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TUNED MASS DAMPER WITH INVERTED PENDULUM FOR REDUCING SEISMIC RESPONSE

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Abstract

Long-period ground motion caused large responses for high-rise buildings in Tokyo and Osaka area, which were far away from the epicenter, during the 2011 off the Pacific coast of Tohoku Earthquake. In the near future, further extreme seismic events such as Nankai trough earthquake are expected in Japan.

To reduce the seismic response of super high-rise buildings due to such destructive large long-period ground motion, a tuned mass damper (TMD) in combination with pendulum and inverted pendulum has been developed. The newly developed TMD with inverted pendulum (TMD-IP) has the following features;

- Particularly effective to reduce the continuously repeated response of super high-rise buildings caused by the destructive long-period ground motions.
- The size including the height of the equipment is smaller compared to the conventional pendulum type TMD.
- The natural period adjustment of TMD-IP can be easily performed by mutually moving the weights between the pendulum and the inverted pendulum.
- The installation can be done without closing the building, because there needs little reinforcement inside the buildings.
- The safety of TMD-IP against unexpected large earthquakes is improved when equipped with the fail-safe platform (FSP) as option.

In this paper, after the concept and the characteristics of the TMD-IP is described, the response control effect is discussed using both exact and approximate numerical model. As results, in the design of the whole building-TMD-IP system, the TMD-IP can be modeled as a linear one-degree-of-freedom model, and the conventional optimal design method of TMD can be applied.

Also, newly proposed fail-safe system has been developed to prevent the huge mass of TMD from being overloaded and damaged against larger seismic motion than design level. The developed fail-safe system mounts the TMD on the building via the FSP instead of directly mounting the TMD on the building. This makes it possible to prevent damage to the TMD-IP at the time of excessive seismic motion without sacrificing the response control performance.

To verify the dynamic characteristic of the TMD-IP and the performance of FSP, two-dimensional shaking table tests were conducted using prototype scaled TMD-IP model. As results, it was confirmed that the dynamic characteristics of the TMD-IP were obtained as equal to its theoretical values and the natural period of TMD-IP can be easily adjusted by mutually moving the weights between the pendulum and the inverted pendulum. It was also confirmed that the increase of TMD-IP displacement was suppressed by the operation of FSP with the increase of the building response. So, the FSP was found to function as a fail-safe system against the excessive displacement.

Furthermore, to establish a method that can represent the complex behavior of inverted pendulum, finite element analysis was performed considering geometric nonlinearity. As results, good agreement was found between the finite element analysis results and the experimental results.

Keywords: structural control; tuned mass damper; high-rise building; long-period ground motion; fail-safe system

1. Introduction

Long-period ground motion caused large responses for high-rise buildings in Tokyo and Osaka area, which were far away from the epicenter, during the 2011 off the Pacific coast of Tohoku Earthquake. In the near future, further extreme seismic events such as Nankai trough earthquake are expected in Japan.

One of the technologies to cope with this issue is installing tuned mass dampers (TMD) on the top of those super high-rise buildings. Actually, several types of TMD which are effective for seismic response have been applied for a newly constructed building [1] and existing buildings as seismic upgrade [2,3].

To reduce the seismic response of super high-rise buildings due to such destructive large long-period ground motion, TMD in combination with pendulum and inverted pendulum has been developed. The newly developed TMD with inverted pendulum (TMD-IP) has the following features;

- Particularly effective to reduce the continuously repeated response of super high-rise buildings caused by the destructive long-period ground motions.
- The size including the height of the equipment is smaller compared to the conventional pendulum type TMD.
- The natural period adjustment of TMD-IP can be easily performed by mutually moving the weights between the pendulum and inverted pendulum.
- The installation can be done without closing the building, because there needs little reinforcement inside the buildings.
- The safety of TMD-IP against unexpected large earthquakes is improved when equipped with the fail-safe platform (FSP) as option.

In this paper, after the concept and the characteristics of the TMD-IP is described, the response control effect is discussed using both exact and approximate numerical model. Also, newly proposed fail-safe system has been developed to prevent the huge mass of TMD from being overloaded and damaged against larger seismic motion than design level. The developed fail-safe system mounts the TMD on the building via the FSP instead of directly mounting the TMD on the building. This makes it possible to prevent damage to the TMD-IP at the time of excessive seismic motion without sacrificing the response control performance.

To verify the dynamic characteristic of the TMD-IP and the performance of FSP, two dimensional shaking table tests were conducted using prototype scaled TMD-IP model. Furthermore, to establish a method that can represent the complex behavior of inverted pendulum, finite element analysis was performed considering geometric nonlinearity.

2. Outline of TMD-IP

2.1 Basic theory of TMD-IP

Fig. 1(a) shows the basic dynamic model of TMD-IP proposed in this paper. There is an example of TMD in combination with pendulum and inverted pendulum [4], where the pendulum and the inverted pendulum connect each other with vertical springs and vertical dampers. However, the proposed TMD-IP has dampers installed horizontally between the rod of the inverted pendulum and the base. This TMD-IP has following features;

- Dampers are installed horizontally to have TMD damping linear from small to large displacement.
- Dampers are attached to the rod of inverted pendulum lower position than the mass of inverted pendulum to have displacement of damper smaller than that of mass. So, existing oil dampers can be used and no need to develop long stroke oil dampers.

