



BRB design optimization for a large arena structure based on generalized response spectrum analysis

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Abstract

This paper presents an application study of the proposed damper optimization method based on generalized response spectrum analysis (GRSA) for an actual large music arena (a free-form spatial structure which does not satisfy a rigid-floor assumption and where displacement responses disproportionately distribute in a story) in order to minimize both of the story drift response and the number of the buckling restrained braces (BRBs). The proposed method enables more efficient design process than trial-and-error approaches with time-consuming non-linear response history analysis. In this paper, the efficiency of the proposed method is verified and discussed through the comparison of an empirical brute force method and the standard damper design method. The optimal solution produced by the proposed method has approximately 20% less steel tonnage of BRBs than that obtained from both of the these conventional design methods, while the seismic performance is equal to or better than the others. According to this study, the configuration of the substructure where BRBs and the conventional braces are arranged in adjacent stories is the most effective configuration for preventing both of the buckling of the conventional braces, reducing story drift response and reducing the number of BRBs though practitioners should be still careful to the possibility of the damage concentration.

Keywords: Elasto-plastic damper, Response spectrum analysis, Complex eigenvalue analysis, Complex stiffness.

1. Introduction

In countries of high seismic hazard, to ensure immediate occupancy after a large earthquake, it is a common design practice to employ energy dissipating dampers such as buckling restrained braces (BRBs). While buildings with dampers can achieve high seismic performance, the construction costs due to these devices can easily increase. Therefore, it is important to optimize the number and the size of dampers so as to provide an economically efficient design. Particularly in Japan, this has led to a spurt in demand for optimal design methods for dampers that can be applied not only to conventional multi-story buildings but also to substructures of free-form spatial structures. While standard damper design methods for conventional multi-story buildings have already been established by many researchers, such as the equivalent linearization method (EL method) proposed by Kasai [1, 2], spatial structures do not satisfy a rigid-floor assumption (the basis of the EL method) and have complex vibration characteristics. Thus, practicing engineers tend to rely on the trial and error approach using time-consuming nonlinear response history analysis (NLRHA) to obtain an acceptable damper layout and damper sizes. With the advancement in technology and computing, numerical optimization methods for dampers have also been studied by many researchers (e.g. Takewaki [3] or Apostolakis et al. [4].) However, since these optimization methods require multiple NLRHAs, these are not necessarily suitable for optimizing damper placement and damper size in three-dimensional (3D) models with complicated vibration characteristics. In contrast to these optimization strategies that use NLRHA, Terazawa et al. [9, 10] proposed an optimal damper design strategy combining generalized response spectrum analysis (GRSA) with metaheuristic optimization algorithms. GRSA is a computational routine wherein complex eigenvalue analysis and response spectrum analysis are iteratively performed. The proposed method is expected to be a huge improvement from the current damper design methods in terms of speed and economic efficiency. However, the effectiveness of the proposed method for the design of



dampers in substructures of free-form spatial structures has not been verified yet. This paper presents an optimal damper design approaches using GRSA for actual free-form spatial structures. This method is applied in the design of buckling-restrained braced frames (BRB) employed in a large music arena, currently in its preliminary design stage, and has a roof with a maximum span of 150m.

2. Outline of the study

2.1 Overview of the project

The subject of this study is a music arena with a maximum span of about 150m, planned to be constructed in Japan. The building area of this arena is about 30,000m², and the estimated seating capacity is about 20,000. The plan is a diamond shape as shown in Fig. 1, and the seats are spread radially around the stage. The substructure has 7 stories and the total height is 40m. The building foundation is made of reinforced concrete (RC), the main structure is a steel moment-resisting-frame, and the roof consists of steel trusses along the Y-direction. As shown in Fig. 1, the substructure consists of a “Stage frame” where the stage is to be placed, a “Right wall frame” and a “Left wall frame” that cantilever from the ground with a maximum height of 40 m, and a “Stand frame” which consists of three floors made of RC where the audience seats will bear the seats. The roof frame protrudes about 10m, is designed to be a steel frame that weighs about 200kg/m². Fig. 2 shows the weight of each frame and the percentage of the roof weight supported by each frame. The weight of the Stand frame is the largest, about five times that of other frames. The roof weight including finishing materials is about 77MN, and the Stand frame supports about half of that.

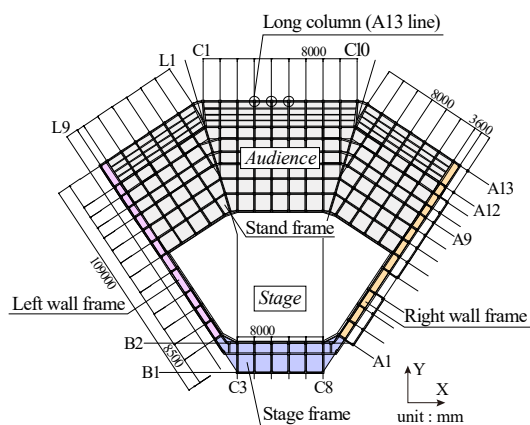


Fig. 1 1st floor plan of arena

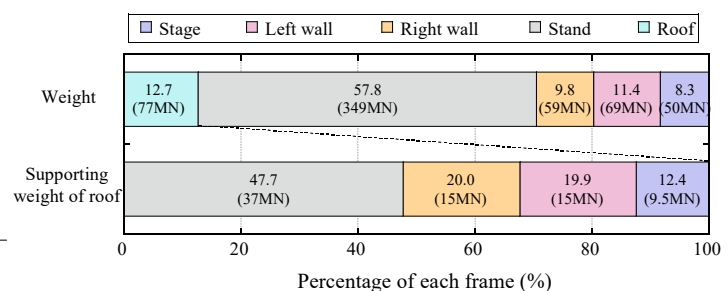


Fig. 2 Percentage of each frame weight

2.2 Analysis model

Fig. 3 shows the 3D analysis model of the arena. The columns and beams are modeled as beam elements and the conventional braces (CBs) are modeled as truss or beam elements. BRBs are modeled as link elements with bilinear hysteresis characteristics. In this study, material nonlinearity is considered only for BRBs, and the other members remain elastic. Fig. 4 shows the acceleration response spectra of the three input ground motions. These observed waves were spectrally matched following [7], and are amplified based on the ground characteristics of the construction site of the arena. The matched waves are the same as those used in the actual design and are denoted as Kobe, Hachinohe, and Random respectively.

In this study, three models are defined for comparing the seismic performance. The first model, denoted as “BF (Bare frame) model”, has no braces. The second model, denoted as “CBF (conventional brace frame) model”, is a dual system of a rigid frame with 442 conventional braces (CBs). SN490 steel and a section H-400×400×13×21 mm was assigned to each of the braces. CBFs were placed at all possible locations in architectural constraints. The third model, denoted as “BRBF model” is a dual system of a rigid frame with 358 BRBs in total, each having a yielding axial force of 3,000kN located in the same locations as conventional braces in the CBF model. Fig. 5 shows the location of each brace for each frame in the CBF



model and the BRBF model. In this study, uni-directional NLRHAs are performed in X and Y directions in Midas iGen (ver. 875) for these models and GRSA-based optimization results. Rayleigh damping of 2% was assigned to the first two dominant modes.

