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OPTIMAL DESIGN OF A TUNED MASS DAMPER INERTER (TMDI) FOR VIBRATION CONTROL OF DAMPED STRUCTURES

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Abstract

A tuned mass damper inerter (TMDI) is a passive vibration absorber which reduces the response of a structure to earthquake ground motions. The TMDI has been shown to outperform the classical tuned mass damper (TMD) in reducing the vibration response of structures. The TMDI couples the TMD with an inerter that is a linear device developing a resisting force proportional to the relative acceleration of its ends. The TMDI utilizes the mass amplification effect of the inerter to improve the performance of a TMD or achieve the same performance with a reduced mass of the TMD, thus achieving a lightweight solution. It has been shown previously that the TMDI reduces the maximum response of a harmonically excited undamped single-degree-of-freedom (SDOF) system and the displacement variance of a white-noise excited undamped SDOF system. In the present study, the optimum TMDI design parameters i.e., structural frequency ratio and damping ratio are obtained for structures equipped with TMDI, while considering the inherent damping. For this purpose, numerical methods are used to determine the optimum parameters which correspond to the minimum response of a damped main mass under harmonic excitations and the minimum response variance of a damped main mass subjected to earthquake ground motions obtained by using a stochastic model. The seismic excitation considered is represented by a spectral model taken from the literature and is an improved random process model of earthquake ground motions developed based on the model proposed by Kanai-Tajimi. A particular site in Japan is chosen for which artificial non-stationary ground motion acceleration records were generated and used to evaluate the parameters of the spectral model. The proposed method involves the solution of a nonlinear system of differential equations for the determination of the optimum design parameters of an SDOF system equipped with a TMDI. In addition, the performance of a TMDI as compared to a tuned mass damper was also evaluated. It was concluded that the structural damping considerably reduces the response of a structure equipped with TMDI and slightly affects the optimum damping and frequency of the TMDI. It was also shown that the incorporation of the inerter enhances the performance of the TMD in reducing the vibration response of a SDOF subjected to earthquake ground motions.

Keywords: inerter; power spectral density model; stochastic excitations; tuned mass damper; vibration absorber.

1. Introduction

Passive vibration absorber devices have been widely used for structural motion control. Tuned mass dampers (TMD) are an example of passive vibration absorbers, which consist of a mass mounted to a main structure using a linear spring and a viscous damper in parallel. Considerable research showed the effectiveness of a TMD in reducing the maximum displacement of an undamped primary mass subjected to harmonic excitations and the displacement variance under stochastic excitations [1, 2].

While the TMD shows good performance when appropriately tuned, it has been shown that it is more effective with a higher mass ratio (e.g. [3]) but increasing the mass ratio leads to an additional mass on the structure.

The inerter was introduced in mechanical networks [4] as a linear device with an internal resisting force F proportional to the relative acceleration between its ends,

$$F = b\left(\frac{d^2u_2}{dt^2} - \frac{d^2u_1}{dt^2}\right), \tag{1}$$

where $u_i\,(i=1,2)$ are the displacements of the ends and b is the inertance. To realize the inerter, the rack and pinion mechanism was first proposed [4]. One end of the inerter is connected to the casing and the other end to a rack whose translational movement causes the rotation of a flywheel. The flywheel then converts the translational kinetic energy into rotational movement. The kinetic energy E_k stored by the flywheel is expressed as,

$$E_k = \frac{1}{2}b\left(\frac{du_2}{dt} - \frac{du_1}{dt}\right)^2. \tag{2}$$

The general expression of the inertance with multiple gears used can be written as,

$$b = m_f \frac{r_f^{\,2}}{r_{pf}^{\,2}} \prod_{i=1}^{n} \alpha_i^2 \geq m_f\,, \tag{3}$$

