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Evaluation Method for Cumulative Plastic Deformation of Hysteretic Dampers Installed in SMRF

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Abstract

Hysteretic dampers installed on a response control structure absorb seismic energy and effectively control the damage to a frame by yielding under small inter-story deformations. When designing a structure with hysteretic dampers, it is important to estimate the maximum deformation and the cumulative plastic deformation of the dampers relative to ground motions. Specifically, in the early stage of design, simple methods are useful for a preliminary study to determine the characteristics of dampers. The present study discusses the disadvantages of the existing simple methods and proposes a new method. Subsequently, the accuracy and applicability of the proposed method are investigated via numerical examples.

Most extant simple methods [1] are based on two-spring model, namely, a single degree-of-freedom (SDOF) model that consists of two parallel springs. In a two-spring model, the frame of a response control structure and all dampers are represented by a single elastic shear spring and a single bilinear elastic-plastic shear spring, respectively. The cumulative plastic deformation of the dampers in each story of the frame is estimated by distributing the response of the elasto-plastic spring of the two-spring model based on the story-wise stiffness distribution of the structure. In the models, it is implicitly assumed that all dampers yield at the same time. However, this type of stiffness distribution significantly depends on the characteristics of the input ground motions. Furthermore, the contribution of higher modes is generally not considered in the methods.

In the study, we propose a multi-spring model in which dampers in each story are represented via a single bilinear elastic-plastic shear spring as opposed to using a single spring in a two-spring model. The study also considers higher modes in which responses are estimated via the multi-spring model for higher modes. By using the multi-spring model, the modal response of the dampers in each story of a response control structure is estimated based on the responses of the SDOF model. The modal analysis is extended to estimate the total hysteretic energy of each story is estimated by superposing the hysteretic energy absorbed by the corresponding elastic-plastic shear spring of each mode. To consider the interaction among modal responses after yielding, we further propose appropriate adjustments to the stiffness and equivalent mass of multi-spring models for modes higher than the first mode.

Using several structural models, it is demonstrated that the accuracy and variability of the estimation are extensively improved when compared that by Ito and Kasai [1] and especially for tall buildings in which it is not possible to ignore the contribution of higher modal responses.

Keywords: hysteretic damper, cumulative plastic deformation, SDOF model

1. Introduction

The current Japanese Building Standards Law requires that "the building does not collapse or otherwise fail due to an extremely rare large-scale snowfall, windstorm, earthquake, or other events." However, there are significant potential demands on the continuous use of buildings without considerable damage and maintaining their values even if they are subject to very strong ground motions such as those at the Great East Japan Earthquake. "Performance-based design" is indispensable in terms of implementation in the near future.

A method to maintain the function of a building that is subjected to a strong ground motion involves absorbing seismic energy to the structure by dampers installed in Steel Moment Resisting Frame (SMRF). When designing a structure with hysteretic dampers (hereafter referred to as "response control structure"), it is important to estimate the maximum deformation and cumulative plastic deformation of dampers when they are subjected to a strong ground motion [1].

Takahashi and Akiyama proposed empirical formulae to estimate maximum deformation and cumulative plastic deformation of the hysteretic dampers installed on shear-type multi-story frames with low secondary stiffness ratio, which is the ratio between the stiffness of the bare frame and that of the entire frame, i.e., $p = 0 – 0.2$ [2]. Takeuchi proposed empirical formulae to estimate the responses when $p = 0.5 – 2$ [3]. Ogawa and Hirano estimate the responses on the basis of energy balance [4]. However, in the aforementioned studies, the characteristics of a response control structure including its natural period and the duration of ground motions are not considered.

Ito and Kasai considered the effect of the secondary stiffness ratio, maximum ductility factor, natural period of a response control structure, and duration of ground motions to propose formulae to estimate cumulative plastic deformation on the basis of several seismic response analyses using two-spring model, namely, a single degree-of-freedom (SDOF) system consisting of two parallel springs as shown in Fig. 1. In a two-spring model, the bare frame of a response control structure is represented by a single elastic shear spring, and all hysteretic dampers as a whole are represented by a single bilinear elasto-plastic shear spring. The cumulative plastic deformation of the dampers in each story of the frame is estimated via distributing the response of the elasto-plastic spring of the two-spring model based on the story-wise stiffness distribution of the structure. In the model, it is implicitly assumed that all dampers yield at the same time. However, this type of stiffness distribution significantly depends on the characteristics of the ground motion. Additionally, the contribution of higher modes is generally not considered in these methods.

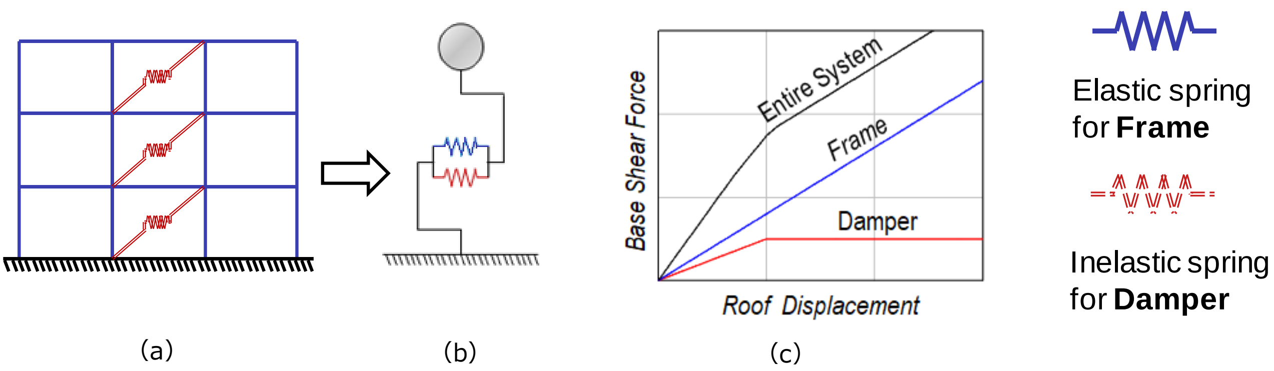


Fig. 1 – Oscillator equivalent to multi-story frame with hysteretic dampers (2S model)

