DEVELOPMENT OF SEMI-ACTIVE TUNED MASS DAMPER ADAPTABLE TO A STRUCTURE'S PERIOD FLUCTUATION

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Abstract

In Japan, there have been increasing applications of tuned mass damper (TMD) to existing high-rise buildings for seismic upgrading against large earthquakes. Most of them have been applied to steel structures, because RC and SRC structures have the characteristics that their natural period is extended under large earthquakes. Once the natural period is extended, it does not return to original period, so a TMD with tuning deviation cannot achieve the required damping performance.

To use TMD for structures with period fluctuation, the following two methods have been mainly studied. The first is to install multiple TMDs. This method has the advantage that can be designed as a passive system, but its damping efficiency is lower than that of a perfectly tuned single TMD. The second method is to use variable springs for TMD, and adapting TMD's period by changing stiffness of the variable springs. In this case, the damping efficiency is better than that of multiple TMDs. However, for high-rise buildings, the required weight becomes several thousand tons, and it is too difficult to realize a mechanism that can support the weight stably and change stiffness arbitrarily.

To solve these problems, the authors proposed a semi-actively controlled TMD that is adaptable to the period fluctuation of the target structure. This TMD consists of a weight, two linear springs in series and a variable dashpot. The resonance frequency of the proposed TMD is controlled by changing the damping coefficient of the variable dashpot. This system is suitable for large TMD, since it does not require the operation of the stiffness elements. However, it was not revealed how to find the adequate TMD settings for the structure with period fluctuation and how to control the TMD during earthquakes.

In this paper, first, the configuration and design method of the proposed TMD are described. Its response control performance is comprehended using random vibration theory. Second, the control method of the proposed TMD is described. A completely new control method which based on the concept of energy absorption is proposed. Third, the results of earthquake response analyses are demonstrated. The superior response control performance of the proposed TMD is confirmed by comparing other types of conventional TMDs. Finally, the experimental results of the shaking table tests are shown. We made a specimen and its control system, and confirmed their expected behavior.

Keywords: Tuned mass damper; Semi-active control; Variable damper; Control algorism, Shaking table test

1. Introduction

In 2013, we developed a large TMD for seismic upgrading against long-period earthquakes, and applied it to an existing 55-story steel building. Its response control performance as expected against earthquakes and strong winds was confirmed by analyzing the observation records [1]. Seismic reinforcement by TMD has the advantages that the influence on users and the damage to the building's appearance can be minimized. However, most of them have been applied to steel structures so far. Because RC and SRC structures have a characteristics that cracks are generated under large earthquakes, which extends their natural periods. Once the natural period is extended, it does not return to the initial value, so the ordinary TMD cannot achieve its expected performance.

To use TMDs for structures with period fluctuation, the following two methods have been mainly studied. One is to install multiple TMDs. This method has the advantage that we can design them as passive systems. However, since all the weights cannot be synchronized to the fluctuating natural period, the damping efficiency is lower than that of a perfectly tuned single TMD. The other is to configure the TMD with a variable spring and a variable dashpot, and adapt them to fluctuating natural period (hereinafter called KC-variable TMD) [2]. In this case, the damping efficiency is better than that of multiple TMDs. However, for high-rise buildings, the required weight becomes several thousand tons, and it is too difficult to realize a mechanism that can support this weight stably and change the stiffness arbitrarily.

This paper proposes a feasible TMD system that is adaptable to the period fluctuation of the target structure. First, we examine the control performance of conventional TMDs against period fluctuation. Second, the mechanism of the semi-active controlled TMD (hereinafter Proposed TMD) is proposed. This TMD system is modeled using a complex stiffness model and a simple method for determining the system parameters is proposed. The performance of the Proposed TMD is compared with those of conventional systems using random vibration theory. Third, we propose a control method for the proposed TMD. We explain the algorithm, and confirm its validity by simulation analyses against earthquake ground motions. Finally, the results of shaking table tests are described. We verified the tuning mechanism of the Proposed TMD experimentally, and tested the behavior of the control system.

2. Performance of conventional TMD against period fluctuation

2.1 Mechanism of Proposed TMD

This chapter discusses the relation between the control performance and tuning deviation using conventional TMD models. Fig.1(a) shows a Single TMD model. To focus on the basic principle, the applied structure is assumed to be a SDOF no-damping model.

First, we set some parameters to investigate the tuning deviation of the TMD. Since the ratio of the TMD's mass m to the main system's mass M is an important parameter that determines the TMD's control performance, the mass ratio μ is defined as

$$\mu = \frac{m}{M} \tag{1}$$

The parameter η, which represents the period fluctuation of the main system is defined as

$$\eta = \frac{T}{T_0} = \frac{\Omega_0}{\Omega} \tag{2}$$

where T_0 is the initial natural period of the main system and T is its fluctuating natural period. Similarly, Ω_0 and Ω are the circular frequency before and after period fluctuation. Since the target structure of this paper is RC structures, the investigated range of η is approximately 1 to 2.

Next, the method of evaluating the TMD's control performance is described. According to random vibration theory [3], the main system's average response σ under stationary excitation is expressed as

$$\sigma = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|H_{X_0}(ip)\right|^2 S_0(p)dp} \tag{3}$$

where p is the circular frequency of the excitation, $S_0(p)$ is the power spectral density (PSD) of the input ground motion, and $H_{X_0}(ip)$ is the displacement transfer function of the main system against the acceleration of the input ground motion. We assume that the input ground motion is steady white noise with PSD = 1.0, and the main system's initial period T_0 is 1.0 (sec). Under this condition, we evaluate the TMD's control performance by average response σ obtained from the above equation.