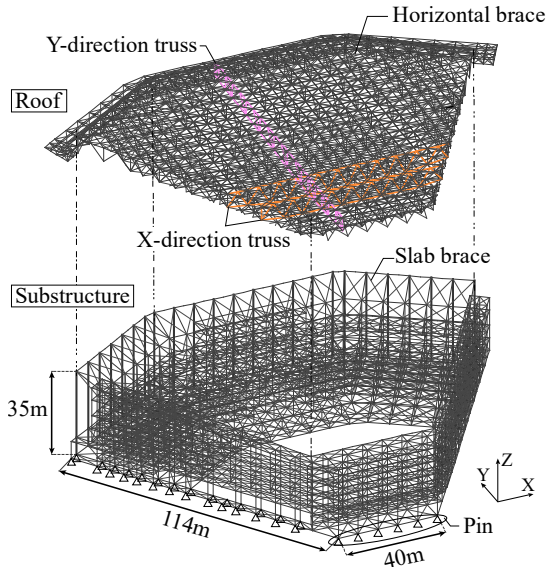


Fig. 3 Analysis model

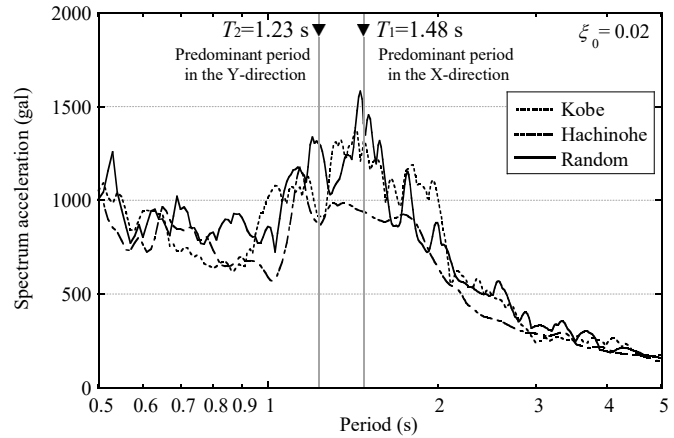


Fig. 4 Response spectra of input ground motions

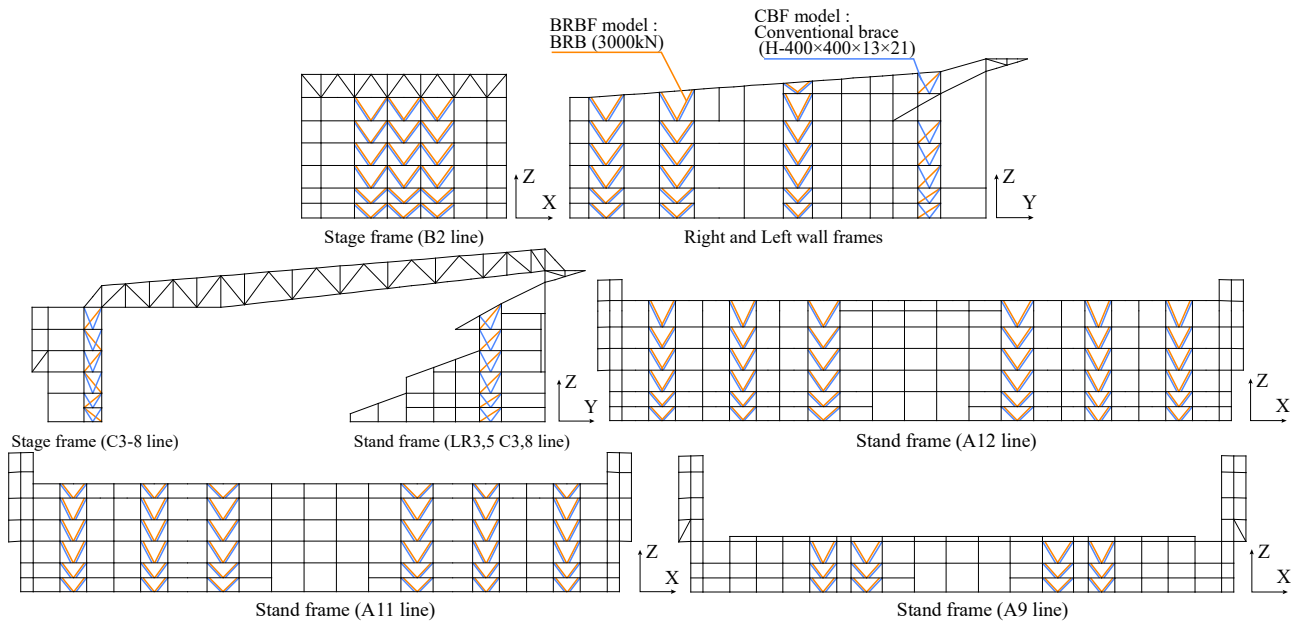


Fig. 5 Brace configurations in CBF model and BRBF model

2.3 NLRHA results of BF model

Fig. 6 shows the maximum story drift ratio (SDR) of each frame described in Fig. 1. The maximum SDR in the X-direction was observed in the Stand frame where the drift in the 5th story reached about 4%. The maximum vertical response of the roof frame is about 800gal regardless of the input direction of the seismic wave. These results show that the seismic performance of the BF model is insufficient. In particular, the maximum SDR of the Stand frame with seats is large, and to limit this, conventional braces and BRBs are required in the substructure. The response (drift) control of the substructure is discussed in the later sections. As can be seen from these results, the response from the Random wave is the maximum and is only



discussed in the subsequent sections for brevity.

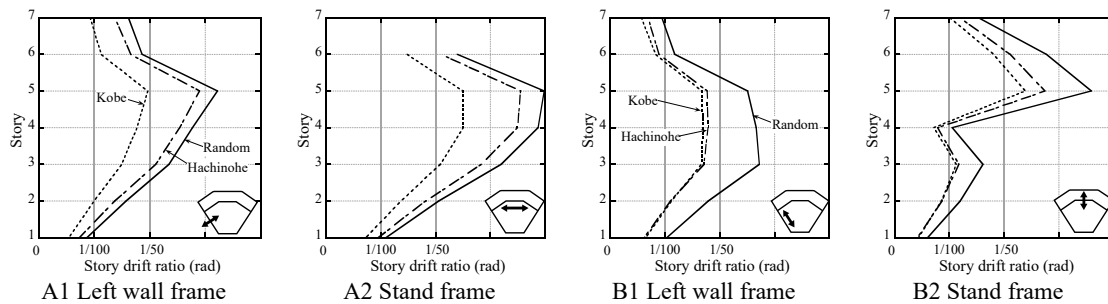


Fig. 6 Maximum responses of BF model, A: Input in X-direction, B: Input in Y-direction

2.4 Response validation using GRSA

In Section 4, where the numerical damper optimization results are discussed, the seismic response is evaluated using GRSA. GRSA is used as the analysis engine for the optimization algorithm. In GRSA, BRB is modeled as a complex stiffness element to simulate amplitude-dependent elasto-plastic damping. The seismic response is evaluated by the modified complete quadric combination method [7]. The detailed procedure is described in the author's papers [5, 6]. The accuracy of GRSA is verified using the CBF model and the BRBF model introduced in Section 2.2. Fig. 7 shows the maximum SDRs of these two models obtained from both NLRHA and GRSA. For both the models, the GRSA results agree well with the NLRHA results in the X-direction. The buckling DCR is calculated by dividing the maximum compressive force by the short-term allowable capacity as per AIJ design standard [9]. Regardless of the seismic input direction, the variation in the evaluation is found to be within 20%.