Kang and Mori proposed the use of multi-spring (MS) model in which hysteretic dampers in each story are represented via a single bilinear elasto-plastic shear spring to estimate the maximum response of a response control structure [5]. The MS model is used to estimate the modal response of the dampers in each story based on the responses of the corresponding spring in the MS model, as shown in Fig. 2. To consider the higher modes, Furukawa et al. [6] proposed a simplified method to estimate the maximum inter-story drift of a multi-story SMRF with hysteretic dampers by using an Inelastic Model Predictor (IMP) [7], which corresponds to an extended version of SRSS that considers the maximum displacement of the elasto-plastic SDOF system equivalent to the first mode and first modal shape after yielding.

The objective of the study is to propose a method for estimating cumulative plastic deformation of hysteretic dampers installed in SMRF by using MS models equivalent to each mode of a response control structure while considering the effect of higher modal responses. When estimating maximum inter-story drift, there is no significant difference in considering whether the modal response that is higher or equal to the second mode is elastic or inelastic. On the contrary, when estimating the cumulative plastic deformation, the inelastic behavior of higher modes should be considered. The accuracy and applicability of the proposed method are investigated using numerical examples.

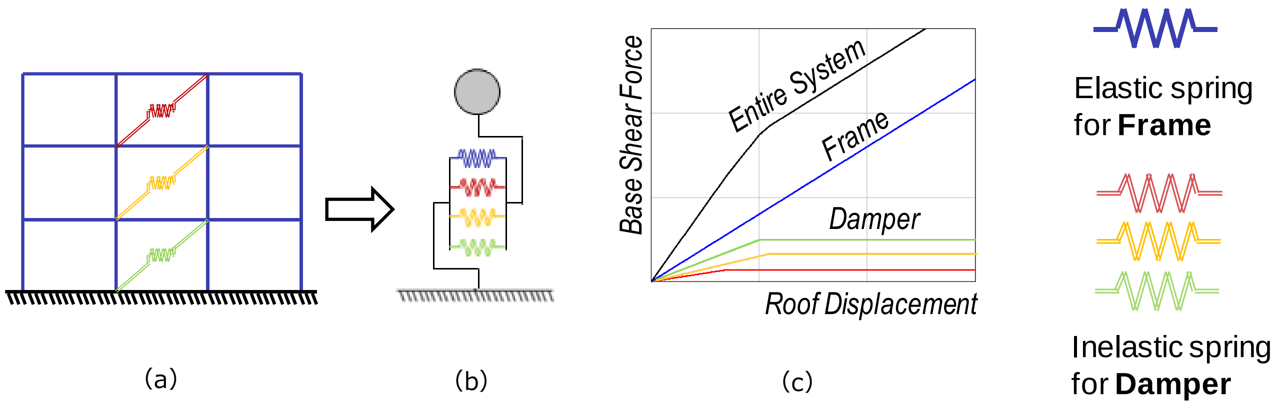


Fig. 2 – Oscillator equivalent to multi-story frame with hysteretic dampers (MS model)

2. Proposed Equivalent SDOF System (MS Model)

In an MS model equivalent to a response control structure, as shown in Fig. 2 (a), the frame is represented by a single elastic shear spring and dampers in each story are represented by a single bilinear elasto-plastic shear spring, as shown in Fig. 2 (b). The force–displacement characteristics of the springs are schematically shown in Fig. 2 (c).

2.1 Effective mass and effective height based on the *j*th mode equivalent MS model

An effective mass, $\overline{m}_j$, and effective height, $\overline{H}_j$, of the MS model are defined as follows:

$$\overline{m}_j = \frac{(\sum_{i=1}^{n} m_i \cdot \phi_{j,i}^{E})^2}{\sum_{i=1}^{n} m_i \cdot \phi_{j,i}^{E}} \quad (1)$$

$$\overline{H}_j = \frac{\sum_{i=1}^{n} (m_i \cdot \phi_{j,i}^{E} \cdot \sum_{k=1}^{i} h_k)}{\sum_{i=1}^{n} m_i \cdot \phi_{j,i}^{E}} \quad (2)$$

where $\phi_{j,i}^{E}$ denotes the *j*th elastic mode vector of the *i*th story obtained from the eigenvalue analysis (EVA) of the response control structure and m_i and h_i denote the mass and height of the *i*th story of the structure, respectively.

2.2 Elastic spring equivalent to bare frame

In the MS model, the properties of the elastic spring equivalent to the response control structure without dampers (simply referred to as "bare frame" hereafter) are defined as follows:

Initial stiffness of the elastic spring equivalent to the bare frame is defined as follows:

$$_{f}\bar{k}_j = \left(\frac{4\pi^2 \cdot \bar{m}_j}{{T_j}^2}\right) - \sum_{i=1}^{n} {}_{d}\bar{k}_{j,i} \quad (3)$$

Here, T_j denotes the natural period of the MS model, and this is equal to the fundamental natural period of the response control structure that is estimated as follows:

$$T_j = 2\pi\sqrt{\frac{\bar{m}_j}{{}_{f}\bar{k}_j + \sum_{i=1}^{n} {}_{d}\bar{k}_{j,i}}} \quad (4)$$