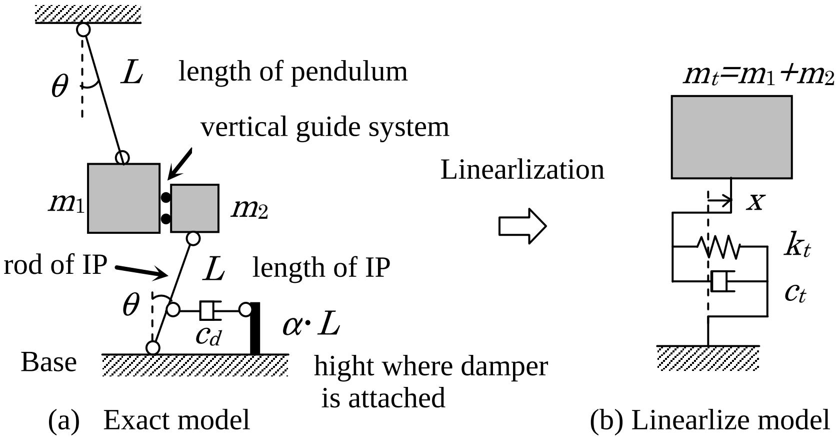


Fig. 1 – Theoretical model of TMD-IP

Here, the length of pendulum and inverted pendulum are assumed to be equal as L, the equation of motion on pendulum angle θ of this model for free vibration is described as follows.

$$(m_1 + m_2)L\ddot{\theta} + c_d\alpha^2 L\,cos^2\,\theta \cdot \dot{\theta} + (m_1 - m_2)g\,sin\,\theta = 0 \quad (1)$$

where m_1: mass of pendulum, m_2: mass of inverted pendulum, c_d: damping coefficient of damper, αL: the hight of inverted pendulum rod where the damper is attached, g: gravity acceleration. Also, it is assumed as $m_1 > m_2$ for the system to be stable.

Since Eq. (1) includes $sin\,\theta$ and $cos\,\theta$, the system shows geometric nonlinear characteristics. To linearize the equation, it is assumed that the pendulum angle is small as $\theta < 30°$ and $\sin\theta \cong \theta$, $\cos\theta \cong 1$. Then, the equation of motion can be rewritten as follows.

$$(m_1 + m_2)L\ddot{\theta} + c_d\alpha^2 L \cdot \dot{\theta} + (m_1 - m_2)g\theta = 0 \quad (2)$$

The natural period of the TMD-IP is obtained as follows.

$$T = 2\pi\sqrt{\frac{m_1+m_2}{m_1-m_2} \cdot \frac{L}{g}} \quad (3)$$

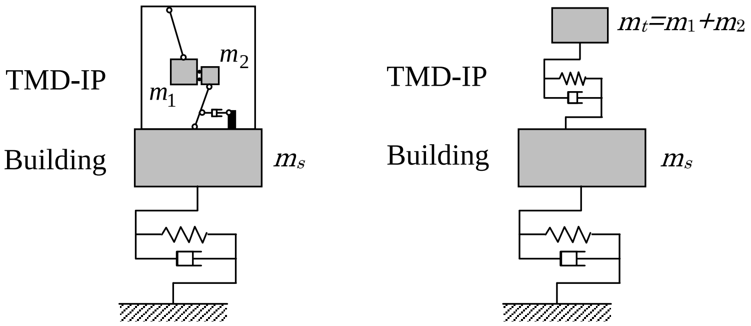
The Eq. (3) shows that the natural period of TMD-IP is $\sqrt{(m_1 + m_2)/(m_1 - m_2)}$ times longer than that of conventional pendulum, which is $T = 2\pi\sqrt{L/g}$. So, in designing TMD-IP, the length of pendulum can be shorter and the whole system can be made compact. It is especially effective for designing TMD for structures with long natural period such as super high-rise buildings.

Also, according to Eq. (3), the natural period of TMD-IP can be easily adjusted by changing the rate of m_1 and m_2 without changing the length of pendulum which is needed for conventional pendulum type TMD.

2.2 Optimum design of TMD-IP

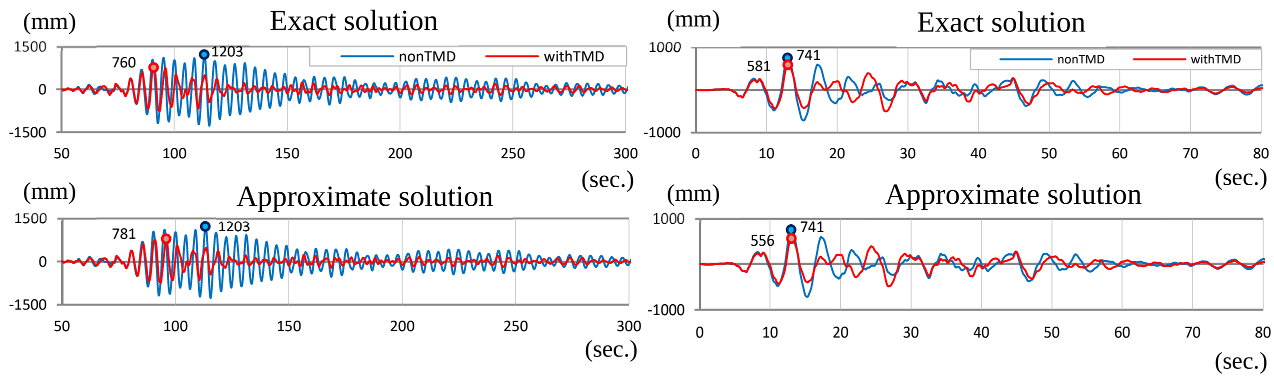
Replacing parameters in Eq. (2) as follows: $m_t = m_1 + m_2$, $c_t = c_d\alpha^2$, $k_t = (m_1 - m_2)\,g/L$, $x = L\theta$, the equation of motion of TMD-IP is described as Eq. (4) and can be treated as single degree of freedom model shown in Fig. 1(b).

