SEISMIC CONTROL OF A REINFORCED CONCRETE CHIMNEY USING TUNED MASS INERTER SYSTERM

L. Zhang[(1)], S.T. Xue[(2)], R.F. Zhang[(3)]

[(1)] *PhD candidate, Department of Disaster Mitigation for Structures, Tongji University, Shanghai 200092, China, zhangli24@tongji.edu.cn*
[(2)] *Professor, Department of Disaster Mitigation for Structures, Tongji University, Shanghai 200092, China, Xue@tongji.edu.cn*
[(3)] *Associate Professor, Department of Disaster Mitigation for Structures, Tongji University, Shanghai 200092, China, zhangruifu@tongji.edu.cn*

Abstract

The tuned mass dampers have been applied and proven effective for the seismic response mitigation of the reinforced concrete chimneys. However, a large additional mass is generally needed for the application of tuned mass damper, which may be inappropriate due to the additional moment action. In this study, a lightweight vibration mitigation device, which is called the tuned mass inerter system (TMIS), is used for the seismic response mitigation of the chimney. The TMIS is composed of a spring element, a parallel inerter subsystem and a tuned mass element. In the TMIS, the parallel inerter subsystem is applied for providing the mass enhancement effect and also the energy absorption and dissipation effect, and the spring element is set for tuning the mass element. The motion governing equation of the chimney structure with TMIS is given firstly. A performance demand-based optimization design method is proposed for the seismic response suppression of the reinforced concrete chimney using the TMIS. A benchmark RC chimney model is used to exemplify the proposed method. It is concluded that the application of TMIS in tall chimney is effective for seismic vibration control and for the reduction of physical mass in comparison to the traditional tuned mass damper.

Keywords: inerter system; reinforced concrete chimney; tuned mass; seismic mitigation; optimal design;

1 Introduction

Reinforced concrete (RC) chimneys are usually tall and slender, and believed to be vulnerable under the horizontal loads [1, 2]. In order to increase the reliability of this structure, the additional tuned mass damper (TMD) has been adopted for the design or retrofit of chimney [3, 4]. However, a large additional mass is usually needed for the application of TMD.

Recently, the inerter, which can provide a large inertance with a small physical mass, is introduced and proven effective for the structural vibration control [5]. The inerter is a two-terminal element whose reaction force is proportional to the relative acceleration of the both ends of the inerter element [6]. This device has been adopted to civil engineering for reducing the structural vibration response [5, 7-9]. By incorporating into the mechanical elements (i.e., spring element and damping element), many types of devices using inerter element with different layouts have been proposed and investigated for its application in structures [10, 11]. Additionally, Zhang et al. [12-14] proposed a series of tuned mass inerter systems (TMISs) based on tuning mass lightweight, including the tuned inerter mass system (TIMS) for suppressing the vibration of floor under the human-induced excitation, the tuned parallel inerter mass system (TPMIS) for mitigating the vibration of wind turbine tower and the tuned liquid inerter system (TLIS) for reducing the oscillatory motion. Although a number of researches have been conducted for the inerter in civil engineering, the application of the inerter-based device in tall and slender chimneys still need to be studied due to the special and important role of a chimney in some industries.

In this study, a lightweight vibration mitigation device, called tuned mass inerter system (TMIS), is used for the seismic response mitigation of the RC chimney. The TMIS is composed of a spring element, a parallel inerter subsystem and a tuned mass element. The numerical model of RC chimney with TMIS is first given. The optimal design method is proposed for the RC chimney with TMIS. A benchmark model of RC chimney is adopted to verify the proposed design method. And also, a comparative study is performed between the chimney with the traditional TMD and the TMIS to illustrate the advantages of TMIS for the seismic mitigation of the RC chimney.