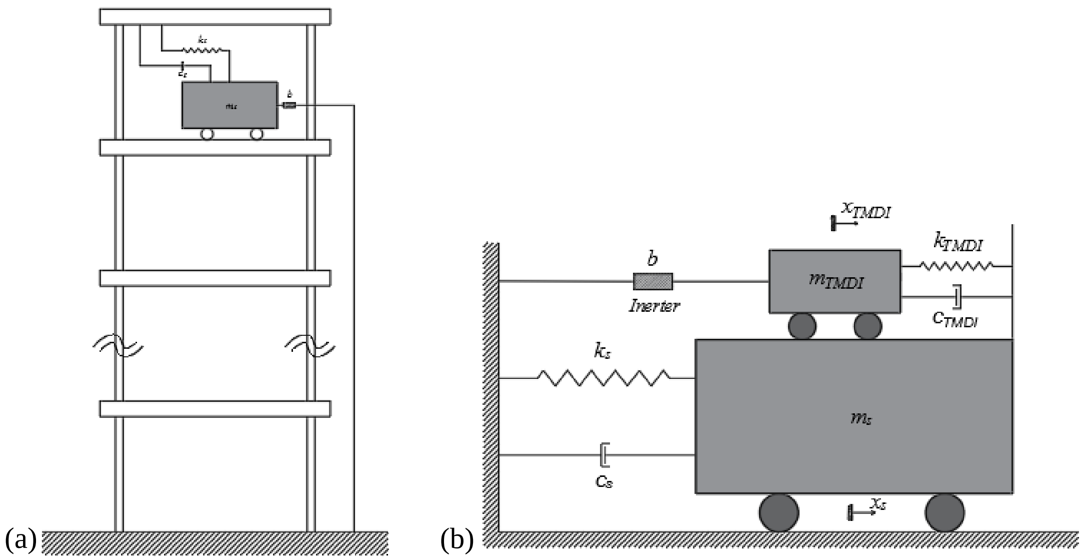

Fig. 1 – (a) Schematic of a tuned mass damper inerter (TMDI) in a structure; (b) SDOF primary system equipped with TMDI.

where r_f and r_{pf} are the radius of the flywheel and the flywheel pinion, respectively, α_i are the gear ratios, and m_f is the mass of the flywheel.

Increasing the number of gears will multiply the inertance by the square of the gear ratio. It can be seen from Eq. (3) that even modest ratios of α result in an inertance that is orders of magnitude larger than the physical mass of the flywheel, and thus the concept of mass amplification.

In combination with TMD, the mechanical oscillations can be reduced without increasing the physical mass of a TMD. The novel passive vibration control configuration is introduced and termed as tuned mass damper inerter (TMDI) by Marian and Giaralis [5]. The TMDI is designed to decrease the maximum displacement of structures under harmonic base excitations and the maximum response variance of structures under stochastic base excitations [5, 6, 7, 8].

The effect of the structural damping of the main structure on the response has been investigated previously for the case of the classical TMD [9, 10] but not for the case of TMDI. Hence, the present study focuses on deriving the optimum design parameters of a TMDI considering the inherent damping of the main structure. For this purpose, a numerical optimization method is adopted and the TMDI optimum design parameters are obtained in terms of the normalized TMDI mass, the normalized inertance and the structural inherent damping ratio. Two input excitations are considered: harmonic excitations and earthquake ground motions.

2. Structural Model of a SDOF System Equipped with TMDI

A primary structure consisting of a SDOF dynamical system is modeled by a mass m_s connected to the ground by a linear spring of stiffness k_s and a linear viscous damper with a damper constant c_s. The arrangement of a TMDI on the main structure is shown in Fig. 1, where m_{TMDI}, k_{TMDI} and c_{TMDI} denote the mass, stiffness and damper constant of the TMDI, respectively. The governing equations can be written in matrix form as,

$$\begin{bmatrix} m_{TMDI}+b & 0 \\ 0 & m_s \end{bmatrix}\begin{bmatrix} \ddot{x}_{TMDI} \\ \ddot{x}_s \end{bmatrix}+\begin{bmatrix} c_{TMDI} & -c_{TMDI} \\ -c_{TMDI} & c_s+c_{TMDI} \end{bmatrix}\begin{bmatrix} \dot{x}_{TMDI} \\ \dot{x}_s \end{bmatrix}+\begin{bmatrix} k_{TMDI} & -k_{TMDI} \\ -k_{TMDI} & k_s+k_{TMDI} \end{bmatrix}\begin{bmatrix} x_{TMDI} \\ x_s \end{bmatrix}=-\begin{bmatrix} m_{TMDI} \\ m_s \end{bmatrix}a_g, \quad (4)$$

where a_g is the acceleration applied to the base of the main structure. Applying Fourier transform to the governing equations, the complex frequency response function (FRF) $G_s(\omega)$ is obtained in the domain of frequency ω as,

$$G_s(\omega)=\frac{\omega_s^4}{\Delta}\left[-\frac{\bar{b}+\bar{m}_{TMDI}}{\bar{m}_{TMDI}}\omega^2+(1+\bar{m}_{TMDI})\omega_{TMDI}^2+2i\xi_{TMDI}(1+\bar{m}_{TMDI})\omega_{TMDI}\omega\right], \quad (5)$$

where

$$\Delta=\left[\frac{\bar{b}+\bar{m}_{TMDI}}{\bar{m}_{TMDI}}\left(\omega^2-\omega_s^2\right)\omega^2+\omega_{TMDI}^2\left(\omega_s^2-\omega^2\right)-\left(\bar{b}+\bar{m}_{TMDI}\right)\omega_{TMDI}^2\omega^2-4\xi_s\xi_{TMDI}\omega_{TMDI}\omega^2\omega_s\right]$$
$$+2i\omega\left[\xi_s\omega_s\left(\omega_{TMDI}^2-\frac{\bar{b}+\bar{m}_{TMDI}}{\bar{m}_{TMDI}}\omega^2\right)+\xi_{TMDI}\omega_{TMDI}\left(1-\omega^2-\left(\bar{b}+\bar{m}_{TMDI}\right)\omega^2\right)\right], \quad (6)$$