2.3 Inelastic spring equivalent to hysteretic dampers [5]

The displacement of the equivalent SDOF system, $\bar{\delta}_{j,i}$, is expressed using the *j*th mode inter-story drift of the *i*th story of the response control structure, δ_i, as follows:

$$\bar{\delta}_{j,i} = \begin{cases} \dfrac{\delta_i}{\phi_{j,i}^E \cdot \Gamma_j^E} & (i = 1) \\ \dfrac{\delta_i}{(\phi_{j,i}^E - \phi_{j,i-1}^E) \cdot \Gamma_j^E} & (i > 1) \end{cases} \quad (5)$$

Here, $\phi_{j,i}^E$ is obtained from the EVA of the response control structure and Γ_j^E denotes the participation factor of the bare frame and is expressed in Eq. (6) as follows:

$$\Gamma_j^E = \frac{\sum_{i=1}^{n} m_i \cdot \phi_{j,i}^E}{\sum_{i=1}^{n} m_i \cdot (\phi_{j,i}^E)^2} \quad (6)$$

Force–displacement characteristics of the dampers installed on the *i*th story and the *i*th inelastic spring of the proposed SDOF system are shown in Fig. 2 (c). Specifically, it is assumed that the energy absorbed by the hysteretic dampers in the *i*th story of the response control structure is equivalent to that by the *i*th inelastic spring in the MS model. Based on the assumption and Eq. (6), the properties of the *i*th inelastic spring in the MS model are expressed as follows:

$$_{d}\bar{\delta}_{yj,i} = \begin{cases} \dfrac{_{d}\delta_{y,i}}{\phi_{j,i}^E \cdot \Gamma_j^E} & (i = 1) \\ \dfrac{_{d}\delta_{y,i}}{(\phi_{j,i}^E - \phi_{j,i-1}^E) \cdot \Gamma_j^E} & (i > 1) \end{cases} \quad (7)$$

$$_{d}\bar{k}_{j,i} = \begin{cases} {}_{d}k_i \cdot (\phi_{j,i}^E)^2 \cdot (\Gamma_j^E)^2 & (i = 1) \\ {}_{d}k_i \cdot (\phi_{j,i}^E - \phi_{j,i-1}^E)^2 \cdot (\Gamma_j^E)^2 & (i > 1) \end{cases} \quad (8)$$

$$_{d}\bar{\mu}_{j,i} = {}_{d}\mu_i \quad (9)$$

$$_{d}\bar{\alpha}_{j,i} = {}_{d}\alpha_i \quad (10)$$

Here, ${}_{d}\bar{k}_{j,i}$ and ${}_{d}k_i$ denote elastic stiffness of the *i*th inelastic spring in the MS model and dampers installed on the *i*th story of the response control structure, respectively; ${}_{d}\bar{\delta}_{yj,i}$ and ${}_{d}\delta_{y,i}$ denote the yield displacement of the *i*th inelastic spring and dampers on the *i*th story, respectively; ${}_{d}\bar{\mu}_{j,i}$ and ${}_{d}\mu_i$ denote the ductility factor of the *i*th inelastic spring and dampers on the *i*th story, respectively; ${}_{d}\bar{\alpha}_{j,i}$ and ${}_{d}\alpha_i$ denote post-elastic stiffness ratio of the *i*th inelastic spring and dampers in the *i*th story, respectively. Fig. 3(a) shows the relationship between the story shear force and inter-story drift of the damper in the *i*th story of the response control structure. It is assumed that the areas denoted by horizontal, diagonal, or vertical lines in Fig. 3 (a) are equal to the corresponding shaded area in Fig. 3 (b). This in turn reveals the relationship between shear force and drift of the *i*th spring in the MS model.

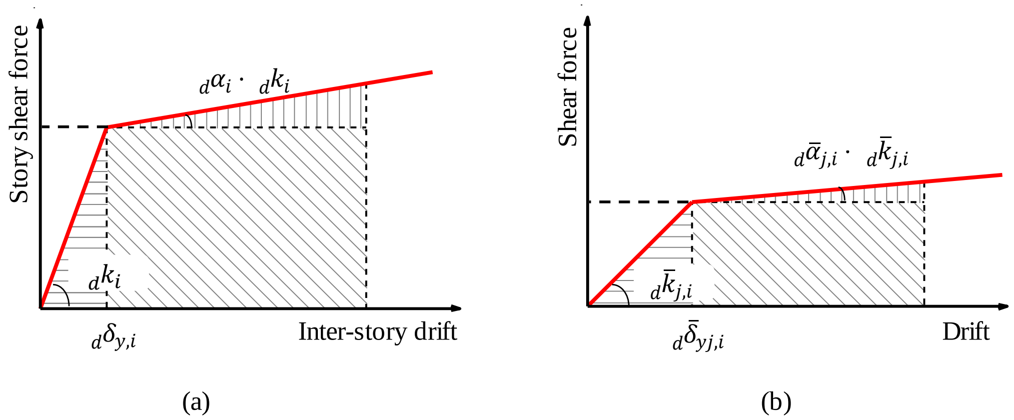


Fig. 3 – Relationship between (a) Story shear force and inter-story drift of the damper in the *i*th story of response control structure and (b) shear force and drift of the *i*th spring in the MS model

3. Evaluation Method for Cumulative Plastic Deformation by Using the MS Model

To investigate the accuracy of the proposed method, two-dimensional steel moment resisting frames are considered for numerical examples.

