DIFFERENTIAL EVOLUTION FOR THE OPTIMAL DESIGN OF TUNED MASS DAMPERS IN A BUILDING OF MEDELLIN CITY

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Abstract

This paper presents a numerical investigation on the optimal design parameters for a tuned mass damper (TMD) using a metaheuristic optimization based on the differential evolution method to reduce the dynamic effects in a structural system caused by seismic loads. It was used for the study the structural model of the *Cantagirone Tre Piu* Building, which is a 144 m height structure, representing the tallest residential building in Medellin. The tuning process is focused on the optimization of three strategic parameters that are analyzed individually as objective functions: minimization of the horizontal peak displacements; minimization of the root mean square (RMS) response for displacements and minimization of the horizontal peak floor acceleration. Eight seismic acceleration records were employed to simulate seismic actions; hence the optimization process and the building performance was evaluated. As part of the results, the dynamic response of the building is significantly enhanced for the three optimization cases studied, improving the values obtained with the optimal parameters derived from other investigations up to 20%. Finally, based on the results obtained in this research, practical design recommendations are provided for TMDs systems with 2% and 5% of attached mass respect to the total mass of the structural system.

Keywords: Tuned mass damper; metaheuristic optimization; differential evolution; optimum design parameters.

1. Introduction

Infrastructure development is a notable feature in countries with emerging economies, as a result, the design, and subsequent construction of high-rise buildings is becoming more common. Countries like Colombia, present great uncertainty regarding the seismic risk, therefore, conventional design methodologies based on stresses, and forces may result insufficient for such structures [1]. Passive control systems are widely accepted by the engineering community, among which the tuned mass damper (TMD) is one of the most studied, and tested devices for the control of vibrations, showing favorable reductions for wind-induced vibrations. However, less significant decreasing has been observed under seismic excitations, this behavior is related to the fact that the design procedure for TMDs, is generally based on works where the tuning is carried out for single degree of freedom systems subject to mono-frequency excitations, as harmonic loads [2], white noise processes [3], or frequency domain analysis [4], which by their characteristics correspond better with dynamic wind loads than seismic action.

Since the early 2000s, novel optimization methodologies based on numerical iterative methods, have been utilized to obtained the tuning parameters for TMDs. Classic bio-inspired metaheuristics have been employed for that purpose such as particle swarm optimization [5,6]; ant colony [7,8]; bat algorithm [9]; and cuckoo search [10]. Furthermore, genetic algorithms, and more recently fuzzy logic combined with machine learning, have been applied by other authors for optimization approaches [11-14]. Nevertheless, some of these techniques represent high computational complexity, and besides, most of these works still face the optimization problem considering loads that do not reproduce seismic action appropriately.

In this paper, a novel metaheuristic based on the differential evolution method (DEM) is applied to a numerical model derived from the 144 m high *Cantagirone Tre Piu* Building, which is the tallest residential building in Medellin Colombia, to solve the TMD optimization problem. Unlike previous works, where the optimization algorithms use equations for the objective function, DEM is combined with an elastic time history analysis in the time domain, in which three strategic parameters are analyzed individually as objective functions: minimization of the horizontal peak displacements, minimization of RMS displacement values, and minimization of the horizontal peak floor acceleration; the model is subjected to eight seismic acceleration records (Chile, El Centro, Italy, Kobe, Loma Prieta, Mexico, Northridge, and Virginia). The results obtained by the approach proposed herein are compared against those obtained by Den Hartog [2], Warburton [3] and Sadek *et al*. [4], showing significant improvements of up to 20% reduction of the structural response. Based upon these results, practical design recommendations are provided for TMDs systems with 2% and 5% of attached mass respect to the total mass of the structural system.

2. TMD design and optimization procedure

The original idea of the TMD was introduced by Frahm [15] as a vibration absorber with no damping associated, to control periodic resonance vibrations. Later, Ormondroyd [16] developed the theory of the dynamic vibration absorber, including viscous damping to the system, to be effective under different frequencies of random vibrations. From there, the design process has been based on finding the optimal frequency ratio (f), and damping ratio (ζ_d) represented by Eq. (1), and Eq. (2):

$$f=\frac{\omega_d}{\omega_1}=\frac{\sqrt{k_d/m_d}}{\omega_1} \tag{1}$$

$$\zeta_d=\frac{c_d}{2\omega_d m_d} \tag{2}$$

where ω_d, ω_1, k_d, m_d, and c_d are the optimum frequency of the TMD, fundamental circular frequency of the main structure, TMD stiffness coefficient, TMD mass, and TMD damping coefficient respectively.