and where the normalized TMDI mass $\bar{m}_{TMDI}$, the normalized inertance $\bar{b}$, TMDI frequency ω_{TMDI} and damping coefficients ξ_{TMDI} and ξ_s are defined by,

$$\bar{m}_{TMDI} = \frac{m_{TMDI}}{m_s},\ \bar{b} = \frac{b}{m_s},\ \omega_{TMDI} = \sqrt{\frac{k_{TMDI}}{m_{TMDI}}},\ \xi_{TMDI} = \frac{c_{TMDI}}{2m_{TMDI}\omega_{TMDI}},\ \xi_s = \frac{c_s}{2m_s\omega_s}, \tag{7}$$

respectively. The dynamic amplification factor $|G_s(\omega)|$, defined as the ratio of the amplitude of vibration of the main mass to the amplitude of the input vibration is,

$$|G_s(\omega)| = \sqrt{\frac{\left[(1+\bar{m}_{TMDI})\bar{\omega}_{TMDI}^2 - \frac{\beta+\mu}{\mu}\bar{\omega}^2\right]^2 + 4\xi_{TMDI}^2(1+\bar{m}_{TMDI})^2\bar{\omega}_{TMDI}^2\bar{\omega}^2}{A^2+4\bar{\omega}^2B^2}}, \tag{8}$$

where

$$\bar{\omega}_{TMDI} = \frac{\omega_{TMDI}}{\omega_s},\ \bar{\omega} = \frac{\omega}{\omega_s}, \tag{9}$$

$$A = \bar{\omega}_{TMDI}^2\left(1-\bar{\omega}^2\right) + \frac{\bar{b}+\bar{m}_{TMDI}}{\bar{m}_{TMDI}}\left(\bar{\omega}^2-1\right)\bar{\omega}^2 - \left(\bar{b}+\bar{m}_{TMDI}\right)\bar{\omega}_{TMDI}^2\bar{\omega}^2 - 4\xi_s\xi_{TMDI}\bar{\omega}_{TMDI}\bar{\omega}^2, \tag{10}$$

$$B = \xi_s\left(\bar{\omega}_{TMDI}^2 - \frac{\bar{b}+\bar{m}_{TMDI}}{\bar{m}_{TMDI}}\bar{\omega}^2\right) + \xi_{TMDI}\bar{\omega}_{TMDI}\left(1-\bar{\omega}^2-\left(\bar{b}+\bar{m}_{TMDI}\right)\bar{\omega}^2\right). \tag{11}$$

3. Design of TMDI for SDOF Systems under Harmonic Excitations

For damped main systems, the fixed point method of Den Hartog [11, 12] cannot be used to derive the equations for the optimum parameters as it can be easily proven that the response curves for damped main systems do not pass through two fixed points for a fixed mass and tuning frequency of the TMDI. Therefore, the optimum parameters are determined by minimizing the response peak for harmonically excited systems. To this end, an iterative numerical search method plots the different response curves, as shown in Fig. 2(a), for a specified structural damping ratio ξ_s and normalized TMDI mass $\bar{m}_{TMDI}$, and a set of tuning frequency ratios $\bar{\omega}_{TMDI}$, tuning damping ratios ξ_{TMDI} and normalized inertance $\bar{b}$. For a fixed value of $\bar{b}$ and $\bar{\omega}_{TMDI}$, the maximum response amplitude for each value of ξ_{TMDI} are stored, then the minimum among them is selected. Then, a loop on $\bar{\omega}_{TMDI}$ will assign to each value of $\bar{\omega}_{TMDI}$ the minimax response and its corresponding ξ_{TMDI}. Finally, another loop on $\bar{b}$, followed by one on $\bar{m}_{TMDI}$, will determine for each value of $\bar{b}$ and $\bar{m}_{TMDI}$ respectively, an optimum $\bar{\omega}_{TMDI}$ and ξ_{TMDI} which lead to the minimization of the peak response. This search method is implemented using JAVA programming language.

The dynamic amplification factor $|G_s(\omega)|$ i.e., the amplitude of transfer function, for damped main systems decreases significantly with higher structural damping and normalized inertance as shown in Fig. 2(b). Therefore, it is possible to enhance the performance of a TMDI by increasing the inertance instead of the physical mass of the TMD itself. The plots of Fig. 3 show the optimum parameters i.e., frequency ratio and damping ratio, in terms of normalized inertance for a fixed normalized TMDI mass and structural damping. By using curve fitting techniques, the explicit equations for the optimum frequency ratio $\bar{\omega}_{TMDI}$ and the optimum damping ratio ξ_{TMDI} are expressed as,

