PERFORMANCE OF MULTIPLE TUNED MASS DAMPERS

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1. Introduction

Igusa and Xu [1] proposed a new design concept for the TMD to suppress vibration of a dynamical system, termed Multiple Tuned Mass Dampers (MTMD). Due to the limitation of detuning of single TMD, to increase the robustness of the response MTMDs are been used quite effectively[2,3]. As we have seen from the literature that people have used different objective functions as well as different optimization techniques with certain assumptions. The basic configuration of the MTMD is a large number of small oscillators whose natural frequencies are distributed around the natural frequency of a controlled mode of the structure[4,5,6]. The primary structure is subjected to the harmonic ground acceleration. The optimum parameters of the MTMD system are obtained corresponding to the minimum of the maximum response of the main structure using an inbuilt function fmincon in MATLAB. It is done by three different method – Equal mass, Equal stiffness and New mehod. Also variation of optimum parameters of MTMD with the number of TMDs and the mass ratio of MTMD and the main system is observed. Then, explicit formulae for the optimum parameters of the optimum parameters are obtained using the function Solver in Excel and error analysis on the proposed expressions is conducted. The aim is to improve the performance of the MTMD system compared to the previous studies.

2. Methodology

The MTMD system configuration consists of a main system supported by n number of TMDs with different dynamic characteristics as shown in Fig. 1 and is externally excited by a harmonic force. The main system is idealized as a single lumped mass characterized by the stiffness, ks, the damping constant, cs, and the mass, ms[7]. It is assumed that the damping in the main system is of viscous type specified by the damping ratio, ξs(i.e., $\xi_s = c_s/2\sqrt{m_s k_s}$). Similar to the main system, the parameters of the jth TMD are the stiffness, kj, the damping constant, cj, and the mass, mj. The main system and each TMD are modelled as a single degree-of-freedom system under harmonic excitation so that the total d.f. of the combined structural system are n+1[8,9].

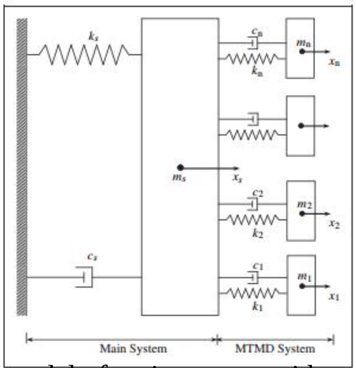

Fig.1- Structural model of main system with multiple tuned mass dampers

Three methods have been used to derive the optimum parameters of the MTMD system–Equal Mass method, Equal Stiffness method and New method. Optimization was done using an inbuilt function fmincon in MATLAB. The distribution of natural frequencies in all the three methods over a specified band-width is kept linear. In Equal Mass method, mass of each TMD is kept same and derived by dividing the total mass of the structure by the number of TMDs while in Equal Stiffness method, distribution of natural frequencies of the MTMD is obtained by keeping the stiffness constant but varying the mass of each TMD[10]. In the new method there is no restriction on the stiffness, mass or damping ratios of the n dampers. Hence, the number of variables in the optimization of the response of the structure under external excitation are 3*n (n variables for n masses of the dampers, n variables for the n damping ratios of the dampers, n variables for the n frequencies of the n dampers).

The natural frequency, w_j (i.e., $w_j = \sqrt{k_j - m_j}$) of the j^{th} TMD is expressed by

$$\omega_j = \omega_T \left[1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right] \tag{1}$$

for all the three methods.

$$\omega_T = \sum_{j=1}^{n} \omega_j / n \tag{2}$$

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \tag{3}$$

where ωT is the average frequency of all MTMD; and β is the non-dimensional frequency band-width of the MTMD system. Note that Eq. (1) indicates that the distribution of the natural frequencies of the MTMD system over a specified band-width is linear.

As suggested by Xu and Igusa [11] that the manufacturing of the MTMD with uniform stiffness is simpler than that with varying stiffness. As a result, the distribution of natural frequencies of the MTMD is obtained by keeping the stiffness constant but varying the mass of each TMD (i.e., $k_1 = k_2 = ... k_n = k_T$) in Equal Stiffness Method.

The damping constant of the jth TMD is expressed as

$$c_j = 2\, m_j\, \xi_T\, \omega_j \tag{4}$$

where ξ_T is the damping ratio kept constant for all the MTMD.

Total mass of the MTMD system is expressed by the mass ratio defined as

$$\mu = \frac{\sum_{j=1}^{n} m_j}{m_s} \tag{5}$$

where μ is the mass ratio of the MTMD system.

Tuning frequency ratio of the MTMD system is expressed by

$$f = \frac{\omega_T}{\omega_S} \tag{6}$$

where ω_s (i.e., $\omega_S = \sqrt{k_S/m_S}$) is the natural frequency of the main system.

The main system is excited by the harmonic force expressed by

$$F(t) = F_O e^{i\omega t} \tag{7}$$

where F(t) is the excitation force at the main mass; F_0 is the amplitude; **ω** is the circular frequency; and $i = \sqrt{-1}$.

