transfer function which is defined as,

$$\boldsymbol{H}_D(\omega) = -(\boldsymbol{K} + \mathrm{i}\omega\boldsymbol{C} - \omega^2\boldsymbol{M})^{-1}\boldsymbol{M}\boldsymbol{r} \tag{4}$$

Similarly, the transfer function of absolute acceleration $\boldsymbol{H}_A(\omega)$ is given as,

$$\boldsymbol{H}_A(\omega) = (\boldsymbol{1} + \omega^2(\boldsymbol{K} + \mathrm{i}\omega\boldsymbol{C} - \omega^2\boldsymbol{M})^{-1}\boldsymbol{M}\boldsymbol{r}) \tag{5}$$

The mean square response of displacement can be written as follows.

$$\sigma_D^2 = \int_{-\infty}^{\infty} |\boldsymbol{H}_{\boldsymbol{D}}(\omega)|^2 \, S_g(\omega) d\omega \tag{6}$$

In Equation 6, $S_g(\omega)$ shows the power spectral density (PSD) function of ground acceleration, $\ddot{x}_g(t)$.

3. Optimization Problem and the Proposed Method

In this study, mean square of displacement response, σ_D^2, which is obtained from random vibration theory, is chosen as an objective function. This objective function can be given as,

$$f_1(m_{d1j}, c_{d1j}, k_{d1j}) = \sigma_D^2 \qquad (j=1,2,\ldots,N) \tag{7}$$

The upper and lower limit design parameters of the j[th] TMD can be expressed as passive constraints,

$$0 \leq m_{d1j} \leq \bar{m}_d \tag{8}$$

$$0 \leq \sum_i^N m_{1j} \leq \bar{m}_d \tag{9}$$

$$0 \leq c_{d1j} \leq \bar{c}_d \tag{10}$$

$$0 \leq k_{d1j} \leq \bar{k}_d \tag{11}$$

where $\bar{m}_d$, $\bar{c}_d$ and $\bar{k}_d$ are the upper bounds of mass, damping and stiffness coefficients of the jth TMD, respectively. Thanks to the intervals of $0 \leq m_{d1j} \leq \bar{m}_d$ and $0 \leq \sum_i^N m_{dj} \leq \bar{m}_d$, the mass quantity and location of each TMD on the floor is calculated. In this study, the DE algorithm [20] is used to obtain the global solution.

4. Numerical Example

To carry out optimum MTMD design and their optimum location, the proposed method is applied considering peak interval in the transfer function. In order to calculate optimum MTMD parameters to obtain their placement on a SDOF system, the objective function which is mean-square response of floor displacement is minimized by using the DE algorithm. If the mass or stiffness parameters of the floor are almost close to zero, TMD on the floor is eliminated. Thanks to this elimination, the optimum location of MTMD is assigned. In order to understand the performance of this design, the structure-MTMD system is tested under El Centro (NS) earthquake acceleration record and the results are compared with a SDOF system without TMD. The mass, stiffness and damping coefficients of the SDOF system are m_1 =12x10^4 kg, k_1 =2.5x10^7 N/m and c_1 =69.282 x10^7 Ns/m, respectively. The natural frequency of the structure is ω_{s1} =14.43 rad/s. In this example, lower and upper values of frequencies for the first mode are chosen in between 12 rad/s and 16 rad/s. In this range, the PSD value is taken as $S_g(\omega)$=0.066 m^2/s^3. Except for this range, it is evaluated as zero. Firstly, two TMDs are placed on the SDOF structure. The mass, stiffness and damping matrices of structure-TMD system can be expressed as follows.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_{d11} & 0 \\ 0 & 0 & m_{d12} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} k_1 + k_{d11} + k_{d12} & -k_{d11} & -k_{d12} \\ -k_{d11} & k_{d11} & 0 \\ -k_{d12} & 0 & k_{d12} \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} c_1 + c_{d11} + c_{d12} & -c_{d11} & -c_{d12} \\ -c_{d11} & c_{d11} & 0 \\ -c_{d12} & 0 & c_{d12} \end{bmatrix} \quad \text{“} \tag{14}$$