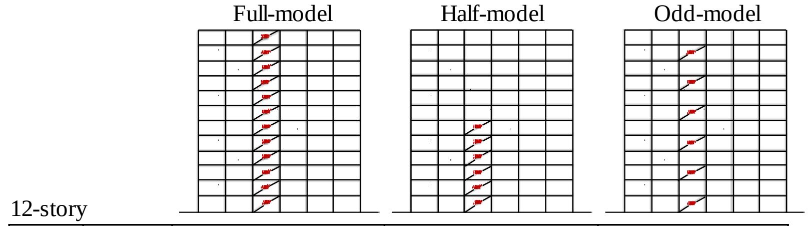
3.1 Building models and earthquake ground motion

A 12-story and 20-story steel moment resisting frames are designed by following the current Japanese seismic requirements for strength and inter-story drift ratio, as shown in Fig. 4. The fundamental natural periods of the bare frames correspond to 2.17 s and 2.89 s, respectively. Hysteretic dampers are installed on all the stories (Full-model), bottom-half stories (Half-model), or at odd stories (Odd-model). The stiffness of the dampers in the *i*th story is determined via the stiffness ratio where κ is defined as the ratio of the damper stiffness to the story stiffness of the bare frame. The yield displacement of the dampers in the *i*th story is determined via the drift ratio, υ, which is defined as the ratio of the inter-story drift of the story when the yield of the dampers in the story is similar to that of the bare frame. In the study, stiffness ratios corresponding to 0.5 and 2 and drift ratios corresponding to 0.25 and 0.75 are considered. The fundamental natural period of the frame models with dampers is also shown in Fig. 4.

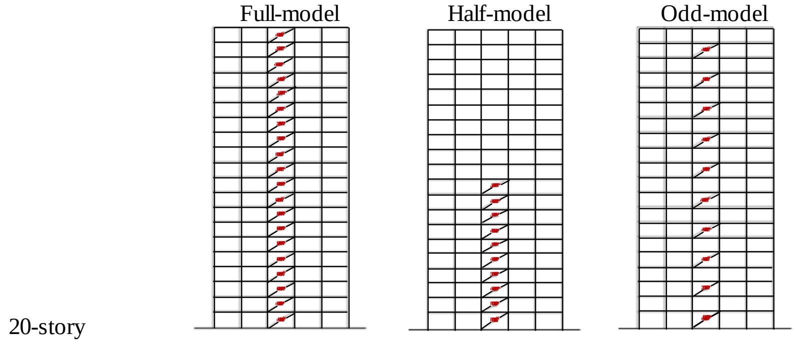
Non-linear dynamic analysis (NDA) is performed using the NDAs program SNAP ver.5. Stiffness-proportional damping is considered with a 2% damping ratio. Additionally, 98 observed ground motion records and 90 simulated ground motion records are used to investigate the proposed method. The records include ground motion recorded at nearby-field sites in the U.S. and Japan during the Kobe Earthquake and Tohoku Earthquake [7] and simulated earthquake ground motion [9].

3.2 Consideration of higher modal responses using MS models

When damping is ignored, the total hysteretic energy absorbed by the dampers in the *i*th story is estimated by superposing hysteretic energies absorbed by the *i*th inelastic shear spring of all MS models equivalent to each mode [8], as shown in Fig. 5 (a). However, if damping cannot be ignored, then superposing energies underestimate the response at the upper stories of the structure as shown in Fig. 5 (b). It is observed that in the MS models equivalent to the modes higher than or equal to the second mode, energy is absorbed as damping energy at the upper stories and not as hysteric energy. In reality, modal responses interact with each other and a modal response can become inelastic under modal displacement that is less than its yield displacement.



		Full-model	Half-model	Odd-model
κ=2	ν=0.25	T_1 =1.274	T_1 =1.558	T_1 =1.604
κ=0.5	ν=0.75	T_1 =1.788	T_1 =1.895	T_1 =1.955



		Full-model	Half-model	Odd-model
κ=2	ν=0.25	T_1 =1.695	T_1 =2.135	T_1 =2.170

Fig. 4 – Arrangement of dampers and elastic first-order natural period

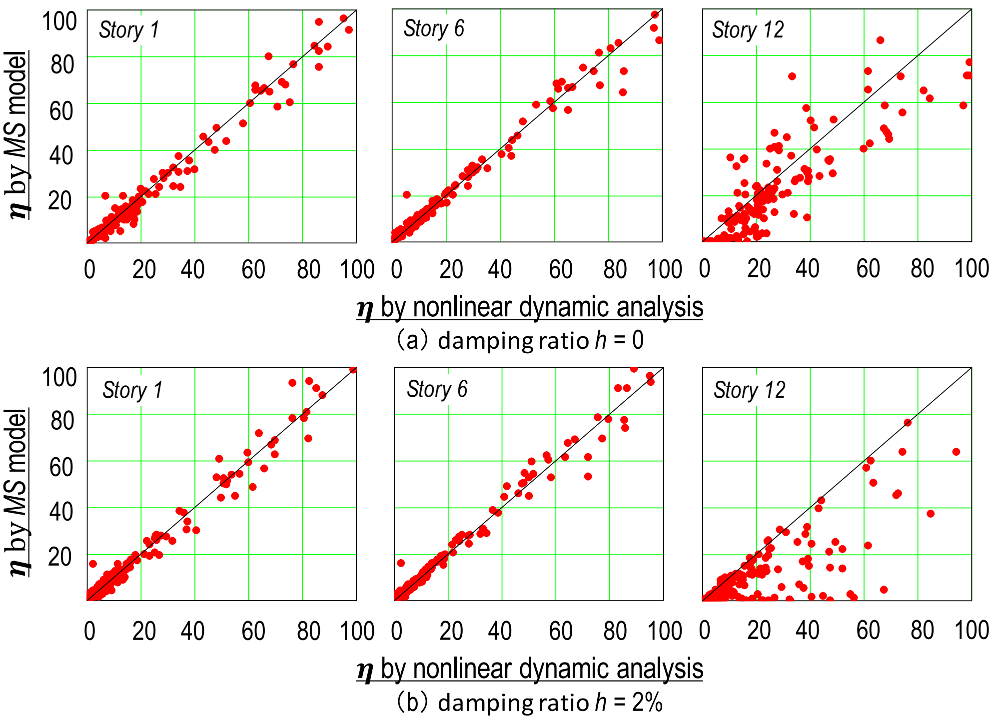


Fig. 5 – Analysis result of 12-story Full-model (κ= 2, υ = 0.25)