The (n+1) equations of motion for system shown in Fig. 1 are expressed as [12]

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \tag{8}$$

Where

$$\{X\} = \{x_S, x_1, x_2, \dots \dots x_n\}^T \tag{9a}$$

is (n+1) vector for displacement response relative to the base and {F} is force vector

$$\{F\} = \{F(t), 0, 0, \dots \dots 0\}^T \tag{9b}$$

The matrices, [M],[C] and [K], are expressed as

$$[M] = \mathrm{diag}[m_s, m_1, m_2, \dots\dots, m_n] \tag{10}$$

$$[C] = \begin{bmatrix} c_s + \sum c_j & -c_1 & -c_2 & . & . & -c_n \\ & c_1 & 0 & . & . & 0 \\ & & c_2 & . & . & 0 \\ & & & . & . & . \\ & & & & . & . \\ symm & & & & & c_n \end{bmatrix} \tag{11}$$

$$[K] = \begin{bmatrix} k_s + \sum k_j & -k_1 & -k_2 & . & . & -k_n \\ & k_1 & 0 & . & . & 0 \\ & & k_2 & . & . & 0 \\ & & & . & . & . \\ & & & & . & . \\ symm & & & & & k_n \end{bmatrix} \tag{12}$$

The steady state harmonic response of the system is obtained by substituting

$$X(\omega) = (\omega^2 M - i\omega[C] + K)^{-1}) F_0 \tag{13}$$

Since the matrices [M] , [C] and [K] have non-zero terms along diagonal, the first row and first column, the inverse in the above equation can be evaluated by Cramer's rule.

3. Response to harmonic excitation

The amplitude of the steady state harmonic displacement of the main system, $x_s(\omega)$, to the excitation, $F(t) = F_0\ e^{i\omega t}$, is expressed by [12]

$$x_s(\omega) = [k_s - i\omega c_s - \omega^2 m_s - i\omega z(\omega)]^{-1} F_0 \quad (14)$$

$$z(\omega) = i\omega \sum_{j=1}^{n} \frac{m_j(\omega_j^2 - 2i\omega\omega_j\xi_j)}{\omega_j^2 - 2i\omega\omega_j\xi_j - \omega^2} \quad (15)$$

The amplitude of the displacement of the main system is expressed in the normalized form as

$$R = \frac{|x_s(\omega)|}{F_0/k_s} \quad (16)$$

The objective function in all the three methods is to minimize the maximum value of the response of the structure, R and find the corresponding optimum parameters. The maximum of the response, R of the structure is Dynamic Magnification Factor (DMF) evaluated as

$$DMF = \frac{1}{\sqrt{Re^2(z) + Im^2(z)}} \quad (17)$$

Where

$$Re(z) = 1 - \beta^2 - \beta^2 \sum_{j=1}^{n} \mu_j \frac{\left(\frac{\beta_j^2}{\beta^2} - 1\right) + 4\xi_j^2}{\left(\frac{\beta_j}{\beta} - \frac{\beta}{\beta_j}\right)^2 + 4\xi_j^2} \quad (18)$$

$$Im(z) = 2\beta\xi_s + \beta^2 \sum_{j=1}^{n} \mu_j \frac{\frac{2\xi_j\beta}{\beta_j}}{\left(\frac{\beta_j}{\beta} - \frac{\beta}{\beta_j}\right)^2 + 4\xi_j^2} \quad (19)$$

Where, $\mu_j = \frac{m_j}{m_s}$ is the mass ratio of each TMD
$\beta = \omega/\omega_s$ frequency ratio, is the ratio of the frequencies of the external force and structure
$\beta_j = \omega_j/\omega_s$ is the ratio of the frequencies of the jth TMD and structure

DMF for an undamped main system depends upon β_j, ξ_j and μ_j. β_j depends upon ω_j which again depends upon tuning frequency ratio because $f = \frac{\omega_T}{\omega_s}$ and frequency bandwidth of MTMD system, β (from Eq. 1). $\mu = \frac{m_j}{m_s}$ is obtained by m_j which is obtained as follows:

In Equal Mass method, $m_j = \frac{\mu\, m_s}{n}$, where $\mu = \frac{\sum_j^n m_j}{m_s}$ is assumed a suitable value whereas in equal stiffness method, the stiffness is kept constant but with varying the mass of each TMD, which means $k_1=k_2=...k_n=k_T$
Constant k_T can be found out by following expression,

$$k_{T=}\frac{\mu * m_s}{\sum_{j=1}^{n} 1/\omega_j^2} \tag{20}$$

And mass of each TMD can be found out by

$$m_j = \frac{k_T}{\omega_j^2} \tag{21}$$

In equal mass and equal stiffness method, tuning frequency ratio($f = \frac{\omega_T}{\omega_s}$), non-dimentional frequency bandwidth ($\beta = \frac{\omega_n - \omega_1}{\omega_T}$) and damping ratio(ξ_j) is kept variable their optimum values are found wheras in new optimization method, frequencies, damping ratios and masses of all TMDs are kept variables and their optimum values are found. As the number of variables in optimization was very high, previous method of numerical search technique is not an effective method to find the optimum parameters. Optimization was done using inbuilt optimization function *fmincon* in MATLAB in all the three methods. This is a constrained minimization problem. We used the interior point algorithm for finding out the optimum parameters of the MTMD system. This algorithm uses either a direct newton step or conjugate gradient step at each iteration for solving the approximate problem.

