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TIME-VARIANT RELIABILITY-BASED DESIGN OPTIMIZATION OF TUNED VISCOUS MASS DAMPER UNDER NONSTATIONARY SEISMIC EXCITATION UTILIZING KRIGING SURROGATE MODEL

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Abstract

The time-variant reliability-based design optimization (tRBDO) is considered for tuned viscous mass damper (TVMD) equipped linear building frames subject to seismic excitations, modeled as nonstationary Gaussian random processes, while accounting for parametric uncertainty. The TVMD is a recently proposed linear passive dynamic vibration absorber for the earthquake protection of building structures coupling viscous and tuned-mass dampers with an inerter device. The frequency and damping ratio, acting as a TMD mass amplifier, are treated as design variables. The time-variant reliability criteria are adopted for quantifying the structural performance under the nonstationary seismic excitations, expressed through the probability of occurrence of different failure modes related to the trespassing of acceptable thresholds for the adopted performance variables. The design-under-uncertainty problems that employ the time-variant reliability criteria as a design constraint and consider its estimation through stochastic simulation. This leads to a high computational burden in each optimization cycle. Therefore, the development of simple accurate, and efficient methods for estimation of the extreme response of dynamical systems subjected to nonstationary random excitations is discussed in the present paper. A framework relying on Kriging surrogate modeling is proposed here to alleviate the computational burden and provides an accurate depiction of the possible model outcome. The surrogate model is formulated to approximate the system response (surrogate model output) with respect to both the design variables and the uncertain model parameters (surrogate model input). The total probability theorem is then applied to calculate the time-dependent probability of failure. So that it can simultaneously support the uncertainty propagation and the design optimization, adopting a differential evolution (DE) strategy for the latter. The tRBDO problem is solved using this approximation for evaluating the reliability constraints over the entire design domain and identifying the feasible region satisfying them. The proposed method is demonstrated using a benchmark 10-story frame structure equipped with TVMD. It is shown that this approach is able to accurately evaluate time-variant reliability and efficiently carry out the tRBDO involving nonstationary stochastic processes, minimizing the number of simulations for the expensive system model.

Keywords: time-variant reliability analysis; design optimization; nonstationary stochastic excitation; adaptive Kriging model

1. Introduction

The optimal design of the energy dissipation device for structural systems is still a challenge considering the inherent uncertainty of material properties, geometry parameters and loadings. In recent years, different optimal design methods for energy dissipation devices are proposed under uncertain seismic input. In order to reduce the complexity of the problem, a simplified approach is usually adopted to represent the performance objectives in terms of the variance value of displacement or inter-story shift response of the building [1]. However, the response characteristics associated with structural performance is the extreme of the response, especially the exceeding of the acceptable threshold corresponding to the bounds of structural safety in engineering applications. When considering uncertain earthquake input, many studies use simple random models (e.g., stationary white noise or Kanai-Tajimi model) to describe earthquake input [2, 3]. However, previous studies show that the reliability of structures is underestimated if the non-stationarity of excitation is not considered. In spite of the conceptual and numerical complexity caused by time variability, it is still necessary to carry out design optimization based on time-varying reliability to ensure the safety level of energy dissipation structures throughout their service life. In addition, uncertainties in structural parameters, such as those including stiffness, mass and damping, may have a greater impact on the response than the uncertainties in the excitation according to previous study [4]. Therefore, in practical design problems, the robustness of the system to uncertainty must be considered as a design problem to avoid performance degradation and dissonance effects.

In the past decade, significant progress has been made in overcoming the limitations of the reliability-based design of buildings with energy dissipation devices. Chakraborty [5] optimized the TMD parameters considering its dynamic reliability by using the first-order perturbation analysis method for the single-DOF structural system with interval seismic power spectrum parameters and structural parameters. Yu et al. [6] simultaneously considered the uncertainty of structural parameters and TMD parameters and carried out a robust optimization design of TMD parameters based on the dynamic reliability of structural systems as the constraint condition. Taflanidis and Gao [7] proposed a simulation-based framework for risk assessment and probabilistic sensitivity analysis of isolated structures, which explicitly incorporates uncertainties in excitation and structural models. Recently, Scozzese et al. [8] proposed a reliability-based optimization (RBO) framework and concluded that ignoring the combination of uncertainties related to damper design parameters may increase the seismic risk of buildings.

