



AN ANALYTICAL MODEL FOR TIN RUBBER BEARINGS UNDER LARGE SHEAR DEFORMATION

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Abstract

Seismic isolation protects a structure from the destructive effects of an earthquake by separating it from the ground utilizing seismic devices such as rubber bearings in the building foundation. Rubber bearings with energy dissipating lead has typically been used as a seismic device. However, while lead rubber bearings have the advantage of having a damping function, the use of lead in the damping material may cause environmental problems. According to RoHS, etc., lead-free products have already made advances in all industrial fields, and practical application of lead-free work is necessary. We developed tin rubber bearings which use tin for damping material due to the considerations above.

A tin plug is inserted in the center of the isolator, and provides damping by deforming plastically when the isolator moves laterally in an earthquake. Tin has properties of low melting point and recrystallization approximately equivalent to lead, but the yield stress is high at 1.7 times that of lead, which means energy absorption efficiency is high.

The bilinear model is used in the design of conventional rubber bearings with plugs. These equations are generally applied to horizontal deformation up to 250% shear strain. However, the ultimate behavior of large deformation and high compressive stress over 250% has not been extensively studied.

In addition to large shear deformation, the change in shear stiffness due to high compressive stress is also an important behavior to consider when base isolated buildings are subjected to extreme ground motions.

In this study, we examined an analytical model that can reproduce the mechanical behavior up to the large deformation region of tin rubber bearings. First, a load test was conducted using compressive stress as the parameter to determine restoring force characteristics, and based on this, a restoring force model evaluation formula was created. Subsequently, the dynamic behavior was predicted at large deformation by applying this restoring force model to the dynamic model considering the P- Δ effect and geometrical nonlinearity. In this study, we apply the Kikuchi-Aiken Model^[1] to create a shear restoring force model, and developed a new restoring force model and a dynamic model that can reproduce the behavior up to the large shear deformation region of tin rubber bearings.

Keywords: Seismic Isolation, Tin Rubber Bearings, Analytical Model, Large Shear Deformation Test



1. Introduction

Tin rubber bearings are products jointly developed by three companies: Aseismic Devices CO.,LTD., Sumitomo Metal Mining Siporex Co., Ltd. and SWCC SHOWA CABLE SYSTEMS CO.,LTD. A modified bilinear model considering the strain dependence as restoring force characteristic is generally applied to tin rubber bearings as well as to lead rubber bearings. However, the modified bilinear model has an application limit strain up to approximately 250%, but when the strain exceeds the limit, the model is not able to reproduce extreme behaviors due to strong nonlinearity and high compressive stress such as hardening occurring at the time of huge deformation. This situation is similar to high damping rubber bearings and lead rubber bearings using a modified bilinear model. With this model, it is impossible to trace with precision the behavior of seismic isolation buildings under seismic motions, which are prone to produce excessive response displacement under long-period ground motion.

Recently, it has been pointed out that the damping performance of seismic isolation components deteriorates with a repeated deformation due to long-period ground motion, in which case the response displacement of the seismic isolation building is feared to increase further. Under these conditions, the restoring force model of the seismic isolation components that can simulate a large shape has been demanded, and the Kikuchi-Aiken Model has come to be widely used.

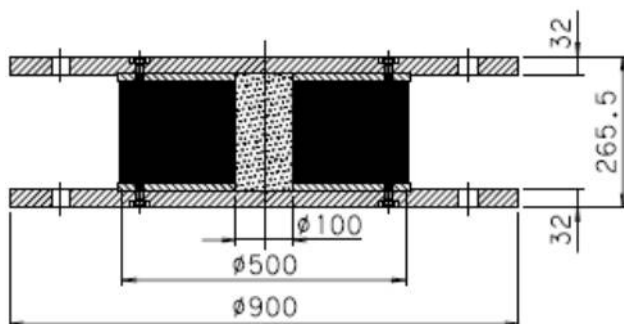
In this study, we applied the Kikuchi-Aiken Model to the tin rubber bearing, and created a shear restoring force model that is able to reproduce the behavior up to a large-scale deformation region.