$$\bar{\omega}_{TMDI}(\bar{m}_{TMDI}, \bar{b}, \xi_s) = \frac{1}{1+\bar{b}+\bar{m}_{TMDI}}\sqrt{\frac{(1+\bar{m}_{TMDI})(2-\bar{m}_{TMDI})-\bar{m}_{TMDI}\bar{b}}{2(1+\bar{m}_{TMDI})}} + f_1\xi_s + f_2\xi_s^2 + f_3\xi_s^3 + a_{18}\xi_s^4, \quad (12)$$

$$\xi_{TMDI} = \sqrt{\frac{\bar{b}^2\bar{m}_{TMDI} + 6\bar{m}_{TMDI}(1+\bar{m}_{TMDI})^2 + \bar{b}(1+\bar{m}_{TMDI})(6+7\bar{m}_{TMDI})}{8(1+\bar{m}_{TMDI})(1+\bar{b}+\bar{m}_{TMDI})\left[2+\bar{m}_{TMDI}\left(1-\bar{b}-\bar{m}_{TMDI}\right)\right]}} + h_1\xi_s + h_2\xi_s^2 + h_3\xi_s^3 + b_{18}\xi_s^4, \quad (13)$$

where

$$f_1 = \left(a_1 - a_2\bar{m}_{TMDI} + a_3\bar{m}_{TMDI}^2\right)\bar{b} + \left(a_4 + a_5\bar{m}_{TMDI}\right)\bar{b}^2 + a_6\bar{m}_{TMDI} + a_7\bar{m}_{TMDI}^2 + a_8, \quad (14)$$

$$f_2 = a_9 - a_{10}\bar{b} + a_{11}\bar{b}^2 + a_{12}\bar{m}_{TMDI} + a_{13}\bar{m}_{TMDI}\bar{b} + a_{14}\bar{m}_{TMDI}^2, \quad (15)$$

$$f_3 = a_{15} + a_{16}\bar{b} + a_{17}\bar{m}_{TMDI}, \quad (16)$$

$$h_1 = \left(b_1 + b_2\bar{m}_{TMDI} + b_3\bar{m}_{TMDI}^2\right)\bar{b} + \left(b_4 + b_5\bar{m}_{TMDI}\right)\bar{b}^2 + b_6\bar{m}_{TMDI} + b_7\bar{m}_{TMDI}^2 + b_8, \quad (17)$$

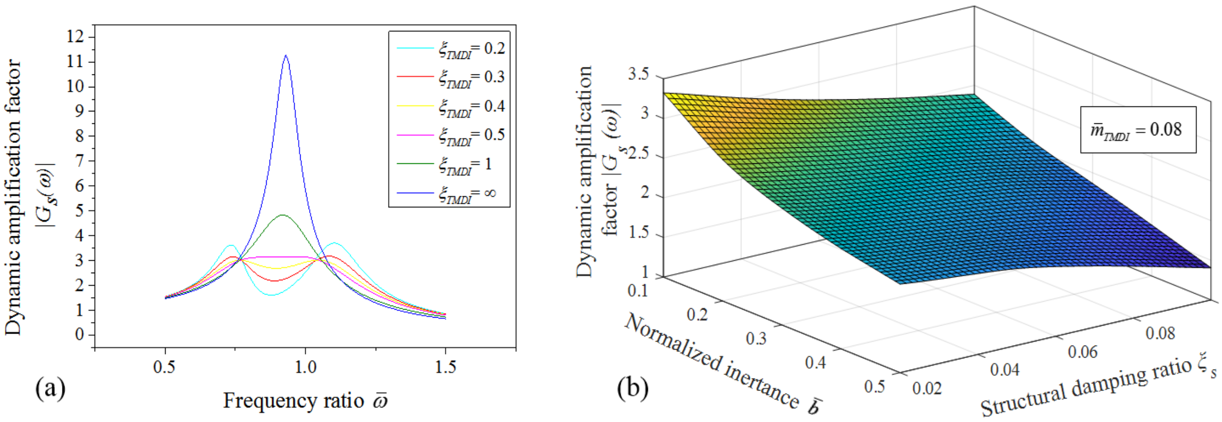

Fig. 2 – (a) Response curves of a damped SDOF system with a TMDI for different tuning damping ratio; (b) Dynamic amplification factor in function of the normalized inertance and the structural damping ratio.