We need to define the lower and upper boundary on the limits of the variables, that is, tuning frequency ratio $f = \frac{\omega_T}{\omega_s}$, non-dimentional frequency bandwidth $\beta = \frac{\omega_n - \omega_1}{\omega_T}$ and damping ratio(ξ_j) in case of Equal Mass and Equal Stiffness method whereas the frequencies, damping ratios, masses of the dampers in the New Method. We also need to start from an initial guess estimate of the variables with the constraint that the total mass of all the dampers is equal to the mass ratio*mass of the structure. Frequencies initial guess was estimated assuming a linearly distributed frequency of the dampers with an initial guess of bandwidth and the initial guess tuning frequency ratio of the dampers. Initial guess for the masses of the dampers was that they have equal mass in case of the New Method. There is a possibility of reaching a local minima for the optimum parameters. To ensure that we reach a global minima these all lower bounds, upper bounds and the initial guess estimated were run for different values and then from the minimum value of the responses, we obtained all the optimum parameters in achieving the objective. In some cases for larger number of dampers, we also need to have good estimate of initial guess for the masses of the dampers. Therefore, in some cases assuming a variable mass, and a linear distribution of frequency and equal damping ratio was used to find the initial guess for the masses of the dampers.

4. Results

Different optimum parameters i.e. maximum response, average of frequencies, range of the frequency, and average damping ratio of the dampers for different mass ratios of the MTMD system for number of dampers equal to 2,3 and 5 by New method, Equal mass method and Equal Stiffness method are presented in Tables 1-6. Individual masses, frequencies and damping ratios of each TMD for different number of TMDs for mass ratio, μ equal to 0.01 and 0.05 are presented in Table 7-8. Optimum parameters by New method, Equal mass method and Equal Stiffness method for different number of TMDs for mass ratio, μ=0.01 and 0.05 are presented in Table 9. Different comparison plots of the dynamic magnification factor with the frequency ratio of the excitation frequency and structural frequency by the three method for different number of TMDs and mass ratio, μ=0.01 are shown in Fig. 2-7. Optimum response of structure and optimum parameters of TMDs are plotted aginst different number of TMDs for mass ratio, μ=0.01 and 0.05 in Fig. 8-11. Optimum response of structure and optimum parameters of TMDs are plotted aginst different mass ratios for n=2,3and 5 in Fig.12-15. Individual frequencies and damping ratios are plotted with their respective number of TMDs in Fig.16-17.

Table 1 Different optimum parameters by New method, Equal stiffness method and Equal mass method for μ=0.01 and μ=0.05

μ=0.01	NEW METHOD					EQUAL STIFFNESS					EQUAL MASS			
n	R^{opt}	f_{avg}^{opt}	f_{range}^{opt}	ξ_{avg}^{opt}	n	R^{opt}	f^{opt}	β^{opt}	ξ_T^{opt}	n	R^{opt}	f^{opt}	β^{opt}	ξ_T^{opt}
2	12.5488	0.99131	0.0589	0.0401	2	12.54319	0.993051	0.059298	0.040744	2	12.55694	0.991767	0.059113	0.040597
3	11.89827	0.99251	0.0856	0.0322	3	11.94432	0.994418	0.0861	0.03187	3	12.12691	0.992809	0.08343	0.032601
5	11.32305	0.993036	0.111873	0.024388	5	11.44527	0.995701	0.111636	0.024022	5	11.67373	0.993588	0.110833	0.024164
7	11.06423	0.992902	0.125587	0.020419	7	11.27982	0.996217	0.122902	0.021103	7	11.52738	0.99394	0.124021	0.020427
9	10.94505	0.993157	0.132843	0.018765	9	11.20847	0.996628	0.130741	0.018497	9	11.45563	0.995647	0.128228	0.018706
11	10.85223	0.99293	0.138784	0.017057	11	11.17812	0.996838	0.135328	0.01725	11	11.41906	0.995797	0.132288	0.017915
μ=0.05	NEW METHOD					EQUAL STIFFNESS					EQUAL MASS			
n	R^{opt}	f_{avg}^{opt}	f_{range}^{opt}	ξ_{avg}^{opt}	n	R^{opt}	f^{opt}	β^{opt}	ξ_T^{opt}	n	R^{opt}	f^{opt}	β^{opt}	ξ_T^{opt}
2	5.7241	0.96536	0.1313	0.0814	2	5.706922	0.967299	0.135205	0.084119	2	5.722485	0.960121	0.128758	0.089342
3	5.40679	0.96641	0.1900	0.0678	3	5.457206	0.973143	0.190865	0.068336	3	5.653559	0.964654	0.175654	0.072355
5	5.1451	0.96804	0.2443	0.0516	5	5.224664	0.979142	0.246461	0.052251	5	5.509966	0.964267	0.238626	0.058426
7	5.048555	0.969763	0.2728	0.043013	7	5.141832	0.981845	0.272218	0.045265	7	5.489639	0.963488	0.271617	0.05093
9	4.99366	0.969926	0.2859	0.041872	9	5.106845	0.983479	0.287741	0.040916	9	5.489714	0.96688	0.276012	0.049003
11	5.008765	0.960725	0.2850	0.045281	11	5.091167	0.98499	0.300208	0.036635	11	5.485763	0.965184	0.282927	0.050234