2. Loading test of rubber bearings

The large shear deformation test was carried out using compressive stress as a parameter for the tin rubber bearing.^[2]

2.1 Test method

The specimen used is rubber 500 mm in diameter and 97.5 mm thick, with a tin plug 100 mm in diameter. The cross-section of the specimen is shown in Fig. 1. One specimen was used for the three compressive stress levels of 1 MPa, 15 MPa, 30 MPa respectively. The test compressive stress 15 MPa was used as the reference compressive stress, 1 MPa was set to eliminate the effect of compressive stress as shear characteristic, and 30 MPa was set to confirm the behavior when compressive stress is high. For shear strain, 4 cycles in the form of a triangular wave 5 mm/s were given for each compressive stress to determine the deformations of $\pm 50\%$, $\pm 100\%$, $\pm 150\%$, $\pm 200\%$, $\pm 250\%$, $\pm 300\%$, $\pm 400\%$ (3 mm/s only for large deformations 300%, 400%). For all of the specimens, initially the standard performance test was carried out by applying the standard compressive stress of 15 MPa.



Shear modulus (MPa)	$G=0.392$
Rubber diameter (mm)	$\phi 500$
Tin Plug diameter (mm)	$\phi 100$
Total rubber thickness (mm)	$3.75\text{mm} \times 26 = 97.5\text{mm}$
S_1 : Primary shape factor	32.0
S_2 : Secondary shape factor	5.1

Fig. 1 – Specimen



2.2 Test results

Fig. 2 shows superimposed graphs of horizontal loading-horizontal displacement relationship for compressive stress of each test. It can be confirmed that all test compressive stress is a stable hysteresis loop up to shear strain $\pm 300\%$. Looking at the test results of compressive stress of the two other levels from the reference compressive stress 15 MPa, the compressive stress 1 MPa has a slightly thinner hysteresis loop. At the compressive stress of 30 MPa, the horizontal load at the shearing strain 400% is observed to decrease at the 2nd and subsequent cycles. Further, for all specimens, no abnormalities such as buckling and breaking were observed.