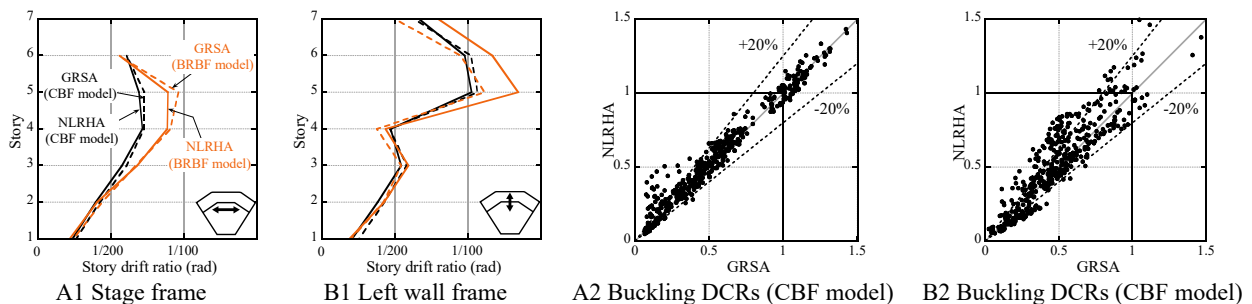


Fig. 7 Comparison of GRSA and NLRHA, A: Input in X-direction, B: Input in Y-direction

3. Damper design using conventional method

3.1 Empirical brute force method (EBF method)

For the preliminary design stage, the arena was designed following the Japanese equivalent lateral force procedure with a response modification factor and the configurations of braces are decided using the EBF method. In this method, a structural engineer manually determines the locations of the conventional braces and the BRBs empirically based on the static push-over analysis performed for the Stage frame, the Right and Left wall frames, and the Stand frame. This EBF model was designed limiting the maximum SDR to 0.5% for serviceability demand targeting 1% SDR for the ultimate level. Fig. 8 shows the configurations of braces on each frame. 239 BRBs and 448 conventional braces are arranged. Fig. 9 shows the maximum SDRs and the buckling DCRs of conventional braces. According to these results, this EBF method can produce an acceptable brace design, though a lot of conventional braces buckle. This is because the ductility of conventional braces is categorized by their slenderness ratio and allowing buckling. As a result, although the EBF method can ensure a certain level of overall seismic performance, it can not guarantee economic efficiency and resilience after a large earthquake. Note that the available brace locations in the optimization study are different from those in the EBF model because of modifications in the actual architectural layout



after the EBF study. The CBF model and the BRBF model which are the initial models of the optimization study have much fewer braces. Therefore, this seismic optimization is under the more severe condition for finding acceptable design, compared with the EBF method.

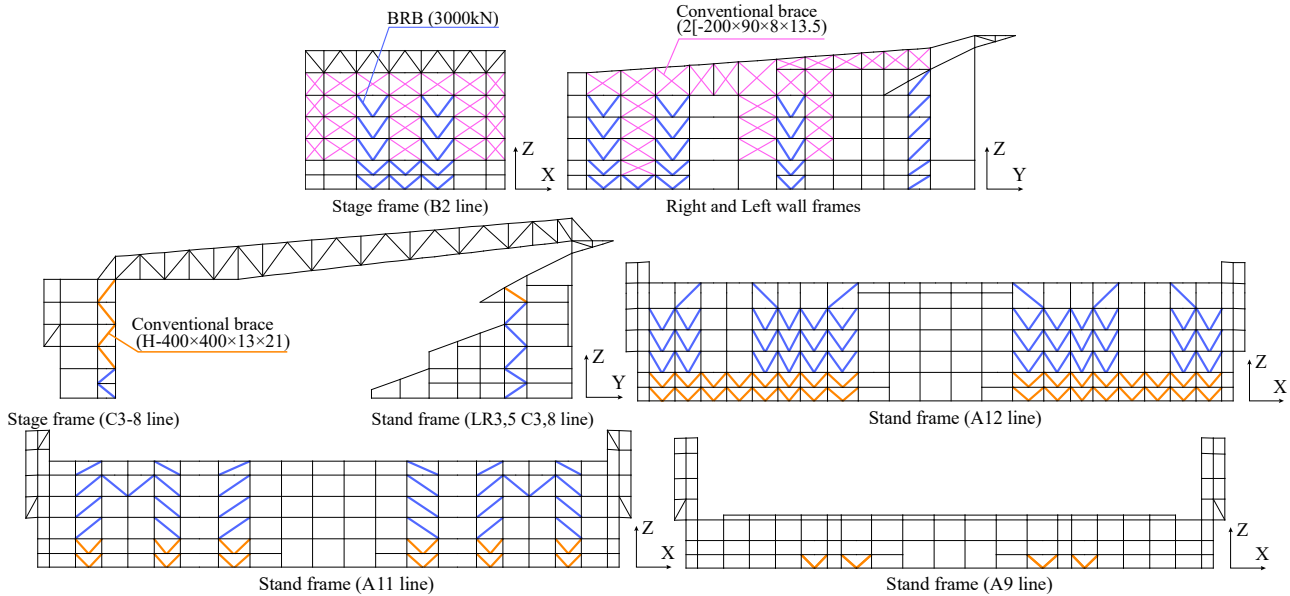


Fig. 8 Brace configurations in EBF model (which is different from the optimization results)

