THE BASE ISOLATION WITH HIGH IMPEDANCE MATERIALS

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Abstract

The goal and purpose of base isolation is to protect the building structure against strong ground motion induced in case of large earthquakes. We would select the stiffness of laminated rubber bearings for a base isolated structure as small as possible, because the response acceleration of super structure decreases as the stiffness becomes smaller. Recently, there has been an increasing attention focused on the risk of damage of structures possibly caused by the long duration earthquakes with long period component. We did not notice the risk of damage of base isolated buildings in case of large earthquakes until March 11, 2011 when the northern part of Japan experienced a huge earthquake with the moment magnitude 9.0. This event forced us to take the damage risk of base isolated structures into serious consideration under a large earthquake with long duration, because it contains a lot of long period component in the frequency domain.

Indeed, there should be a lower boundary of stiffness for laminated rubber bearings as compared with that of super structure. In other words, we should have a rational regulation for their stiffness. The relative displacement of the super structure with respect to the bearings should be small to protect the structure itself when the system is under a large ground motion. The response displacement of the bearing with respect to the ground motion should be also small to evade the risk of collapse of the whole system. With these two different goals satisfied at the same time, the author proposes a method to adjust the damping factor and the stiffness of bearings to the optimal value. The optimum stiffness of bearings is determined according to the natural period of the super structure. The spectrum of the ground motion also plays an important role to determine the appropriate damping factor for the bearings.

1. Introduction

Ever since the idea of base isolation was established and realized more than 30 years ago, we have paid a lot of attention to reduce the absolute acceleration of the super structure of base isolation system. The stiffness of the bearings is so small as compared with the lateral stiffness of the super structure that we think it is better to model it as a single degree of freedom system. Illustration in Fig.1 is the conventionally designed building structure, while the model of base isolation is in Fig. 2. In the past decade there have been accumulated a lot of ground motion data in case of major earthquake events in Japan. The Tohoku earthquake in May 11, 2011 created a huge ground motion that propagated a long distance away from the epicenter and left significant damage on the buildings, houses, and bridges as well. Tall buildings in the business district of the city of Tokyo swayed like a pendulum despite the fact that they are more than 400km away from the epicenter. Because they have small damping factor and the duration of ground motion was so long that they had enough time to reach the steady state response motion.

This event raised a question among engineers and scientists in Japan whether the base isolation method is the appropriate seismic approach to protect tall buildings in case of large earthquakes. It is true that we paid little attention to the deformation capacity of bearings until May 11, 2011, but we should take into consideration that the bearing deformation capacity is another critical factor for the base isolation system. This paper proposes an approach to design a base isolation system with high stiffness bearings and damping coefficient. The question is whether it is possible to find the optimum damping coefficient factor and the optimum modal frequency. If the disturbance ground motion is supposed to be the input signal and the displacement of the system is supposed to be the output signal, the bearings such as laminated rubber materials are the media between the two signals. The concept of impedance in the electric circuit is equivalent to the bearings in the mechanical dynamics of base isolation system. We try to reduce the bearing deformation and the reaction of the structure response at the same time, which is quite similar to the impedance matching of the electric circuit with input and output signals.

2. Definition of the optimum dynamics of the base isolation system

The single degree of freedom model in Fig.1 represents the first mode dynamics of the building structure. It has the circular frequency ω_1 and effective mass m and stiffness k in Eq. (1). The base isolated structure in Fig.2 has the identical first mode dynamics shown in Fig.1. The bearings that support the vertical load of the super structure have the stiffness k_d and damping coefficient c_d. Suppose that the damping coefficient c_d varies from zero to infinity, the circular frequency of the base isolated system ω_{eq} changes from ω_o to ω_1.