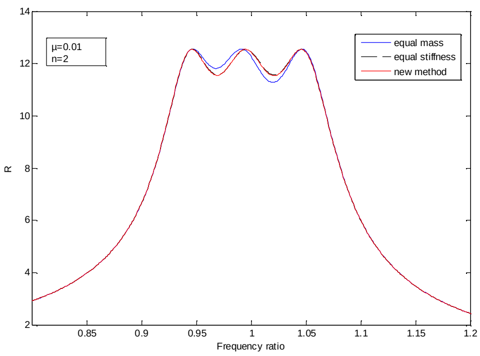

Figure 3 Plot of R versus Frequency ratio for μ=0.01 and n=2 by Equal masss, Equal stiffness and New

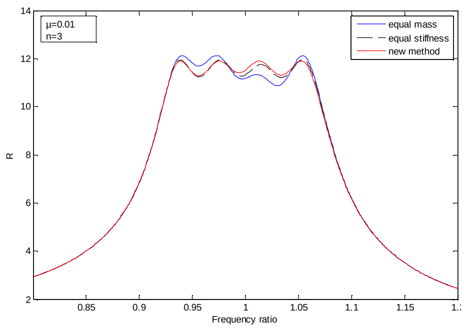

Figure 2 Plot of R versus Frequency ratio for μ=0.01 and n=3 by Equal masss, Equal stiffness and New method

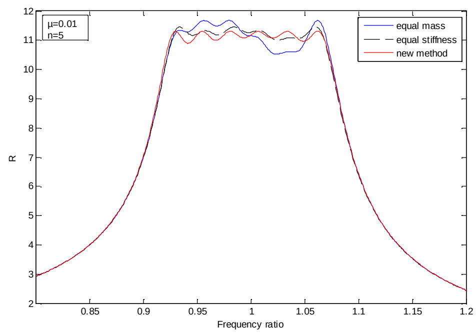

Figure 1 Plot of R versus Frequency ratio for μ=0.01 and n=5 by Equal mass, Equal stiffness and New method

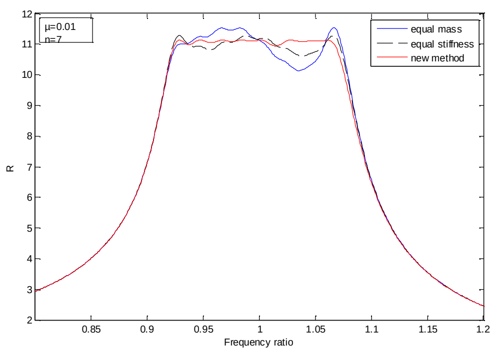

Figure 6 Plot of R versus Frequency ratio for μ=0.01 and n=7 by Equal masss, Equal stiffness and New method

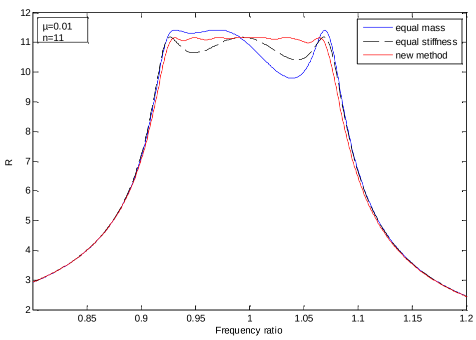

Figure 5 Plot of R versus Frequency ratio for μ=0.01 and n=9 by Equal masss, Equal stiffness and New method

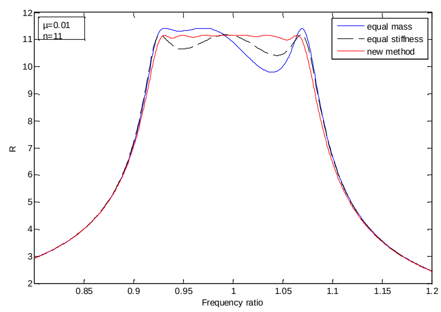

Figure 4 Plot of R versus Frequency ratio for μ=0.01 and n=11 by Equal masss, Equal stiffness and New method