However, considerable researches on the optimization design based on time-varying reliability (i.e. the reliability constraint is time-dependent) have not been developed, mainly because the calculation of time-varying reliability is complicated. The core problem of structural reliability analysis is to calculate the multiple integrals of the joint probability density of the variables over the failure domain. It is impossible to obtain the failure probability by analytical method for large and complex structures. Monte Carlo numerical simulation method is effective for all structures, but it requires a large number of samples and heavy computation. The alternative method of the surrogate model is a research hotspot in this field. The most representative surrogate model is the Kriging model, which is more flexible than a single parameterized model and overcomes the limitations of a non-parameterized model when dealing with higher-dimensional data. In recent years, the adaptive sampling method of Kriging model has been applied to structural reliability problems with high time consuming implicit functions. Bichon [9] et al. proposed an effective global reliability analysis method (EGRA) for structural systems with implicit functional functions, and established the desired feasibility function. Echard et al. [10] proposed a learning function (i.e. U-function) to select new training samples and developed an active learning reliability method (AK-MCS) combining Kriging and original MCS. Hu and Mahadevan [11] used the global sensitivity analysis method to select training samples for the new reliability method based on the adaptive Kriging model. Recently, some studies have applied the adaptive surrogate model method to the research of time-varying reliability in order to reduce the computational burden. For example, Wang and Chen [12] proposed a new adaptive extreme response surface method (AERS) to continuously quantify and to improve the accuracy of the Kriging agent model by using confidence level measures through sequential adaptive sampling.

In this setting, the design optimization for recently developed tuned mass damper (TVMD), a new seismic control system achieving enhanced performance compared to a same-mass TMD by exploiting the mass amplification effect, is proposed in this study. In the field of civil engineering, some scholars have made plenty of researches on the basic theory and application of TVMD. Ikago et al. [13] proposed a parameter design method for TVMD based on the fixed-point theory with the inerter coefficient determined based on experience. Pan et al. [14] suggested that the design of TVMD for single-DOF system should consider the objective of structural performance requirements and the principle of cost optimization. Taflanidis et al. [15] proposed a multi-objective design of inerter-based dampers according to performance-based seismic design using reliability criteria. However, the previous studies neglect the time-variant performance of the TVMD under nonstationary excitation for the computation of reliability constraints.

The major contribution of this study is that it offers a comprehensive trade-off in time-variant reliability-based design optimization without a giant computational burden. The proposed simulation-based tRBDO framework balance the objective of cost and the reliability constraint of performance by using the adaptive Kriging surrogate model. The rest of the paper is organized as follows. Section 2 proposes the general statement of the problem. In Section 3, the crucial methods included in the tRBDO framework is presented. Section 4 shows the illustration of the proposed tRBDO method for TVMD equipped in a multi-DOF structure. Section 5 are some conclusions.

2. Problem statement

In engineering design, the RBDO is to minimize the value of the object function, such as the life-cycle cost (including initial cost, failure cost, and maintenance cost), under the reliability-based constraint condition. In general, the RBDO problems can be expressed as:

$$\begin{aligned} &\min\ f(\mathbf{w},\ \mathbf{d}) \\ &\text{s. t. }\ P_f = \Pr(g_i(\mathbf{w},\ \mathbf{d}) \le 0) \le 1 - R_i,\ \ i = 1,\dots,n \\ &\qquad \mathbf{w}^{\mathrm{L}} \le \mathbf{w} \le \mathbf{w}^{\mathrm{U}} \\ &\qquad \mathbf{d}^{\mathrm{L}} \le \mathbf{d} \le \mathbf{d}^{\mathrm{U}} \end{aligned} \tag{1}$$

where $f(\mathbf{x},\ \mathbf{d})$ is the object function, $g_i(\mathbf{x},\ \mathbf{d})$ is the ith failure mode, and R_i is the ith required reliability target. $\mathbf{w}$ is a vector of design random variables, $\mathbf{d}$ is a vector of design random deterministic variables, and n is the number of constraints, respectively. However, the excitation uncertainty has been considered as well as parameter uncertainty to ensure satisfactory optimal design. Therefore, time-variant RBDO (tRBDO) is introduced to obtain the optimal solution while satisfying the reliability requirements of the system over a period of time:

$$\begin{aligned} &\min\ f(\mathbf{w},\ \mathbf{d}) \\ &\text{s. t. }\ P_f(0,T) = \Pr\left(\exists t \in [0,T],\ g_i(\mathbf{w},\ \mathbf{d},\mathbf{F}(t),t) \le 0\right) \\ &\qquad\qquad \le 1 - R_i,\ \ i = 1,\dots,n \\ &\qquad \mathbf{w}^{\mathrm{L}} \le \mathbf{w} \le \mathbf{w}^{\mathrm{U}} \\ &\qquad \mathbf{d}^{\mathrm{L}} \le \mathbf{d} \le \mathbf{d}^{\mathrm{U}} \end{aligned} \tag{2}$$

where $\mathbf{F}(t)$ represents a vector of stochastic processes. In addition, nonstationary random processes are used to describe earthquake input rather than simple random models (e.g., stationary white noise or Kanai-Tajimi model). Thus, time-related uncertainties are taken into account in RBDO to improve considerably the estimates of the reliability of structural systems.