2 Illustrative modeling

2.1 Illustration of TMIS

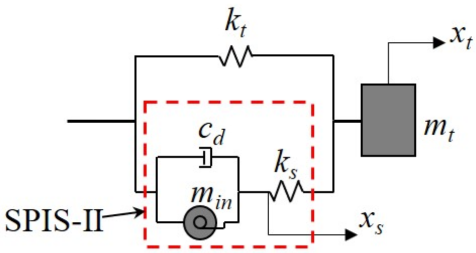


Fig. 1 Model of TMIS

The mechanical model of TMIS is depicted in Fig. 1. The TMIS is presented by replacing the damping element of a TMD with a series-parallel layout II inerter subsystem [15]. This device is composed of three parts: a tuning spring, a tuned mass and a SPIS-II. The inerter element with the mass enhancement effect is employed to decrease the required physical mass of the tuned mass. The additional TMIS is expected to obviously improve the structural performance with a lightweight device.

In the TMIS shown in Fig. 1, k_t and k_s are the stiffnesses of the tuning spring and the spring element in SPIS-II, respectively; c_d denotes the damping parameter; m_t and m_{in} denote the tuning mass and the inertance of the inerter element, respectively; x_t and x_s are the displacement of the tuning mass and the spring in SPIS-II, respectively. For the inerter element, the output force F_{in} is proportional to the relative acceleration of its two terminals.

2.2 Modeling of chimney with TMIS

The numerical model of RC chimney with TMIS is built herein to check its seismic performance. The chimney is modelled as an assemblage of two-dimensional (2D) beam elements with both the translational degrees of freedom (DOF) and rotational DOF at each node, as shown in Fig. 2. To facilitate the numerical analysis of the chimney with TMIS, the rotational DOF of the chimney is condensed. The mass matrix $\mathbf{M}_\mathrm{p}$ of the chimney can be obtained by simply remove the rotational inertia involving the rotational DOF (θ_i in Fig. 2b) of each node. The stiffness matrix $\mathbf{K}_\mathrm{p}$ of the chimney is built by employing the static condensation. The Rayleigh damping is adopted to obtain the damping matrix $\mathbf{C}_\mathrm{p}$ of the chimney. Using the dynamic equilibrium condition, the governing equation of motion for the uncontrolled chimney is expressed in Eq. (1), where $\mathbf{x} = \{x_1 \quad x_2 \quad \cdots \quad x_n\}^T$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the vector of the nodal displacement, velocity and acceleration of the uncontrolled chimney relative to the ground, $\boldsymbol{\iota}_\mathrm{p} = \{1 \quad 1 \quad \cdots \quad 1\}^T$ is the influence coefficient vector, $\ddot{x}_g$ represents the ground motion.

$$\mathbf{M}_\mathrm{p}\ddot{\mathbf{x}} + \mathbf{C}_\mathrm{p}\dot{\mathbf{x}} + \mathbf{K}_\mathrm{p}\mathbf{x} = -\mathbf{M}_\mathrm{p}\boldsymbol{\iota}_\mathrm{p}\ddot{x}_g \tag{1}$$