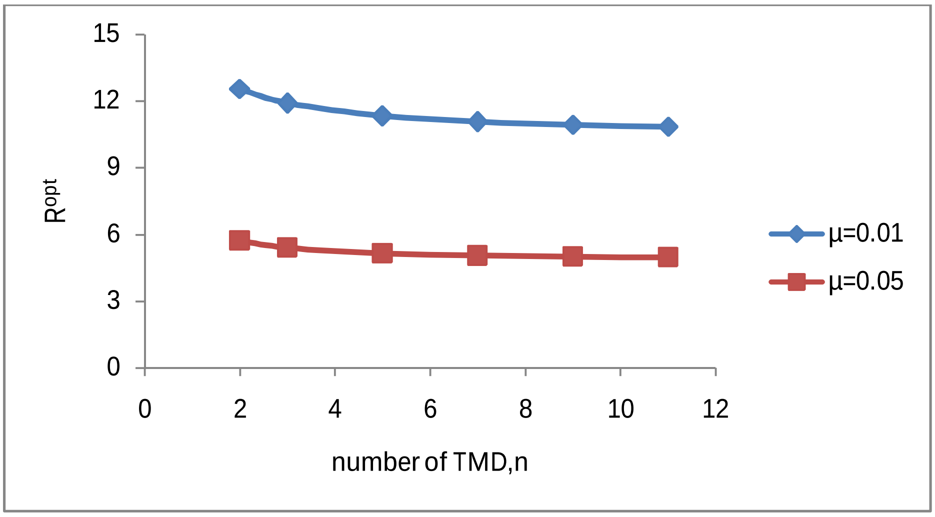

Figure 7 Optimum Maximum Response versus Number of TMDs by New method

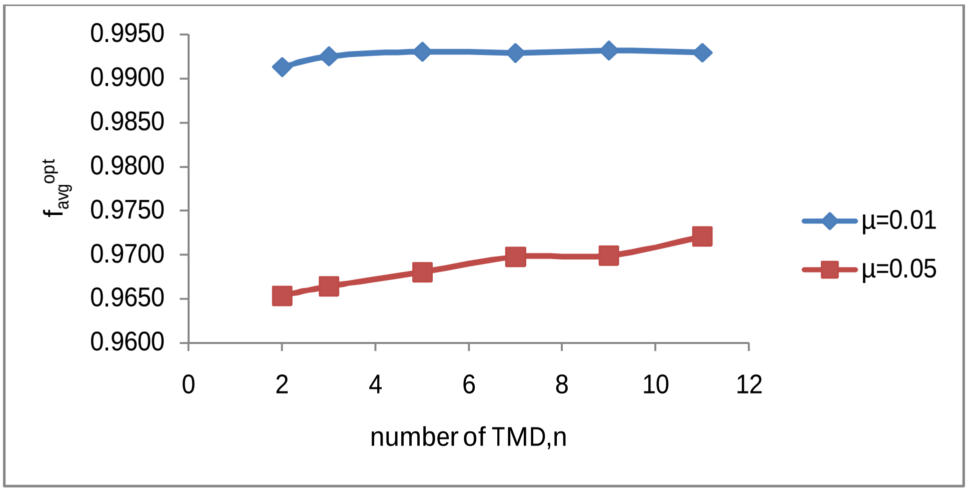

Figure 8 Optimum Average Frequency versus Number of TMDs by New method

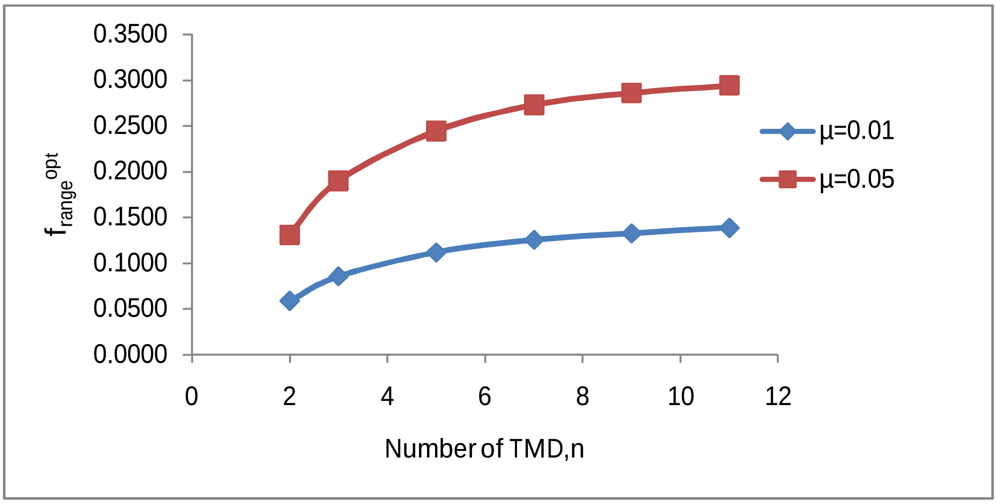

Figure 9 Optimum Frequency Range versus Number of TMDs by New method

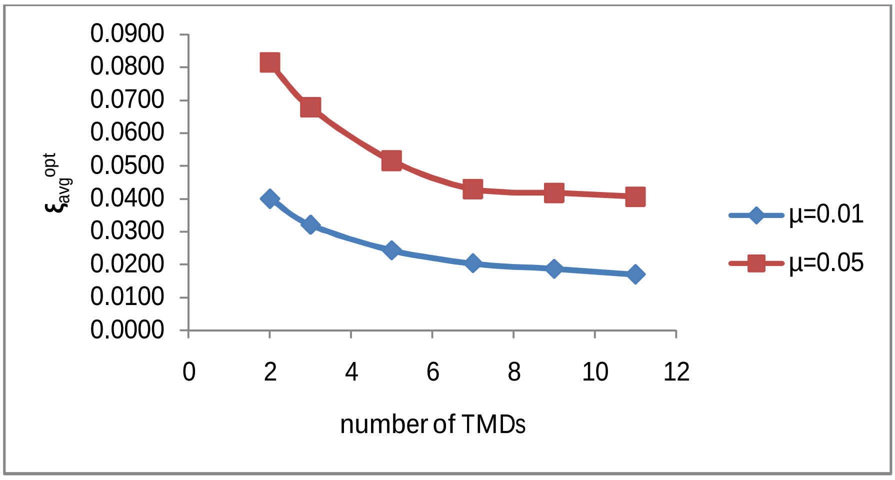