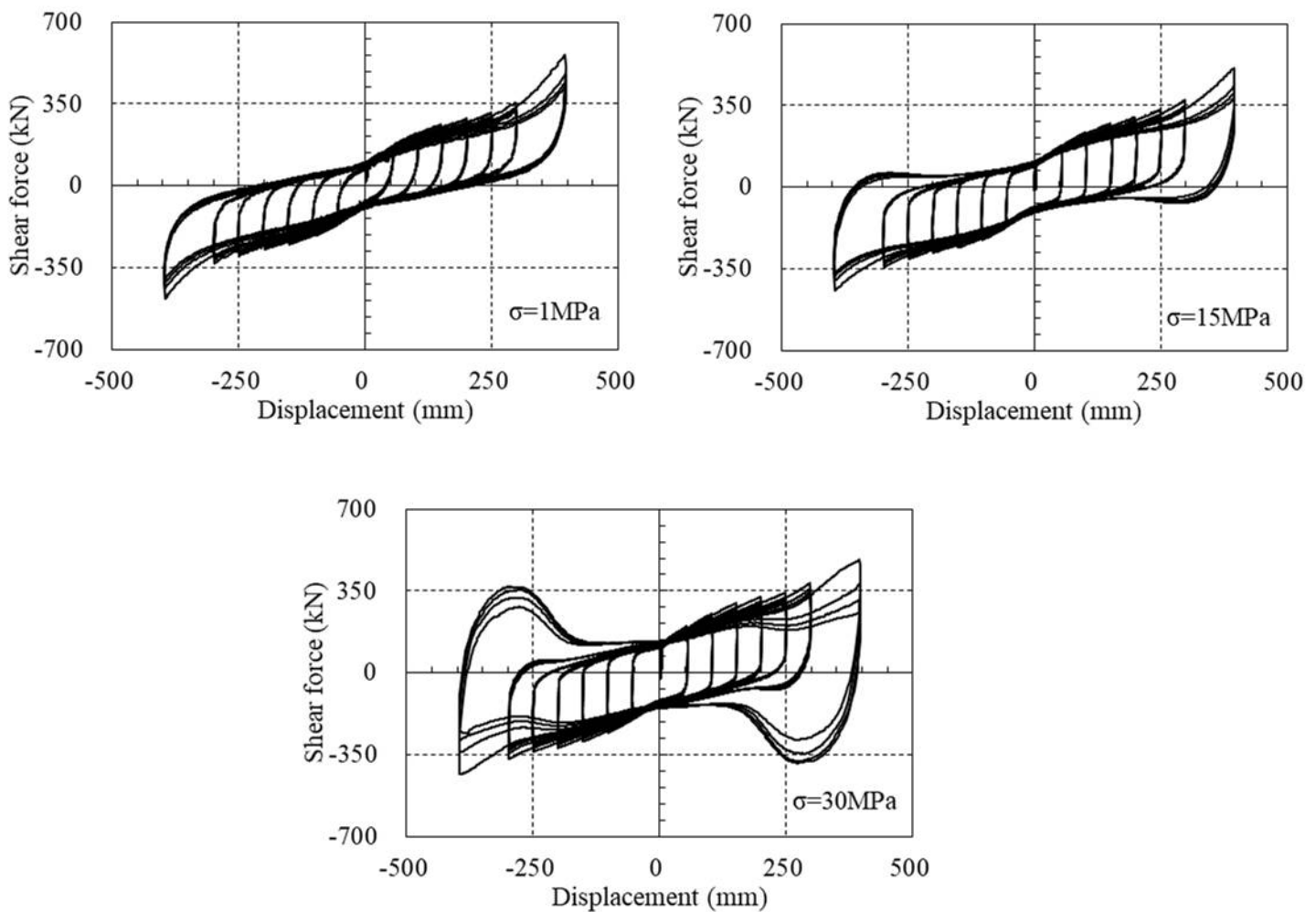


Fig. 2 – Test results for each compressive stress



The calculation method of the secondary stiffness K_2 and the yield load Q_d is shown in Fig. 3. The result for the strain dependence of the secondary stiffness is shown in Fig. 4 and the result of strain dependence of the yield load is shown in Fig. 5.