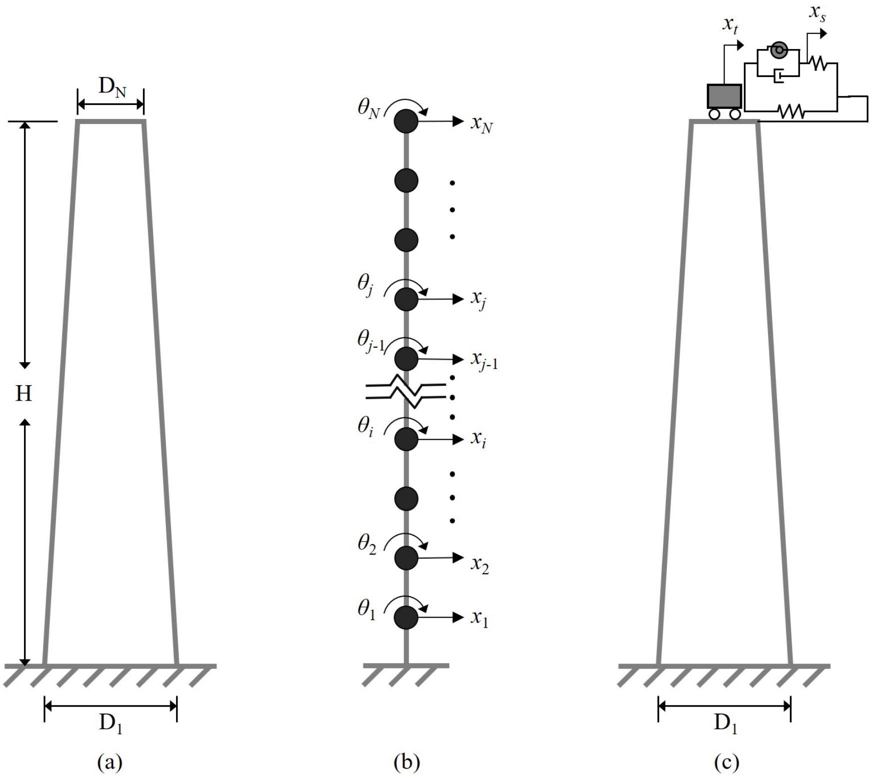


Fig. 2 Models of the chimney: (a) uncontrolled model; (b) lumped mass model; (c) chimney with TMIS;

Considering the common location of TMD in practice, the TMIS is installed at the top of the chimney to mitigate the seismic response as shown in Fig. 2c. The governing equation of motion for chimney with TMIS can be written as follows:

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=-\mathbf{M}_g\boldsymbol{\iota}\ddot{x}_g \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{M}_g$ are the mass, damping, stiffness matrices and the mass matrix the chimney with TMIS, respectively; $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ are the vector of the displacement, velocity and acceleration of the chimney with TMIS corresponding to the ground motion; $\boldsymbol{\iota}$ is the influence coefficient vector of the chimney with TMIS. The matrices mentioned above can be expressed as follows:

$$\mathbf{M}=\begin{bmatrix}\mathbf{M}_p & \mathbf{0}_{N\times1} & \mathbf{0}_{N\times1}\\ \mathbf{0}_{1\times N} & m_{in}+m_t & -m_{in}\\ \mathbf{0}_{1\times N} & -m_{in} & m_{in}\end{bmatrix}_{(N+2)\times(N+2)} \tag{3}$$

$$\mathbf{C}=\begin{bmatrix}\mathbf{C}_p & \mathbf{0}_{N\times1} & \mathbf{0}_{N\times1}\\ \mathbf{0}_{1\times N} & c_d & -c_d\\ \mathbf{0}_{1\times N} & -c_d & c_d\end{bmatrix}_{(N+2)\times(N+2)} \tag{4}$$

$$\mathbf{K}=\begin{bmatrix}\mathbf{K}_p & \mathbf{0}_{N\times1} & \mathbf{0}_{N\times1}\\ \mathbf{0}_{1\times N} & k_t & 0\\ \mathbf{0}_{1\times N} & 0 & k_s\end{bmatrix}_{(N+2)\times(N+2)}+\begin{bmatrix}\boldsymbol{\chi}[k_t+k_s]\boldsymbol{\chi}^T & \boldsymbol{\chi}[-k_t] & \boldsymbol{\chi}[-k_s]\\ [-k_t]\boldsymbol{\chi}^T & 0 & 0\\ [-k_s]\boldsymbol{\chi}^T & 0 & 0\end{bmatrix}_{(N+2)\times(N+2)},\text{where }\boldsymbol{\chi}=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}_{N\times1} \tag{5}$$

$$\mathbf{M}_g=\begin{bmatrix}\mathbf{M}_p & \mathbf{0}_{N\times1} & \mathbf{0}_{N\times1}\\ \mathbf{0}_{1\times N} & m_t & 0\\ \mathbf{0}_{1\times N} & 0 & 0\end{bmatrix}_{(N+2)\times(N+2)} \tag{6}$$

$$\mathbf{X}=\{x_1 \quad x_2 \quad \cdots \quad x_N \quad x_t \quad x_s\}^T \tag{7}$$