Figure 10 Optimum Average Damping Ratio versus Number of TMDs by New method

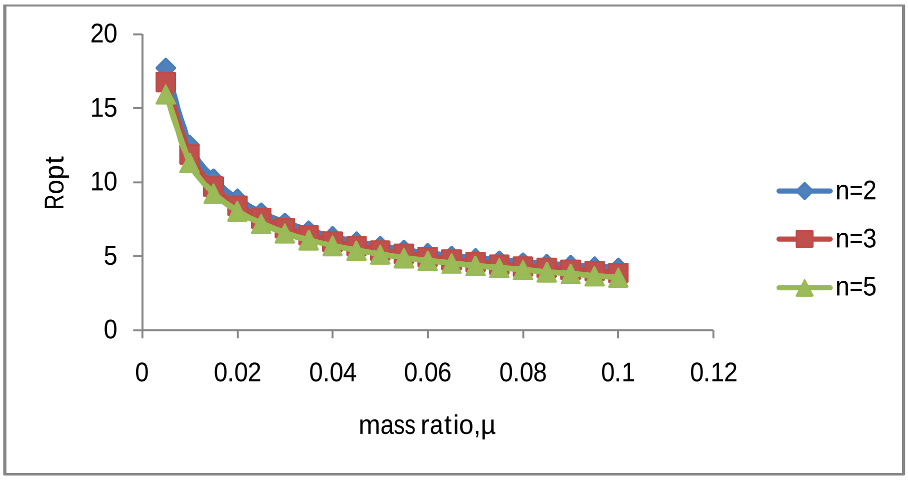

Figure 11 Variation of Optimum Maximum Response versus Mass ratio for different number of TMDs by New Method

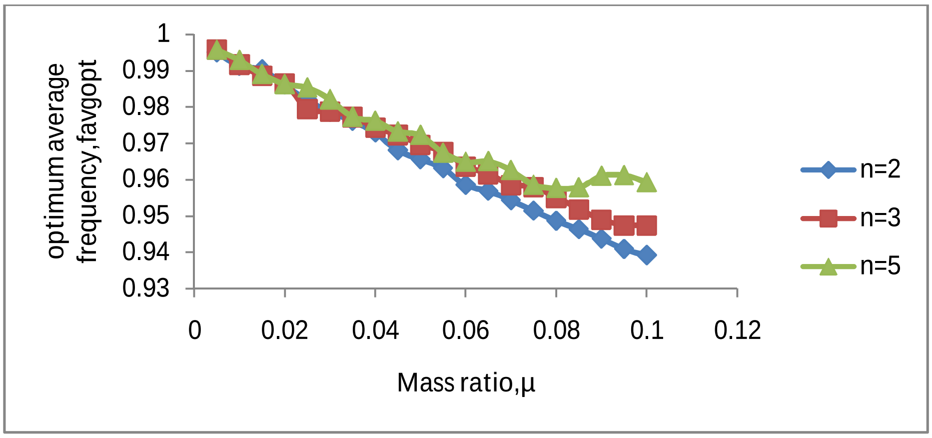

Figure 12 Variation of Optimum Average Frequency versus Mass ratio for different number of TMDs by New Method