The upper and lower limits of TMD parameters can be expressed as follows.

$$0 \leq m_{d1j} \leq 0.6\text{x}10^4 \text{ kg} \quad (j=1,2,\ldots,N) \tag{15}$$

$$0 \leq \sum_i^N m_{1j} \leq 0.6\text{x}10^4 \text{ kg} \quad (j=1,2,\ldots,N) \tag{16}$$

$$0 \leq c_{d1j} \leq 7.5\text{x}10^4 \text{ N.s/m} \quad (j=1,2,\ldots,N) \tag{17}$$

$$0 \leq k_{d1j} \leq 1\text{x}10^6 \text{ N/m} \quad (j=1,2,\ldots,N) \tag{18}$$

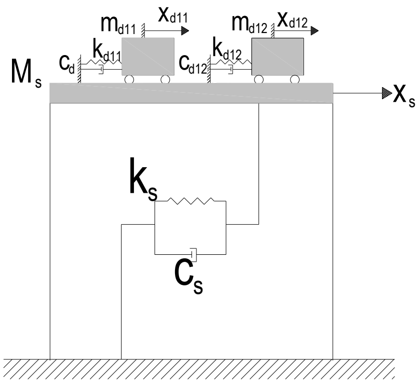

Fig. 2 - SDOF system with two TMDs

Considering this constraint, the objective function which is the mean square of floor displacement σ_D^2 is minimized by using the DE algorithm. After optimization, TMD parameters on the floor are calculated as c_{d1} = 17.36x10^3 Ns/m, c_{d2} =18.671 x10^3 Ns/m, k_{d11} =106.218x10^3 N/m, k_{d12} = 0 N/m, m_{d11} = 0.6x10^4 kg and m_{d12} = 0 kg. Since k_{d12} = 0 and/or m_{d12} = 0 are equal to zero, this TMD on the floor should be eliminated and the optimum location can be carried out. The absolute values of transfer functions on the floor, which are displacement $|H_D(\omega)|$ and acceleration $|H_A(\omega)|$, are shown in Figures 3-4. As can be seen in these figures, the proposed method is quite effective to control the first mode. This design is also tested under the El Centro (NS) ground motion as shown in Figures 5-6 for time histories of floor displacement and acceleration. As can be seen in all these figures, the proposed method is also effective in the reduction of El Centro (NS) ground motion effect.

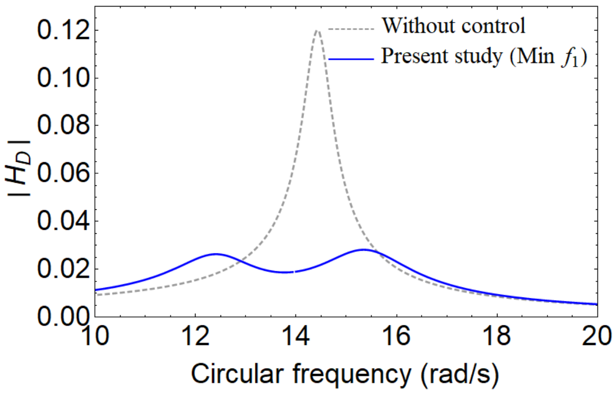

Fig. 3 – Absolute value of displacement transfer function $|H_D(\omega)|$ of the SDOF system with and without TMD due to f_1 minimization