$$\boldsymbol{\iota}=\{\boldsymbol{\iota}_p^T \quad 1 \quad 0\}^T \tag{8}$$

2.3 Seismic response under stochastic excitation

For the seismic design of a structure, the stochastic excitations need to be appropriately considered. In this study, the ground motion $\ddot{x}_g$ is assumed as a white noise with zero mean for the RC chimney with TMIS, and the corresponding power spectral density (PSD) function is expressed as $S(\Omega)=S_0$, where Ω denote the frequency of the excitation. To solve the structural random vibration response, the state space model of the chimney with TMIS under the stochastic excitation is employed. The state space description of Eq. (2) is expressed as

$$\begin{aligned}\dot{\mathbf{x}}_s(t)&=\mathbf{A}_s\mathbf{x}_s(t)+\mathbf{E}_s\ddot{x}_g(t)\\ \mathbf{z}_s(t)&=\mathbf{C}_s\mathbf{x}_s(t)\end{aligned} \tag{9}$$

where $\mathbf{x}_{ss}=[\mathbf{X}^T \quad \dot{\mathbf{X}}^T]^T$ is the state vector involving the nodal displacement and velocity of the chimney with TMIS. The corresponding state space matrices $\mathbf{A}_s$, $\mathbf{E}_s$ and $\mathbf{C}_s$ can be expressed as

$$\mathbf{A}_s=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\quad \mathbf{E}_s=\begin{bmatrix}\mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{M}_g\boldsymbol{\iota}\end{bmatrix}\quad \mathbf{C}_s=\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix} \tag{10}$$

The observation matrix $\mathbf{C}_s$ in Eq. (10) involves the output variables vector $\mathbf{z}_s(t)$ that includes all the nodal displacement and absolute accelerations for the chimney with TMIS relative to the ground. Under the white noise excitation, the output of the system $\mathbf{z}_s(t)$ is also considered a stationary random process with zero mean and the covariance matrix is calculated as

$$\mathbf{K}_{z} = \mathbf{C}_{s}\mathbf{P}\mathbf{C}_{s}^{T} \tag{11}$$

where the matrix $\mathbf{P}$ is the state covariance and can be solve by the algebraic Lyapunov equation expressed as

$$\mathbf{A}_{s}\mathbf{P} + \mathbf{P}\mathbf{A}_{s}^{T} + 2\pi S_{0}\mathbf{E}_{s}\mathbf{E}_{s}^{T} = 0 \tag{12}$$

Then, the variance $\sigma_{z,i}^{2}$ for the system $\mathbf{z}_{s}(t)$ can be obtain at the corresponding diagonal entry of $\mathbf{K}_{z}$.

3 Optimal design of chimney with TMIS

It can be seen, in Fig. 1, there are five parameters to be determined for the seismic design of chimney with TMIS. For convenience, the parameters of TMIS is expressed in a dimensionless form as follows:

$$\mu_{t}=\frac{m_{t}}{M_{T}},\mu_{in}=\frac{m_{in}}{m_{t}},\kappa=\frac{k_{s}}{k_{t}},\lambda_{k}=\frac{\omega_{t}}{\omega_{0}},\xi_{d}=\frac{c_{d}}{2\sqrt{m_{t}k_{t}}} \tag{13}$$

where μ_{t}, μ_{in}, κ, λ_{k} and ξ_{d} are tuned-to-primary mass ratio, inertance-to-mass ratio, the stiffness ratio of TMIS, natural frequency ratio and the damping ratio of TMIS, respectively; ω_{0} and $\omega_{t}=\sqrt{k_{t}/m_{t}}$ are the natural frequency of the chimney and TMIS, respectively; $M_{T}=\sum_{i=1}^{N} m_{i}$ is the total mass of the chimney.

In this study, the stochastic displacement response σ_{z,d_N}^{2} at the top of the chimney with TMIS is employed to evaluate the structural vibration. To estimate the vibration control effect of TMIS mounted on chimney, the stochastic displacement mitigation ratio γ_{N} is defined as

$$\gamma_{N}\left(\mu_{t},\mu_{in},\kappa,\lambda_{k},\xi_{d}\right)=\frac{\sigma_{z,d_N}}{\sigma_{z0,d_N}} \tag{14}$$

where σ_{z,d_N} and σ_{z0,d_N} are the root mean square (RMS) value of the top displacement of the chimney with TMIS and the uncontrolled chimney, respectively. The calculation of σ_{z0,d_N} is similar to the derivation of σ_{z,d_N} described in Subsection 2.3.