To consider the interaction between the 1st modal response and other modal responses, it is proposed to decrease the yield displacement of the elasto-plastic spring in the MS model equivalent to the second modes or higher by a factor of 2/3 for a response control structure with 2% damping, and thus the yield shear force of the spring is unchanged. The equivalent masses of the MS models are adjusted via the following equation such that the natural period of the MS model equivalent to the jth mode remains as equal to the natural period of the jth mode, T_j, of the response control structure.

$$\widehat{m}_j = \left({}_f\bar{k}_j + \frac{3}{2}\sum_{i=1}^{n} {}_d\bar{k}_{j,i} \right)\left(\frac{\mathrm{T}_j{}^2}{4\pi^2}\right) \tag{11}$$

3.3 Consideration of the post yield first mode shape

Modal shapes change after yielding, and a large deformation can be concentrated at a story. However, this type of change cannot be considered by the elasticity mode vector $\phi^E_{j,i}$ and especially a large response can be underestimated [6]. A method to consider the change in the first mode vector via yielding the dampers is proposed.

In IMP (which estimates the maximum inter-story drift ratio of a frame), the first mode vector is estimated based on the story-wise displacement of the frame obtained via pushover analysis corresponding to the maximum displacement response of the equivalent SDOF system. However, a frame does not continue to vibrate with its maximum inter-story drift, and thus the “average” first mode vector, $\phi^I_{1,i}$, after yielding is considered when estimating cumulative plastic deformation. Subsequently, the first elasticity mode vector, $\phi^E_{1,i}$ in Eqs. (1), (2), and (5) – (8) are replaced by the mode vector after yielding $\phi^I_{1,i}$.

It is proposed that the average first mode can be estimated as the average of the elastic mode and mode after yielding, which exhibits the most significant difference from the first elastic mode. The modes after yielding are determined based on the roof drift ratio of the response control structure as obtained by pushover analysis. However, if the roof drift ratio of the response control structure exceeds 0.01, it is considered as 0.01. The average first mode vectors after yielding are shown in Fig. 6 in conjunction with the first elastic mode vectors.

4. Numerical Examples

Figures 7 and 8 show the cumulative plastic deformations of the dampers, η, estimated by the proposed method using elastic mode vector are compared with those obtained by NDA using the 12-story models assuming κ= 2 and υ = 0.25 (Fig. 7) and κ = 0.5 and υ = 0.75 (Fig. 8). The results are denoted by ●, and one-to-one straight lines are denoted by solid lines. The proposed method led to a slight overestimation at the first story although the results generally align along the one-to-one line at other stories. In the same figures, the estimates of η by Ito and Kasai’s method are also denoted by ○. The dispersion of the estimates by Ito and Kasai’s method generally exceed those by the proposed method. At the top story, where the dampers are installed, η in Full-model is overestimated, η in Half-model is slightly underestimated, and η in Odd-model is underestimated via Ito and Kasai’s method.

Figures 9 and 10 show the results of the proposed method using $\phi^I_{1,i}$. A comparison of Figs. 7 and 8 in which $\phi^E_{1,i}$ is used indicates improvements in the estimates of η in the first story of Full-model (κ= 2, υ = 0.25).

Figures 11 and 12 illustrate the result of the 20-story model using $\phi^E_{1,i}$ and $\phi^I_{1,i}$, respectively. The results estimated via the proposed method using elastic mode vector $\phi^E_{1,i}$ indicated that the Full-model evaluation using the proposed method underestimates the first story. The middle parts of the structure are slightly overestimated, and the upper part of the structure is underestimated. Specifically, a large dispersion is observed at the top of the structure. Conversely, in the Ito–Kasai method, in a manner similar to the result of the 12-story model, the variation is large, and the results of the Full-model and Half-model are overestimated in the lower and middle story, and the Odd-model is underestimated. There are improvements in the evaluation accuracy of the first story in which elastic mode vector $\phi^{I}_{1,i}$ is used for the concentrated deformation.