These parameters are deduced from a system in which the main structure is idealized as a single degree-of-freedom (SDOF) system, as illustrated in Fig.1:

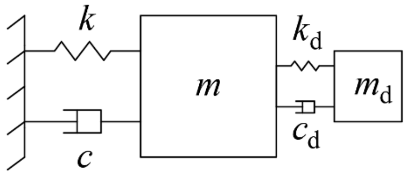


Fig. 1 – Model of SDOF system and TMD

where k, c, and m are the stiffness coefficient, damping coefficient, and mass of the SDOF system.

Several authors have focused on searching the parameters f, and ζ_d, developing closed form expressions to determine these parameters for some types of mono-frequency excitations. Of these works, three of the most outstanding are taken as references to compare against the results obtained through DEM:

Table 1 – *f, and* ζ_d closed form of the compared methods

Author	f	ζ_d	**Excitation type**
Den Hartog [2]	$\frac{1}{1+\mu}$ (3)	$\sqrt{\frac{3\mu}{8(1+\mu)}}$ (4)	Harmonic
Warburton [3]	$\frac{\sqrt{1+\mu/2}}{1+\mu}$ (5)	$\frac{\sqrt{\mu\left(1+3\mu/4\right)}}{\sqrt{4(1+\mu)\left(1+\mu/2\right)}}$ (6)	White noise
Sadek *et al.* [4]	$\frac{1}{1+\mu}\left[1-\zeta\sqrt{\frac{\mu}{1+\mu}}\right]$ (7)	$\frac{\zeta}{1+\mu}+\sqrt{\frac{\mu}{1+\mu}}$ (8)	Frequency domain analyses

where μ is the mass ratio of the TMD, and ζ is the damping ratio of the main structure.

As Eqs. (3)-(8) were developed for a SDOF system, a multi-degree of freedom (MDOF) system must be simplified as a SDOF system. Therefore, the modal mass for the first critical frequency is taken as the mass of the structure in calculating the mass ratio:

$$\mu=\frac{m_d}{\phi_1^{T}\mathbf{M}\,\phi_1}\Phi \tag{9}$$

where ϕ_1 is the first mode shape, **M** is the mass matrix of the MDOF system, and Φ is the amplitude of the first mode shape at the TMD location.

However, adopting these closed forms on the TMD design, in order to control dynamic vibration induced by seismic loads in real building models, may result in an unrealistic idealization. Thus, in order to find the optimal TMD design parameters, DEM is applied. This method, introduced by Storn and Price [17], is a metaheuristic part of evolutionary computing oriented to the optimization problem of real variables in continuous fields. Since the approach proposed in this investigation is aimed to solve the optimization problem for multi-story building models, DEM is combined with an elastic time history analysis in the time domain, in such a way that the system can be subjected to any random excitations such as seismic actions simulated by an accelerogram ($\ddot{\mathbf{u}}_g$), as represents the following movement equation:

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{C}\dot{\mathbf{U}}(t)+\mathbf{K}\mathbf{U}(t)=-\mathbf{M}\{\mathbf{1}\}\ddot{\mathbf{u}}_g(t) \tag{10}$$

where **M**, **C**, and **K** are the mass, damping, and stiffness matrices, modified by the addition of the TMD, as illustrated in Eqs. (11)-(13):

$$\mathbf{M}_{(n+1,n+1)}=\begin{bmatrix} m_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m_{n-1} & 0 & 0 \\ 0 & 0 & \cdots & 0 & m_n & 0 \\ 0 & 0 & \cdots & 0 & 0 & m_d \end{bmatrix} \tag{11}$$

$$\mathbf{C}_{(n+1,n+1)}=\begin{bmatrix} c_1+c_2 & -c_2 & \cdots & 0 & 0 & 0 \\ -c_2 & c_2+c_3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{n-1}+c_n & -c_n & 0 \\ 0 & 0 & \cdots & -c_n & c_n+c_d & -c_d \\ 0 & 0 & \cdots & 0 & -c_d & c_d \end{bmatrix} \tag{12}$$