In this paper, the n-DOF linear vibratory system controlled by a TVMD system with random parameters is considered:

3. Proposed methodology

3.1 Time-variant reliability analysis and total probability theorem

As mentioned before, the estimation of the probability constraint is a crucial undertaking in the optimal design process. In dynamic structural systems, the time-variant probability of failure problem is defined as a first-passage problem that a quantity of interest related to the displacement or velocity responses exceeds a deterministic constant threshold *b*. Considering the random parameters, the calculation of the time-variant probability of failure becomes a challenging problem. Therefore, the cumulative time-variant probability of failure is calculated based on the total probability theorem. The cumulative time-variant probability of failure can be expressed as:

$$P_f(0,T) = \int_{\Omega_{\mathbf{w}}} P(FE/\mathbf{w}) f_{\mathbf{w}}(\mathbf{w}) d\mathbf{w} \tag{3}$$

where FE represents the failure event, $P(FE/\mathbf{w})$ is the conditional time-variant probability, $f_{\mathbf{w}}(\mathbf{w})$ is the joint probability density function of the random vector, and $\Omega_{\mathbf{w}}$ is the support of $f_{\mathbf{w}}(\mathbf{w})$.

In general, the exact computation of this integral is impractical. Thus, simulation methods are applied to calculate the time-variant probability of failure by conditional probabilities with constant parameters for each realization of $\mathbf{w}$ based on the total probability theorem. The equation of cumulative time-variant probability of failure can be rewritten as:

$$P_f(0,T) = \sum_{i=1}^{N} P\{FE/\mathbf{w} = \mathbf{w}_i\} P(\mathbf{w}_i) \quad \mathbf{w}_i \in \Omega_{\mathbf{w}} \tag{4}$$

In this research, the accurate computation of the conditional time-variant probability of failure is presented in Section 3.2 and then the cumulative probability is derived based on a prebuilt Kriging surrogate model as described in Section 3.3. This method can deal with non-normal and correlated random variables without additional computation to satisfy accuracy requirements taking advantage of the total probability theorem.

3.2 Conditional time-variant probability of failure

The dynamic loads such as earthquake ground motions and wind action on structural systems are represented by a nonstationary Gaussian random process. An evolution spectrum process model for nonstationary random excitation has been widely used in engineering:

$$f(\boldsymbol{\theta},t) = A(t,\omega)\cdot x(\boldsymbol{\theta},t) \tag{5}$$

where $\boldsymbol{\theta}$ is a vector of input uncertainty parameter, $A(t,\omega)$ is a deterministic modulation function of t and ω, $x(\boldsymbol{\theta},t)$ is a zero-mean stationary random process. In this way, the spectral characteristics of non-stationary random processes can be described by the usual concepts of energy and frequency. If $A(t)$ is used instead of $A(t,\omega)$, then the non-uniform modulation evolution spectrum is replaced by the time modulation evolution spectrum commonly used in engineering.

The computation of time-variant reliability for structural dynamic response is transformed into the computation of equivalent distribution of extreme value. Based on the evolution spectrum representation theory of nonstationary random processes, a spectrum representation method for simulating nonstationary ground motions is derived:

$$f(t) = \sum_{k=1}^{N} 2\left|A(t,\omega)\right|\sqrt{S_{ff}^{0}(\omega)\Delta\omega}\cos(\omega_k t + \theta_k)$$
$$\Delta\omega = \omega_u / N \tag{6}$$

where ω_u is the calculated cut-off frequency, θ_k is a random phase angle uniformly distributed in the interval $[0,2\pi]$ and independent of each other. This paper only considers the single output problem. According to the mentioned definition of the first-passage problem, the cumulative time-variant probability of failure can be expressed as:

$$P_f(0,T) = \Pr\left(\exists t \in [0,T],\ y(\mathbf{w},\mathbf{d},\boldsymbol{\theta},t) \geq b\right) \quad (7)$$

where $y(\mathbf{w},\mathbf{d},\boldsymbol{\theta},t)$ is the system response.

Due to the high computational cost, applications of existing advanced reliability analysis methods for time-variant reliability analysis is still a challenge. To covert the time-variant problem into the time-invariant one, the extreme response variable is defined as:

$$y_e = \max_{t\in[0,T]} \left\{ y(\mathbf{w},\mathbf{d},\boldsymbol{\theta},t) \right\} \quad (8)$$

For each realization of parameters $(\mathbf{w},\mathbf{d})$, the subset simulation method is used to generate the random samples in the seismic excitation, and the extreme value of the structural response is calculated to obtain the conditional probability of failure. The subset simulation converts small probability of failure to a product of a series of relatively large probability of failure events by introducing intermediate events adaptively. A brief introduction to subset simulation is presented here. A series of nested intermediate failure events, $FE_1 \supset FE_2 \supset ... \supset FE_m = FE$, is defined to express target probability of failure:

$$P_f = P(FE) = P(FE_m | FE_{m-1}) P(FE_{m-1}) = ... = P(FE_1) \prod_{i=2}^{m} P(FE_i | FE_{i-1}) \quad (9)$$

The expression of the intermediate event is similar to the target failure event FE, i.e. $FE_i = \left\{ y_e \geq b_i, i = 1,...m \right\}$ $(b = b_m > ... > b_2 > b_1)$. m is the total number of intermediate events, which can be determined adaptively by set intermediate probabilities $P(FE_i | FE_{i-1})$ as a constant. The improved Metropolis-Hasting algorithm is employed to solve the problem of generating high-dimensional conditional samples in view of its advantages of generating samples that have arbitrary high-dimensional probability distribution.