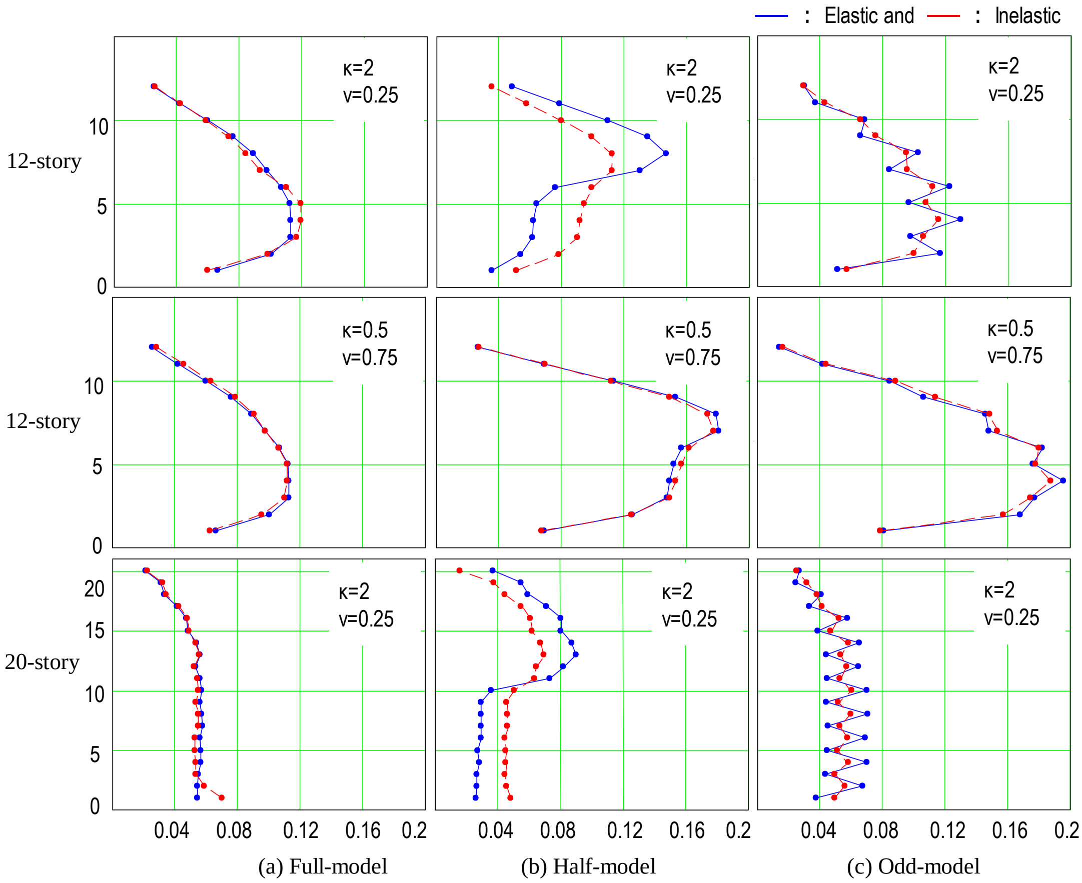


Fig. 6 – Change in first elastic mode vector

5. Conclusions

In the study, we proposed a new evaluation method for cumulative plastic deformation of hysteretic dampers installed in SMRF using MS models equivalent to each mode of the frame. By using MS models in which hysteretic dampers in each story are represented via a single inelastic shear spring, the response of the dampers in each story can be estimated on the basis of the responses of the corresponding spring in the MS models. Also in the proposed method, the interaction among modal responses after yielding by and the change in the first mode vector via yielding the dampers are considered. Using several structural models, it is demonstrated that the accuracy and variability of the estimation are extensively improved when compared that by Ito and Kasai [1] and especially for tall buildings in which it is not possible to ignore the contribution of higher modal responses. However, a large dispersion in the estimates is observed at the top of 20-story model, and further research will be conducted to improve the proposed method.

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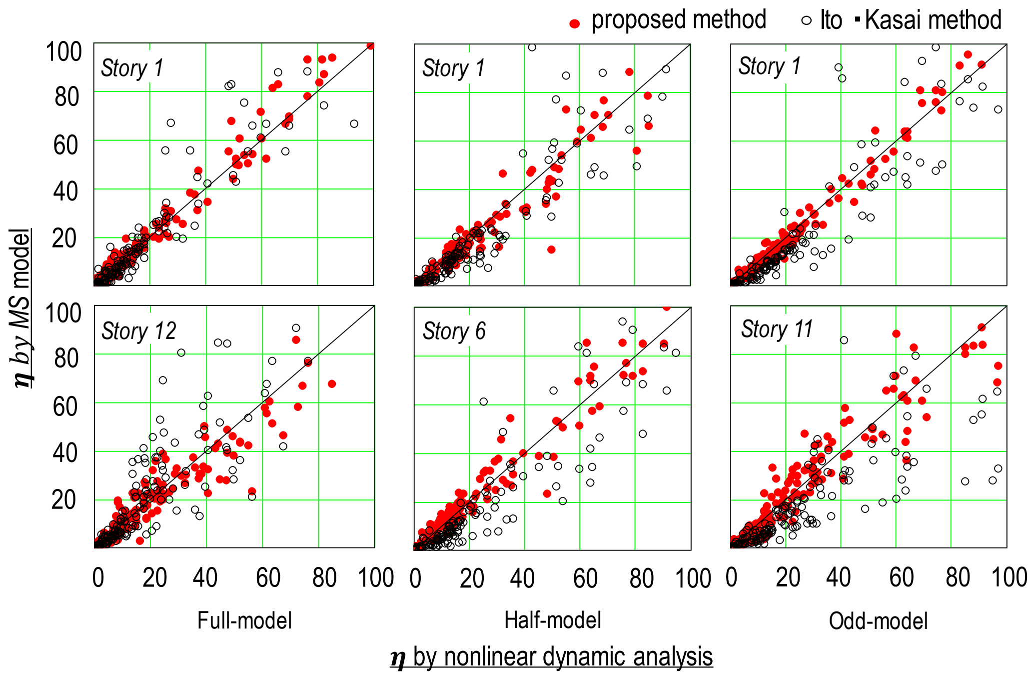

Fig. 7 – Analysis result of the 12-story model (κ= 2, υ = 0.25, $\phi_{1,i}^{E}$)