As is mentioned above, the tuned mass of TMD is limited in its application of the tall slender chimney due to the additional moment action. Hence, the TMIS is proposed as a lightweight device for the seismic control of the chimney. For the optimal design of the TMIS in chimney, the tuned mass (i.e., μ_{t}) should be considered. In this study, γ_{N} and μ_{t} are considered as the objectives in TMIS optimization. The parameters μ_{t}, μ_{in}, κ, λ_{k} and ξ_{d} are the determined variables for optimal design.

To simplify the design process, the dual objectives optimization of TMIS is transformed to a single objectives optimization using the ε-constraint method by setting $\gamma_{N} \le \gamma_{N,\lim}$ as the nonlinear constraint condition, where $\gamma_{N,\lim}$ is the target stochastic displacement mitigation ratio. Additionally, the parameters κ and ξ_{d} in SPIS-II are proposed to be calculated using the improved fix-point theory [5] involving μ_{in} as:

$$\kappa=\frac{\mu_{in}}{1-\mu_{in}},\xi_{d}=\frac{\mu_{in}}{2}\sqrt{\frac{3\mu_{in}}{\left(1-\mu_{in}\right)\left(2-\mu_{in}\right)}} \tag{15}$$

Therefore, the optimal design of the TMIS in chimney can be expressed as:

$$\begin{aligned}&\text{find} && \mu_t,\mu_{in},\lambda_k,\\ &\text{minimize} && \mu_t \\ &\text{subjected to} && \begin{cases}\gamma_N\left(\mu_{in},\mu_k,\mu_t\right)\le\gamma_{N,\lim}\\ \mu_{t,\min}\le\mu_t\le\mu_{t,\max}\\ \mu_{in,\min}\le\mu_{in}\le\mu_{in,\max}\\ \lambda_{k,\min}\le\lambda_k\le\lambda_{k,\max}\end{cases}\end{aligned} \tag{16}$$

where $\mu_{t,\min}$, $\mu_{in,\min}$ and $\lambda_{k,\min}$ are the lower bounds of μ_t, μ_{in} and λ_k, respectively, and $\mu_{t,\max}$, $\mu_{in,\max}$ and $\lambda_{k,\max}$ are the upper bounds of μ_t, μ_{in} and λ_k, respectively.

4 Case study

A benchmark model of RC chimney researched by Datta and Jain [16] and Elias et al. [17] is chosen to illustrate the optimal design of TMIS and perform the comparative study between TMD and TMIS mounted on this chimney. The height of this chimney is 250 m. The chimney is divided into 20 equal beam element and the length of each element is 12.5 m. The identical segment is assumed for each beam element. The modulus of and density of the concrete are assumed as 2.5×10 N/m^{2}and 2400 kg/m^3, respectively. The Rayleigh damping is adopted for the chimney with same damping ratio of 0.05 for all modes. The numerical model of the RC chimney is built as a 20 DOF system referring to Subsection 2.2. The total mass of the RC chimney M_T is 9199 ton. According to the modal analysis result, the fundamental frequency ω_0 of the chimney is calculated as 1.91 rad/s.

In order to improve the reliability of the RC chimney, the TMIS is suggested to mitigate the structural seismic response. For the optimal design of TMIS, $\gamma_{N,\lim}$ is set as 70%. By analyzing the stochastic response of the uncontrolled chimney and chimney with TMIS in frequency domain and solving Eq. (16), μ_t, μ_{in} and λ_k of TMIS are determined as 0.0056, 0.1474 and 0.8828, respectively. For the common application of TMD in structure, the range of tuned mass ratio is usually 1-5%. According to the analysis result, μ_t of TMIS is approximately half of the minimum common value of tuned mass ratio of TMD with $\gamma_{N,\lim}$ equal to 70%. Hence, the TMIS can be recommended as a lightweight device for the seismic mitigation of RC chimney. A detailed comparison between TMD and TMIS is conducted below.