3.3 Adaptive Kriging surrogate model for the probability of failure predicting

In this section, the Kriging surrogate model is constructed for the conditional probability of failure and updated by identifying important samples across parameter domain. Then the cumulative probability of failure is evaluated using Monte Carlo simulation on the resulting Kriging surrogate model considering the probability density function. The proposed time-variant reliability-based design optimization aims to handle reliability constraints for optimal design. Thus, an active learning approach called the AK-MCS method is applied to generate additional training samples located in the corresponding parameter domain near the critical probability of failure $\tilde{P}_f$ to increase the accuracy of the surrogate model. The vector of design random variables $\mathbf{w}$ and the vector of design random deterministic variables $\mathbf{d}$ are expressed uniformly as a vector of design variables $\mathbf{X}$. The normalized index of probability of failure is defined as:

$$g(\mathbf{x}) = \frac{P_f(\mathbf{x})}{\tilde{P}_f} - 1 \quad (10)$$

The details of the proposed adaptive Kriging surrogate model approach are discussed in the following steps.

Step 1. The initial samples $\mathbf{x}^S = \left\{ \mathbf{x}^{(1)},\ldots,\mathbf{x}^{(N_0)} \right\}$ are generated according to a predefined design of experiment scheme. A commonly used sampling approach, Latin hypercube sampling (LHS), is employed in the parameter domain.

Step 2. The computation method presented in Section 3.2 is called to obtain the conditional time-variant probability of failure $P_f(\mathbf{x})$ at the above sample points. Then the corresponding normalized index set $\mathbf{g}^S=\left\{g\left(\mathbf{x}^{(1)}\right),\ldots,g\left(\mathbf{x}^{(N_0)}\right)\right\}$ is calculated to construct the supporting set $\left\{\mathbf{x}^S,\mathbf{g}^S\right\}$.

Step 3. A Kriging prediction model can be developed by:

$$\hat{g}(\mathbf{x})=\mathbf{h}(\mathbf{x})\boldsymbol{\beta}+Z(\mathbf{x}) \tag{11}$$

where $\mathbf{h}(\mathbf{x})$ is defined as the trend of the model, $\boldsymbol{\beta}$ is a vector of the trend coefficients. $Z(\mathbf{x})$ is a stationary Gaussian process with a mean of zero and a covariance given by the correlation function and the variance of the process. The output of the Kriging model at each point is a Gaussian random variable denoted by

$$\hat{g}(\mathbf{x})\sim N(\mu(\mathbf{x}),\sigma^2(\mathbf{x})) \tag{12}$$

Step 4. Generate a large number of MC candidate samples $S=\left\{s^{(1)},\ldots,s^{(N_{MC})}\right\}$. Predict the corresponding normalized index of probability of failure for each candidate sample by Kriging model.

Step 5. Based on a learning function called U-function, the best sample s^* is chosen to be a training point added to the experimental design. The U-function is based on the misclassification, which means the prediction of sign of the Kriging model fails. The U-function is defined as:

$$U(\mathbf{x})=\frac{\left|\mu_{\hat{g}}(\mathbf{x})\right|}{\sigma_{\hat{g}}(\mathbf{x})} \tag{13}$$

The corresponding probability of misclassification is:

$$P_m(\mathbf{x})=\Phi(-U(\mathbf{x})) \tag{14}$$

where Φ is the cumulative density function of a standard Gaussian variable. The next sample point with the highest probability of misclassification is selected from the sample set $S=\left\{s^{(1)},\ldots,s^{(N_{MC})}\right\}$:

$$s^*=\arg\max_{s\in S}U(s)=\arg\max_{s\in S}P_m(s) \tag{15}$$

Step 6. Check whether the convergence criterion is met. The convergence criterion terminates the improvement of the accuracy of probability estimates by stopping the addition of samples to the experimental design. The stopping criterion is given by:

$$\frac{\hat{P}_f^+-\hat{P}_f^-}{\hat{P}_f^0}\le\varepsilon_{\hat{P}_f} \tag{16}$$

where $\hat{P}_f^0=P(\mu_{\hat{g}}(x)\le 0)$, $\hat{P}_f^\pm=P(\mu_{\hat{g}}(x)\mp k\sigma_{\hat{g}}(x)\le 0)$, $\varepsilon_{\hat{P}_f}=10\%$, and $k=\Phi^{-1}(97.5\%)=1.96$ typically. Return to Step 2 if the convergence criterion isn't met.

Step 7. Estimate the probability of failure considering parameter uncertainty through Monte Carlo simulation with the final surrogate model.