$$\omega_1 = \sqrt{\frac{k}{m}} \tag{1}$$

$$\omega_0 = \sqrt{\frac{kk_d}{m(k+k_d)}} \tag{2}$$

$$\omega_0 < \omega_{eq} < \omega_1 \tag{3}$$

We have to set a question. How do we select the stiffness and damping coefficient of the bearings for reducing the response of the base isolation system under an earthquake with long period component? We have to make a dynamic model to answer this question as follows. We specify the equation of motion of the model in Fig. 2 as shown in Eq. (4). There should be a required condition for optimizing the parameters k_d and c_d. For this purpose, we have to specify the ground motion as well. Let us suppose that the ground motion is stationary random process instead of nonstationary random excitation. The spectrum of the random process is supposed to be constant or equivalently white noise. Under this supposition, we could optimize the parameters in a probabilistic sense rather than deterministic one in the time domain. As a result, optimization requires no numerical calculations in the time domain but analytical solutions in the frequency domain.

$$\begin{cases} m\ddot{y} + k(y-z) = -\ m\ddot{x}(t) \\ k(y-z) - k_d z - c_d \dot{z} = 0 \end{cases} \tag{4}$$

where
- $x(t)$: ground motion
- $y(t)$: response displacement of the structure with respect to ground motion
- $z(t)$: response displacement of the bearings with respect to ground motion
- m : effective mass of the structure
- k : effective stiffness of the structure
- k_d : stiffness of the bearings
- c_d : damping coefficient of the bearings

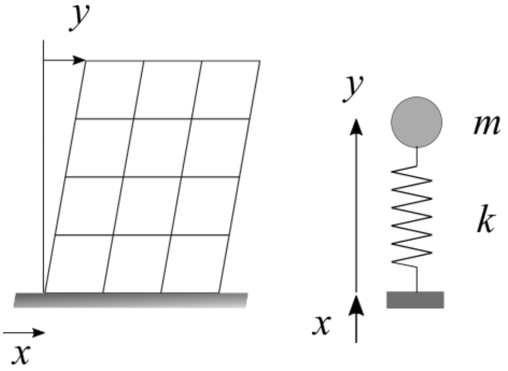

Fig. 1 Normally designed building structure

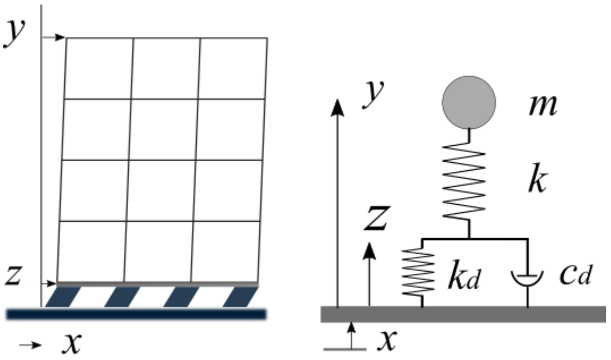

Fig.2 Base isolated structure with bearings

3. Transfer functions of the base isolation system

We introduce next substitution for the following derivations of transfer functions.

$$\omega_d = \sqrt{\frac{k_d}{m}} \qquad and \qquad \omega_\infty = \sqrt{\frac{k + k_d}{m}} \tag{5}$$

$$c_d = 2m\omega_1\eta \tag{6}$$

We define the relative stiffness of the bearings in terms of β or equivalently by Eq. (7).

$$\beta = \frac{k_d}{k} = \frac{{\omega_d}^2}{{\omega_1}^2} \tag{7}$$

With these notations and substitutions on mind, we converge Eq. (4) into Eq. (8) from which we can derive the Laplace Transform as Eq. (9).

$$\begin{cases} \ddot{y} + {\omega_1}^2(y - z) = -\ \ddot{x}(t) = f(t) \\ {\omega_1}^2(y - z) - {\omega_d}^2 z - 2\omega_1\eta\dot{z} = 0 \end{cases} \tag{8}$$

$$\begin{pmatrix} s^2 + {\omega_1}^2 & -{\omega_1}^2 \\ -{\omega_1}^2 & 2\omega_1\eta s + {\omega_1}^2 + {\omega_d}^2 \end{pmatrix} \begin{pmatrix} Y(s) \\ Z(s) \end{pmatrix} = \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{9}$$