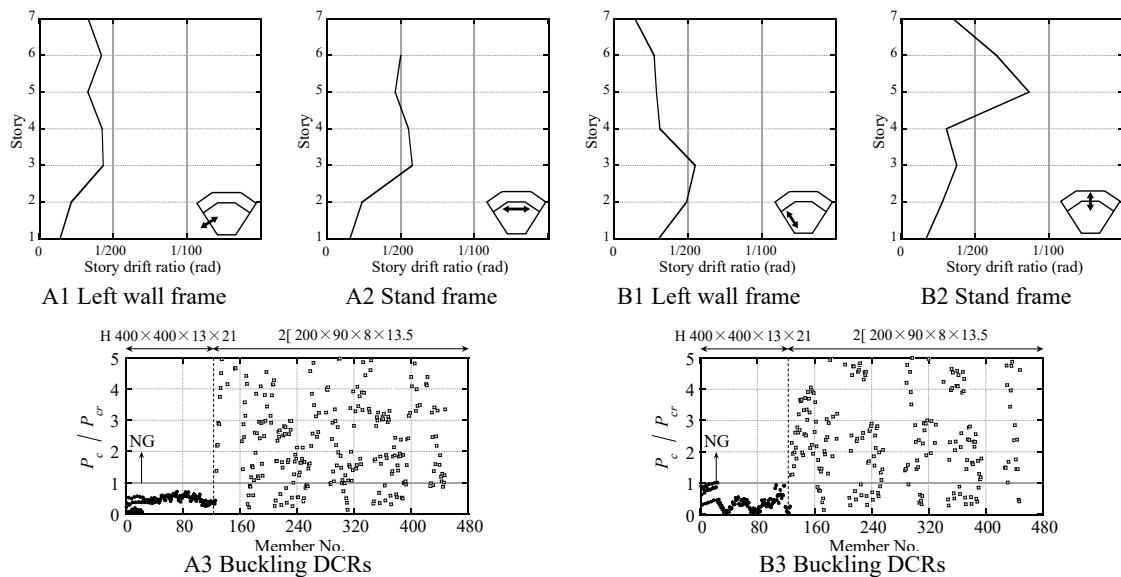


Fig. 9 Maximum responses of EBF model, A: Input in X-direction, B: Input in Y-direction

3.2 Equivalent single-degree-of-freedom-system method (EL method)

The EL method is applied to the separated structural blocks as shown in Fig. 6. The layout of the BRBs is the same as that in the BRBF model. In the EL method, the damper ductility and the damper-to-moment-frame stiffness ratio K_a/K_f that satisfies the target drift reduction rate is firstly determined using the ESDOF system [2]. The K_a/K_f selected in the ESDOF system is then distributed to the MDOF system so that the story drift ratio and the ductility of each story are the same. The story shear force distribution is obtained using A_i distribution, particularly in Japan. Here, no BRBs are arranged in the story if K_a is 0 or less. Fig. 10 shows the sizes of BRBs in the EL model and the maximum SDRs. In total, 274 BRBs are arranged. However, when the wave is input in the Y-direction, the maximum SDRs of the Stand frame exceed about 1.4%.

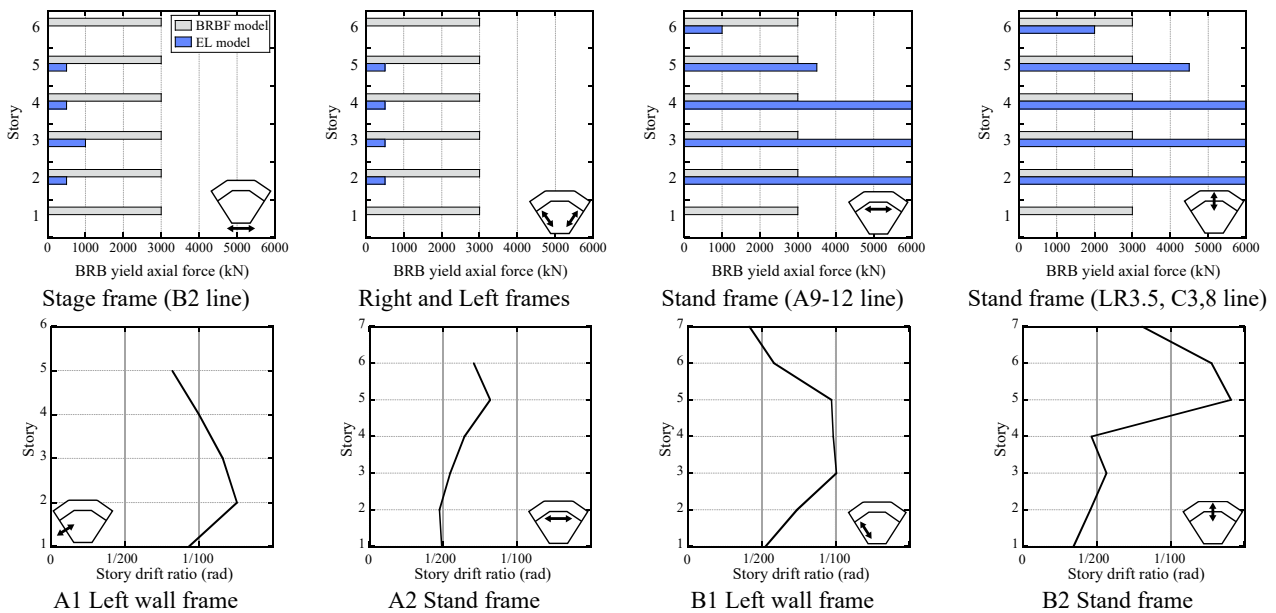


Fig. 10 BRB sizes and maximum responses of EL model, A: Input in X-direction, B: Input in Y-direction

4. Optimal damper design based on GRSA

4.1 Optimization problem statements

Table 1 and Table 2 show the outline of the optimization problems of this study. The procedure of the optimal design for BRBs is divided into two stages. First, the configurations of the conventional braces and the BRBs are optimized using the CBF model as an initial model. For damper layout optimization, two types of problem statements are considered in order to reduce the size of the design space. The first one (Layout-C-1, 2) is to optimize the layout of BRBFs, and the other (Layout-S-1, 2) is to optimize the layout of BRBFs at each story in each of the predetermined groups. After the locations of conventional braces and BRBs are fixed, the BRB sizes are optimized (Size-C-1, 2 and Size-S-1, 2). This procedure is selected so as to reduce the computation time because the possible number of the brace arrangements without any constraint is very large. The simple genetic algorithm is used for the BRB layout optimization, and the particle swarm optimization is used for the BRB size optimization. The objective function *Fitness* as defined in Equation 10 is minimized in this study. *Fitness* is a weighted function of the two response reduction ratios R_d and R_n . R_d is calculated as the reduction ratio of the peak SDR obtained from the optimization result to that obtained from the BF model as shown in Eq. (1) and Eq. (2). Here, SDR_j^X is the peak SDR in the X-direction, SDR_j^Y is the peak SDR in the Y-direction, SDR_{BF}^X is the peak SDR in the X-direction of the BF model, and SDR_{BF}^Y is the peak SDR in the Y-direction of the BF model. R_n is the reduction ratio of the total number of BRBs or the total steel tonnage of BRBs in the optimization solution to those of the BRBs in the BRBF model depending on the problem statement and is defined separately in each case. R_d and R_n are weighted by coefficients m and n . As a penalty, " ϕ " is added into *Fitness* for cases when the buckling DCR of the conventional braces in the optimization solution exceeds 1.0, and the value is 9999.