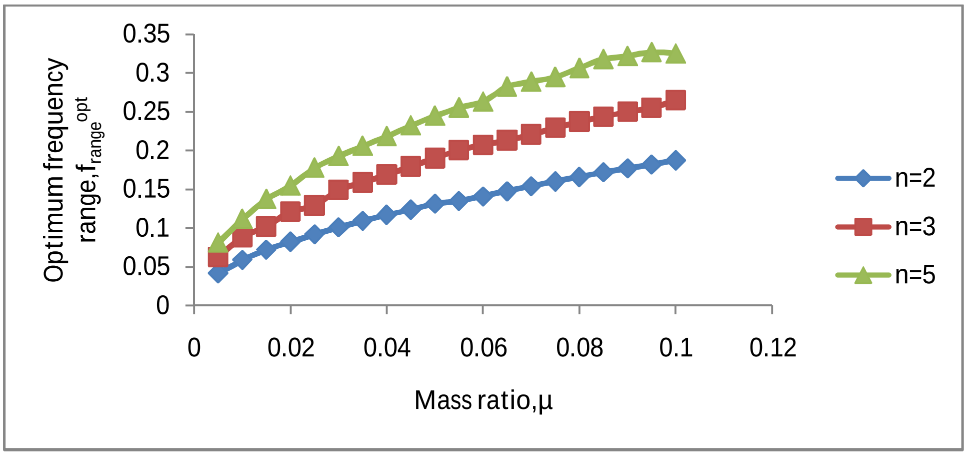

Figure 13 Variation of Optimum Frequency Range versus Mass ratio for different number of TMDs by New Method

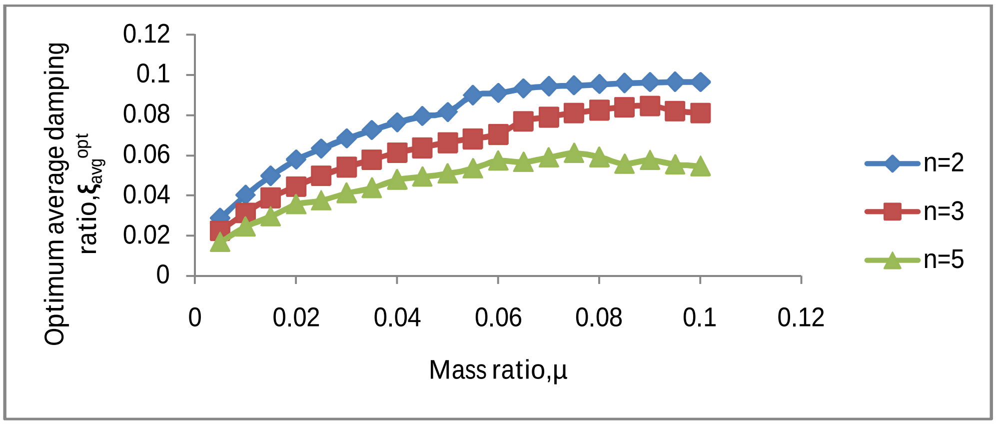

Figure 14 Variation of Optimum Average Damping Ratio versus Mass ratio for different number of TMDs by New Method