Finaly, we derive the transfer function with respect to Y as Eq. (11) .

$$\begin{pmatrix} Y(s) \\ Z(s) \end{pmatrix} = \begin{pmatrix} s^2 + {\omega_1}^2 & -{\omega_1}^2 \\ -{\omega_1}^2 & 2\omega_1\eta s + {\omega_1}^2 + {\omega_d}^2 \end{pmatrix}^{-1} \begin{pmatrix} F(s) \\ 0 \end{pmatrix} \tag{10}$$

$$H_Y(s) = \frac{({\omega_d}^2 + {\omega_d}^2) + 2\omega_1\eta\, s}{2\omega_1\eta\, s^3 + \ ({\omega_d}^2 + {\omega_d}^2)\, s^2 + 2{\omega_1}^3\, \eta\, s\ + {\omega_1}^2{\omega_d}^2} \tag{11}$$

$$Y(s) = H_Y(s)\, F(s) \tag{12}$$

The stationary random disturbance of $F(s)$ is supposed to be constant S_o so that we can obtain the expected power spectrum with respect to Y as Eq. (13).

$$E[Y^2] = \int_{-\infty}^{\infty} H_Y(i\omega)H_Y(-i\omega)S_o\, d\omega \tag{13}$$

Substituting Eq. (11) into Eq. (13), we obtain Eq. (14) by means of residue integral in the complex damain.

$$E[Y^2] = \frac{2\pi S_o}{{\omega_1}^3}\left(\eta + \frac{{\omega_\infty}^6}{4{\omega_1}^4{\omega_d}^2\,\eta}\right) = \frac{2\pi S_o}{{\omega_1}^3}\, G(\eta, \beta) \tag{14}$$

$$G(\eta, \beta) = \ \eta + \frac{{\omega_\infty}^6}{4{\omega_1}^4{\omega_d}^2\,\eta} = \eta + \frac{(1 + \beta\,)^3}{4\,\beta}\,\frac{1}{\eta} \tag{15}$$

We would like to obtain the optimum set of parameters β and η that could minimize the response displacement of the base isolated system under stationary random white noise.

4. The optimum stiffness and damping of the bearings for base isolation system

Our final goal is to minimize the response displacement of the base isolation system so that we could find the optimum set of two parameters by differentiating Eq. (15) with respect to η or β.

$$\frac{\partial G}{\partial \eta} = 1 - \frac{(1+\beta)^3}{4\beta}\frac{1}{\eta^{\,2}} = 0 \tag{16}$$

$$\frac{\partial G}{\partial \beta} = 2\beta + 3 - \frac{1}{\beta^2} = 0 \tag{17}$$

We have found the optimum parameter β from Eq. (17) as well as η from Eq. (16).

$$\beta_{opt} = 0.5 \tag{18}$$

$$\eta_{opt} = \sqrt{\frac{(1+\beta)^3}{4\beta}} = 1.30 \tag{19}$$

Substituting Eq. (18) and (19) into Eq. (14), we obtained Eq. (20).

$$E[Y^2] \geqq \frac{2\pi S_o}{{\omega_1}^3} G_{min}(\eta, \beta) > \frac{5.2\pi S_o}{{\omega_1}^3} \tag{20}$$

There exists the optimum stiffness and damping factor that minimizes the expected power spectrum of the response Y under stationary white noise excitation. In the past study, as the stiffness of the bearings came down, the better performance was expected for base isolation system. It, however, has been proved that it is not necessary to reduce the stiffness and damping factor associated with the bearings more than the specified value of Eq. (18) and (19). If the selection of the optimum parameters β and η is sensitive and has significant effect on the performance of the base isolation system, we must clarify how sensitive it would be. The function $G(\eta,\beta)$ is converted from Eq. (15) into Eq. (21) and (22).