Fig. 11 shows the definition of the design variables $\{x_i\}$ of the layout optimizations. For elevation layout optimization "Layout-C-1," the substructure is divided into 20 BRBFs (a ~ t), and the layout optimization is performed for each of these BRBFs. The design variable x_i is the arrangement of the BRB and includes two states for each of the 20 BRBFs; replacing the CBF with the BRBF and leaving the CBF. For "Layout-C-2," a state of removing a CBF is added to "Layout-C-1." For plan layout optimization for each group "Layout-S-1," the substructure is divided into 7 groups of frames ($gr = \{a, b, c, d, e, f, g\}$). For "Layout-S-2," a state of removing a CBF is added to "Layout-S-1." Fig. 12 shows the definition of the $\{x_i\}$ of the BRB size optimizations. The BRB size optimization is performed for the layout optimization results. As a constraint,



the available sizes of the BRBs is limited to the range ran of standard sizes in Japan, where $ran = \{500\text{kN}, 750\text{kN}, 1,000\text{kN}, 1,500\text{kN}, 2,000\text{kN}, 2,500\text{kN}, 3,000\text{kN}, 3,500\text{kN}, 4,000\text{kN}, 4,500\text{kN}, 5,000\text{kN}, 6,000\text{kN}\}$. If the solution opts for a BRB of size 0kN , a conventional brace (H-400×400×13×21) is assigned.

$$Fitness = mR_d + nR_n + \varphi \tag{1}$$

$$R_d = [\max \{SDR_j^X | j=1,2,\dots,5\} / SDR_{BF}^X + \max \{SDR_j^Y | j=1,2,\dots,5\} / SDR_{BF}^Y] / 2 \tag{2}$$

Table 1 Damper layout optimization problem matrix

Optimization problem	For each BRBF		For each story of groups	
	Layout-C-1	Layout-C-2	Layout-S-1	Layout-C-2
	Only replacing with BRBF	Containing removing CBF	Only replacing with BRBF	Containing removing CBF
Optimize	Damper layout (fixed size)			
Algorithm	Simple genetic algorithm (SGA)			
Minimize	Peak SDR, the number of BRBs			
Coefficient m, n	$m = n = 1.0$	$m = 0.75, n = 0.25$	$m = n = 1.0$	$m = 0.75, n = 0.25$
Variables	$x_i = 0$ or 1	$x_i = 0$ or 1 or -1	$x_i = 0$ or 1	$x_i = 0$ or 1 or -1
The number of variables	20	20	38	38

Table 2 Damper size optimization problem matrix

Optimization problem	Size-C-1	Size-C-2	Size-S-1	Size-C-2
Initial mode	Optimal solution of Layout-C-1	Optimal solution of Layout-C-2	Optimal solution of Layout-S-1	Optimal solution of Layout-S-2
Optimize	Damper size (fixed layout)			
Algorithm	Particle swarm optimization (PSO)			
Minimize	Peak SDR, the total tonnage of BRBs			
Penalty	DCR < 1.0			
The number of variables	38	26	17	14
Subject to	$0\text{kN} \leq x_i \leq 6000\text{kN}$			

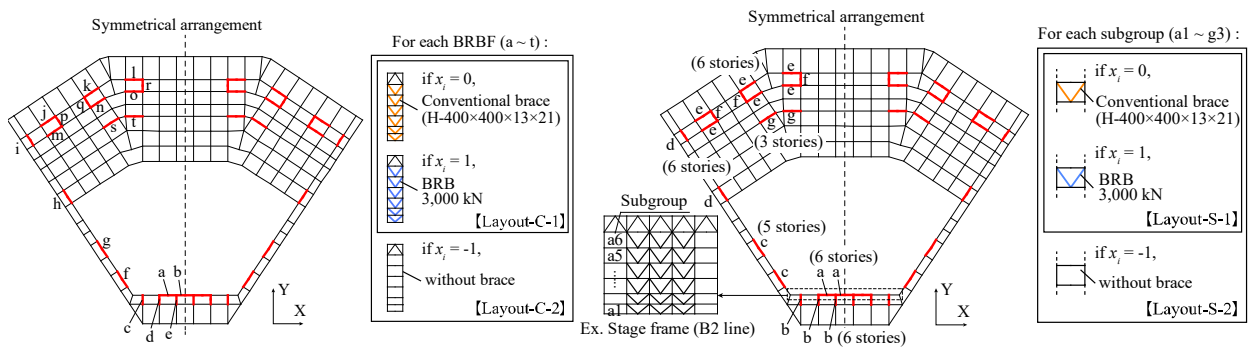


Fig. 11 Definitions of design variables of damper layout optimization

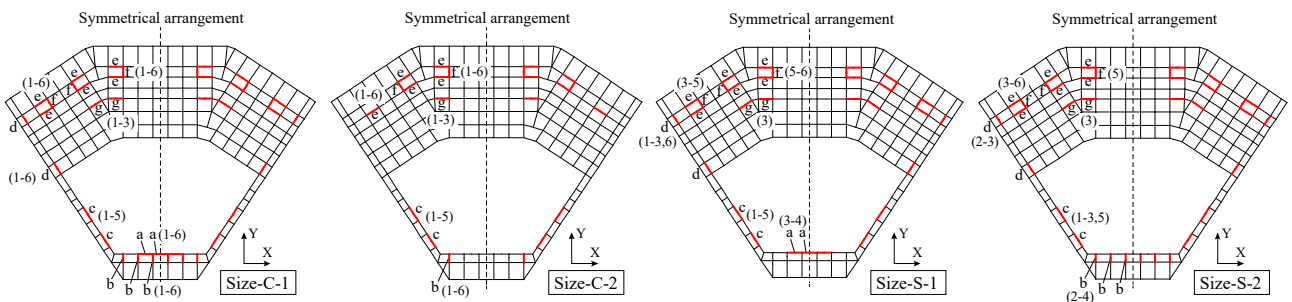


Fig. 12 Definitions of design variables of BRB size optimization (Number in () is the number of stories)



4.2 Damper layout optimization for each vertical BRBF (Layout-C-1, Layout-C-2)

For Layout-C-1, no CBF is placed and BRBFs are assigned instead, and the configurations of the BRBs are the same as those in the BRBF model. Fig. 13 shows the BRB configurations in the solutions obtained from Layout-C-2. The optimal solution has 212 BRBs which is a 41% reduction if compared to the BRBF model.

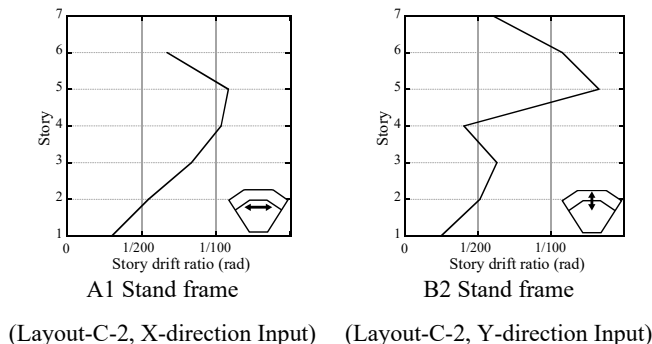
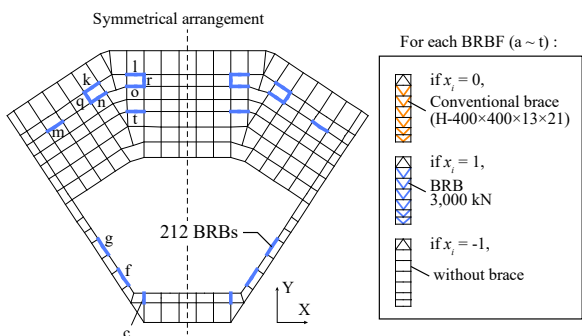


Fig. 13 Layout optimization result (Layout-C-2)

Fig. 14 Maximum SDR responses (Layout-C-2)

4.3 Damper layout optimization for each story of each group (Layout-S-1, Layout-S-2)

Fig. 15 shows the results of the layout optimization (Layout-S-1, Layout-S-2). As shown in Fig. 15A, Layout-S-1 has 176 BRBs which is a 51% reduction if compared to the BRBF model. In the Stage frame and the Stand frame, the BRBs are located exclusively in adjacent stories across the width of the frame, and the conventional braces are located in upper and lower stories. In the previous study [6], similar configurations were obtained as an optimal brace configuration. This result indicates that the configurations similar to mid-level seismic isolation are effective even for large spatial structures to reduce the drift and prevent the brace buckling. Fig. 16 shows the maximum SDRs. Only the response of the Stand frame is about 1.2%.