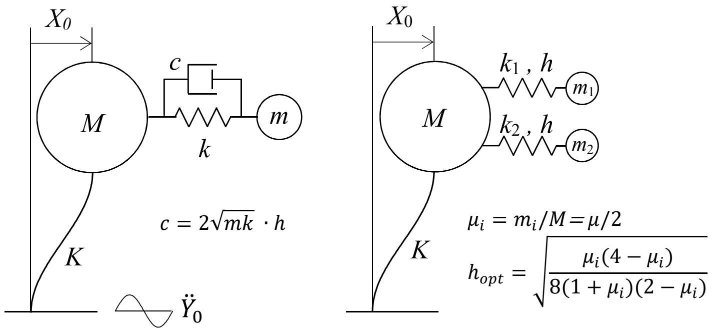
When $S_0(p)$ is steady white noise, TMD's optimum frequency ω_{opt} and damping ratio h_{opt} that minimize the main system's average response can be solved algebraically, and the solutions are expressed as [4]

$$\omega_{opt} = \frac{\sqrt{1-\mu/2}}{1+\mu}\Omega \tag{4}$$

$$h_{opt} = \sqrt{\frac{\mu(4-\mu)}{8(1+\mu)(2-\mu)}} \tag{5}$$

Fig.2 shows the relationship between the average response σ and the period fluctuation η. These results are obtained from the numerical calculation based on Eq. (3). The black line shows the result for a Single TMD whose parameters are set by Eq. (4) and Eq. (5) with respect to the initial period T_0. The yellow line shows the average response of the KC-variable TMD which is the ideal variable TMD that can adapt its stiffness and damping factor arbitrarily. Its parameters are set by Eq. (4) and Eq. (5) with respect to the fluctuating period T. The average response of the Single TMD is equal to that of the KC-variable TMD at η=1.00, but it approximately doubles that of the KC-variable TMD at η=1.30. This result indicates that we cannot expect sufficient performance with a Single TMD with tuning deviation.

Fig.2 also shows the result for the Dual TMD shown in Fig.1(b). We set its total mass equal to that of the Single TMD, and each mass in the Dual TMD is tuned to η=1.00 and η=1.66. The average response of the Dual TMD over a longer period range is smaller than that of a Single TMD, so it can be seen that the Dual TMD is more effective against period fluctuation than a Single TMD. On the other hand, its control performance at η=1.00 is lower than that of a Single TMD. As mentioned above, for a conventional TMD, there is a trade-off between robustness over a wide period range and control performance at tuning condition.



(a) Single TMD (b) Dual TMD

Figure 1 Mechanical model of conventional TMD

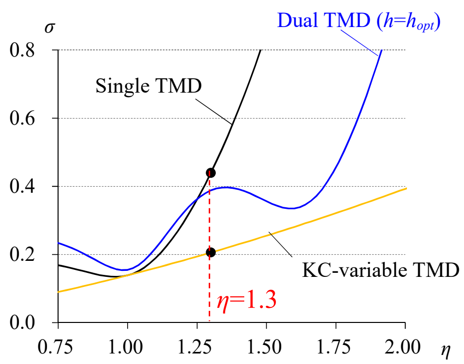


Figure 2 σ-η relation (μ=5%)

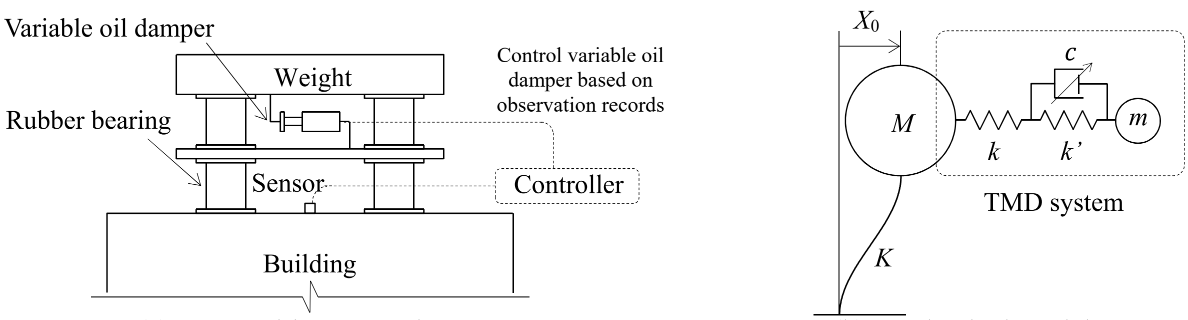
3. Proposition of semi-active TMD adaptable to period fluctuation

In this chapter, we propose a mechanism for a semi-active TMD that can adapt its tuning period only by the control of the variable dashpot. First, we explain the principle of its tuning mechanism using a complex stiffness model, and then describe the design method. Additionally, the control performance of the Proposed TMD is compared with that of other TMDs using random vibration theory.