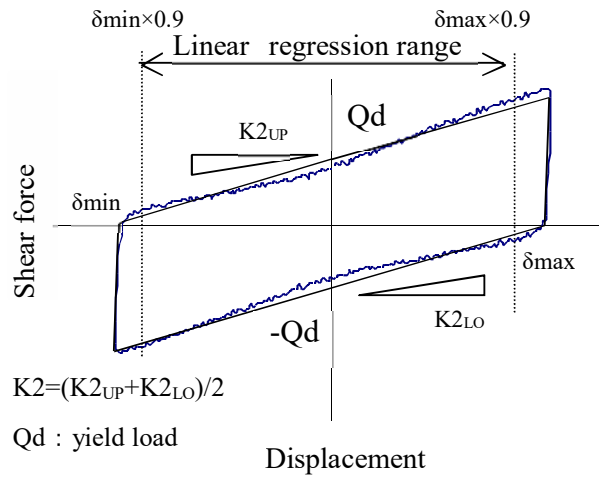


Fig. 3 - Calculation methods of secondary stiffness and yield load

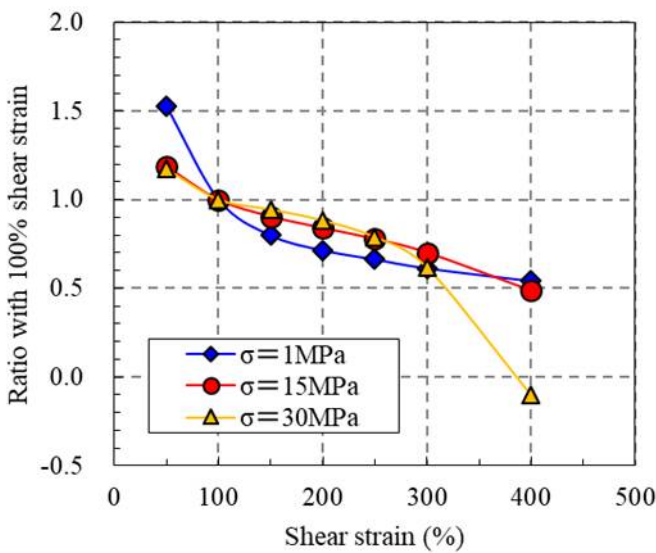


Fig. 4 - Strain dependence of secondary stiffness

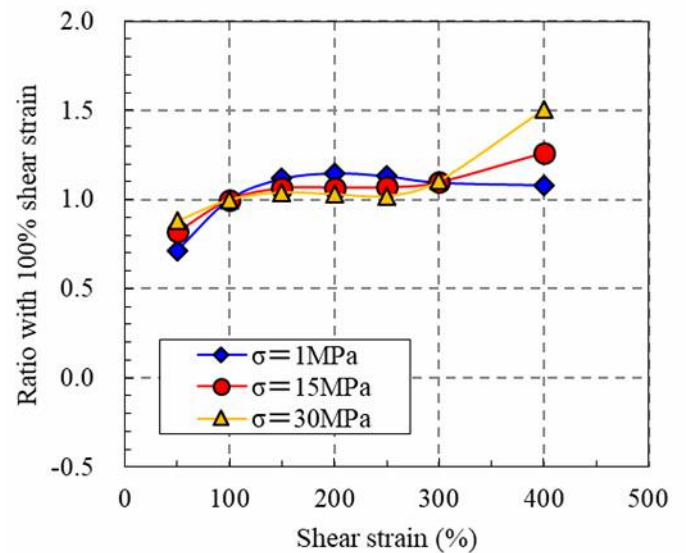


Fig. 5 - Strain dependence of yield load



3. Hysteresis model

For plug insertion type rubber bearings, since it is possible to arbitrarily combine the rubber diameter and the plug diameter, the skeleton curve of the hysteresis model is given by Eq. (1)^[3]

$$F = Q_d + K_d X \quad (1)$$

where, Q_d is the yield load and K_d is the secondary stiffness of the modified bilinear model. Assuming that both Q_d and K_d have a shear strain dependence, as a reference value at the time of 100% shear strain, the correction coefficients CQ_d and CK_d are computed as a function of the shear strain γ , and are evaluated as in Eq. (2) and Eq. (3)

$$Q_d = CQ_d(\gamma) \cdot Q_{d100} \quad (2)$$

$$K_d = CK_d(\gamma) \cdot K_{d100} \quad (3)$$

where, Q_{d100} is the yield load and K_{d100} is the secondary stiffness, respectively at shear strain 100%.

When applying the Kikuchi-Aiken Model to tin rubber bearings, 1 MPa and 15 MPa are used as relevant compressive stress.

The hysteresis curve is expressed by the sum of the nonlinear elastic component F_1 and the hysteresis damping component F_2 as shown in the following equation

$$F = F_1 + F_2 \quad (4)$$

$$F_1 = \frac{1}{2}(1 - u)F_m \{x \pm |x|^n\} \quad (5)$$

$$F_2 = \pm u F_m \left\{ 1 - 2e^{-a(\pm x)} + b(1 \pm x)e^{-c(\pm x)} \right\} \quad (6)$$

where, x is dimensionless displacement ($=X / X_m$), u is the ratio of shear force at zero displacement ($=Q_d / F_m$), and X_m is the peak shear displacement on the skeleton curve, F_m is the peak shear force on the skeleton curve. Parameters a , b , c , n are for manipulating the shape of the hysteresis loop. By changing the parameters in accordance with the shear strain, it is possible to represent the change in the hysteresis loop shape from the spindle shape to the hardening phase. Parameter n represents hardening, and it is possible to increase hardening with $n > 1$. Parameter c is for manipulating the displacement of the maximum point (at the time of positive force) and the minimum point (at the time of negative force) in Eq. (6) of the hysteresis loop. Eq. (7) and Eq. (8) are used to calculate a and b , with the condition that the hysteresis loop area is equivalent to the test result.^[4]

$$\frac{1 - e^{-2a}}{a} = \frac{2u - \pi h_{eq}}{2u} \quad (7)$$

$$b = c^2 \left[\frac{\pi h_{eq}}{u} - \left\{ 2 + \frac{2}{a} (e^{2a} - 1) \right\} \right] \quad (8)$$

where, h_{eq} is an equivalent viscous damping constant.