3.4 Summary of the tRBDO framework

Combining Sections 3.1-3.3, the proposed tRBDO framework considering excitation and parameter uncertainties is summarized, which mainly consists of five critical components: (1) stochastic processes realization and description of excitation uncertainty parameters, (2) stochastic equivalent transformation to time-invariant analysis, (3) computation of conditional time-variant probability of failure for deterministic parameters by subset simulation, (4) adaptive Kriging model surrogating extreme value distribution, (5) optimal design using differential evolution (DE) optimization algorithm. The flowchart of tRBDO framework is shown in Fig. 1.

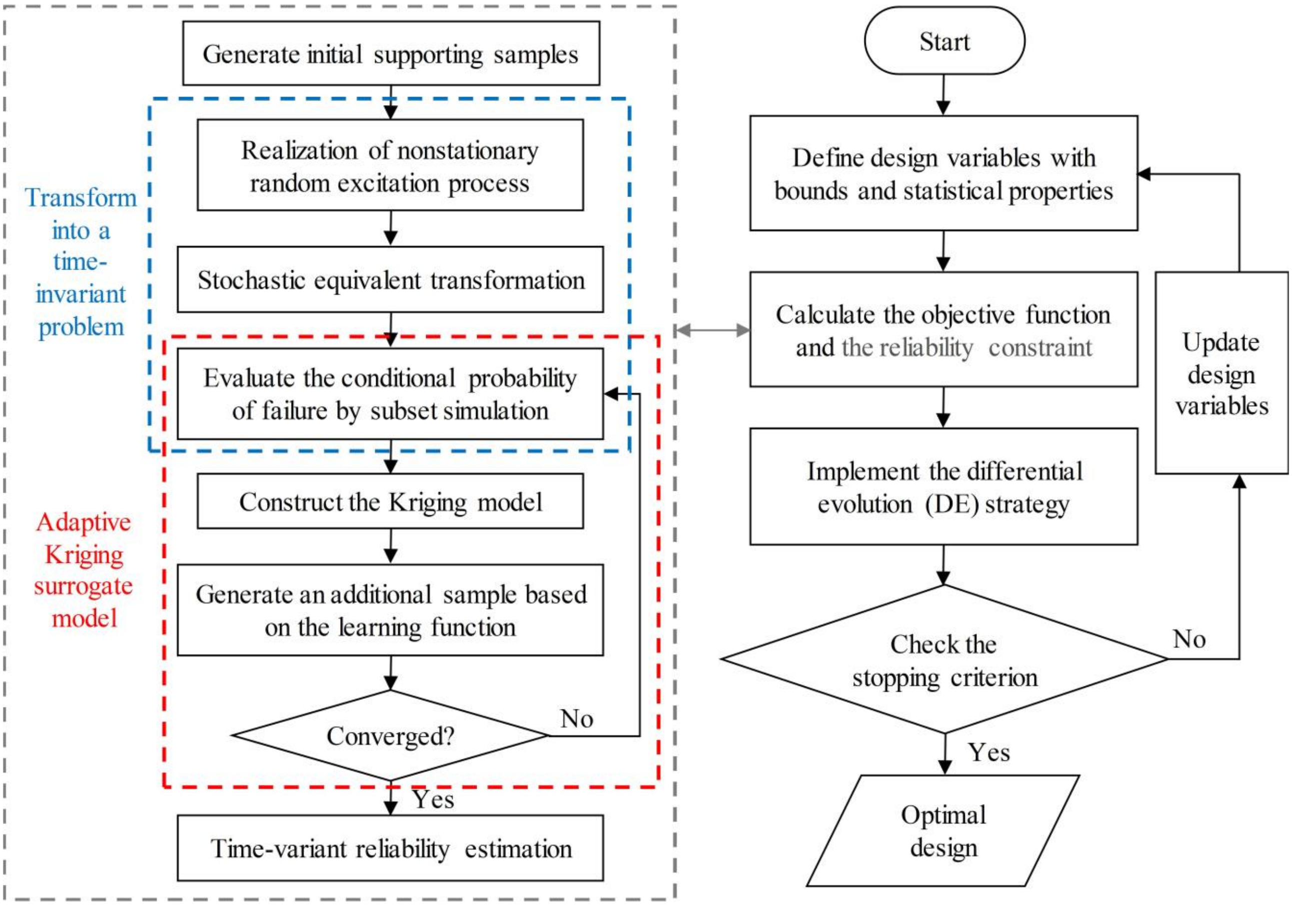


Fig. 1 – The proposed tRBDO framework

4. Illustration of the proposed tRBDO method for TVMD

4.1 Dynamics of primary structure-TVMD system

As shown in Fig. 2, a linear time-invariant *n*-degree-of-freedom primary structure with *n* TVMD in the entire structure is considered in this study. A 10-story benchmark structure is used as the primary structure with properties presented in [16]. The structural system is excited by ground acceleration $\ddot{u}_g(t)$. The equation of motion of the structural system is given by