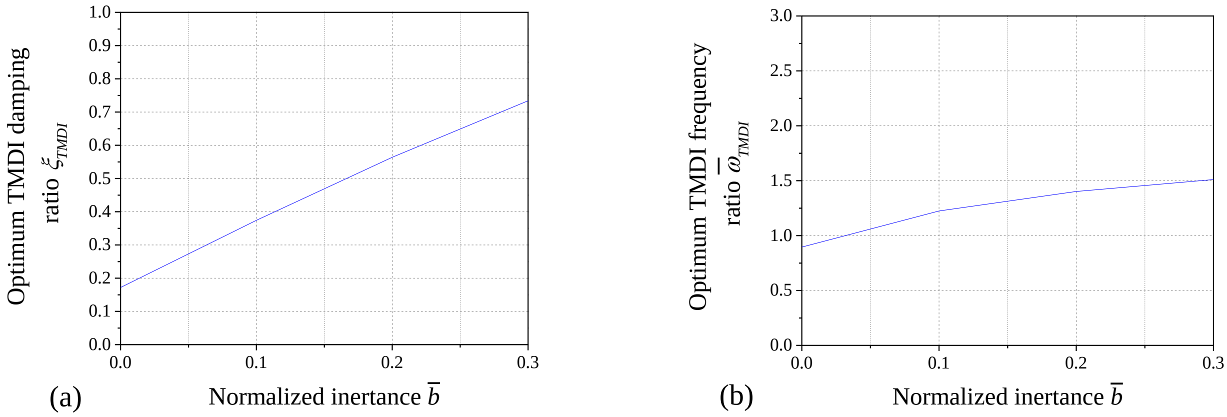

Fig. 3 – (a) Optimum TMDI frequency and (b) damping ratio for a normalized TMDI mass of 8% and structural damping ratio of 2%.

$$h_2 = b_9 + b_{10}\bar{b} + b_{11}\bar{b}^2 + b_{12}\bar{m}_{TMDI} + b_{13}\bar{m}_{TMDI}\bar{b} + b_{14}\bar{m}_{TMDI}^2 \,, \tag{18}$$

$$h_3 = b_{15} + b_{16}\bar{b} + b_{17}\bar{m}_{TMDI} \,, \tag{19}$$

$$a_i = \{305.3,\ -1370.4,\ 2738.3,\ -169.37,\ 365.27,\ -1780.5,\ 8247.8,\ 112.8,\ -2986.19,\ -4286.1,$$
$$1215.9,\ 25444.4,\ 8829.8,\ 61324.6,\ 318883.6,\ 17953.9,\ -109507.6,\ 120240.7\},$$
$$b_i = \{-0.1,\ 40.9,\ -168.2,\ -1.1,\ -19.1,\ 15.8,\ 76.3,\ 1.1,\ -20.7,\ 9.0,\ 17.5,\ 268.3,\ -88.6,\ 1964.4,$$
$$270.6,\ -142.3,\ -620.3,\ -11730\}.$$

4. Design of TMDI for SDOF Systems under Seismic Excitations

4.1 Earthquake ground motions for Hino, Tottori

The Kanai-Tajimi model [13, 14] assumes that the ground acceleration is an ideal white noise at bedrock level, which is filtered by the overlaying soil layers. The soil is modelled by a mass m_g connected to the bedrock by a Kelvin-Voigt contact element (Fig. 4). The elastic spring and viscous damper of the Kelvin-Voigt model reflect the visco-elastic property of the ground. In order to make the Kanai-Tajimi model non-stationary Fan and Ahmadi [15] proposed that the predominant ground frequency is considered as a time dependent function and introduced an amplitude envelope function $e(t)$. The coupled system of differential equations can be written as,

$$\frac{d^2 x_f}{dt^2} + 2\xi_g \omega_g(t)\frac{dx_f}{dt} + \omega_g^2(t)x_f = -n(t) \,, \tag{20}$$

$$\frac{d^2 x_g}{dt^2} = -\left(2\xi_g \omega_g(t)\frac{dx_f}{dt} + \omega_g^2(t)x_f\right)e(t) \,, \tag{21}$$

where $n(t)$ is a white noise process, $x_f(t)$ is the filtered response, ξ_g is the damping ratio, $\omega_g(t)$ is the frequency of the ground. The parameters ξ_g and $\omega_g(t)$ are obtained from previous earthquake records or the geological features of the site. The white noise process is a stationary Gaussian process whose expected values are

$$E[n(t)] = 0 \,, \tag{22}$$

$$E[n(t_1)n(t_2)] = -2\pi S_0 \delta(t_1 - t_2) \,, \tag{23}$$

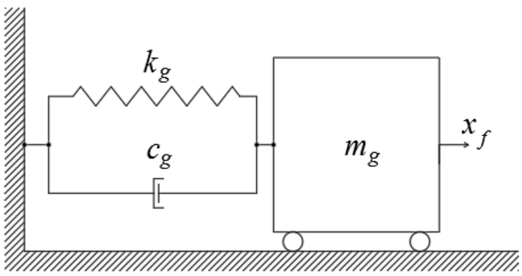

Fig. 4 – Oscillator representing the soil layers between the bedrock and the ground surface.

where δ is the Dirac delta function and S_0 is the power spectral density at the bedrock level (constant power spectrum).