$$G(\eta, \beta) = \eta + \frac{(1+\beta\,)^3}{\beta}\frac{1}{4\eta} = \eta + \frac{f(\beta)}{\eta} \tag{21}$$

$$f(\beta) = \frac{1}{4}\left(\beta^2 + 3\beta + 3 + \frac{1}{\beta}\right) \tag{22}$$

As we have already discussed that function $f(\beta)$ is minimized when β is 0.5. Function $f(\beta)$ is shown in Fig. 3, from which we understand that $f(\beta)$ varies from 1.69 to 2.16 while β is between 0.2 and 1.0 .

$$G(\eta, \beta) = \eta + \frac{f(\beta)}{\eta} > \eta + \frac{f(0.5)}{\eta} = \eta + \frac{1.69}{\eta} = g(\eta) \tag{23}$$

According to Eq. (23) we can define function $g(\eta)$ that gradually decreases from 5.93 as η increases from 0.3 until it comes to 1.3 that is the optimum damping factor or equivalently Eq. (19). As compared with ordinary base isolation system, bearing stiffness is relatively high and damping coefficient is several times larger than ordinary values for conventially designed base isolation structures. It is also interesting that the power spectrum of the response is not sensitive to the deviation of the parameters from the optimum values.

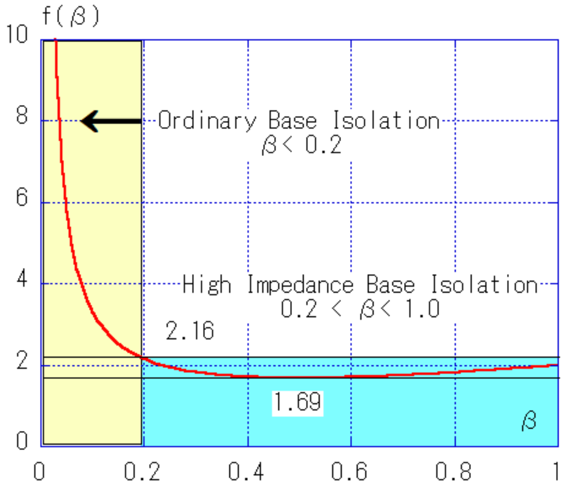

Fig. 3 Function $f(\beta)$

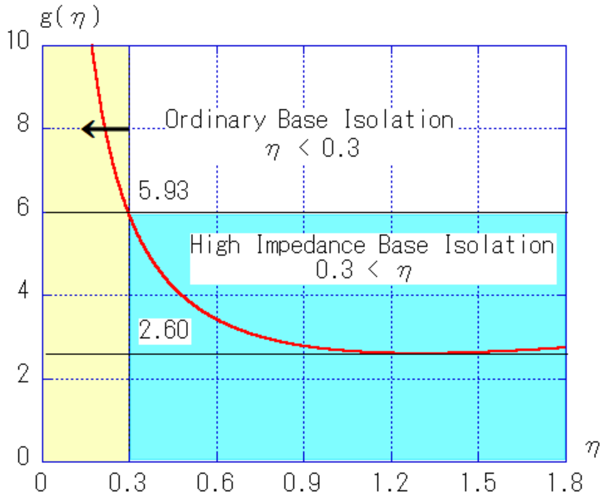

Fig. 4 Function $g(\eta)$

5. Comparison with the conventionally designed base isolation system

We have obtained the optimum parameters of bearings for high impedance base isolation system. They are so different from those values conventionally designed in the past application projects. For example, we selected the stiffness parameter β in Fig.3 as small as possible. In fact, β is set less than 0.2 for ordinary base isolation systems. As far as damping factor η is concerned, it is usually less than 30 % or 0.3 in Fig.4. Here arises a natural question: how much is the performance of the high impedance base isolation compared with the conventionally designed base isolation? We discuss the performance of the high impedance base isolation as compared with a single degree of freedom model with the same natural frequency and damping factor. For this purpose, we evaluate the power spectrum of SDOF under white noise excitation of S_o. The transfer function of SDOF model is as follows.