$$m_t\ddot{x} + c_t\dot{x} + k_t x = 0 \quad (4)$$



(a) Exact model (b) Approximate model

Fig. 2 – Analytical model of the whole building and TMD-IP system



(a) Displacement responses of building for NKT (b) Displacement responses of building for KKJ

Fig. 3 – Comparison between exact and approximate solutions

Assuming the building model as single degree of freedom, the whole building-TMD-IP system can be expressed as two degrees of freedom model as shown in Fig. 2(b). Then, the optimal tuning period T_{opt} and the optimal damping h_{opt} of TMD-IP can be derived from the following conventional TMD optimal design method [5].

$$T_{opt} = (1 + \mu) \cdot T_s \quad (5)$$

$$h_{opt} = \sqrt{3\mu/8(1 + \mu)} \quad (6)$$

where T_s: natural period of the building, μ: the ratio of mass of TMD-IP to that of the building.

2.3 Seismic performance of TMD-IP and evaluation of approximate model

The seismic performance of TMD-IP is evaluated using seismic response analysis. Two cases are performed and compared. The first one is the exact solution applying Eq. (1) directly to the analysis model. The second one is the linearilzed approximate solution applying Eq. (4) to the analysis model. The building is modeled as single degree of freedom for both cases.

The parameters used in the numerical analysis are as follows: mass of the building $m_s = 5000t$, natural period of the building $T_s = 4.5s$, damping of the building $h_s = 0.02$, mass ratio $\mu = m_t/m_s = 0.1$, total mass of TMD-IP $m_t = 500t$ including mass of pendulum $m_1 = 414t$ and inverted pendulum $m_2 = 86t$, the length of pendulum and inverted pendulum $L = 4.0m$, the hight where damper is attached $\alpha = 0.3$. According to the optimal design method of TMD, Eq. (5) and Eq. (6), the natural period and damping of TMD-IP are

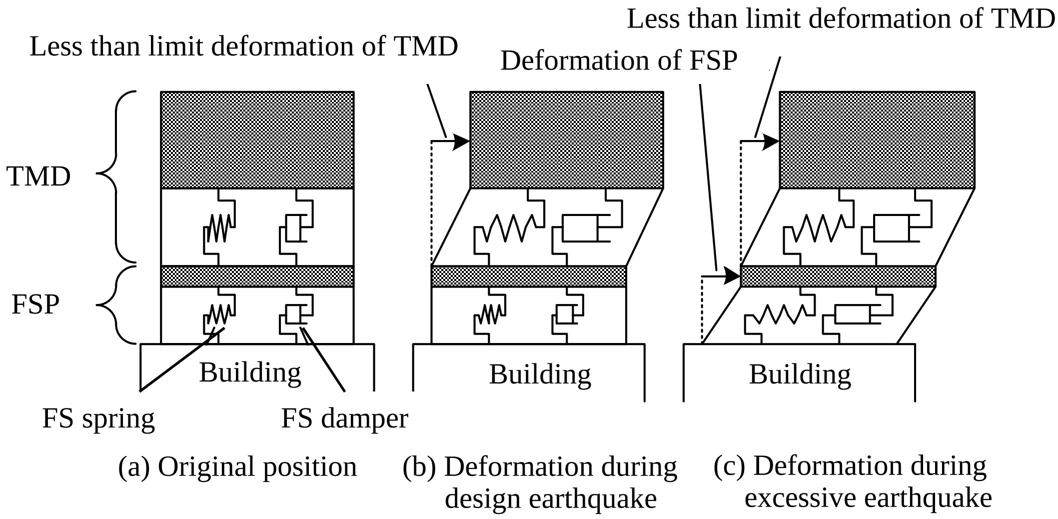


Fig. 4 – Basic Idea of FSP

determinded as follows: $T_t = 4.95s$, $h_t = 0.185$. As for input ground motion, the artificial site wave derived for Osaka plain during expected Nankai trough earthquake (NKT), and Kokuji-ha wave (KKJ), which is the design input ground motion generated according to the design spectrum of the Building Standard Law of Japan are used for the seismic response analysis. The response without TMD-IP is also examined.

The results are shown in Fig.3 and considerations are as follows.

- Applying TMD-IP, large response reduction performance can be obtained for NKT, which is long-period ground motion, while TMD-IP is also effective for KKJ.
- The defferense between the exact solution model and the approximate solution model are enough small. So, TMD-IP can be modeled as linear single degree of freedom in the range where the pendulum angle is less than 30°.

3. Outline of FSP

If a building equipped with a TMD encounters an excessive ground motion that exceeds the design level, it may lead to a serious accident such as an increase in building response due to damage to the TMD or a drop of the TMD mass. Therefore, the fail-safe mechanism is indispensable for the TMD as a countermeasure against the excessive seismic motion.

As the fail-safe mechanism, a method has been proposed in which the additional mass is suddenly braked when the TMD device is about to exceed the limit deformation [2]. In using such devices, the performance of TMD may decrease after sudden braking, which may lead to an increase in building response.

In this paper, a method is proposed to prevent TMD from being damaged in the event of an excessive ground motion and to suppress the deterioration of seismic performance by mounting TMD in a building via an FSP, instead of mounting TMD directly on the building.

3.1 Configuration of FSP

The FSP has a role of transmitting the vertical load of the TMD mounted on the upper part to the building, and of transmitting the horizontal control force between the TMD and the building. Fig.4 shows a conceptual diagram of the operation of the TMD and FSP in the horizontal direction. The FSP is composed of an FS spring and an FS damper and exhibits the fail-safe mechanism by being designed so that the sum of the horizontal forces of both devices does not exceed the maximum control force of TMD.