3.1 Mechanism of Proposed TMD

Fig.3(a) shows the typical composition of the Proposed TMD. This TMD consists of two piled up rubber bearings and a variable damper installed parallel to one side. In this system, the dominant vibration frequency of the building's top floor is identified online during an earthquake motion, and the TMD is tuned by controlling the variable oil damper. Fig.3(b) shows a mechanical model of the Proposed TMD. In this TMD, the stiffness ratio of the two springs is an important parameter that determines the adaptable period range, so we define it as λ expressed by the following equation.

$$\lambda = \frac{k'}{k} \tag{6}$$



(a) Composition example (b) Mechanical model

Figure 3 Proposed TMD

3.2 Principle and design method of Proposed TMD

To comprehend the principle of the tuning mechanism, we focus on the TMD system described in Fig.3(b), and the relation between the damping coefficient of the variable dashpot and the resonance frequency of the TMD system is investigated.

Considering the harmonic external force $F = e^{ipt}$, the complex stiffness of the TMD system's supporting part k^*, which is composed of k, k' and c is expressed as

$$k^* = \frac{\lambda k^2 + kcpi}{(1+\lambda)k + cpi} \tag{7}$$

Assuming the resonance frequency is ω_e, the equivalent stiffness k_e under resonance vibration is expressed by

$$k_e = \mathrm{Re}[k^*] = \frac{{g_e}^2 + \lambda(1+\lambda)}{{g_e}^2 + (1+\lambda)^2} \cdot k \tag{8}$$

where g_e is a non-dimensional parameter as follows.

$$g_e = \frac{c\omega_e}{k} \tag{9}$$

Substituting Eq. (9) into the resonance condition formula ${\omega_e}^2 = k_e / m$, the resonance frequency ω_e is expressed by

$$\frac{\omega_e}{\omega} = \sqrt{\frac{{g_e}^2 + \lambda(1+\lambda)}{{g_e}^2 + (1+\lambda)^2}} \tag{10}$$

where ω is a parameter

$$\omega = \sqrt{k/m} \tag{11}$$

By solving Eq. (10), the relation between the damping coefficient of the variable dashpot and the resonance frequency of the TMD system can be obtained in a closed form as

$$\frac{\omega_e}{\omega} = \sqrt{\frac{g_0^2 - (1+\lambda)^2 + \sqrt{g_0^4 - 2g_0^2(1-\lambda^2) + (1+\lambda)^4}}{2g_0^2}} \tag{12}$$

where g_0 is a non-dimensional parameter expressed by following equation. Hereinafter g_0 is called as non-dimensional damping coefficient.

$$g_0 = \frac{c\omega}{k} = \frac{c}{\sqrt{mk}} \tag{13}$$

On the other hand, the equivalent damping ratio of the TMD system under resonance vibration is expressed as

$$h_e = \mathrm{Im}\left[\frac{k^*}{2k_e}\right] = \frac{g_e}{2g_e^2 + 2\lambda(1+\lambda)} = \frac{(\omega_e/\omega)g_0}{2(\omega_e/\omega)^2 g_0^2 + 2\lambda(1+\lambda)} \tag{14}$$

Fig.4 shows the relation between resonance frequency ω_e and non-dimensional damping coefficient g_0 obtained from Eq. (12). Fig.5 shows the relation between the equivalent damping ratio h_e and g_0 obtained from Eq. (14). The resonance frequency of the TMD system can be controlled by changing the damping coefficient of the variable dashpot, but then the equivalent damping ratio of the TMD system also changes.

Next, the design method of the Proposed TMD is explained. First, we set the mass ratio μ and the stiffness ratio λ. In the Proposed TMD, as for other types of TMDs, μ determines the control performance. On the other hand, λ determines the adaptable range as shown in Fig.4. In this study, μ is set to 5% and λ is set to 0.5. Fig.6 shows the design concept of the Proposed TMD [5]. The vertical axis shows ω_e and h_e, and the horizontal axis shows g_0. h_{opt} in this figure is the optimum dapming ratio obtained from Eq. (5), and it can be seen that there are two points (A, B) that satisfy this condition. Here, we set the stiffness of the Proposed TMD at point A to tune to the initial natural period of the main system. This setting is suitable for the TMD of the main system, in which the period extends as for an RC and SRC structures, because the natural period of the TMD can be extended by decreasing the damping coefficient of the variable dashpot.