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=-\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{17}$$

where $\mathbf{u}(t)$ consists of a 10-dimensional displacement vector of the primary system relative to the ground and a vector of each damper in each story:

$$\mathbf{u}(t)=\left\{\mathbf{u}_p{}^T(t),\mathbf{u}_{d1}(t),\ldots,\mathbf{u}_{dn}(t)\right\} \tag{18}$$

Because the TVMD system is activated by the inter-story relative displacement instead of the ground motion, the influence coefficient vector is:

$$\mathbf{r} = \{\underbrace{1,\ldots,1}_{10},\underbrace{0,\ldots,0}_{10}\}^T \tag{19}$$

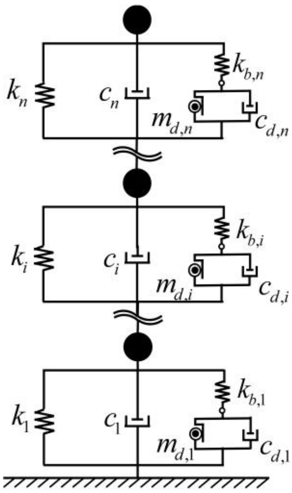


Fig. 2 – Analytical model of the structural system

In the aforementioned equation of motion, $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$ are mass, stiffness, and damping matrices consists of the primary structure parameters, $m_{p,i}$, $k_{p,i}$, $c_{p,i}$, and the TVMD parameters, $m_{d,i}$, $k_{b,i}$, $c_{d,i}$, respectively. Considering the calculation efficiency, the simplified equivalent SDOF system incorporated with the TVMD presented by [16] is selected. The additional mass ratio λ is specified as 0.1. The distribution of the additional masses is proportional to the distribution of the primary structure stiffnesses:

$$m_{d,i} = \frac{\lambda}{{}_1\Omega_p} k_{p,i} \tag{20}$$

Other TVMD parameters can be calculated by:

$$k_{b,i} = \omega^2 m_{d,i} \tag{21}$$

$$c_{d,i} = 2\zeta\omega m_{d,i} \tag{22}$$

4.2 Adaptive Kriging model for the probability of failure

The optimal design of the TVMD parameters, angular frequency ω, and damping ratio ζ, are obtained by the proposed tRBDO method for a given additional mass ratio. In addition, the angular frequency and damping ratio are considered as Gaussian random variables governed by the statistical properties μ_ω and $\sigma_\omega = 0.05\mu_\omega$, μ_ζ and $\sigma_\zeta = 0.05\mu_\zeta$, with respect to the variation of properties of the TVMD system. The nonstationary random excitation is defined by the evolution spectrum process model with time modulation function:

$$g(t) = \begin{cases} (t/t_a)^2 & 0 \le t \le t_a \\ 1 & t_a \le t \le t_b \\ \exp[-\beta(t-t_b)] & t_b \le t \le T \\ 0 & t > T \end{cases} \tag{23}$$

where T=15s, Δt=0.02s, t_a=1s, t_b=7s, and β=0.35. The power spectrum density function is:

$$S_{ff}(\omega)=\frac{\omega_g^4+4\xi_g^2\omega_g^2\omega^2}{\left(\omega^2-\omega_g^2\right)^2+4\xi_g^2\omega_g^2\omega^2}S_0 \tag{24}$$

where $\omega_g=4\pi$, $\xi_g=0.6$, and $S_0=0.0156$. The admissible design space is taken to be [2.5,5.5] for the nominal value of ω and [0,0.4] for ζ, concerning the expected operational range of the TVMD parameters. A total of 10 above-ground story drifts variables with thresholds taken as 1.25% of story height, which represent failure modes for the structure system, are used in the optimal design problem as reliability constraints. The supporting points of Kriging surrogate model for cumulative time-variant probability of failure at $T=15\text{s}$, i.e. $P_{f,15\text{s}}$, obtained from the proposed reliability method considering both excitation and parameter uncertainties by UQLAB toolbox is shown in Fig. 3, consisting of 40 initial samples and 20 additional samples. The locations of new samples indicate the ability to exploit the region of interest focusing on the vicinity of the upper bound on probability of failure defined as 10^{-3}.