$$\mathbf{K}_{(n+1,n+1)}=\begin{bmatrix} k_1+k_2 & -k_2 & \cdots & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{n-1}+k_n & -k_n & 0 \\ 0 & 0 & \cdots & -k_n & k_n+k_d & -k_d \\ 0 & 0 & \cdots & 0 & -k_d & k_d \end{bmatrix} \tag{13}$$

where m_i, k_i, and c_i are the mass, lateral stiffness, and damping of i^{th} floor (i=1, 2…n). In addition, **U** is defined as the response vector for the structure:

$$\mathbf{U}=\{u_1 \quad u_2 \quad \ldots u_{n-1} \quad u_n \quad u_d\}^T \tag{14}$$

where u_d is the displacement of the TMD, and the dots on **U** represent the derivatives of the function in time.

The applied procedure for DEM can be summarized in four fundamental steps: Initialization, mutation, crossing, and selection. For initialization, a population of solution vectors with a size between 5 to 10 times the dimensions of the function to be optimized is usually taken. In this sense, TMD optimization is a two-dimensional problem, in which the search is focused on finding the optimal f and ζ_d parameters, for a fixed m_d value. In order to search for design parameters in accordance with practical engineering, the search domain is defined as:

$$0.50 \leq f \leq 2.00 \tag{15}$$

$$0 \leq \zeta_d \leq 0.50 \tag{16}$$

Now, in the mutation a randomly chosen population vector is disturbed with the proportional difference of two randomly chosen vectors defined in Eq. (17):

$$w_i=v_1+F(v_2-v_3) \tag{17}$$

where w_i is the mutated vector for each ith iteration; v_1, v_2 and v_3 are randomly chosen vectors of the previous iteration and F is the mutation constant that meets the conditions $F > 0$, and $F \in [0,1]$.

Next, crossing is applied, generating a z_i vector according to Eq. (18), that comes from the combination of the v_i y w_i vectors positions subject to a probability of crossing or recombination.

$$z_i(j)=\begin{cases} w_i(j) \text{ if } rand \leq C_r \\ v_i(j) \text{ otherwise} \end{cases} \tag{18}$$

where *rand* represents a randomly chosen real number between 0 and 1 and C_r the crossing or recombination constant, that meets the conditions $C_r > 0$, and $C_r \in [0,1]$.

Finally, selection is achieved by evaluating the z_i vector in the cost function. The cost function programmed in the algorithm consists of evaluating the ratio between the controlled response via TMD and the uncontrolled response of the system focused at the n^{th} degree of freedom. This ratio is defined as the performance index (PI); hence the improvement obtained is inversely proportional to the value of PI:

$$PI=\frac{\max(n^{\text{th}}\text{ DOF controlled response})}{\max(n^{\text{th}}\text{ DOF uncontrolled response})} \tag{19}$$

To evaluate the uncontrolled response, the dynamic analysis described in Eq. (10) is performed, but the additional DOF (n+1) added by the TMD is removed. The structural response evaluation of the TMD equipped system is based on three strategic parameters that are analyzed individually as objective functions: minimization of the horizontal peak displacement, minimization of the RMS displacement response, and minimization of the horizontal peak floor acceleration. Therefore, there are three optimization approaches defined by:

$$F_{obj1}=\max|u_n| \tag{20}$$

$$F_{obj2}=RMS(u_n) \tag{21}$$

$$F_{obj3}=\max|\ddot{u}_n| \tag{22}$$

If a better result is obtained after evaluating the cost function, the vector either goes to the next generation, otherwise, the previous vector is retained and then the process is repeated until the convergence of the cost function is achieved. The flowchart in Fig.2 describes the optimization methodology:

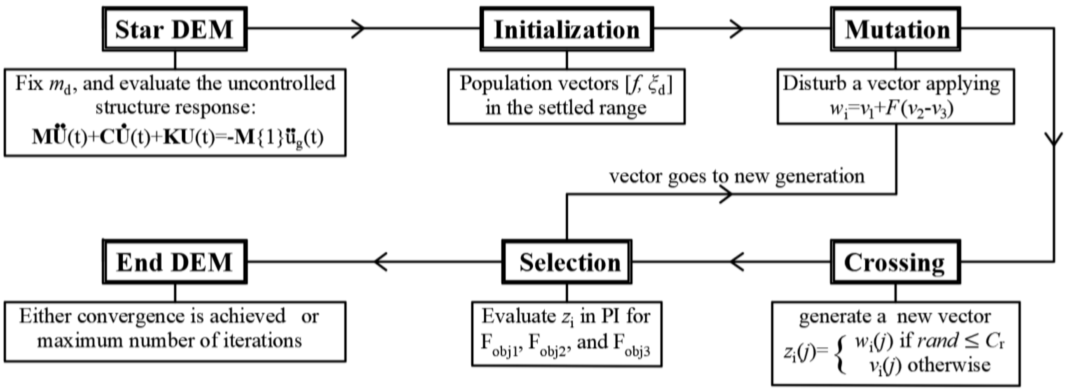
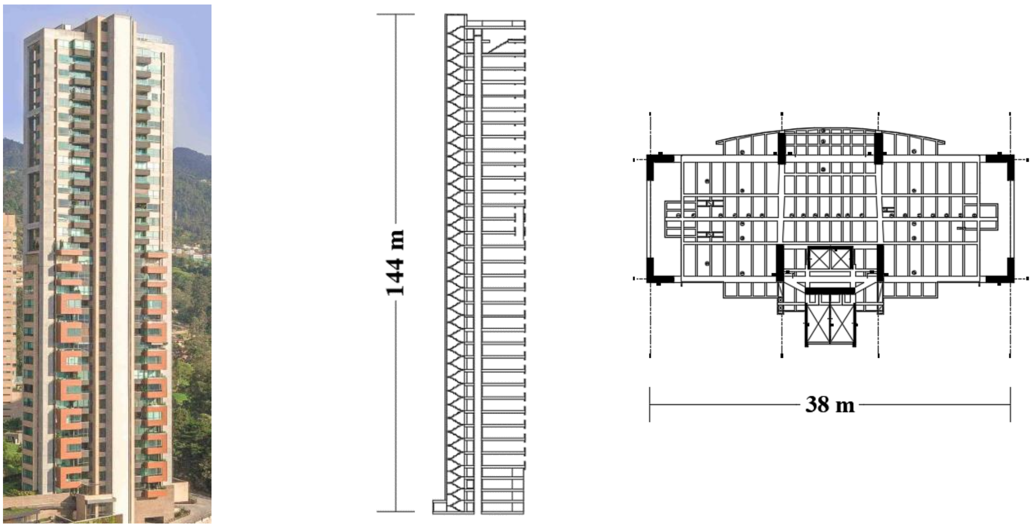


Fig. 2 – Flowchart of the optimization methodology

3. Case-study and earthquake input data

3.1 Case Study

Fig.3 (a) shows an actual view of the *Cantagirone Tre Piú* building. The 37-story building was built with prestressed concrete, and it has a total of 144 m height, which can be seen in Fig. 3(b). The lateral force resisting system consists on resistant moment frames combined with structural walls located at the building corners to increase the structure stiffness. The building is more flexible in the longitudinal direction, in-plan view, which spans over 38 m, as shown in Fig.3 (c), which is why this direction was selected for the analysis in the TMD study.



(a) *Cantagirone Tre Piú* Building (b) Elevation (c) Plan: Analysis direction

Fig. 3 – Case study building

A typical frame was analyzed, with a total mass of 8053.60 Mg distributed at each story of the superstructure according with the architectural distribution. The resulting mass, and stiffness matrix are 37x37 size, considering 37 horizontal degrees of freedom (one at each level), obtained by assuming in-plane infinitely rigid floor diaphragms and applying static condensation on the remaining vertical and rotational degrees of freedom. The damping matrix, **C**, was evaluated using Rayleigh's method for 5% structural damping in the first and second vibration modes. To characterize the model, a modal analysis was performed with a routine developed in Matlab Simulink [18]. Values of period (*T*), frequency (*F*), circular frequency (ω), and mass participation in the direction of analysis (UX), are shown in Table 2 until the 6th vibration mode, for which a mass participation greater than 90% is achieved.

Table 2 – Modal analysis of the case-study building

Mode	*T* [s]	*F* [Hz]	ω [rad/s]	UX	Σ UX
1	5.4692	0.1828	1.1488	0.6800	0.6800
2	1.6425	0.6088	3.8254	0.1100	0.7900
3	0.8276	1.2083	7.5917	0.0496	0.8396
4	0.5033	1.9869	12.4838	0.0341	0.8737
5	0.3364	2.9722	18.6752	0.0262	0.8999
6	0.2416	4.1385	26.0030	0.0210	0.9209

Fig.4 shows the first three mode shapes for the analysis direction modelled.