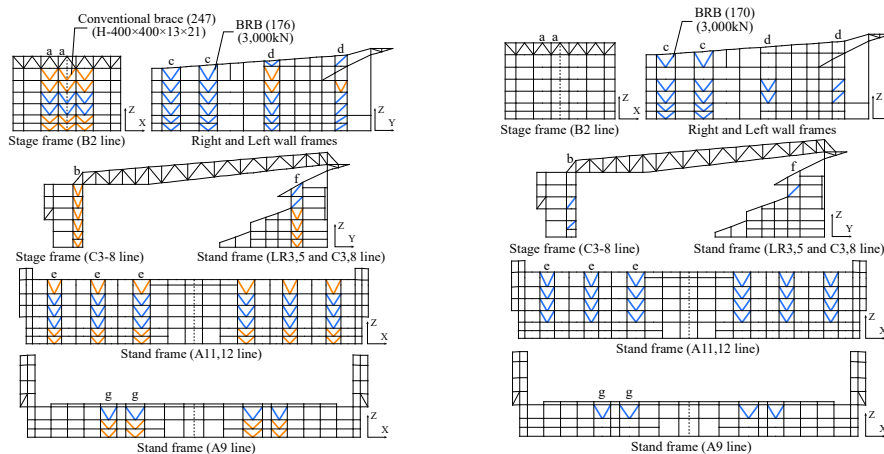


Fig. 15 Layout optimization result, A: Layout-S-1, B: Layout-S-2

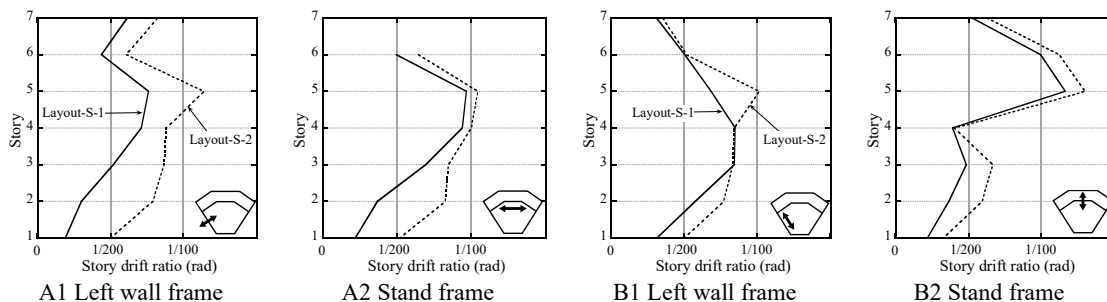


Fig. 16 Maximum SDR responses (Layout-S-1, Layout-S-2), A: Input in X-direction, B: Input in Y-direction



4.4 BRB size optimization for each BRBF (Size-C-1, Size-C-2)

Fig. 17 shows the obtained optimal brace configuration (Size-C-1). The number of BRBs in Size-C-1 is 226 which is a 37% reduction if compared to the BRBF model. As shown in Fig. 17, the configurations similar to mid-level seismic isolation are again produced as an optimal solution (regardless of the definitions of the design variables). This result implies that this configuration is the most effective to prevent the buckling of the conventional braces and to reduce the displacement of the substructure. Fig. 18 shows the comparison of the BRB sizes. Although the sizes of most BRBs are 3,000kN or less, the sizes of the BRBs in 4th and 5th stories of the Stage frame are more than 4,000kN. The total steel tonnage of the BRBs of Size-C-1 is about 46% of that of the BRBF model. For Size-C-2, the total steel tonnage of the BRBs is about 93% of that of Layout-C-2. While the BRBs with sizes up to 6,000kN are arranged in the Stand frame by the EL method, smaller BRBs were arranged in the optimal solution by the proposed method considering the dynamic characteristic of the whole structure.