$$\ddot{y} + 2\omega_{eq}\eta_{eq}\dot{y} + \omega_{eq}{}^2 y = -\ \ddot{x}(t) = f(t) \tag{24}$$

$$H_Y(s) = \frac{1}{s^2 + 2\omega_{eq}\eta_{eq}\ s\ + \omega_{eq}{}^2} \tag{25}$$

$$Y(s) = H_Y(s)\ F(s) \tag{26}$$

The power spectrum with respect to Y under white noise S_o is calculated in the same manner as Eq. (13). Finally, we obtained Eq. (27) for SDOF model.

$$E[Y^2] = \frac{\pi S_o}{2\omega_{eq}{}^3\eta_{eq}} \tag{27}$$

The fist modal frequency ω_{eq} of the high impedance base isolation with β_{opt} and η_{opt} is approximately Eq. (28), which is substituted into Eq. (20).

$$\omega_{eq} = \sqrt{\frac{\omega_o{}^2 + \omega_1{}^2}{2}} \cong 0.809\omega_1 \tag{28}$$

Eq. (20) is equivalent to the power spectrum of SDOF under stationary white noise excitation.

$$\frac{5.2\pi S_o}{{\omega_1}^3} = \frac{\pi S_o}{0.3632{\omega_{eq}}^3} = \frac{\pi S_o}{2{\omega_{eq}}^3 \eta_{eq}} \tag{29}$$

The equivalent damping factor η_{eq} that we derive from Eq. (29) represents the performance of the high impedance base isolation.

$$\eta_{eq} = 0.186 \tag{30}$$

In summary, it is not necessary to set the bearing stiffness and damping factor excessively small in order to improve the seismic protection performance of base isolation. It is also important to notice that the appropriate bearing stiffness and damping coefficient depends on the original structure dynamics rather than spectrum of the ground motion. Therefore, the low rise building structure should be isolated with much higher impedance bearings.

6. Ground motion data on March 11, 2011

In the following chapters, we demonstrate several numerical calculations with the intension of comparing the performance of high impedance base isolation with conventionally designed base isolation under nonstationary random disturbances. The ground motion data at the campus of Tokyo City University on March 11 in 2011 has a long duration more than 300 seconds and contains a lot of long period component. The time history in Fig. 5 is the acceleration data of the ground motion in North-South direction, while response velocity spectrum in Fig. 6 shows its flat spectrum over the low frequency domain. The location of the campus is more than 400 km away from the epicenter. The peak acceleration and the velocity in the North-South direction are 129.0 cm/s^2 and 17.7 cm/s, respectively. The response velocity spectrum over the low frequency domain is almost constant that is 20 cm/s when the damping factor is 20 %. In other words, if the ground intensity is 5 times greater than the original motion, the response velocity of ordinary base isolation is almost 100 cm/s. If the system natural period is 6 second, the response displacement is approximately 100 cm or more. This number is almost double as compared with the conventionally designed base isolation system. There has been an increasing attention focused on the risk of damage of tall building structures and base isolation system in case of large earthquake event, even if the intensity level of the ground motion is the same as used be.