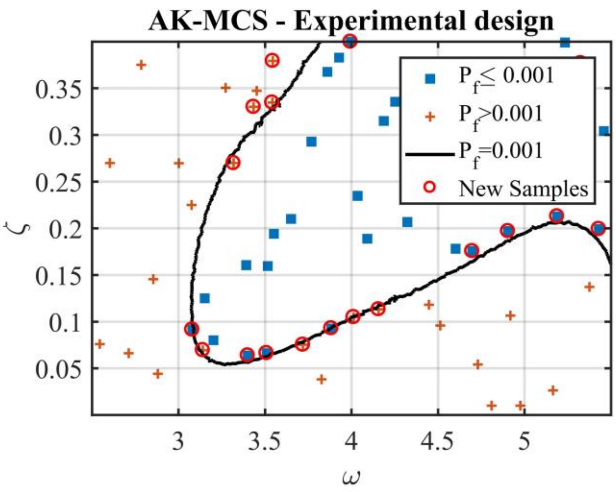
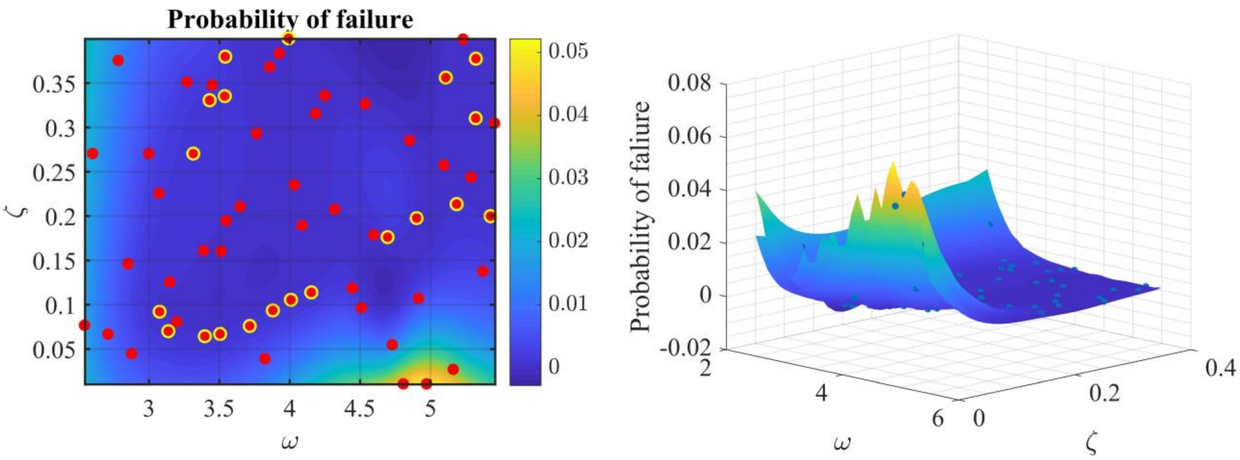


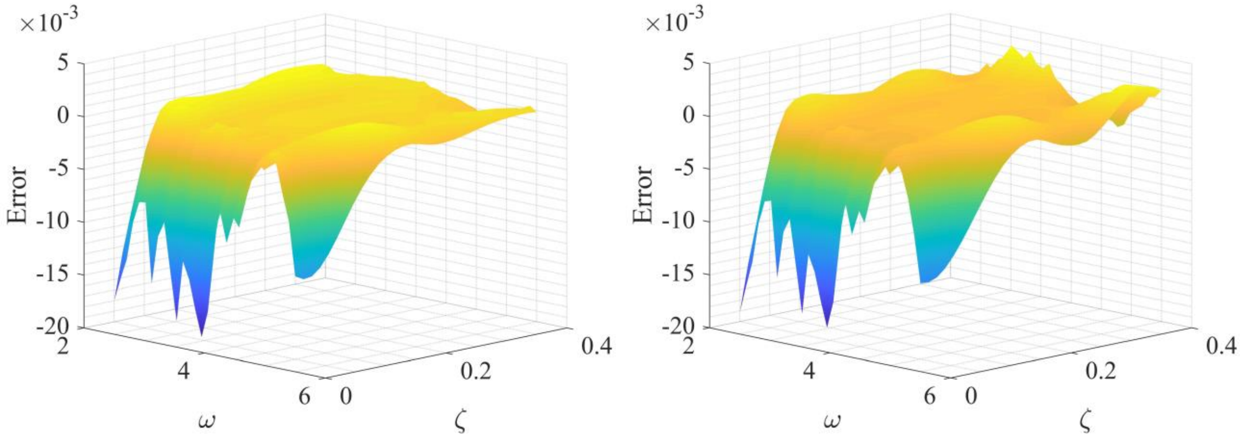
Fig. 3 – Experimental design of adaptive Kriging model

The most robust approach, crude Monte Carlo simulation with 10^6 samples is used to verify the accuracy of the constructed adaptive Kriging model. The graphical visualization of the adaptive surrogate model with supporting samples represented by solid dots and MC simulation results are shown in Fig. 4. The adaptive surrogate model matches the results of MC simulation well in limited regions of interest by reducing the computational burden in areas which are scarce in data. Furthermore, error in estimating probability of failure of both the adaptive Kriging model and the Kriging model without adaptive learning strategy based on 60 LHS samples is compared in Fig. 5. The adaptive Kriging model in better agreement with the MC simulation results compared with the ordinary Kriging model, especially in the region around the critical probability of failure. It is observed that the maximum error insignificant region (less than 10 times of critical probability of failure) is well within 10%, which is considered adequate for all practical purposes.