Using the coupled differential equations and a moving time window technique [16, 17], artificial ground records are generated for the NS component of the 2000 Tottori earthquake using the record of Fig. 5 [18]. The ground motion duration corresponds to the window containing the 5-95% of the Arias intensity,

$$I_A = \frac{\pi}{2g}\int_0^T a^2(t)dt\,. \tag{24}$$

The first 7.62 s of the record is taken and using a time averaging procedure, which uses a time window moving along the record, the standard deviation of the ground acceleration is obtained from

$$\sigma(t) = \left(E[X^2] - E[X]^2\right)^{0.5}. \tag{25}$$

The standard deviation is then fitted to a smooth algebraic equation expressed by a third order Gaussian. Then, the amplitude envelope function is given by,

$$e(t) = C_0\sigma(t)\,. \tag{26}$$

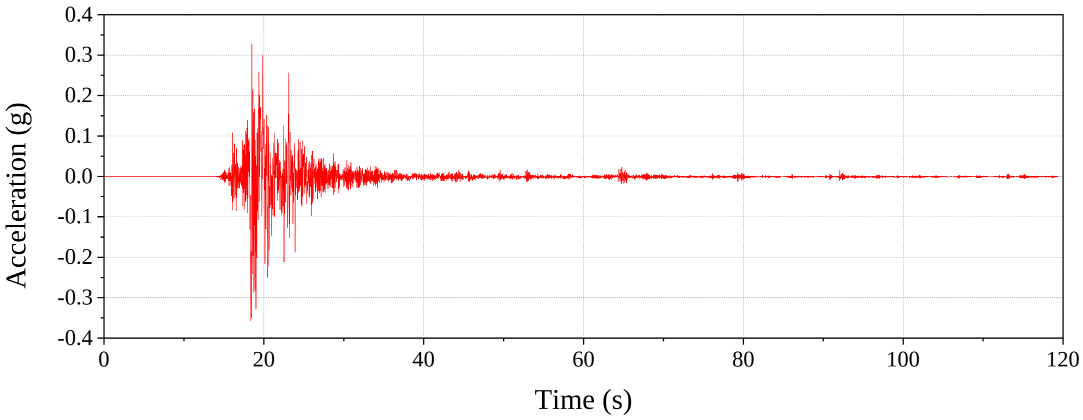

Fig. 5 – Ground motion record for the 2000 Tottori earthquake (M_ω 7.3), Hino station, Japan.

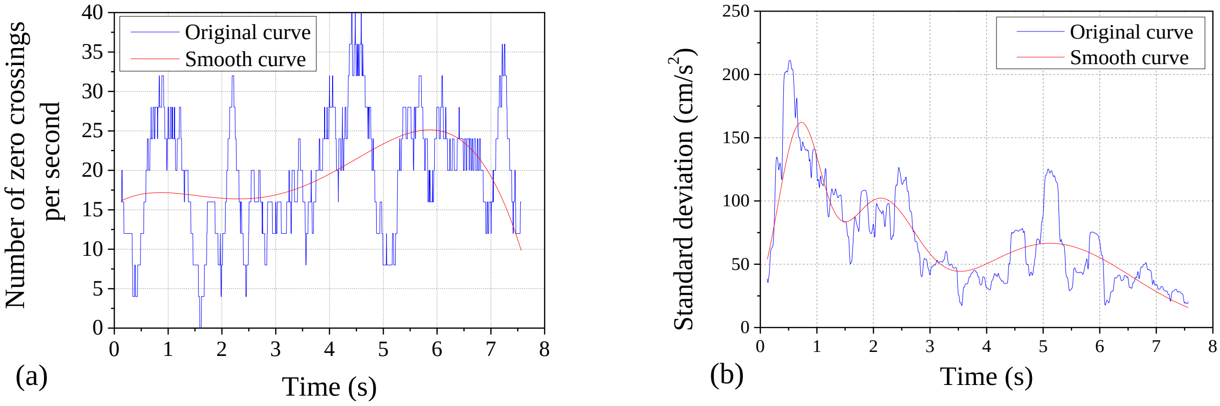

Fig. 6 – (a) Standard deviation, and (b) rate of zero crossing for the 2000 Tottori earthquake (M_ω 7.3) ground motion record.