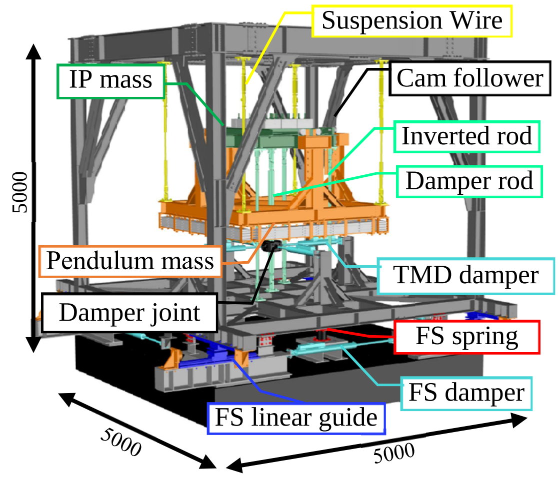


Fig. 5 – Schematic of test specimen

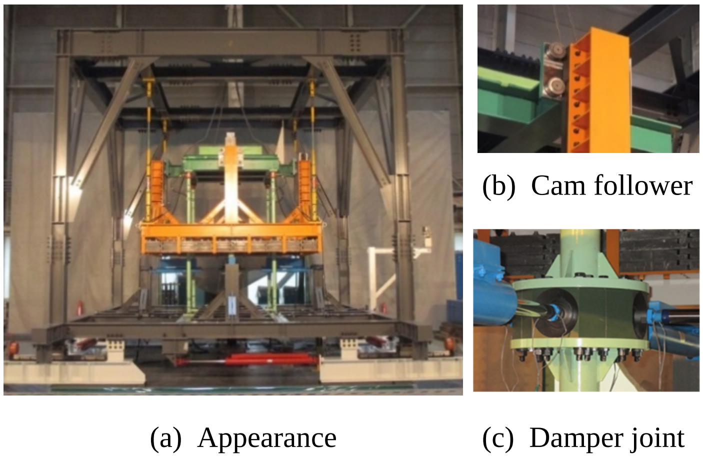


Fig. 6 – Photos of test specimen

Table 1 – Specfication of test specimen

(a) TMD-IP

Mass	Pendulum: IP: Total:	4.46t 1.74t 6.20t
Pendulum Length	Pendulum IP	2m 2m
Linear natural period		4.29sec.
Maximum horizontal displacement		±1000mm
Maximum pendulum angle		±30°

(b) FSP

FS mount	Cross-shaped linear guide ±500mm 4 units
FS damper	Oil damper ±500mm 4 units Relief force F_r: 5kN Primary dampig: propotional to square of velocity Secondary damping: 5kN/(m/s)
FS spring	Laminated rubber bearing ±1200mm 1 unit with 2 layers Stiffness: 19 kN/mm / unit

3.2 Operation of FSP

Within the range of the design seismic motion, the range where TMD is less than the limit deformation determined by the mechanism of the spring and the damper, etc., the FS damper keeps a substantially fixed state, and the FSP hardly deforms, and only the TMD operates as shown in Fig.4(b).

When the building encounters excessive ground motion exceeding the design level and the control force of the TMD approaches the maximum control force, FSP starts to deform reaching the relief area where the damping coefficient of the FS damper decreases, while transmitting control force to the building as shown in Fig. 4(c).

Since the force acting on the TMD from the building, which is equal to the control force acting on the building from the TMD, is limited by the operation of the FSP, collision and damage due to excessive response of the TMD, and the rapid increase of building responses such as accelerations and story shear forces can be prevented. Also, the energy is absorbed by the FS damper while the FSP is operating, so there is no raid reduction in seismic performance of the building[6].

4. Experimental verification

In performing the detailed design of the TMD-IP device, it is necessary to confirm that the pendulum and the inverted pendulum move as expected and have a natural period derived from the thoery described in Chapter