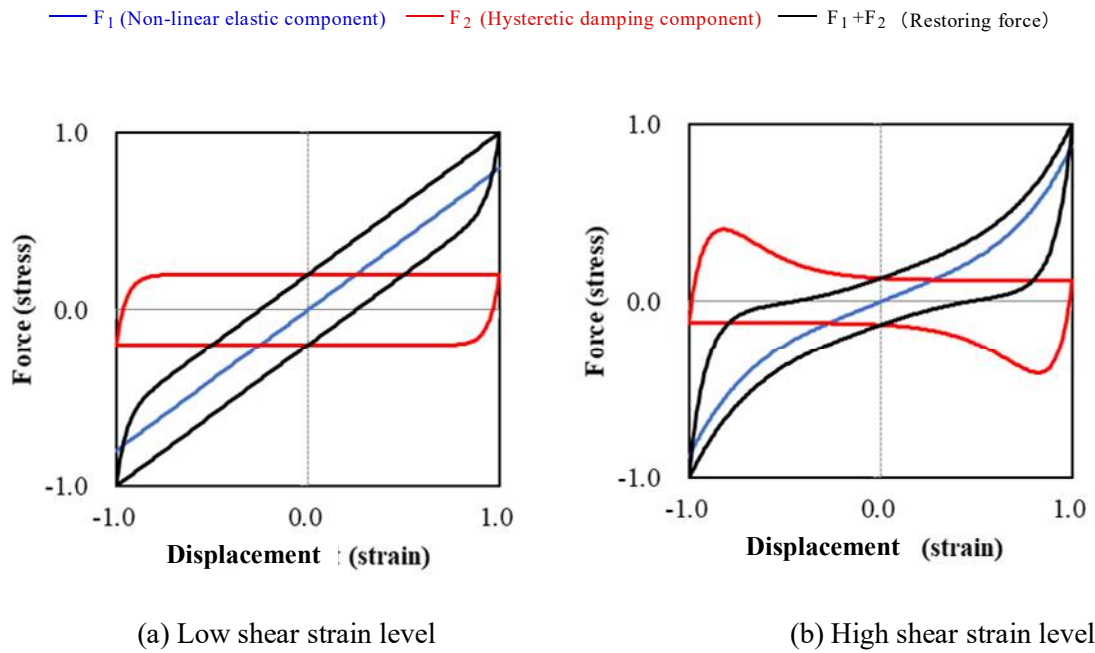


Fig. 6 - Normalized hysteresis loops: Kikuchi-Aiken Model

Initially, the primary stiffness, secondary stiffness and yield load of the binary model at shear strain 100% are determined by the following equations

$$\text{Primary stiffness : } K_1 = 112 \cdot K_2 \quad (9)$$

$$\text{Secondary stiffness : } K_2 = (G \cdot A_r) / H_r \quad (10)$$

$$\text{Yield load : } Q_d = \sigma_p \cdot A_p \quad (11)$$

K_2 and Q_d in any shear strain of γ is determined by the following equations

$$K_2(\gamma) = CK_2(\gamma) \cdot K_2 \quad (12)$$

$$Q_d(\gamma) = CQ_d(\gamma) \cdot Q_d \quad (13)$$



The equation for setting parameters for each compressive stress is as follows.

Compressive stress: 1 MPa

$$CK_2(\gamma) = 1.5835 - 0.00082262\gamma - 0.18229\gamma^2 + 0.037927\gamma^3 \quad (0.05 \leq \gamma \leq 4.0) \quad (14)$$

$$CQ_d(\gamma) = 0.82 \text{ (const.)} \quad (0.05 \leq \gamma \leq 4.0) \quad (15)$$

$$h_{eq}(\gamma) = \eta(0.28268 + 0.0086434\gamma - 0.010423\gamma^2) \quad (0.05 \leq \gamma \leq 4.0) \quad (16)$$

$$n(\gamma) = 1.0 \quad (0.05 \leq \gamma < 2.64) \\ = -2.77641 + 1.4325\gamma \quad (2.64 \leq \gamma < 4.0) \quad (17)$$

a: to be determined by Eq. (7), where a = 28.895(const.) with $\gamma > 1.1$

b: to be determined by Eq. (8), where b = 0.0(const.) with $\gamma \leq 1.1$

Compressive stress: 15 MPa

$$CK_2(\gamma) = 1.7491 - 0.17576\gamma - 0.08086\gamma^2 + 0.019298\gamma^3 \quad (0.05 \leq \gamma \leq 4.0) \quad (18)$$

$$CQ_d(\gamma) = 0.82 \text{ (const.)} \quad (0.05 \leq \gamma \leq 4.0) \quad (19)$$

$$h_{eq}(\gamma) = \eta(0.4072 - 0.085276\gamma + 0.0098413\gamma^2) \quad (0.05 \leq \gamma \leq 4.0) \quad (20)$$

$$n(\gamma) = 1.0 \quad (0.05 \leq \gamma < 2.73) \\ = -1.6624 + 0.97788\gamma \quad (2.73 \leq \gamma < 4.0) \quad (21)$$

a: to be determined by Eq. (7), where a = 29.015(const.) with $\gamma > 0.55$

b: to be determined by Eq. (8), where b = 0.0(const.) with b = 0.0(const.)