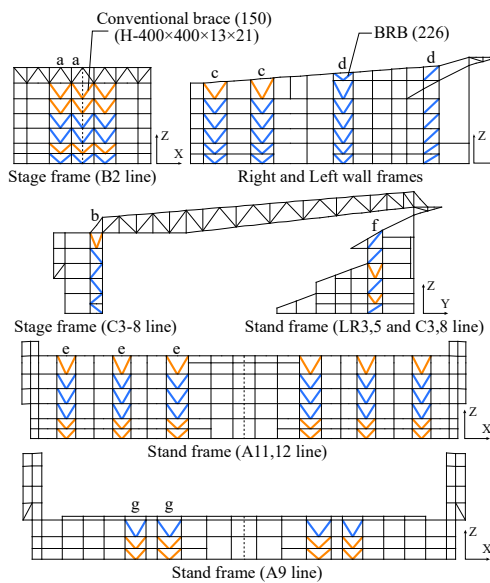


Fig. 17 Layout optimization result, A: Size-C-1

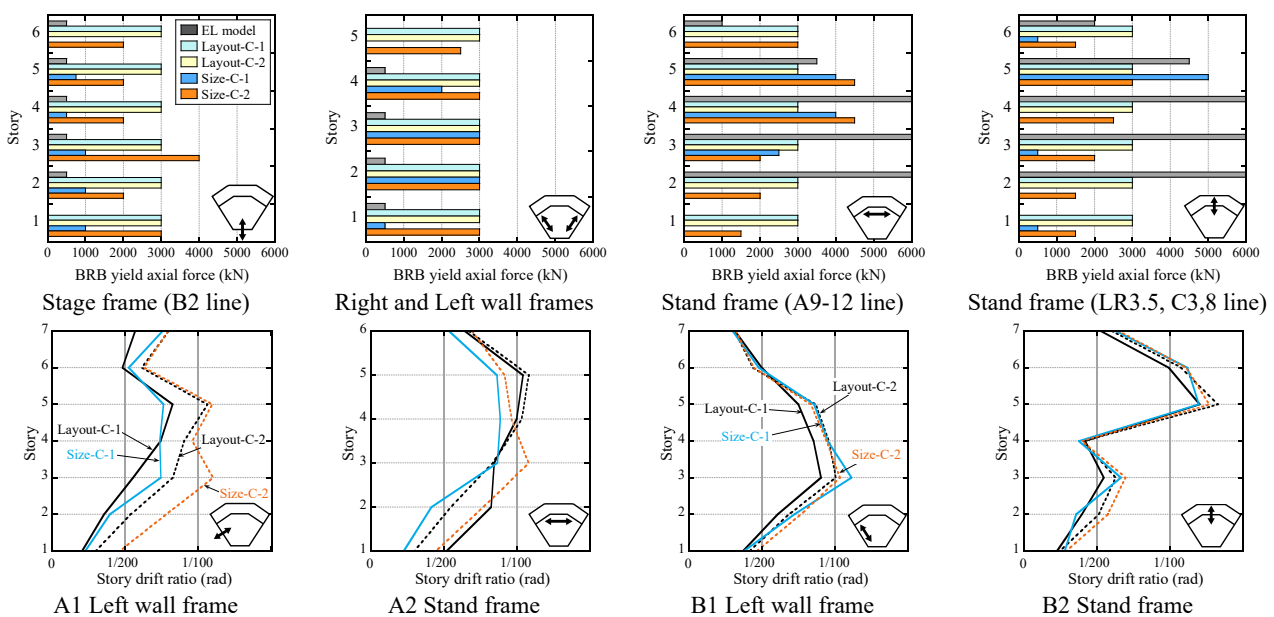


Fig. 18 BRB sizes and maximum SDR responses, A: Input in X-direction, B: Input in Y-direction



4.5 BRB size optimization for each story of each group (Size-S-1, Size-S-2)

Fig. 19 shows the comparison of the BRB sizes. The number of BRBs in Size-S-1 reduced to 156 which is an 11% reduction from Layout-S-1 by replacing BRBs with the conventional braces in the upper stories of the Right and Left wall frames. The total steel tonnage of the BRBs remains 97% of Layout-S-1. In Size-S-2, while the sizes of the BRBs in the Stage frame are reduced, BRBs with sizes of 4,000kN or more are placed in the Stand frames, and the total steel tonnage of the BRBs is about 95% of that of Layout-S-2. As shown in Fig.19, the maximum SDR response of Size-S-1 is less than 1%, and the response of the Stand frame in the Y-direction is reduced to around 1% compared with the response of Layout-S-1. The responses of the other solutions are almost the same as that of the optimal solution obtained from the layout optimization.

According to the results in Section 4.4 and 4.5, it can be seen that the BRB size optimization can produce solutions that have an equal to or lesser drifts than that of the layout optimal solution despite the reduced steel tonnage of BRBs. Furthermore, the drift response of the solutions obtained from the proposed optimization method is smaller than that of the solution obtained from the EL method in Section 3. According to these results, the EL method does not provide the most optimal solution and further studies are required to extend its application to the large free-form spatial structures. In contrast, the proposed design method can produce solutions with good seismic performances.

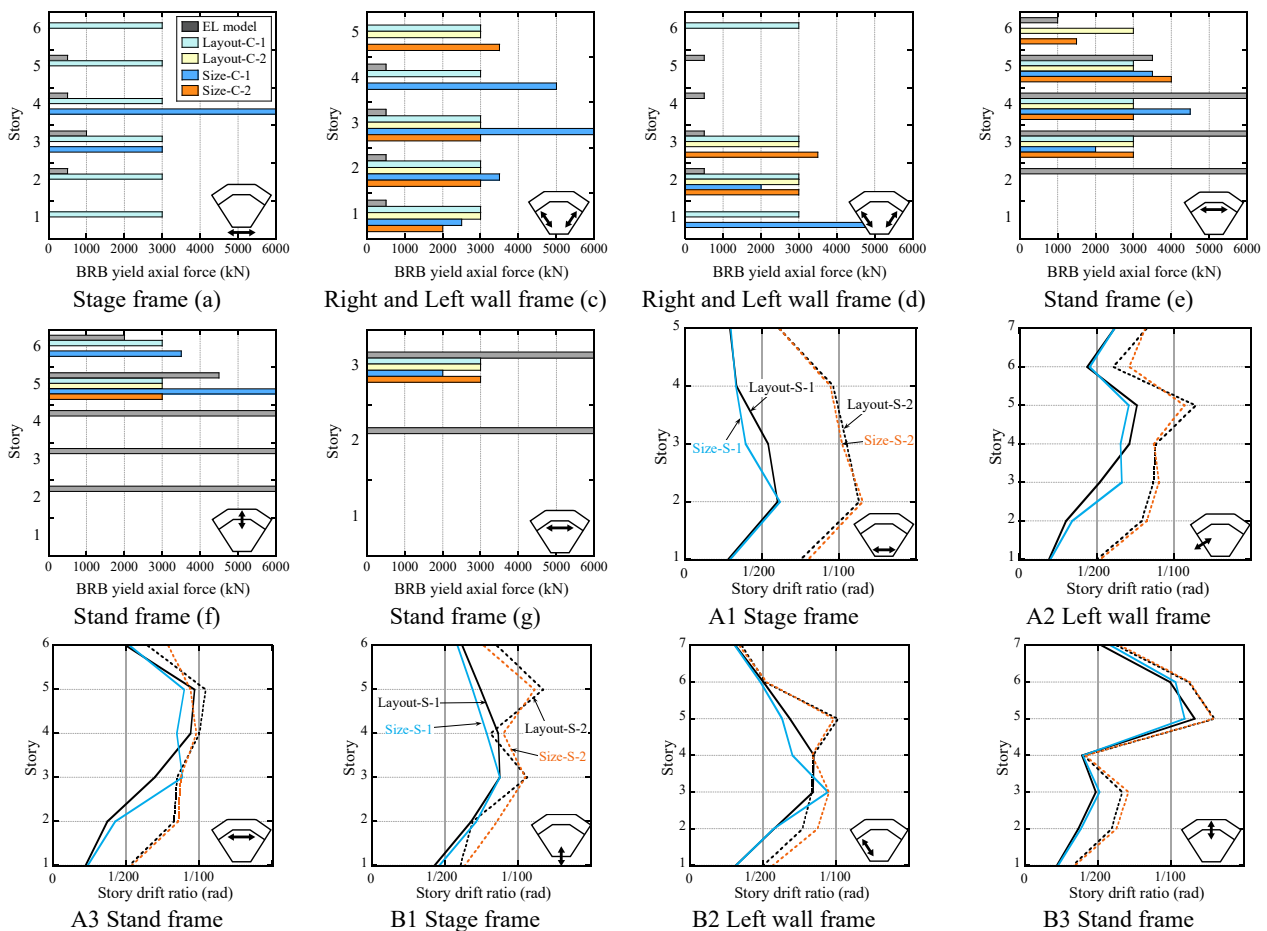


Fig. 19 BRB sizes and maximum SDR responses, A: Input in X-direction, B: Input in Y-direction

4.6 Comparison of damper design methods

Table 3 summarizes the number and the total steel tonnage of BRBs and the seismic response characteristics



of the solutions obtained by each damper design method in Section 3 to 4. For the maximum SDR, the EBF model has the best performance because many conventional braces and BRBs are introduced compared with the others. The maximum response of the EL model is about 1.4% which is the largest. While most of the conventional braces buckled in the EBF model, the proposed optimal design method prevents buckling of the conventional braces by defining the penalty in the Fitness for braces with DCR more than 1. The total steel tonnage of the BRBs of the EBF model is about 73% of that of the BRBF model. In the EL model, the number of BRBs is 274 which is a 23% reduction from the BRBF model. In contrast with these solutions, the number of BRBs in the optimal solutions is 156~226 which is about 37~56% reduction if compared to the BRBF model and the total steel tonnages of BRBs are reduced to about 46~59%. In addition, while the equivalent modal damping ratios ξ_{eq} of the EBF model and the EL model are about 2.9~3.9 %, those of the solutions obtained from the proposed optimal design method are about 3.1~5.7%. Fig. 20 shows the comparison of the number of braces and the maximum SDR. In the BRB layout optimization, Layout-S-1 shows the best performance despite having a lesser number of BRBs. In the BRB size optimization, Size-S-1 shows the best performance in terms of both the SDR and the number of BRBs. Note the proposed optimal design method consumed a run time of only 48 hours (for each case) to produce the optimal solution.