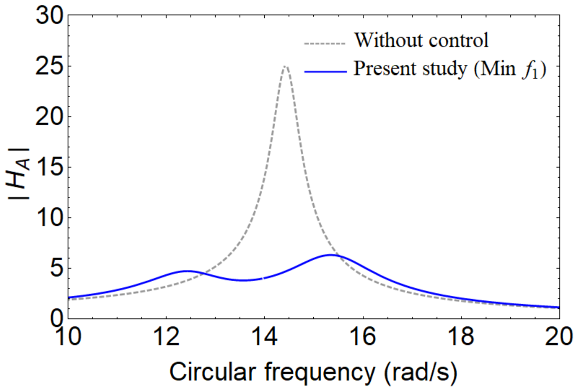

Fig. 4 – Absolute value of acceleration transfer function $|H_A(\omega)|$ of the SDOF system with and without TMD due to f_1 minimization

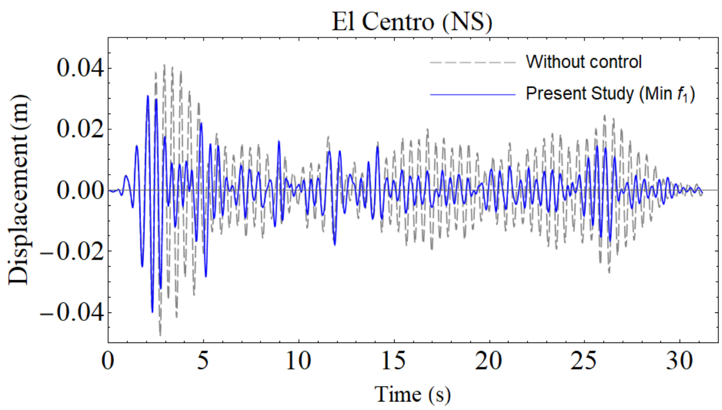

Fig. 5 – Displacement time history of the SDOF system with and without TMD under El Centro (NS) ground motion

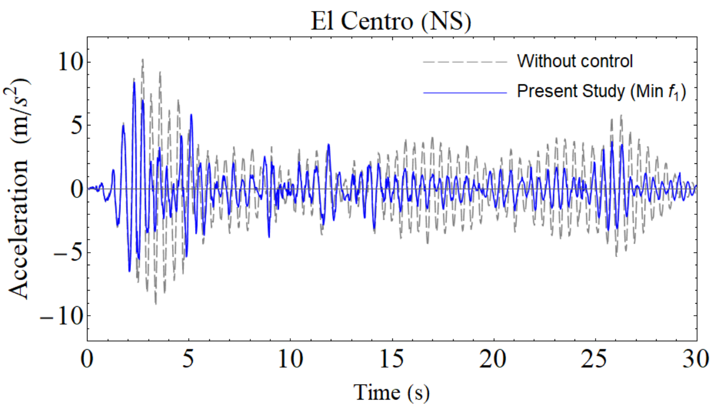

Fig. 6 –Acceleration time history of the SDOF system with and without TMD under El Centro (NS) ground motion

5. Conclusions

In this study, an optimum design method is proposed in order to determine the optimal location and find optimum parameters of multiple tuned mass dampers (MTMDs). The conclusions can be expressed as follows.

1) In this paper, MTMD parameters (mass, stiffness and damping) are obtained as a closed-form in the objective function in order to minimize the mean square of the floor displacement under the stochastic excitation. Differential Evolution (DE) algorithm is used in order to minimize the objective function under some constraints.

2) Thanks to the DE algorithm, the nonlinear objective function can be optimized to find optimum MTMD parameters successfully.

3) Optimum location of MTMDs and their mass quantities have not been widely studied in the literature. Therefore, the proposed method provides the optimal location and design of MTMDs. If TMD mass parameter or TMD stiffness parameter converges to zero on the floor, this TMD is automatically eliminated so that the optimum location of MTMD is executed.

4) The proposed method is very effective to decrease the absolute values of displacement and acceleration transfer functions.

5) In order to test the success of the proposed method, the results are tested under El Centro (NS) ground motion in the time domain. The results show that the proposed method is very effective to reduce the displacement and acceleration responses on the floor.

6. References

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