$$\eta = \frac{h_{eq,bl}}{h_{eq,500std}} \quad (22)$$

where, η is a correction factor for conforming to any combination of rubber diameter and plug diameter, $h_{eq,bl}$ is an equivalent viscous damping constant at 100% shear strain of the tin rubber bearing, and $h_{eq,500std}$ is an equivalent viscous damping constant of the standard diameter product when the plug diameter is 20% of the rubber diameter, at the time of 100% shear strain. They are determined by Eq. (23).

$$h_{eq} = \frac{2}{\pi} \cdot \frac{Q_d}{Q_d + K_2 H_r \gamma} \left\{ 1 - \frac{Q_d}{(K_1 - K_2) H_r \gamma} \right\} \quad (23)$$



4. Results of the analysis

Fig. 7 shows a comparison of the experimental results and analytical results by the Kikuchi-Aiken Model. It was confirmed that the test results of compressive stress of both sides are satisfactorily reproduced up to 400% shear strain.

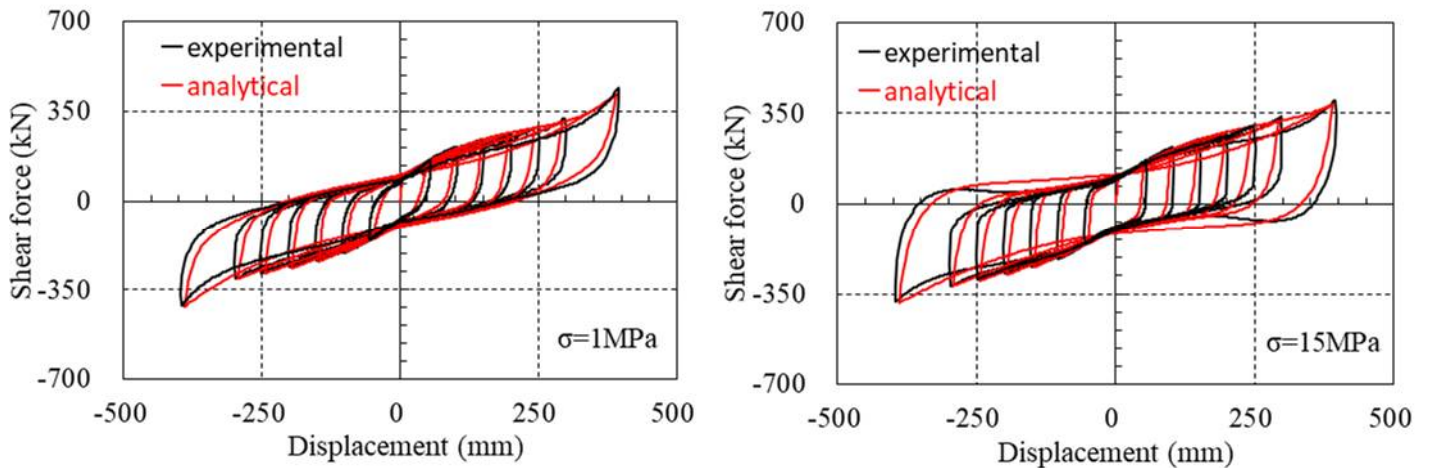


Fig. 7 - Comparison of the experimental results and analytical results

5. Conclusion

For tin rubber bearings, we have created a new restoring force model capable of supporting deformation of up to 400% shear strain. This value corresponds to twice the application limit strain of 250% of the previous modified bilinear model, capable of satisfactorily reproducing the hysteresis loop of strong nonlinear status up to 400%. The current model is designed to respond to both 15 MPa standard compressive stress and 1 MPa in order to eliminate minor compressive stress, but our future mechanical model will be combined with a parallel axis spring model in order to deal with any compressive stress and varying compressive stress.

6. References

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