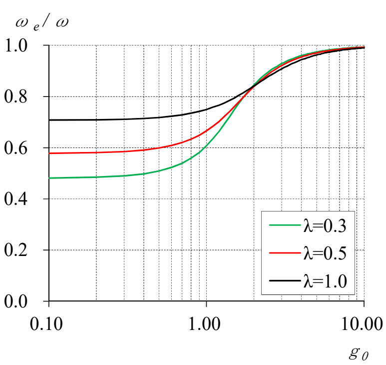


Figure 4 Resonance frequency of the TMD system

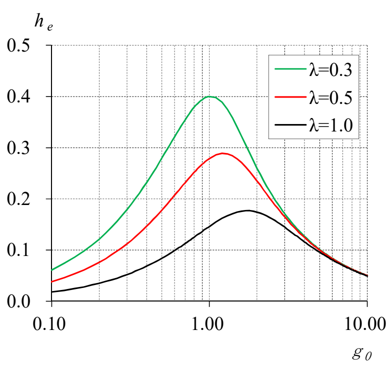


Figure 5 Equivalent damping factor of the TMD system

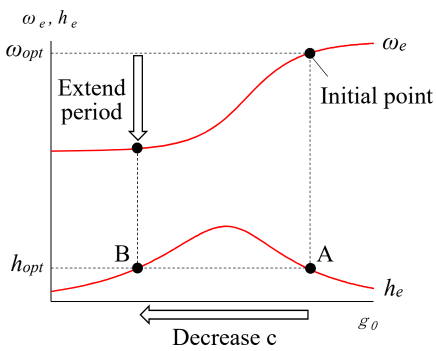


Figure 6 Design concept of Proposed TMD

3.3 Control performance of Proposed TMD

The control performance of the Proposed TMD against the main system's period fluctuation is compared with those of other TMDs: Single, Dual and KC-variable. Table 1 shows the setting parameters. For the Proposed TMD, we conduct numerical studies with non-dimensional damping coefficient g_0 as a variable, and find the optimum damping coefficient g_{opt} that minimizes average response of the main system. Fig.7 shows g_{opt} and Fig.8 shows the average response of the main system. It can be seen that the average response with the Proposed TMD is smaller than that with the Single and Dual TMDs over a wide period range. Additionally, we consider 3-mode control as shown in Fig.7 to simplify the control. Fig.8 also shows the average response of 3-mode control, and it is almost matches to the result of g_{opt}. From the result, it is confirmed that the Proposed TMD can cover a wide period range without impairing the control performance at tuning condition, and 3-mode is enough to control with sufficient performance.

Table 1 Setting parameters of each TMD

Case	Single	Dual	KC-variable	Proposed
Mass ratio	μ=0.05	μ_1=0.025 μ_2=0.025	μ=0.05	μ=0.05
TMD's Frequency	ω_{opt} for T_0	ω_{opt} for $1.66T_0$ *1 ω_{opt} for T_0 *1	ω_{opt} for T (adapting)	Set by Fig.6 (λ=0.5)
Damping factor	h_{opt}	$2h_{opt}$ *1 $2h_{opt}$ *1	h_{opt}	Set by Fig.7

*1 Calculated by individual mass ratio: μ_1 or μ_2

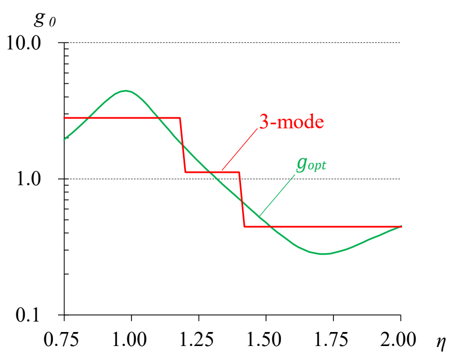


Figure 7 Optimum damping coefficient of Proposed TMD

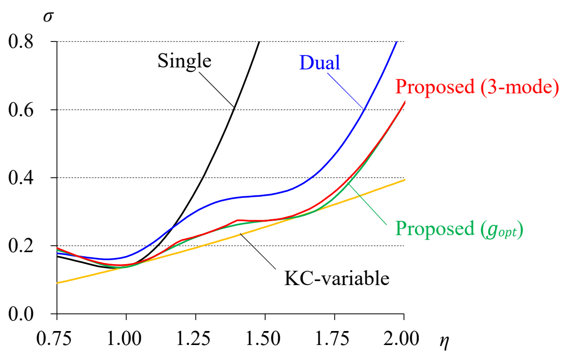


Figure 8 σ-η relation of main system (μ=5%)

4. Control strategy

4.1 Concept of control strategy

In controlling the proposed TMD, the accuracy, stability and reactivity are important to keep TMD's tuning against fluctuating building's condition under seismic vibration. It is also important in application to reduce the number of measuring points and computational complexity.

To solve these problems, we propose a new control method as shown in Fig.9. In this method, first, we get the acceleration records of the top floor, and apply a low pass filter to remove higher mode vibrations. Next, the filtered wave is inputted to the virtual TMDs that correspond to each mode of the proposed TMD. For example, when the Proposed TMD has 3 control modes, 3 virtual TMDs are prepared and dynamic analyses of them are performed in parallel. The duration of these dynamic analyses are short and the mode that maximizes the absorbed energy in the duration is selected. Finally, the damping coefficient of the variable oil damper is controlled to the selected mode. This operation is carried out real time, and repeated all time.

This control method is based on the idea that it is more important tuning the TMD to the dominant period of the vibration than tuning it to the building's natural period. By focusing on the TMD's absorbing energy, stable search of the dominant period during an earthquake is realized. This method is physically clear in terms of selecting the state with the highest energy absorption efficiency from the modes that the TMD can take.

In addition, the proposed method has an advantage in application. The control method in previous studies require the real-time period identification of the building. When we try to identify the building's fluctuating natural period accurately, it is necessary to consider the influence of various noises and higher mode vibrations, and a large number of measurement points are required to remove them. On the other hand, the proposed method needs only one measurement point to control, and is easy to apply since the wiring in the building can be omitted.

4.2 Control algorism

In the control, first, we should determine W_L that is the duration of a short term dynamic analysis. According to our preliminary study, the appropriate value of W_L is about 4 to 8 times of the building's initial period to balance stability and reactivity of the control. Here, E_i: the absorbed energy of the virtual TMD corresponding to i-th mode, is expressed as

$$E_i = \int_{t_0}^{t_0+W_L} c_i \cdot \dot{\delta}(t)^2 \, dt \tag{15}$$

where t_0 is the start time of the short term dynamic analysis, c_i is the damping coefficient of the i-th mode, and $\dot{\delta}(t)$ is the velocity of the dashpot deformation. In the control, we select the mode that maximize the E_i.