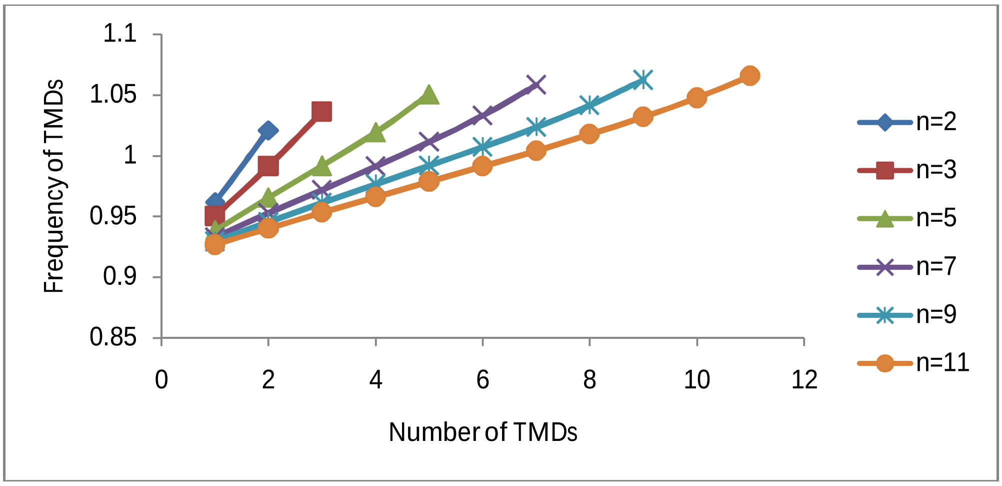

Figure 15 Frequency of TMDs versus Number of TMDs

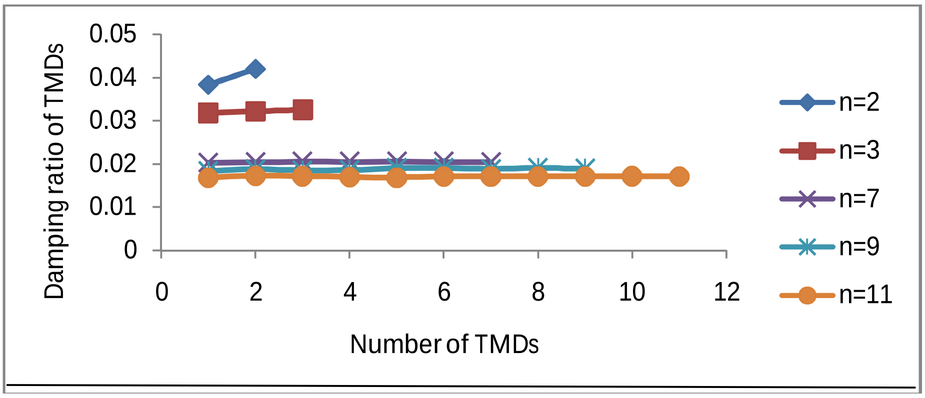

Figure 16 Damping Ratio of TMDs versus Number of TMDs

5. Explicit formulae for optimum parameters for New Method

Since the structural system considered is relatively complicated, therefore, it is extremely difficult to get the exact closed form expressions for the optimum parameters of MTMD system theoretically. However, the explicit formulae for optimum parameters can be obtained by applying the curve fitting technique, using the optimum parameters of the MTMD obtained previously. The optimum parameters of the MTMD system and response of the main structure will be function of the mass ratio, μ, and number of the MTMDs, n [13,14]. This implies that suitable curves shall be selected which is function of the two parameters (i.e.,μ and n). After several trials and errors the following expressions of the optimum parameters are found in Bandivadekar and Jangid (2012a) to give the minimum error for the optimum parameters.

$$R^{opt} = \sqrt{\frac{2}{\mu}\left(1+\frac{\mu}{2}\right)} + (a_1 + a_2\sqrt{\mu} + a_3\mu)\sqrt{\frac{1}{\mu}\left\{a_4\left(\frac{1}{\sqrt{n}}-1\right) + a_5\left(\frac{1}{n}-1\right) + a_6\left(\frac{1}{n\sqrt{n}}-1\right)\right\}}$$

$$f^{opt} = \left(\frac{1}{1+\mu}\right) + (a_1 + a_2\sqrt{\mu} + a_3\mu)\sqrt{\mu}\left\{a_4\left(\frac{1}{\sqrt{n}}-1\right) + a_5(n-1) + a_6(\sqrt{n}-1)\right\}\frac{1}{\sqrt{n}}$$

$$\beta^{opt} = (a_1 + a_2\sqrt{\mu} + a_3\mu)\sqrt{\mu}\left\{a_4\left(\frac{1}{\sqrt{n}}-1\right) + a_5(n-1) + a_6(\sqrt{n}-1)\right\}\frac{1}{\sqrt{n}}$$

$$\xi^{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} + (a_1 + a_2\sqrt{\mu} + a_3\mu)\sqrt{\mu}\left\{a_4\left(\frac{1}{\sqrt{n}}-1\right) + a_5\left(\frac{1}{n}-1\right) + a_6(\sqrt{n}-1)\right\}$$

The coefficient values in the expressions are given in Table, which are obtained using minimizing the sum of square error by Solver function in Excel.

Table 10 Coefficients values in the expressions of the optimum parameters for the New Method for Undamped main system

Coefficients	R^{opt}	f_{avg}^{opt}	$\beta^{opt}/f_{range}^{opt}$	ξ_{avg}^{opt}
a1	0.435442	0.097908	1.177999	0.220455
a2	-0.12349	-0.31723	-0.11811	-0.46002
a3	0.112151	5.747784	-0.27524	1.346828
a4	-0.95219	-0.3936	0.097039	-2.87821
a5	2.959278	-0.02839	-0.07505	3.795511
a6	-1.27837	0.433564	2.021126	

5 Conclusions

1. Different comparison plots of the dynamic magnification factor with the frequency ratio of the excitation frequency and structural frequency by the three method for different number of TMDs are plotted. It is found that DMF is a continuous and differentiable function of the frequency ratio. The number of the minima peaks is equal to the number of dampers and hence the number of maxima is equal to number of dampers +1.

2. For both, a particular mass ratio,μ and the number of dampers,n the most effective and robust method is the new method followed by equal stiffness method and the least as equal mass method. Comparing the new method, equal stiffness method and equal mass method, we get that the new method is the most effective method among the three due these following two reasons:

   a. R value, that is, the maximum response is the least for the new method.

   b. The degree of flatness is most in the new method, hence the sensitivity is the least making it most robust.

3. It is also observed that with the increase of number of MTMD the optimum damping ratio and displacement of the main system decreases whereas the optimum band- width and tuning frequency increases. On the other hand, the optimum damping ratio and band-width increases and tuning frequency and corresponding displacement of the main system decrease with the increase of the mass ratio of the MTMD system.

4. Frequency of TMDs are linearly distributed over a frequency range around the natural frequency of the structure and Damping Ratios are almost constant.

5. Comparison of optimum parameters by optimization method in MATLAB and explicit expressions for different mass ratios of the multiple tuned mass dampers for n=5 showed that there is good agreement between the optimum parameters by the two approaches. The maximum error for any value of ξT, ,βopt , fopt and Ropt is observed to be 10.171, 4.86, 0.70 and 3.36%, respectively. The magnitude of error for optimum damping ratio is relatively more. This is due to the fact that the optimum damping ratio of the MTMD system is sufficiently low.

4. Acknowledgements

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5. Copyrights

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