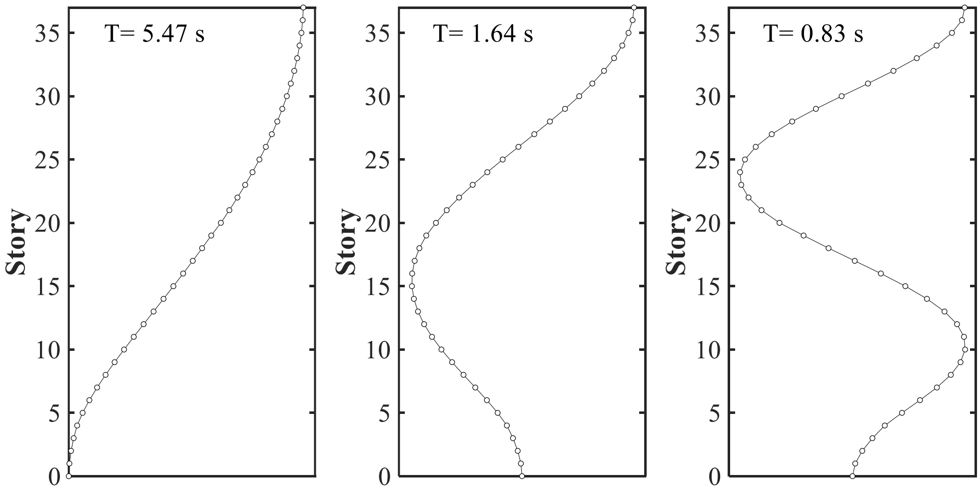


Fig. 4 – First three mode shapes of the modelled building

3.2 Earthquake input data

A total of eight seismic acceleration records, were selected to simulate the seismic action in the optimization approach. The accelerograms are numbered in Table 3 and described by the name of the event, peak ground acceleration (PGA), and duration.

Table 3 – Acceleration records used in optimization approach.

Seismic Record	Event name	PGA [g]	Duration [s]
1	Chile	0.3627	56.35
2	El Centro	0.3188	31.16
3	Italia	0.9280	40.00
4	Kobe	0.6791	29.99
5	Loma Prieta	0.4720	40.00
6	Mexico	0.1712	180.00
7	Northridge	0.8306	47.82
8	Virginia	0.4536	40.96

These acceleration records present very diverse frequency content, which turns into an ideal condition to achieve a more realistic tuning process. The frequency content is illustrated in Fig.5 using the frequency spectrum of the earthquakes, calculated by applying the Fast Fourier Transform (FFT).