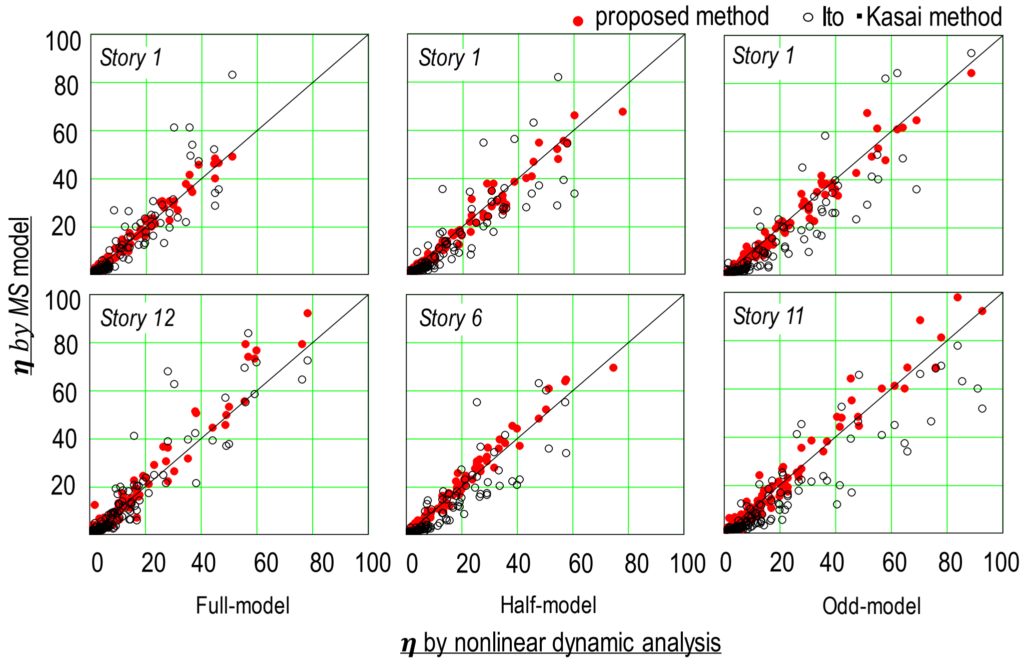

Fig. 8 – Analysis result of the 12-story model (κ= 0.5, υ = 0.75, $\phi^{E}_{1,i}$)

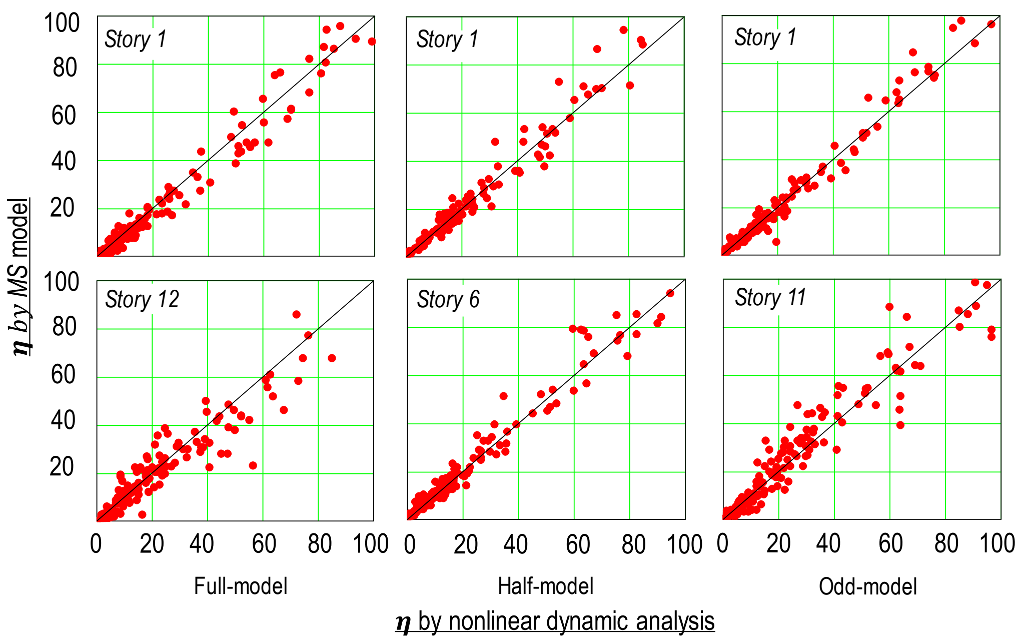

Fig. 9– Analysis result of the 12-story model (κ= 2, υ = 0.25, $\phi^{I}_{1,i}$)

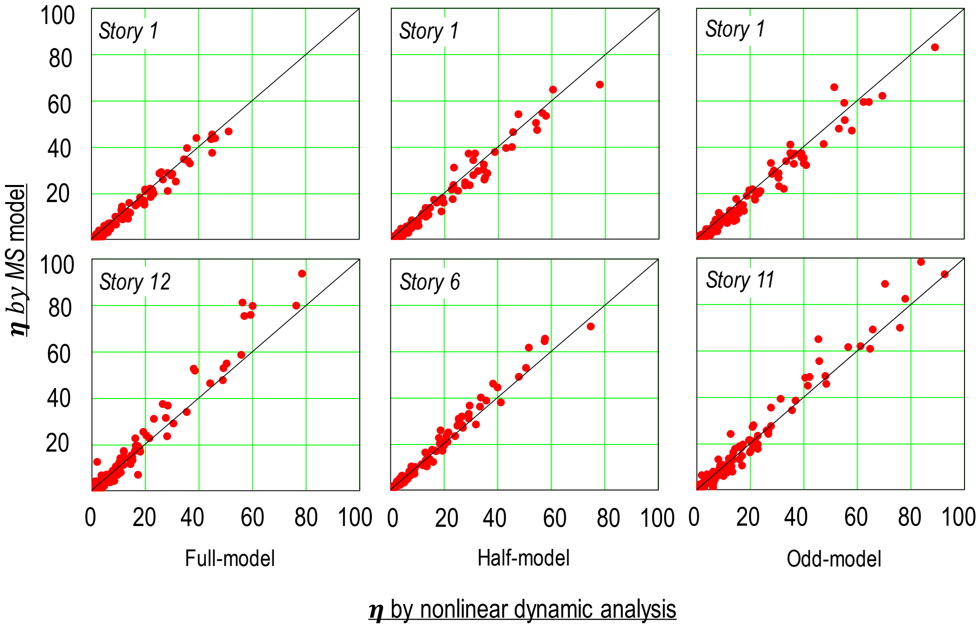

Fig. 10 – Analysis result of the 12-story model (κ= 0.5, υ = 0.75, $\phi^{I}_{1,i}$)