In the above method, the stability and reactivity are in a trade-off relationship, because the longer the analyses duration, the higher the stability but the worse the reactivity. So we propose control algorism that uses overlapping analytical windows as shown in Fig.10. Here, W_D is the overlap length of the analytical windows. The shorter the W_D, the finer the control, but the number of the analyses performed in parallel increases. However, since the analysis model is simple (2DOF), and the number of control modes is a few (about 3), computation can be performed in a feasible time.

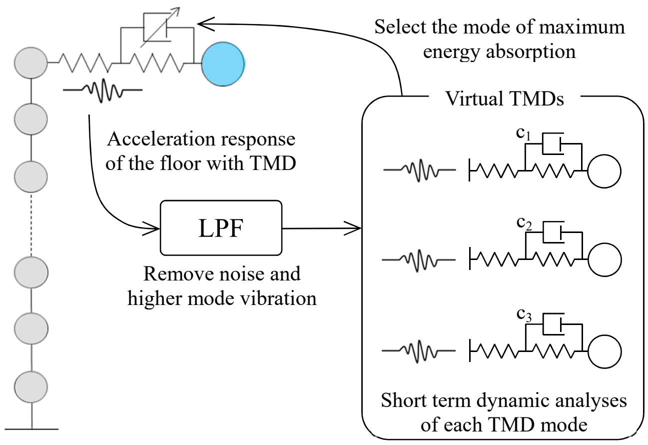


Figure 9 Concept of control method

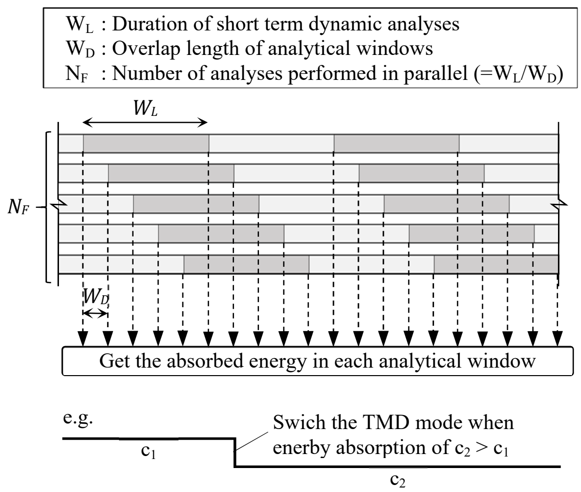
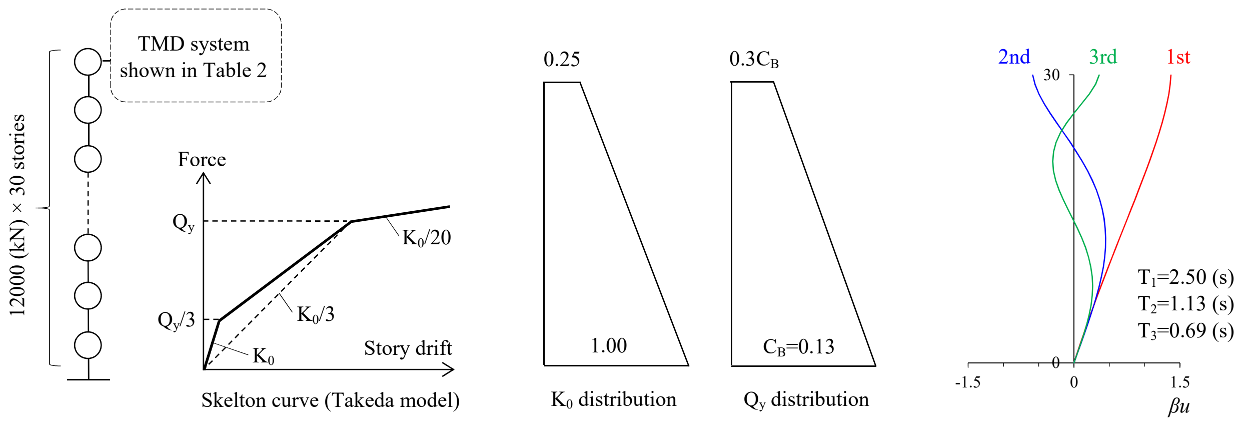


Figure 10 Analytical Windows

5. Simulation analysis

5.1 Analytical model

Fig.11 shows the characteristics of the applied structure. A 30-story lumped mass model as shown in Fig.11(a) is employed. Each story is modelled by a shear spring with restoring force characteristics of degrading tri-linear model (Takeda-model). Fig.11(b) shows the distribution of the initial stiffness and the yield strength. Fig.11(c) shows the eigenmodes and natural periods. The damping ratio of the main structure is 3%.