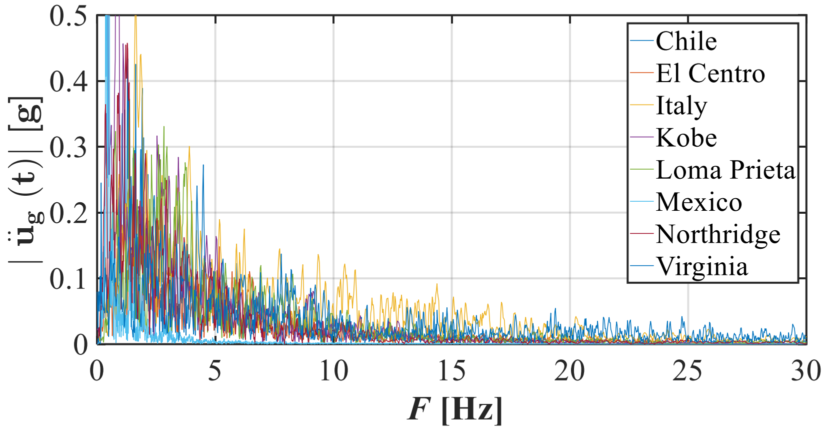


Fig. 5 – Seismic records FFT

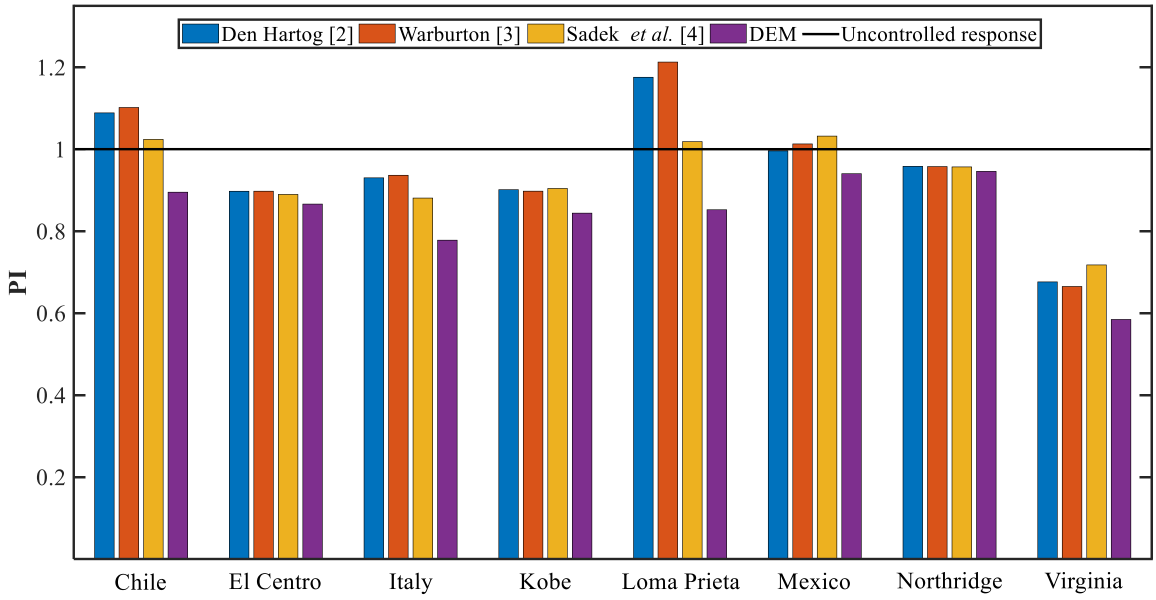
4. Results and discussion

Optimum results obtained after applying DEM, are summarized in Table 4. The optimization was carried out considering fixed m_d values of 161.07 Mg and 402.68 Mg, which represent 2% and 5% mass ratios with respect to the structural system mass, respectively, for a total of 48 numerical evaluations.