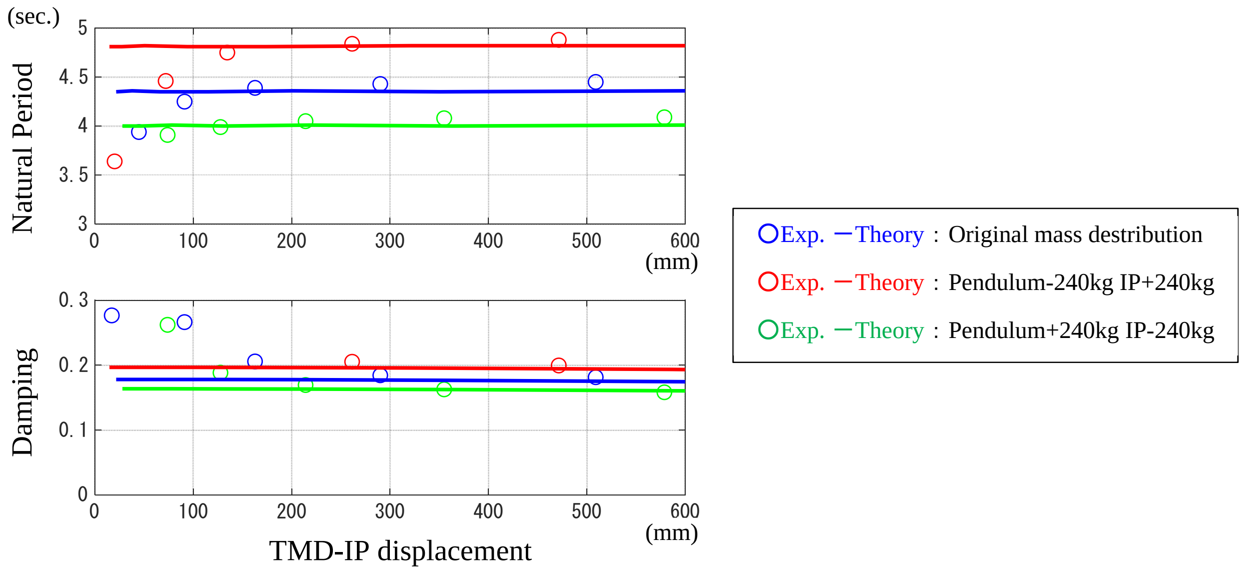


Fig. 7 – Natural period and damping due to TMD-IP displacement

2. In addition, since the actual device has a two-dimensional system that moves in two horizontal directions, it is necessary to confirm that there is no interaction effect from the orthogonal direction.

Therefore, a prototype scaled model of TMD-IP with FSP is manufactured and the dynamic charactericties are verified by shaking table test. Here, the outline and the experimental results are described.

4.1 Outline of test specimen

The test specimen was designed assuming a building with a primary natural period of about 4 seconds. When trying to realize a TMD with a period of 4 seconds using only a pendulum, the length of the pendulum would be 4 m. However, by using an inverted pendulum to be TMD-IP, the test specimen was realized with a pendulum length of 2 m, which is half the length of conventional pendulum type TMD.

Fig. 5 shows a schematic diagram of the test specimen, and Fig. 6(a) shows the appearance of the specimen. Table 1 summarizes the specifications of the test specimen.

The pendulum mass of 4.46t is supported by four suspension wires (φ22.4mm) with length of 2m from the upper frame. The inverted pendulum mass of 1.74t is supported by four inverted rods (φ60.5x4mm) with length of 2m from the lower frame. The pendulum and the inverted pendulum are integrated horizontally by cam followers arranged on the four sides of the pendulum and the inverted pendulum frame, allowing relative movement only in the vertical direction as shown in Fig. 6(b). Universal joints are inserted at both ends of both the suspension wire and the inverted rod so that the device can move smoothly in two horizontal directions.

As the damping of this TMD-IP, two oil dampers were installed in each of two horizontal directions, for a total of four. These oil dampers are attached via ball bearings to a damper rod that does not support the load of the inverted pendulum provided in the center of the inverted pendulum frame as shown in Fig. 6(c). The attached position of the oil damper in the vertical direction is 0.6 m below the lower part of the 2 m damper rod. As a result, the required damper stroke is 0.3 m for the designed maximum stroke of the pendulum and the inverted pendulum of 1 m where the pendulum angle is 30 °, resulting in a compact damper design.

As for fail-safe mechanism, an FSP is provided below the lower frame and the entire frame is supported by cross-shaped linear guides. Relief type oil dampers are used to switch the operation of the FSP and laminated rubber bearings are used as the restoration mechanism. The characteristics of the relief type oil dampers are such that it has the largest possible damping characteristic up to the relief load by using the characteristic proportional to the square of the velocity, and the damping coefficient becomes as small as possible after the relief load.

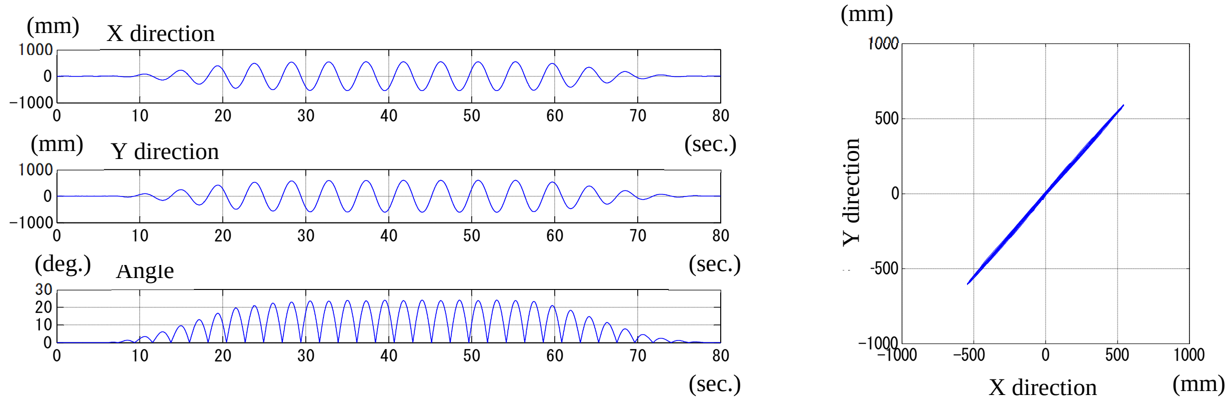


Fig. 8 – TMD-IP dsiplacement due to bi-directional sinusoidal in-phase excitation