(a) Model and restoring force (b) Stiffness and strength distribution (c) Eigenmodes

Figure 11 Characteristics of applied structure

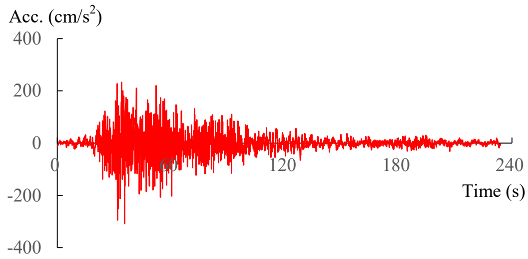
Table 2 shows the analytical cases. We set each type of TMD to the top floor as shown in Fig.11(a), and investigate their response control performance. The total weights of each TMD are set to the same value and their effective mass ratio is 5%. In the case of KC-variable TMD, we applied real-time identification method using mode decomposition and ARX model [6], and control the stiffness and damping factor according to Eq.(4) and Eq. (5). This is a less feasible system due to the difficulty in tuning mechanism and identification method, but it is included in the analytical case for comparison.

Table 2 Setting parameters of each TMD

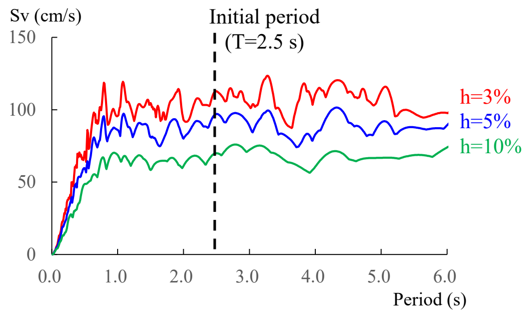
Case	Single	Dual	KC-variable	Proposed
TMD's Mass (Mass ratio)	7300 kN (μ=5%)	m_1=3650 kN m_2=3650 kN (Total μ=5%)	7300 kN (μ=5%)	7300 kN (μ=5%)
TMD's period	2.66 sec.	T_1=4.28 sec. T_2=2.58 sec.	Adapting by Eq.(4)	Set by Fig.6 (λ=0.5)
Damping ratio	0.110	h_1=0.157 h_2=0.157	adapting by Eq.(5)	Fig.7 (3-mode) (W_L=10 s, W_D=1 s)

5.2 Input earthquake ground motion

Fig.12(a) shows the acceleration time history of the input ground motion, and Fig.12(b) shows its velocity response spectrum. It is a simulated earthquake ground motion: Kokuji-wave of Hachinohe EW phase by Japanese code. In this study, three levels: 20%, 50% and 100% are considered and the response control performance of the TMDs against each input level is investigated.



(a) Acceleration time history



(b) Velocity response spectrum

Figure 12 Input ground motion (Kokuji-wave of Hachinohe EW phase)

5.3 Analytical result

(1) Time history

Fig.13 shows the time histories of the building's natural period. They are obtained by system identification method using ARX model. The larger the input level, the longer the building's natural period, and it becomes about 3.6 sec (η = 1.44) under 100% input. Fig.14 shows the displacement time history of the top floor under 100% input. It can be seen that the displacement of "Proposed TMD" is smaller than that of "without TMD" in the latter half of the earthquake ground motion, even though the natural period of the main structure is extended. Thus we confirmed that the proposed TMD is effective even when the period of the target structure is extended.

(2) Maximum response

Fig.15 shows the maximum displacement of the main structure. The response control performance of the Single TMD is as high as that of the KC-variable TMD in 20% input, because the amplitude is small and the natural period of the building does not extends. The performance of the Single TMD decreases as the input level increases due to the tuning deviation. Focussing on the results of the Dual TMD, its response control performance is lower than that of the Single TMD in 20% input, but higher in 100% input. The performance of the Proposed TMD is higher than that of the Dual TMD in every input level and that of the KC-variable TMD is the highest. However, we consider KC-variable TMD is less feasible, because of the difficulty in tuning mechanism and identification method which request a lot of observation points. From these results, we confirmed the validity and feasibility of the Proposed TMD against the period fluctuation of a main structure.

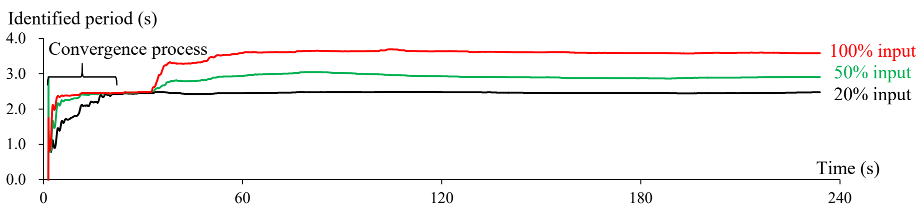


Figure 13 Identified natural period (Proposed TMD)