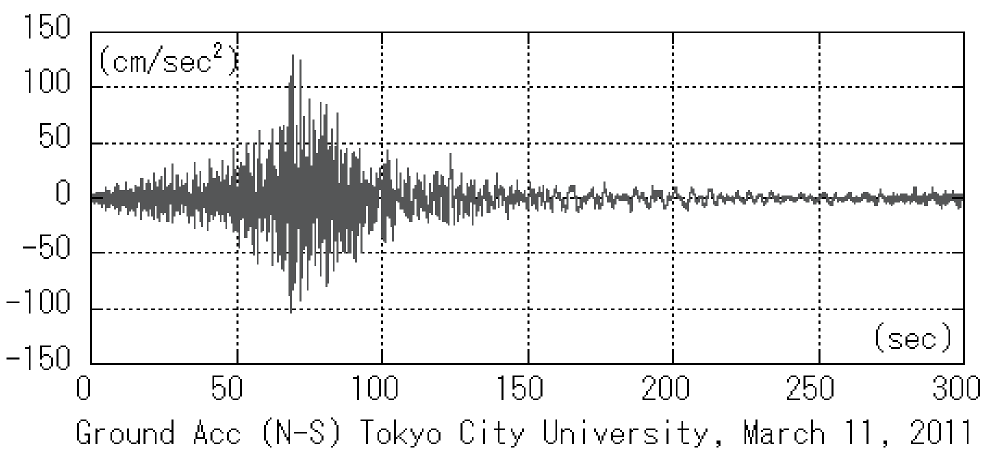

Fig. 5 Ground acceleration at Tokyo City University on March 11 in 2011 N-S direction

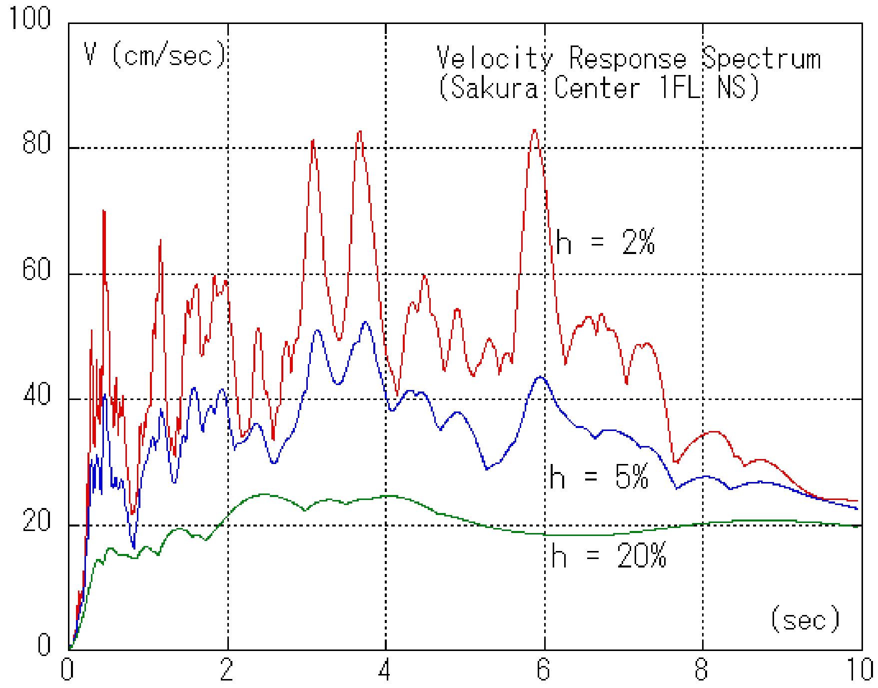

Fig. 6 Velocity Response Spectrum of the data on March 11, 2011

7. Numerical calculation

There are two example case studies conducted in this chapter. We selected the original structure circular frequency ω_1 is 2.0 rad/sec, while the parameter β is set to the optimum value 0.5. The damping factor η is also set to the optimum value 1.3 according to Eq. (19). As a result, the equivalent SFOF model has the circular frequency ω_{eq} = 1.63, and damping factor η_{eq} = 0.186. This is the case 1 study. The optimum parameters obtained from Eq. (1), (2), (18), (19), (28), and (30) are in Table 1, where two examples, CASE 1 and 2, are explicitly given. The peak response values are numerically calculated and shown in Table 2, while the corresponding equivalent SDOF model with the equivalent frequency and damping factor are in Table 3. As compared with Table 2 and 3, we confirmed that Eq. (28) and (30) give us the appropriate performance evaluation for the high impedance base isolation. Comparing Fig. (7) and (8), we notice that the deformation capacity required for base isolation material is reduced significantly, even though the ground motion contains a lot of long period component. The high impedance base isolation shows us the feasibility and capability to reduce the risk of damage of base isolation system.