(a) (b)

Fig. 4 – Graphical visualization of: (a) adaptive Kriging model, (b) adaptive Kriging model and MC simulation



(a) (b)

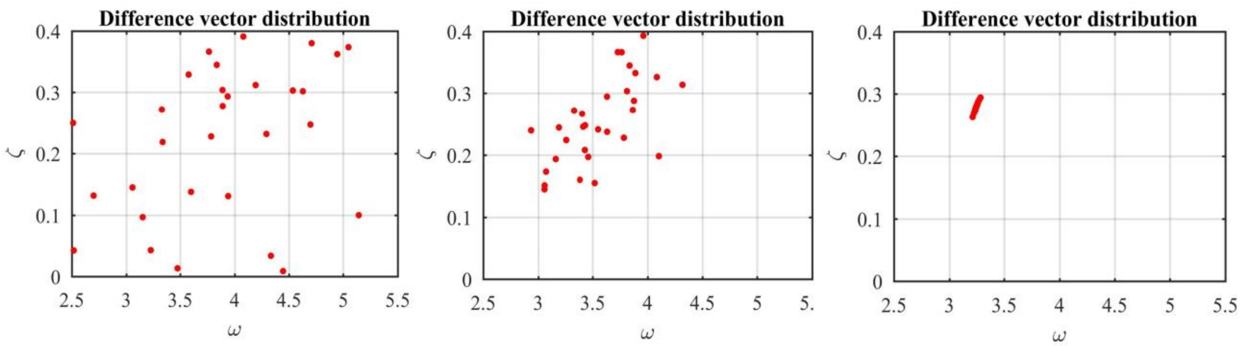
Fig. 5 – Error of surrogate model: (a) adaptive Kriging model, (b) ordinary Kriging model

4.3 The TVMD optimal design using reliability criteria

The proposed tRBDO approach is applied for robust design of the TVMD considering uncertainty to both excitation and parameters. The tRBDO problem in this application is defined as

$$\begin{aligned} \min\ & \sigma_{F_{\max}}(\omega,\ \zeta, t) \\ \text{s. t.}\ & P_{f,15\text{s}}(\omega,\ \zeta) \le 10^{-3} \\ & \omega^{\text{L}} \le \omega \le \omega^{\text{U}} \\ & \zeta^{\text{L}} \le \zeta \le \zeta^{\text{U}} \end{aligned} \tag{25}$$

where $\sigma_{F_{\max}}$ represents the maximum variance of the TVMD forces by using the solutions of pseudo excitation method directly. The DE algorithm is integrated into the tRBDO framework. Because the DE algorithm has been well developed, the detailed implementation is not presented here. The above problem is solved using DE with a population size of 30 for 60 generations. Fig. 6 shows the convergence of difference vector distribution by satisfying the reliability constraint. From this figure, it may be noticed that the optimal design parameters are close to the region corresponding to the critical probability of failure 10^{-3}. In addition, according to the aforementioned tRBDO framework, the best vector in each iteration is added to the experimental design of the surrogate Kriging model. Eventually, the optimal TVMD parameters are $\omega^{opt}=3.2449$ and $\zeta^{opt}=0.2805$, with the objective function $\sigma_{F_{\max}}=1.3472\times10^{6}$. In order to check the accuracy of the proposed surrogate-model-based computation method for reliability criteria, MC simulation is employed to calculate the precise probability of failure based on the optimal parameters. The result of the MC simulation validation is $P_{f,15s}(\omega^{opt}, \zeta^{opt})=9.998\times10^{-4}$. It can be concluded that the proposed tRBDO method using adaptive Kriging model performs accurately and efficiently in presence of excitation and parameter uncertainties.



(a) (b) (c)

Fig. 6 – Difference vector distribution: (a) 2nd iteration, (b) 10th iteration, (3) 20th iteration

5. Conclusions

A time-variant reliability-based design optimization framework of TVMD whose parameters are modeled by probabilistic variables for a seismically excited structure system is presented. The seismic excitation is represented as an evolution spectrum process model of nonstationary random process. In addition, the variance of performance of the TVMD is caused by the uncertainties associated with the physical properties. The distribution of the extreme value instead of the corresponding stochastic process at each supporting point for Kriging model is estimated by the subset simulation. The time-variant problem is transformed into a time-invariant problem. Then, the framework employs an adaptive Kriging surrogate model technique to estimate the probability of failure in parameter domain over the time interval under consideration. The reliability-based constraint is calculated by the MC simulation adopted on the adaptive Kriging model for better approximation. The accuracy of the probability of failure prediction and the optimization result is verified by comparing that with crude MC simulation respectively. The proposed method has advantages in the following two aspects:

(1) The surrogate-model-based method alleviates the overall numerical efforts by reducing the number of evaluations of the limit state function which is implicit with respect to time significantly for time-variant reliability estimation when uncertainties in the parameter and the excitation are significant.

(2) The active learning technique improves the accuracy of reliability prediction using the same number of samples. The adaptive Kriging model for reliability constraint in optimal design deals with the difficulty to assess small probability events and inability to select important samples for enrichment.

6. Acknowledgements

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