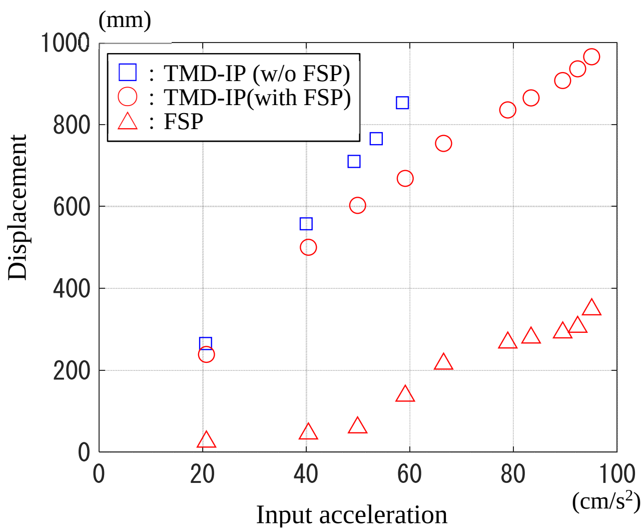


Fig. 9 – Relation between input acceleration and TMD-IP/FSP displacement

4.2 Results of experiments

4.2.1 Natural period and damping

Fixing the FSP mechanically, the shaking table is oscillated with single sinusoidal wave with a period of 4 seconds which is near the natural period of TMD-IP. Each peak value of the amplitude is extracted from the free vibration wave of the pendulum displacement, and the relationship between the amplitude level and the natural period, and that between the amplitude level and the damping are obtained. Fig. 7 shows the results. The figure also shows the results of the natural period and damping when the weight of 240 kg is mutually moved between the pendulum and the inverted pendulum from the standard mass distribution of the pendulum and the inverted pendulum in Table 1. Each solid line indicates a theoretical value obtained by using the exact solution of Eq. (1).

As results, when the pendulum amplitude is less than 100 mm, the natural period is short and the damping increases due to the influence of the cam follower friction, etc., but for larger amplitudes, it was confirmed that the natural period and the damping well agree with the theoretical values. It was also confirmed that the period could be easily adjusted by mutually moving the weight between the pendulum and the inverted pendulum.

4.2.2 Bi-directional movement

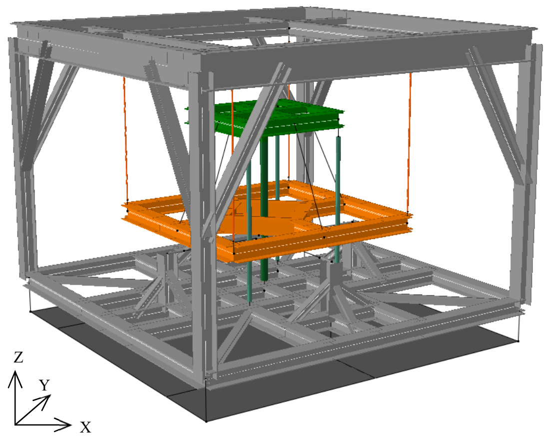


Fig. 10 – Analysis model

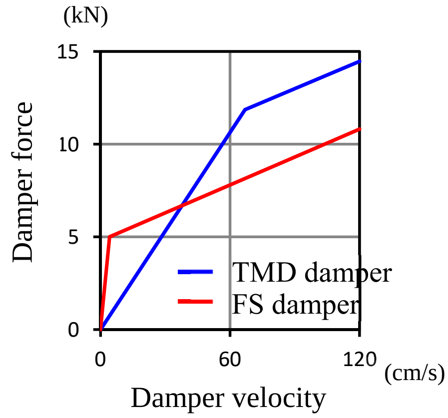


Fig. 11 – F-V Characteristic of oil dampers

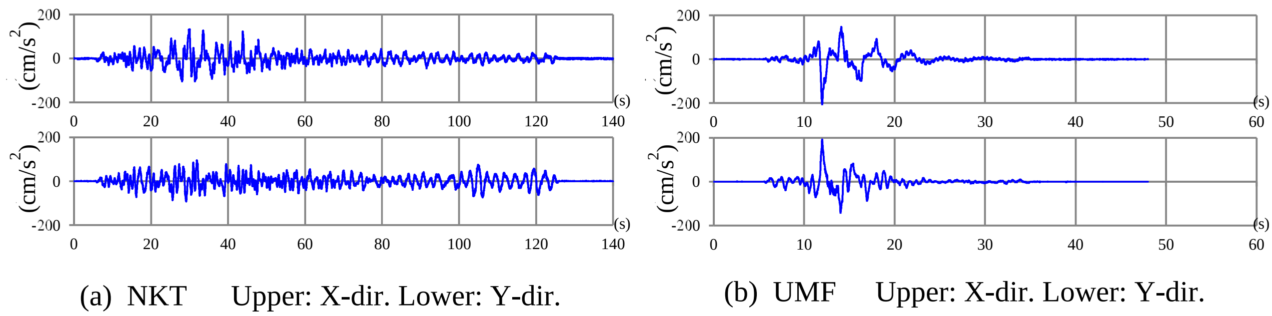


Fig. 12 – Time histories of input acceleration

The shaking table was excited with a sinusoidal wave with a period of 4.5 seconds which is near the natural period in two horizontal directions in phase and out of phase with 90 °. It was also excited with different period of 4 seconds and 4.5 seconds for each direction. Fig. 8 shows the time history and the horizontal bi-directional displacement Lissajous when excited by an in-phase sinusoidal wave with a period of 4.5 seconds. It was confirmed that the TMD-IP moved smoothly even in two horizontal oscillation.