Table 3 Comparison of damper design methods

Design method	-		EBF method		EL method		GRSA-based damper optimization method															
							BRB layout optimization						BRB size optimization									
							For each BRBF			For each story			For each BRBF			For each story						
							Replacing with BRB	Removing CB	Replacing with BRB	Removing CB	Replacing with BRB	Removing CB	Replacing with BRB	Removing CB	Replacing with BRB	Removing CB	Replacing with BRB	Removing CB				
Model name	BRBF		EBF		EL		Layout-C-1	Layout-C-2		Layout-S-1		Layout-S-2		Size-C-1	Size-C-2		Size-S-1		Size-S-2			
Input direction	X	Y	X	Y	X	Y	X		Y		X		Y		X		Y		X		Y	
Equivalent damping ratio ξ_{eq} (%)	5.1	4.1	5.5	2.9	3.9	3.8	5.6	4.7	4.6	3.2	5.7	4.6	4.5	4.4	5.3	4.5	5.6	4.3	5.6	4.7		
Peak SDR(%)	0.97	1.20	0.58	0.86	1.30	1.40	1.29	1.30	0.97	1.20	1.10	1.30	0.92	1.20	1.30	1.27	0.90	1.09	1.15	1.29		
Buckling of conventional braces (CBs)	-	-	273	308	-	-	-	0	0	-	-	4	0	-	0	0	-	0	0	-		
The number of CBs	0	0	448	0	0	0	0	247	0	150	0	267	0									
The number of BRB	358	239	274	212	176	170	226	212	156	170												
Cost (%) (Total steel tonnage of BRBs)	100	73	60	59	52	49	46	56	50	46												

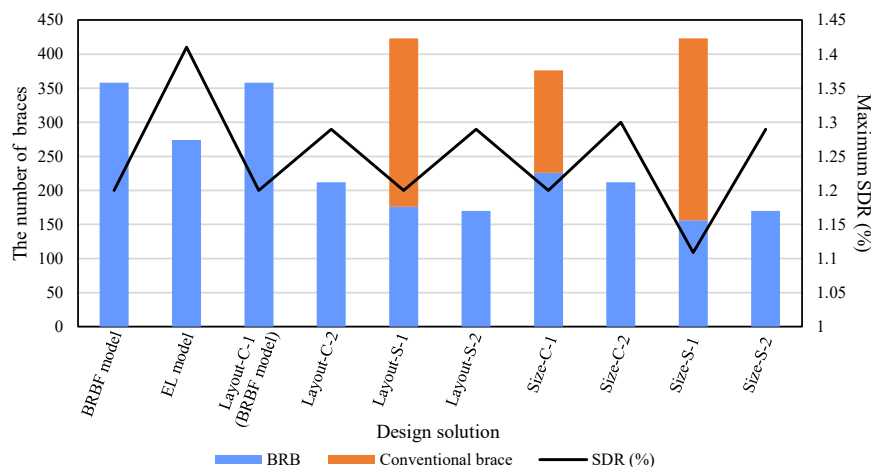


Fig. 20 Comparison of the number of braces and maximum SDR



5. Conclusion

As a conclusion, the following results were obtained.

- 1) Although the EBF method can produce the configurations of the conventional braces and the BRBs which satisfy the target drift 1% limiting by using static push-over analysis, buckling failure of conventional braces was allowed. Moreover, it is necessary for the trial and error approach using the time-consuming NLRHA or nonlinear static pushover analysis to obtain an acceptable brace layout. In contrast, the proposed design method can produce optimal solutions that have a good seismic performance in practical time by defining the penalty with buckling demands in conventional braces.
- 2) From the solution obtained from the EL method, the number of BRBs is 274 which is a 23% reduction if compared to the BRBF model and the maximum SDR is about 1.4%. This method cannot consider the interaction between the frames of the substructure during the earthquake because it is applied to each frame individually, and as a result, BRBs with larger sizes are arranged in the Stand frame (with large mass and stiffness). In contrast, the proposed optimal design method can produce a better configuration in which the number of BRBs is 156~226 which is a 37~56% reduction if compared to the BRBF model and the maximum SDR of the solution is reduced to about 1%.
- 3) The solution obtained from the proposed method has approximately 20% less steel tonnage of BRBs than that obtained from other design methods, while the seismic performance is equal to or better than the others. This is because this method incorporates the effects of other major economic parameters such as the number and steel tonnage of BRBs.
- 4) Similar to mid-level seismic isolation in multi-story structures, the configuration of the substructure where BRBs and the conventional braces are arranged in adjacent stories is the most effective configuration for preventing the buckling of the conventional braces and reducing drift response.

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7. References

- [1] K. Kasai, Y. Fu, A. Watanabe (1998): Passive control systems for seismic damage mitigation, *Journal of Structural Engineering*, **124** (5), 501-512.
- [2] The Japanese society of seismic isolations (2013): *Design and construction manual for passive control system 3rd edition* (in Japanese.)
- [3] I. Takewaki (1997): Optimal damper placement for minimum transfer functions, *Earthquake engineering and structural dynamics*, **26** (11), 1113-1124.
- [4] G. Apostolakis, G. F. Dargush (2010): Optimal seismic design of moment-resisting steel frames with hysteretic passive devices, *Earthquake engineering and structural dynamics*, **39** (4), 355-376.
- [5] Y. Terazawa, T. Takeuchi (2018): Generalized response spectrum analysis for structures with dampers, *Earthquake spectra*, **34** (3), 1459-1479.
- [6] Y. Terazawa, T. Takeuchi (2019): Optimal damper design strategy for braced structure based on generalized response spectrum analysis, *Japan architectural review*, **2** (4), 477-493.
- [7] Notification of the Ministry of Construction No. 1457 (2002), *The building standard law of Japan* (in Japanese.)
- [8] R. Sinha, T. Igusa (1995): CQC and SRSS methods for non-classically damped structures, *Earthquake engineering and structural dynamics*, **24** (4), 615-619.
- [9] Architectural Institute of Japan (2017): *Design standard for steel structures – Based on allowable stress concept –*