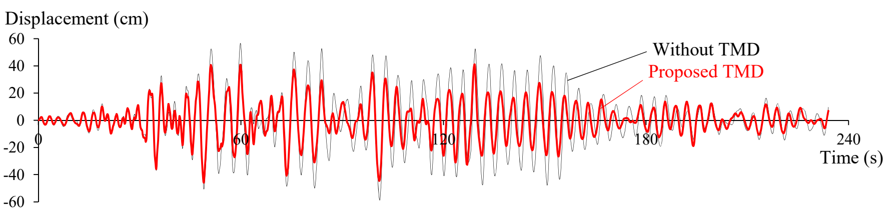
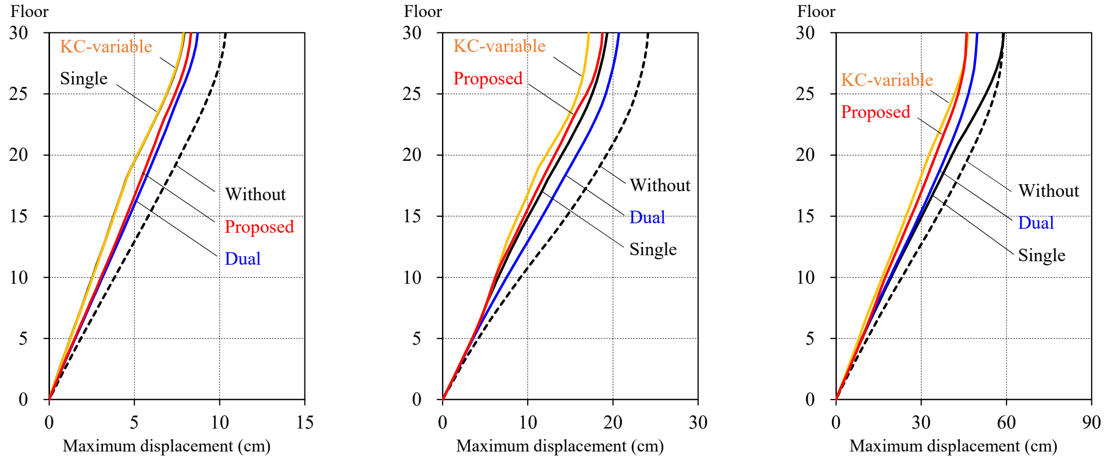


Figure 14 Displacement time history of top floor (100% input)



(a) 20% input (b) 50% input (c) 100% input

Figure 15 Maximum response displacement under each input levels

6. Shaking table test

6.1 Specimen and control system of Proposed TMD

To confirm the tuning mechanism of the Proposed TMD experimentally, we planned shaking table tests. Fig.16 shows the configuration of the test specimen. It consists of a concrete weight (400 kN), 12 rubber bearings, and 2 variable oil dampers. Oil dampers are arranged orthogonally to each other for 2-way experiment. Fig.17(a) shows the picture of the whole specimen, and Fig.17(b) shows variable oil dampers. This specimen is designed as a TMD for a 30-story RC building, though due to the vibration capacity of the shaking table, the weight is scaled down to 1/10 and the natural period is shortened to half of real scale. According to the free vibration test without oil dampers, the natural period of the specimen is 2.1 s.

We also set up the control system. The sensor on the table mesures 2-way acceleration and sends them to the controller. This controller works with aforementioned algorism. It judges the necessity of controlling variable oil dampers and sends the control signal. This process is performed automatically in real time.

In the mesurement, acceleration sensors are set on the shaking table and on the weight. Displacement sensors are installed to four sides of the specimen and the displacement is calcurated by averaging same direction records. Additionally, we install pressure sensors to mesure load of the oil dampers. The control signals to oil dampers are constantly monitored and the behaviour of the controller is checked.

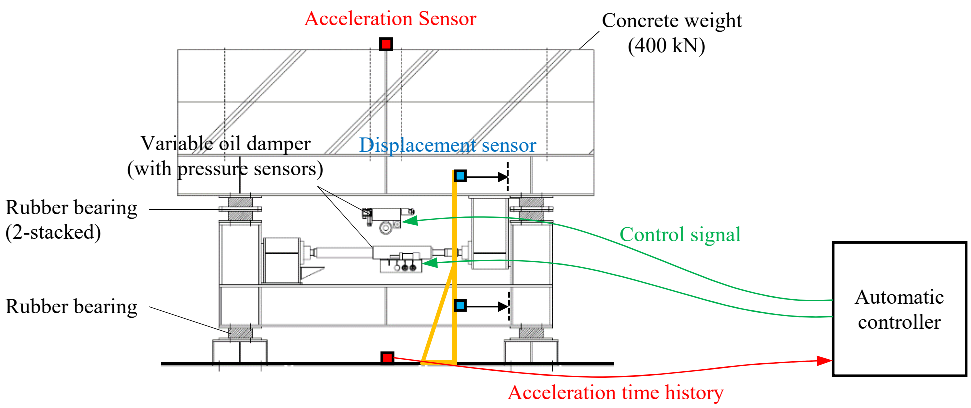
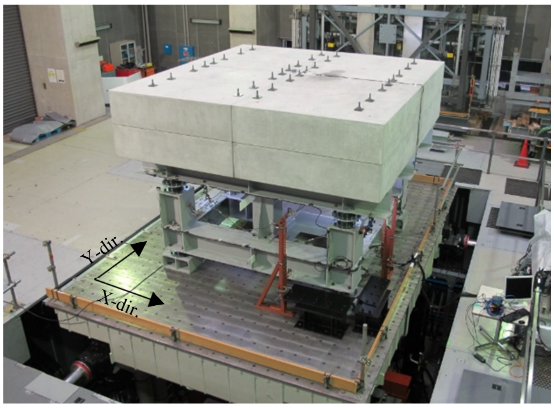
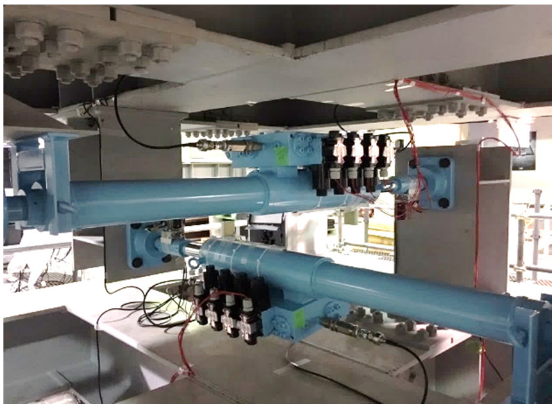