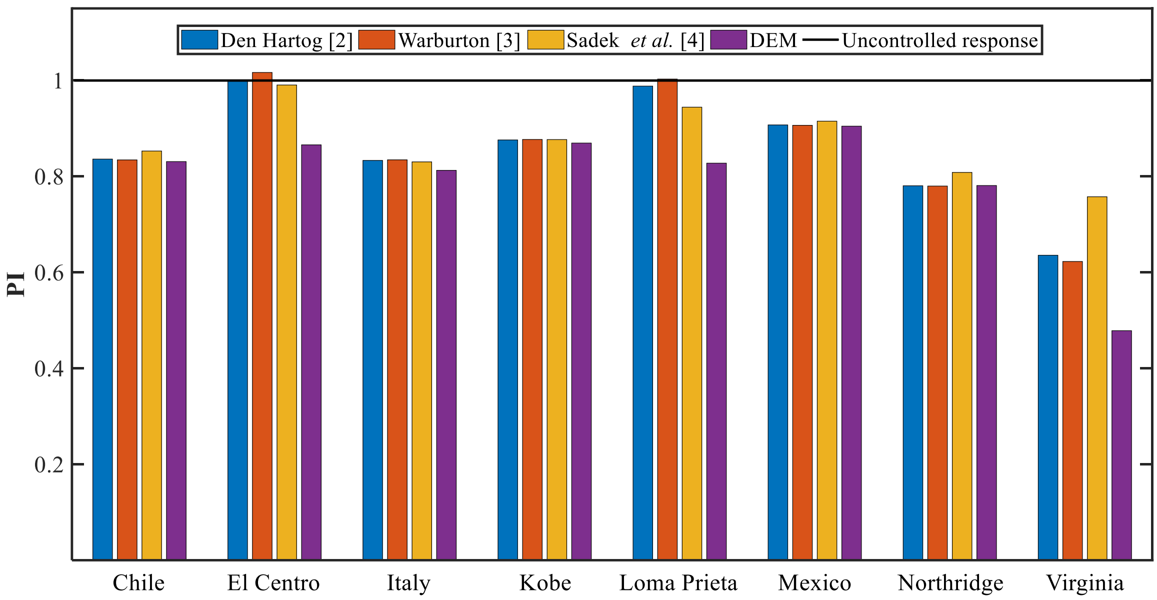
Table – 4 Optimum results

μ	F_{obj1}			F_{obj2}			F_{obj3}			*Seismic record*
	f	ζ_d	PI	f	ζ_d	PI	f	ζ_d	PI	
2%	1.2589	0.0376	0.8960	1.0052	0.1255	0.8308	1.9914	0.2400	0.8714	1
	1.9231	0.4432	0.9151	0.8036	0.038	0.8657	1.9446	0.4795	0.9076	2
	1.9932	0.4776	0.8639	1.0586	0.1508	0.8125	1.9926	0.4728	0.9255	3
	1.3434	0.0077	0.9314	1.0201	0.1686	0.8694	1.9986	0.0856	0.9242	4
	1.9909	0.4518	0.9013	1.9908	0.4728	0.8275	1.9363	0.4824	0.7933	5
	0.7672	0.0640	0.9367	1.0116	0.0918	0.9046	1.2074	0.0008	0.9619	6
	1.5591	0.0159	0.9776	0.9522	0.1376	0.7809	1.9986	0.482	0.8609	7
	0.9154	0.0231	0.7280	0.9154	0.0231	0.4784	1.0539	0.0124	0.7987	8
Avg.	-	-	0.8938	-	-	0.7962	-	-	0.8804	-
5%	1.9986	0.4925	0.8952	0.7433	0.3971	0.8121	1.8758	0.1429	0.8261	1
	1.3442	0.4744	0.8662	0.7485	0.1592	0.8203	1.9753	0.4930	0.8012	2
	1.5954	0.1263	0.7781	1.1583	0.3176	0.6814	1.9926	0.4728	0.8335	3
	1.4076	0.0077	0.8441	1.0608	0.3720	0.7751	1.997	0.2566	0.8515	4
	1.9909	0.4518	0.8524	0.6785	0.4948	0.8285	1.9363	0.4824	0.6851	5
	0.7285	0.1326	0.9404	0.8897	0.2968	0.8829	0.8389	0.0302	0.9501	6
	1.5282	0.0028	0.946	0.9037	0.2114	0.6982	1.9931	0.4972	0.7181	7
	0.9261	0.0108	0.5846	0.9037	0.0808	0.3447	0.9177	0.0027	0.6109	8
Avg.	-	-	0.8384	-	-	0.7304	-	-	0.7846	-

To evaluate the effectiveness of DEM in the tuning process, the PI values of F_{obj1} are compares against the classical optimization methodologies previously presented in Table 1. It is clear in Fig.6 (a) for μ=2%, and (b) for μ=5%, that the reductions in the dynamic response, measured with the value of PI, are in all cases significantly better when DEM is applied.



(a) μ=2%



(b) μ=5%

Fig. 6 – Comparison of PI for F_{obj1}

However, the optimization referred to F_{obj1} focuses only on the reduction of peak displacements, giving less importance to the remaining oscillations. On the other hand, the values computed for f, and ζ_d in F_{obj3}, tend to the previously established limit in Eqs. (15) and (16), which allows confirming that there are no optimal design values in the analyzed domain; in that sense, f and ζ_d parameters obtained in F_{obj2}, present more consistent values from a practical point of view, among the three optimization approaches. Fig.7 shows a time history analysis performed to examine thoroughly the enhancement achieved using f and ζ_d derived from F_{obj2}. The dynamic response of the building is controlled via TMD with μ=5%, and the seismic action is modeled using the Virginia acceleration record. Comparison is made against conventionally TMDs designed with the methodologies described in Table 1.