Table 1 – The Optimum Parameters for the Base Isolation System

	ω_1	ω_o	ω_d	ω_∞	ω_{eq}	m	k	β	η	k_d	c_d
CASE1	2.0	1.15	1.41	2.45	1.63	1.0	4.0	0.5	1.3	2.0	5.20
CASE2	1.0	0.577	0.707	1.22	0.809	1.0	1.0	0.5	1.3	0.5	2.60

Table 2 – Maximum response of the high impedance base isolation

	y (cm)	z (cm)	$\ddot{y}+\ddot{x}$ (cm/s^2)	$\ddot{z}+\ddot{x}$ (cm/s^2)
CASE1	13.4	4.60	43.9	129.2
CASE2	19.5	6.95	16.6	127.3

Table 3 – Maximum response of the equivalent SDOF model

	y (cm)	$\ddot{y}+\ddot{x}$ (cm/s^2)
CASE1	12.7	40.0
CASE2	16.8	12.4

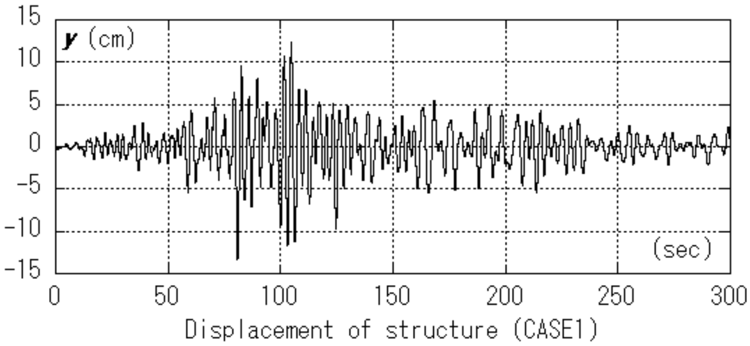

Fig.7 Response displacement CASE 1 under the ground motion of Fig. 5

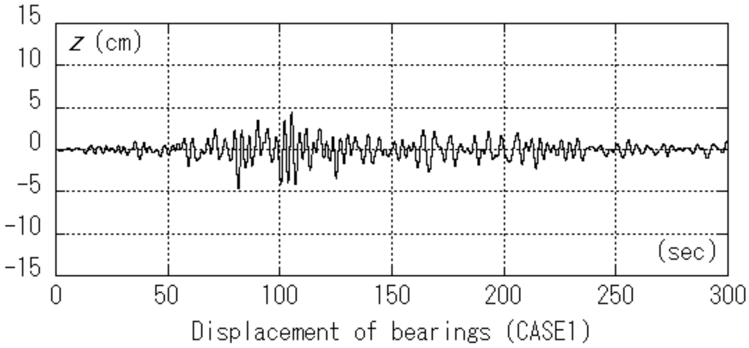

Fig.8 Response displacement CASE 1 under the ground motion of Fig. 5

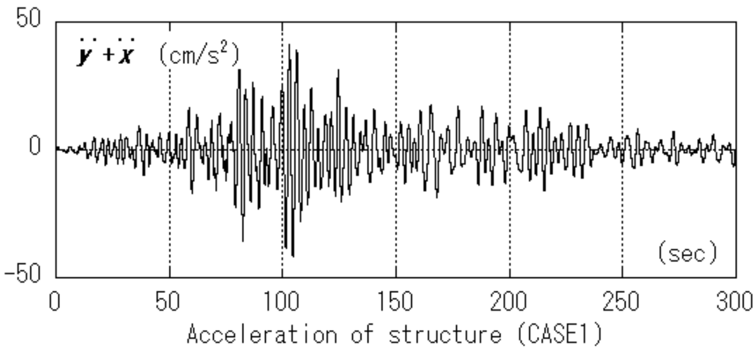

Fig.9 Response acceleration CASE 1 under the ground motion of Fig. 5

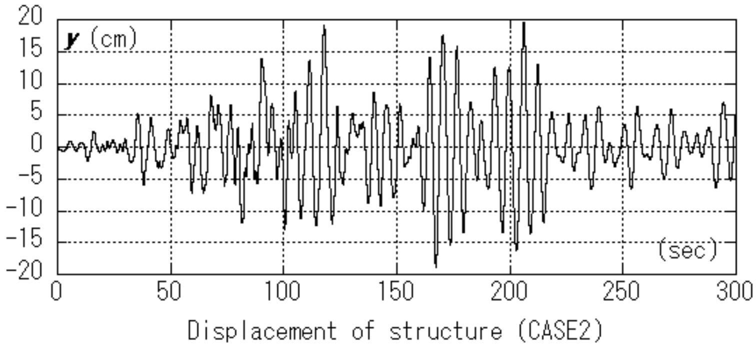

Fig.10 Response displacement CASE 2 under the ground motion of Fig. 5

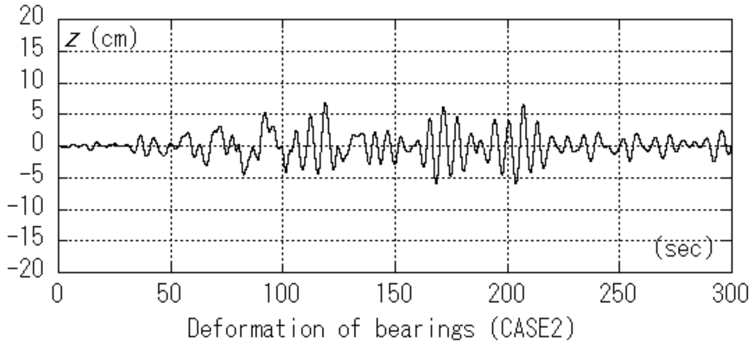

Fig.11 Response displacement CASE 2 under the ground motion of Fig. 5

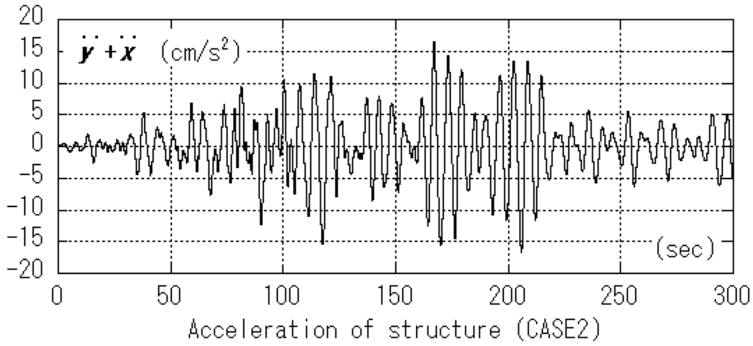

Fig.12 Response acceleration CASE 2 under the ground motion of Fig. 5

8. Conclulsive remarks

This paper reports how to select the stiffness and damping coefficient for the bearing materials that are most appropriate for the base isolation system under large earthquakes with long duration and low frequency component. There are the optimum parameters that could reduce the response displacement of the isolated building structure as small as possible under stationary random disturbances with constant power spectrum. There are also example numerical calculations carried out to demonstrate the validity of the theoretical prediction. The author used the ground motion data at the Tokyo City University on March 11 in 2011 when Tohoku Earthquake took place in the northern part of Japan. The proposed parameters for the base isolation have much higher dynamic stiffness or equivalently high impedance dynamic property.

9. References

[1] Isao Nishimura (2012): The structure health monitoring for identifying the damping performance of building structures under strong ground motion on March 11 in 2011, No.4551, Proceedings of *15th of World Conference on Earthquake Engineering, Lisboa*

[2] Isao Nishimura (2004): Performance evaluation of damping devices installed in a building structure, *Journal of Structural and Construction Engineering, Transactions of Architectural Institute of Japan*, No.579, 22-30