Figure 16 Configuration of specimen and control system



(a) Whole specimen



(b) Variable oil dampers

Figure 17 Picture of test specimen

6.2 Test results

(1) Resonance test

In order to confirm the principle of the tuning mechanism, the resonance tests were performed. Here, we input sine waves to the specimen and measured the response acceleration of the weight in the steady vibration. In this test, the damping coefficient of the variable oil dampers was manually controlled (controller was not used). Fig.18 shows the test results. The vertical axis shows the acceleration transfer function, and horizontal axis shows the frequency of the input sine waves. It can be seen that the specimen has different resonance frequency according to the oil damper settings.

(2) Seismic response wave test

In order to confirm the control operation, the seismic response tests were performed. Here, we employed seismic response wave obtained by preliminary simulation analysis like Fig.11. The response wave of the top floor against Kokuji-wave of Hachinohe EW phase was inputted to X-direction, and that of Hachinohe NS phase was inputted to Y-direction. In this test we employed 3-modes control (c = 6.0, 2.1, 1.0 kNs/cm: design value). Time scale and amplitude of the input waves were adjusted smaller than original for the limit of the specimen and shaking table. Fig.19 shows the displacement of the weight and the energy absorbed by the dampers. The red line indicates the experimental results, and the black line indicates the simulation results assuming that the controller operates correctly. We find that experimental results show good agreement with the simulation results, and the accurate operation of the controller was confirmed.

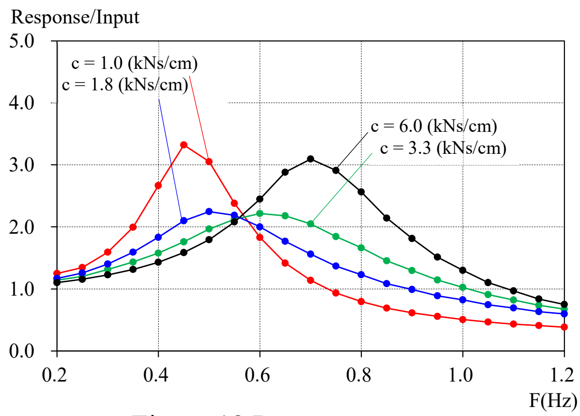
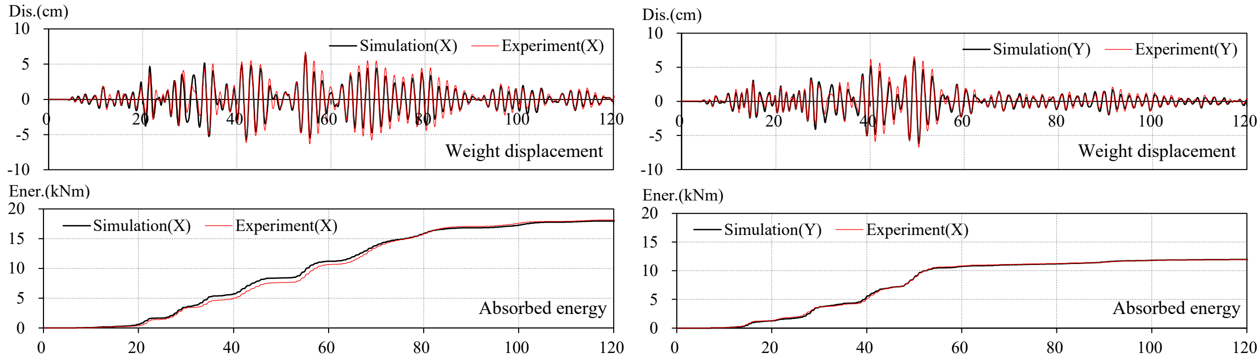


Figure 18 Resonance curve



(a) X-direction (b) Y-direction

Figure 19 Displacement of weight and damper's absorbing energy

7. Conclusions

(1) This paper proposed a mechanism for a semi-active controlled TMD adaptable to period fluctuation of a main structure. It is a reasonable system, since the resonance frequency of this TMD can be changed only by the control of damping coefficient of variable dashpot.

(2) The tuning principle and design method of the Proposed TMD were presented based on a complex stiffness model. Its superior response control performance was confirmed by comparing with other type TMDs using random vibration theory.

(3) The reasonable and feasible control method for the semi-active TMD was proposed. The method is based on the idea maximizing the TMD's absorption energy. The validity of the proposed control method was confirmed by seismic simulation analyses.

(4) Shaking table tests on the scaled specimen were carried out, and the principle of the tuning mechanism is confirmed experimentally. Experiments were not only on the TMD but also on the control system, and the expected behaviour of the control system was confirmed.

8. Acknowledgements

The authors would like to thank Dr. Qian of Senqcia Co. supporting the development of the TMD controller. We also thank Mr. Kaneko and Mr. Minagawa of Kajima Co. for their support in the experiments.

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