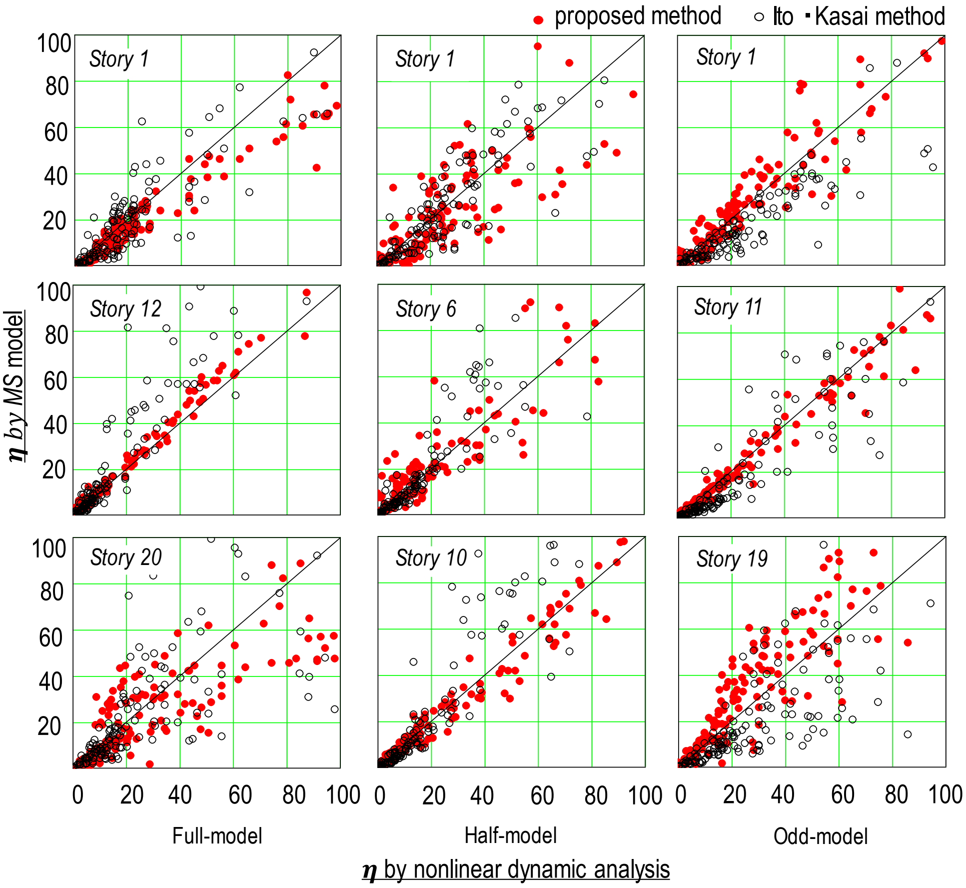

Fig. 11 – Analysis result of the 20-story model (κ= 2, υ = 0.25, $\phi^{E}_{1,i}$)

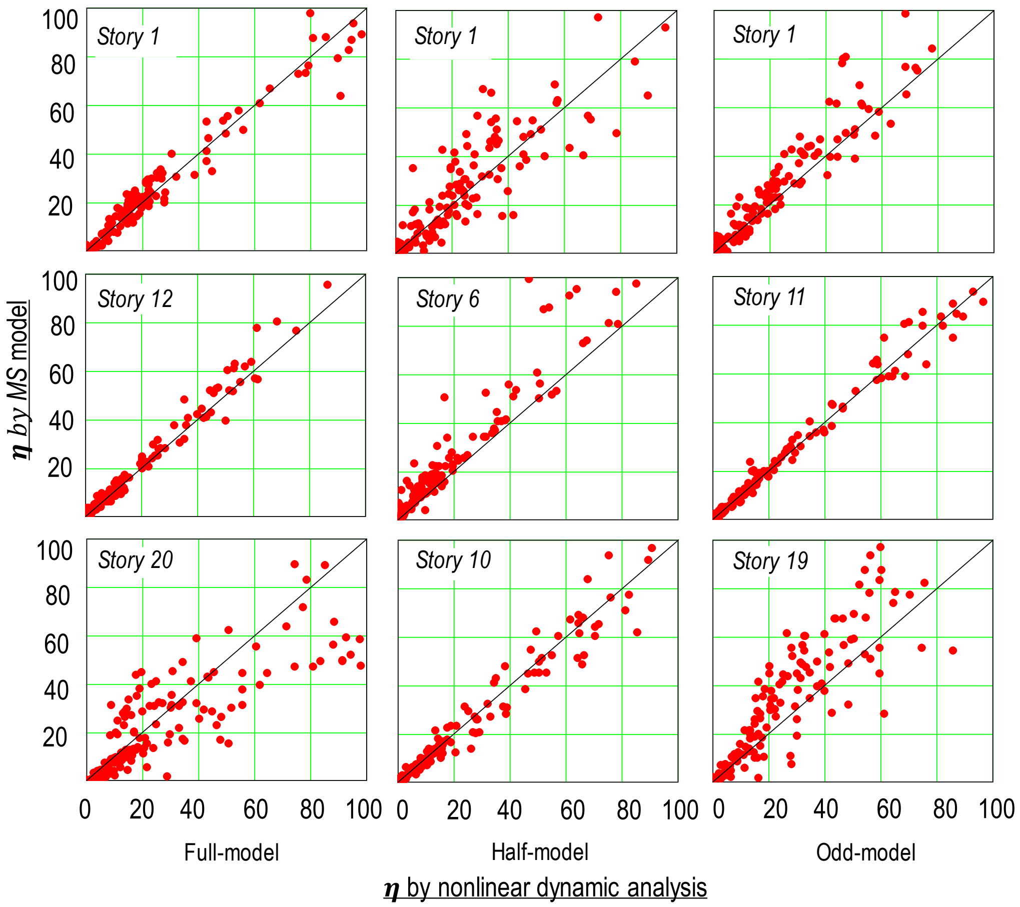

Fig. 12 – Analysis result of the 20-story model (κ= 2, υ = 0.25, $\phi^{I}_{1,i}$)