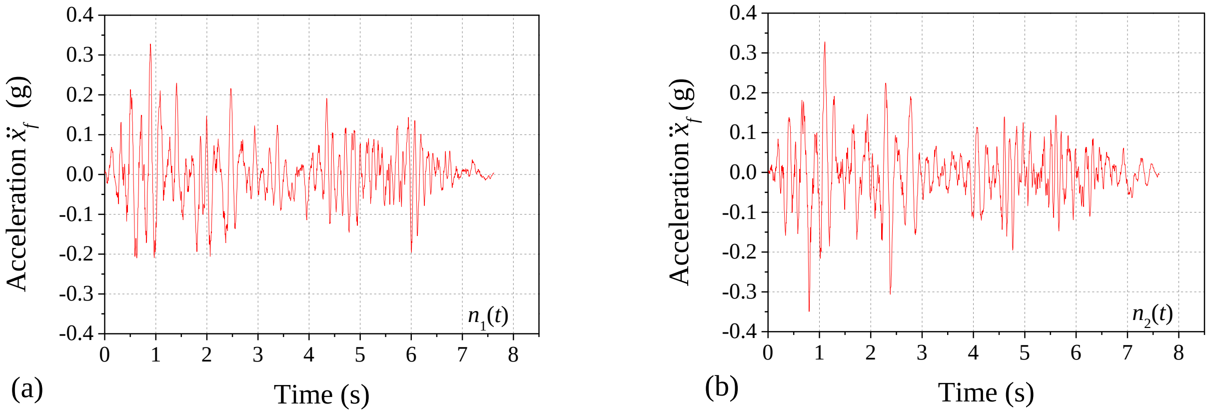

Fig. 7 – Artificial ground motion records, based on (a) $n_1(t)$ and (b) $n_2(t)$, generated for Hino, Tottori.

In order to incorporate the non-stationarity of the frequency content, the zero crossing rate *F(t)* is plotted and smoothened using a fourth order polynomial. The ground frequency function is given by,

$$\omega_g(t) = \pi F(t) . \tag{27}$$

The standard deviation and the rate of zero crossing are shown in Fig. 6.

A white noise process is used to generate the synthetic accelerograms after numerical integration of Eq. (20) and Eq. (21). Two synthetic accelerograms are shown in Fig. 7. The generated records preserve the amplitude and frequency content of the original record.

4.2 Optimum parameters $\bar{\omega}_{TMDI}$ and ξ_{TMDI}

Considering the stochastic excitation represented as a double sided spectral density function by Eq. (28), and applying it to the system of Fig. 1, the variance of the relative displacement of the primary structure is expressed as,

$$\sigma_s^{\ 2} = \int_{-\infty}^{+\infty} \left|G_s(\omega)\right|^2 S_X(\omega) d\omega , \tag{28}$$

where $\left|G_s(\omega)\right|$ is the modulus of the frequency response function defined in Eq. (8) and $S_X(\omega)$ is the power spectral density of the ground motion records. The optimization problem consists of minimizing $\sigma_s^{\ 2}$ subject to $\left[k_{TMDI}, c_{TMDI}\right]$ or equivalently $\left[\bar{\omega}_{TMDI}, \xi_{TMDI}\right]$.

For the numerical evaluation of the integral in Eq. (28), an adaptive Gauss-Konrod method [19] is used. The normalized response variance is plotted as a function of the normalized TMDI mass for different normalized inertance in Fig. 8(a) and as a function of the normalized inertance for different normalized TMDI mass in Fig. 8(b). It can be seen from Fig. 8 that increasing the inertance and/or the TMDI mass decreases the response variance. It is also observed that the mass amplification effect of the TMDI, which allows to increase the effective mass without increasing the physical mass, is more pronounced for lower mass. In addition, in contrast to a TMD ($\bar{b} = 0$), the response variance is more sensitive to the variation of the inertance than that of the TMDI mass.

The optimum TMDI parameters as a function of the normalized inertance and for different normalized TMDI mass are plotted in Fig. 9. The normalized response variance in function of the structural damping ratio for different normalized TMDI mass is plotted in Fig. 10. It can be seen from Fig. 10 that the structural inherent damping of the main mass significantly decreases response variance.

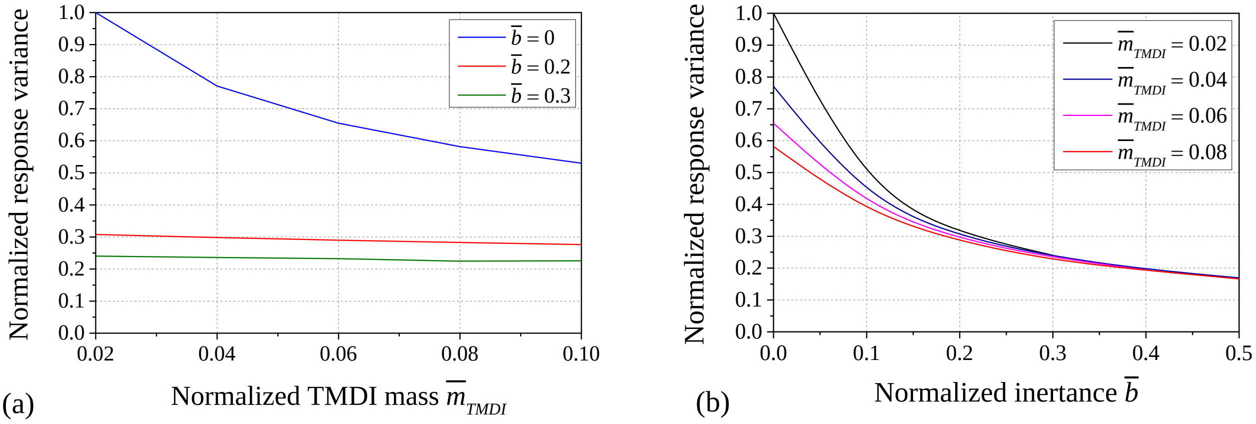

Fig. 8 – Normalized response variance (a) in function of the normalized TMDI mass $\bar{m}_{TMDI}$ for different normalized inertance $\bar{b}$, and (b) in function of normalized inertance $\bar{b}$ for different normalized TMDI mass $\bar{m}_{TMDI}$ and for 4% structural damping.