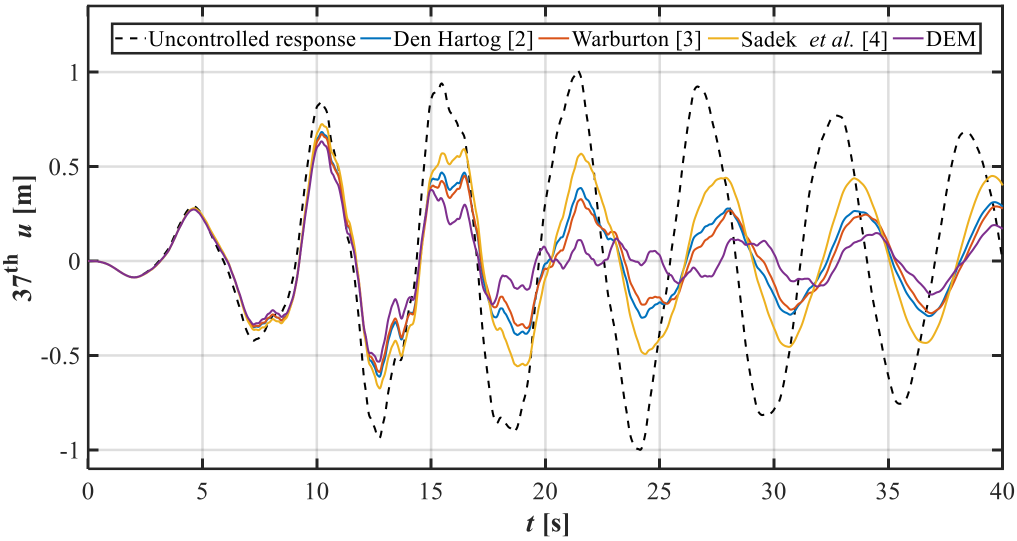


Fig. 7 – Time history analysis for all cases using Virginia ground motion record

The maximum uncontrolled displacement at the 37th degree of freedom computed in the example is 1.01 m. Applying the TMD design with DEM, the controlled displacement was 0.63 m, which represents a 37% reduction, while the best reduction of the peak displacement achieved in the comparison cases was 33%. Now, the RMS displacement response was reduced up to 66% respect to the uncontrolled response with the DEM design TMD, and barely 55% using Warburton methodology, which was the best comparison case. Consequently, the optimization referred to minimizing the RMS displacement response, results in a more feasible option to determine the best-fit design values, that not only reduce effectively the peak displacement but produced a much more stable behavior during the whole seismic action. To confirm this statement, the displacement values for all horizontal degrees-of-freedom in the previously performed time history analysis were registered. Fig.8 shows the RMS computed for these displacements, where it can be noticed that a considerable reduction was achieved not only for the objective degree-of freedom (37th storey level), but for the remaining degrees of freedom as well:

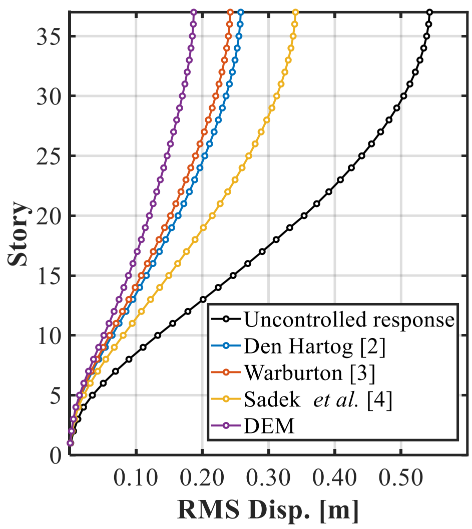


Fig. 8 – RMS displacements at each story level

The average reduction in the RMS displacement response, for all horizontal degrees of freedom, is 63% using the TMD designed with DEM, and 52%, 55%, and 39% for Den Hartog [2], Warburton [3], and Sadek *et al.* [4] methodologies, respectively. These results validate the methodology used in this investigation, in which the top story level was selected as the objective for the three optimization approaches, although the same tendency resulted for the rest of the degrees of freedom.

5. Conclusion

A numerical study based on the application of a novel metaheuristic technique inspired by DEM for the tuning procedure of TMDs in Colombia was presented. The case-study employed for the analysis was derived from what is currently the tallest residential building in Medellin city, subjected to accelerograms of recorded earthquakes. The tuning process focused on the optimization of three strategic parameters that were analyzed individually as objective functions: minimization of horizontal peak displacements, minimization of RMS displacements response, and minimization of horizontal peak floor acceleration. Considering the obtained results and related discussions in previous sections, it may be concluded that among the three optimization approaches, the one relating to minimizing the RMS displacement response, represents the best-fit design values, not only for RMS displacement response, but for the other two response parameters as well. The comparative analysis shows a significant structural response reduction when DEM is applied, with larger reductions than those afforded with conventional tuning methodologies. Finally, the results for all degrees of freedom considered in the model are consistent with the performance of the optimization developed for the top floor degree of freedom.

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