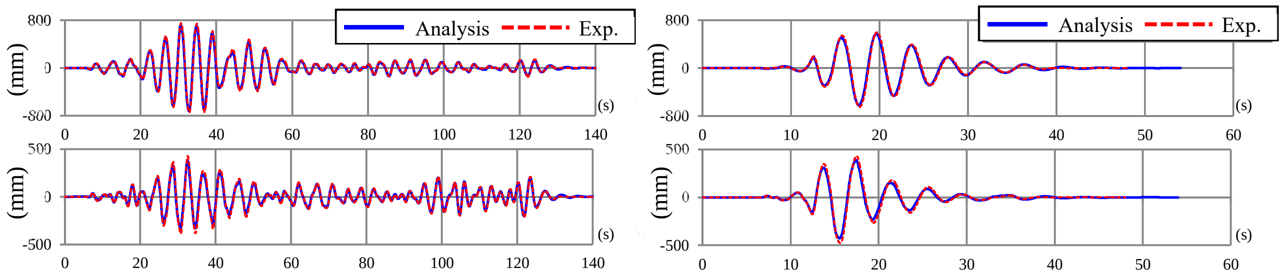
4.2.3 Performance of FSP

Steady sinusoidal wave with a period of 4.5 seconds, which is near the natural period, was applied to the shaking table and the relationship between input acceleration and TMD-IP displacement and FSP displacement was confirmed. The results are shown in Fig. 9.

As shown in Fig. 9, when the FSP is fixed mechanically, without fail-safe system, the pendulum amplitude increases in proportion to the increase in input acceleration. However, when TMD-IP equipped with FSP, the FSP operates and displaces with an increase in input acceleration, resulting in suppressing the increase in TMD-IP displacement. So, it is confirmed that the proposed FSP functioned as a fail-safe mechanism against excessive displacement of TMD-IP.

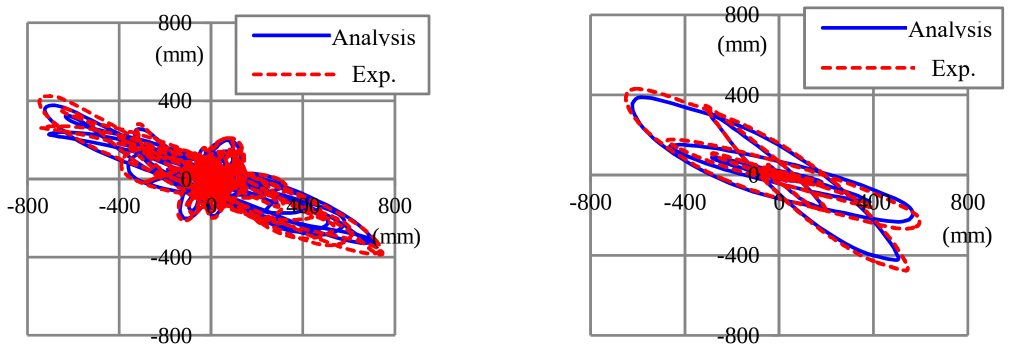
5. Verification of numerical analysis method

The proposed system has been confirmed to behave as expected by the prototype scaled model experiment. However, experimental verification using a scaled test specimen cannot be performed every time TMD-IP is designed. Therefore, it is necessary to establish a method that can reproduce complicated pendulum and inverted pendulum behavior by analytical simulation.



(a) NKT Upper: X-dir. Lower: Y-dir. (b) UMF Upper: X-dir. Lower: Y-dir.

Fig. 13 – Time histories of TMD-IP displacement without FSP



(a) NKT (b) UMF

Fig. 14 – Lissajous of TMD-IP displacement without FSP

Since the behavior of the TMD-IP has a geometric nonlinearity due to large deformation, analysis was performed using a finite element analysis considering the geometric nonlinearity. In this analysis, the exact solution of Eq. (1) is solved for bi-directional horizontal input motion.

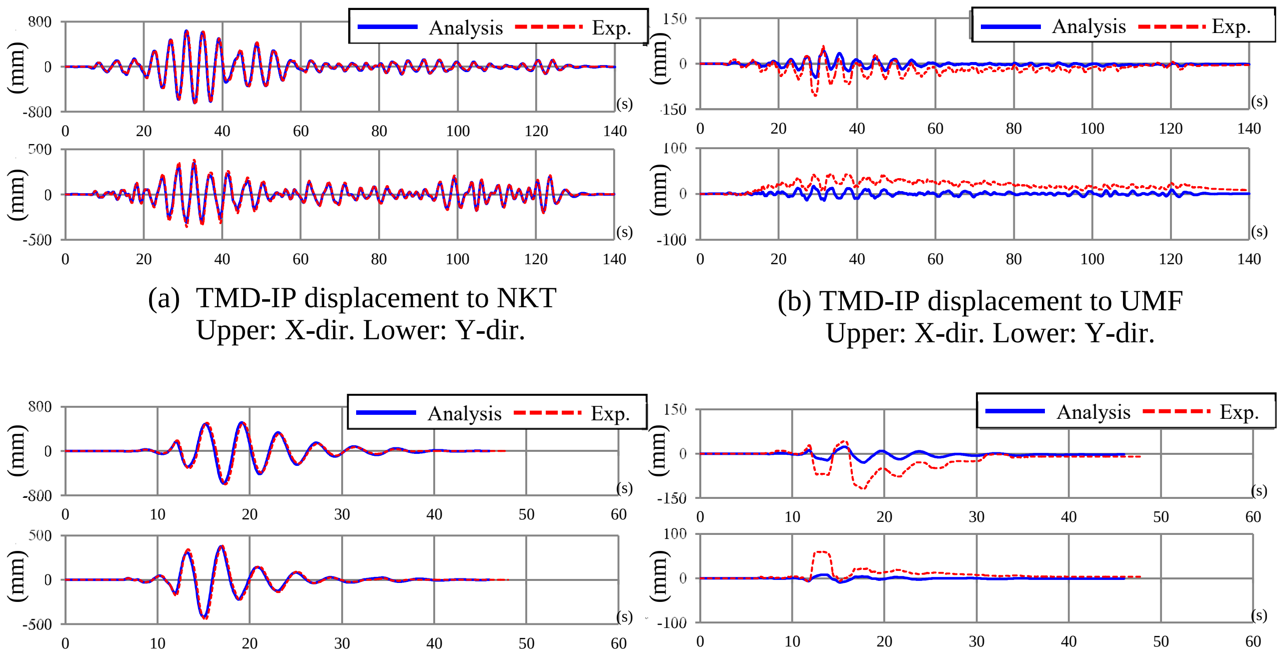
In this chapter, based on the experimental results in Chapter 4, the validity of the numerical analysis was examined by comparing the results with that of shaking table tests for the inputs of the seismic responses at the top of the high-rise building.