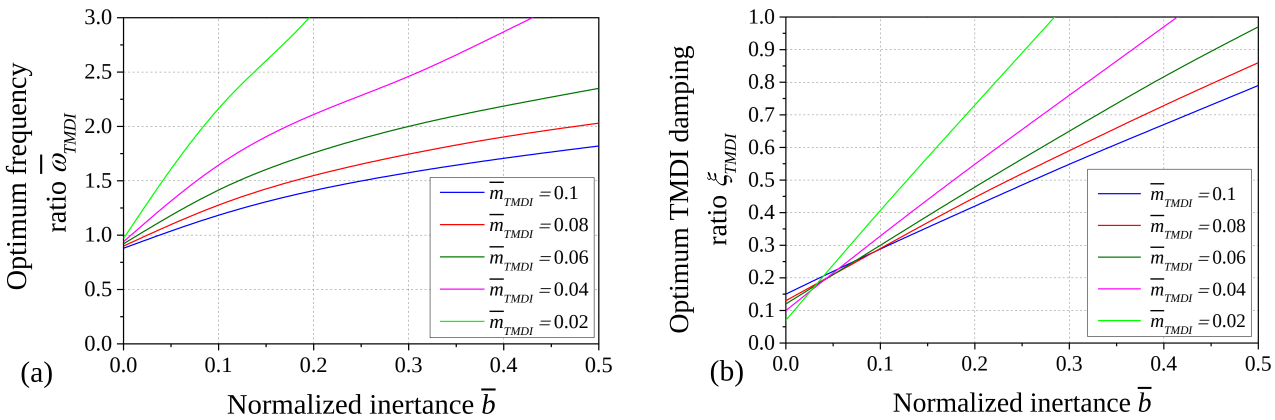

Fig. 9 – (a) Optimum frequency ratio $\bar{\omega}_{TMDI}$, and (b) optimum damping ratio ξ_{TMDI} in function of the normalized inertance $\bar{b}$ for different normalized TMDI mass $\bar{m}_{TMDI}$ and for 4% structural damping.

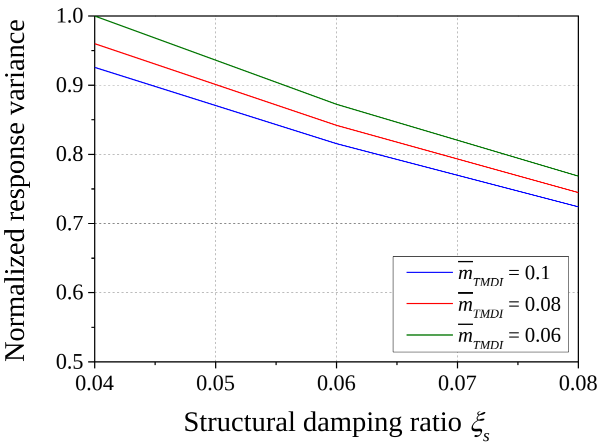

Fig. 10 – Normalized response variance in function of the inherent structural damping ratio ξ_s for different normalized TMDI mass $\bar{m}_{TMDI}$.

In view of the discussed results, the TMDI is proven to be more effective in reducing the response variance of a structure subjected to earthquake ground motions. It can also be seen that the inherent damping of the main structure significantly affects the response variance and thus should be taken into consideration when determining the desired maximum allowable response. The optimum tuning parameters on the other hand are not significantly sensitive to the inherent damping of the main structure.

5. Conclusions

The optimum parameters of a TMDI on a damped SDOF system subjected to harmonic excitations and to earthquake ground motions were obtained. The considered method accounts for the structural damping of the main mass and the earthquake excitations were represented by a power spectral density model. In this regard, the optimum parameters were obtained by deriving the governing differential equations for a damped SDOF system equipped with a TMDI to minimize the fundamental mode of vibration. Referring to the results, it has been concluded that the inerter significantly decreases the response of a SDOF system when subjected to earthquake ground motions, thus resulting in a lower required TMDI mass.

For the evaluation of the optimum TMDI parameters, neglecting the structural damping provides a good estimation of the parameters, when stochastic excitations are considered. But incorporating the structural damping significantly reduces the response variance. Therefore, neglecting its contribution results in an overestimation of the damping forces demand.

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