5.1 Parameters of the analytical model

The analytical model simulating the shaking table test of TMD-IP with FSP is shown in Fig. 10. Each member of the analytical model was modeled by wire elements, and the specifications used in the analysis were the same as in the shaking table test shown in Table 1. Here, the F-V diagram of the oil damper is shown in Fig. 11, where the primary damping of the FS damper is modeled by linear approximation. The boundary conditions were set appropriately simulating the joints.

As the input wave to be used, it is assumed that the TMD-IP is installed at the top of a high-rise building, and response waves in two X and Y directions at the top of the building with a primary natural period of 4.0 and 4.5 seconds are used. Two cases are studied. One is using a response wave to the Nankai trough earthquake (NKT) and the other is a response wave to the Uemachi fault earthquake 3B (UMF), which is one of the largest expected near fault earthquakes in Japan. Fig. 12 shows time history of the acceleration response used as inputs.

5.2 Analytical study on TMD-IP



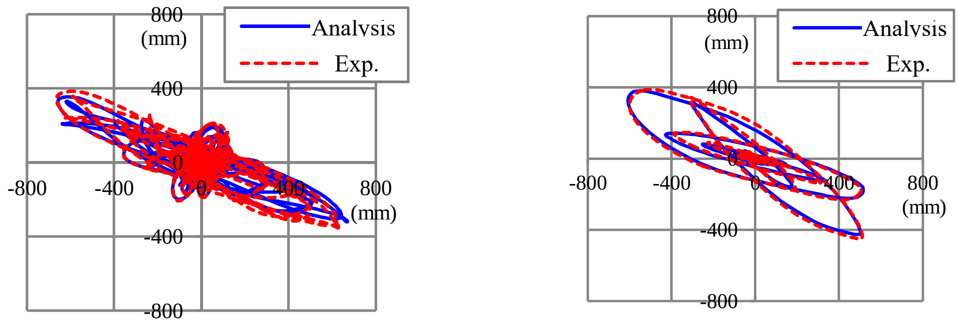
(a) TMD-IP displacement to NKT
Upper: X-dir. Lower: Y-dir.

(b) TMD-IP displacement to UMF
Upper: X-dir. Lower: Y-dir.

(c) FSP displacement to NKT
Upper: X-dir. Lower: Y-dir.

(d) FSP displacement to UMF
Upper: X-dir. Lower: Y-dir.

Fig. 15 – Time histories of TMD-IP and FSP displacement



(a) NKT

(b) UMF

Fig. 16 – Lissajous of TMD-IP displacement with FSP

Fig. 13 shows a comparison of the time histories between analysis and experiment for the TMD-IP displacement response, where FSP is fixed mechanically. Fig. 14 shows its Lissajous. As shown in these figures, the analysis values and the experimental values are well agreed, and it was confirmed that the TMD-IP experiment could be analyzed with high accuracy.

5.3 Analytical study on FSP

As described in Chapter 3, the FSP has been devised as a mechanism to prevent excessive deformation of the TMD when an earthquake larger than the design level occurred. Here, the response reduction effect from the analysis and experiment is studied using the same inputs as previous section.

Fig. 15 shows a comparison of the time histories between analysis and experiment for the displacement response of the TMD-IP and the FSP. Fig. 16 shows its Lissajous. As shown in these figures, the analysis value of the displacement of TMD-IP and the experimental value are well agreed, but not as well for the displacement of the FSP. This may be due to the dispersion of the FS damper used in the experiment and the effect of the linear approximation model of the primary damping of the FS damper. However, the response of the FSP is smaller than that of the TMD. Therefore, the effect caused from such errors on TMD-IP response is limited.

In addition, comparing the results of the TMD-IP and that with FSP, the effect of the FSP on preventing excessive deformation of the TMD-IP can be confirmed even under nonstationary input motion.

6. Conclusions

TMD-IP has been developed to ensure the safety of high-rise buildings in response to long-period ground motions expected during such as Nankai trough earthquake. In addition, an FSP was proposed as a countermeasure against an excessive earthquake for the safety of the TMD-IP. The findings obtained through theoretical studies, prototype scaled model experiments, and finite element analysis are summarized below.

- Since the same natural period can be realized with a shorter suspension length than the conventional pendulum type TMD, it is effective for downsizing the equipment.
- Natural period of TMD-IP can be adjusted by the distribution of the mass on the pendulum and the inverted pendulum, so the natural period can be easily adjusted even after completion.
- In the design of the whole building-TMD-IP system, the TMD-IP can be treated as a linear single degree of freedom system model, and the conventional TMD optimization method can be applied.
- As a countermeasure against an excessive earthquake, installing the proposed FSP can prevent damage to TMD-IP without sacrificing the response control performance.
- The complex behavior of pendulum and inverted pendulum can be accurately reproduced by finite element analysis considering geometric nonlinearity and can be used for detailed design of TMD-IP.

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