4.1 Improvement of TMIS for seismic control

The applications of TMD and TMIS are compared here to investigate their performance in seismic mitigation of RC chimney. The parameters of TMD are obtained using the fixed-point theory [18]. Five design cases (i.e., Case-A, Case-B, Case-C Case-D and Case-E) are set for the TMDs with different tuned mass ratios of 1%, 2%, 3%, 4% and 5%, respectively. Under the white noise, the γ_N of TMD in Case-A to Case-E are calculated in the frequency domain as 78.7%, 78.3%, 80.2%, 82.9% and 86.1%,respectively. The tuned mass ratio of 2% is analyzed as the most effective value for seismic mitigation of Chimney with TMD, and this result is the same with the analysis of Elias [17]. Then, for the comparative study of TMD and TMIS in seismic mitigation of RC chimney, $\gamma_{N,\lim}$ of TMISs in Case-A to Case-E are set to the same with γ_N of TMD to achieve the same seismic control effect and the corresponding parameters of TMISs are obtained with the optimal design recommended above. Besides, in Case-F that the tuned mass ratio of TMD is set to 2% and $\gamma_{N,\lim}$ of TMIS is set to 80% of γ_N in TMD (i.e., $\gamma_{N,\lim}$=62.6%), the corresponding design parameters of TMIS is also calculated.

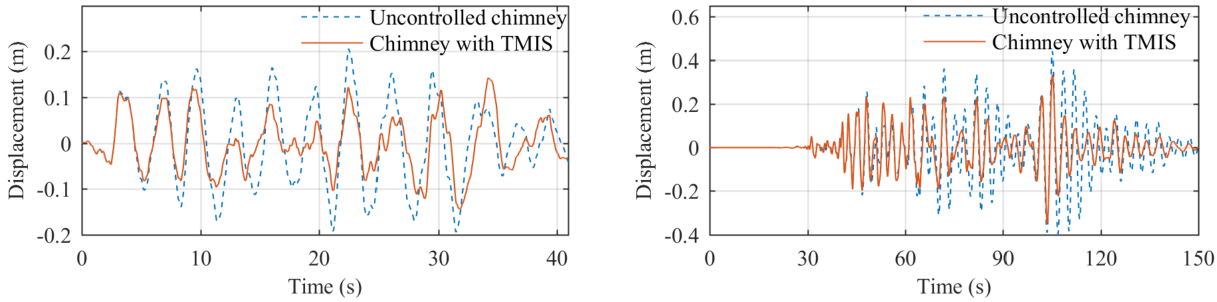
Table 1 Parameters obtained in the comparative analysis

Case ID	Design parameters of TMIS		
	μ_t	μ_{in}	λ_k
Case-A	0.0022	0.0575	0.9498
Case-B	0.0023	0.0600	0.9479
Case-C	0.0018	0.0489	0.9564
Case-D	0.0013	0.0363	0.9662
Case-E	0.0010	0.0249	0.9752
Case-F	0.0130	0.4104	0.6934

For Case-A to Case-E, the design parameters of TMISs are listed in Table 1. It can be seen, under the same seismic mitigation effect, the requirement tuned mass of TMIS is obviously less than that of TMD (Case-A to Case-E). And even when $\mu_{t,TMIS}$ is smaller than $\mu_{t,TMD}$, a better seismic mitigation can be achieved (Case-F). Due to the actual physical mass of the inerter element can be much smaller than the apparent mass (i.e., the inertance) and then ignored, the TMIS is considered a much lighter device than TMD for the vibration control in an RC chimney.

4.2 Time history analysis

The optimal design of TMIS adopted in RC chimney is also checked in the time domain. A white noise excitation with zero mean and a natural seismic wave, the Chi-chi (1999) record, are selected to conduct the time history analysis. The parameters of TMIS is set to achieve $\gamma_{N,\lim}$=70% ,which has been calculated in the optimization above. The top displacement responses under these excitations are obtained using the Newmark's integration method for both the uncontrolled chimney and the chimney with TMIS as shown in Fig. 3. The RMS displacement mitigation ratio γ_N is also calculated for these four excitations to compare with the target performance.



(a) White noise, γ_N=67.4% (b) Chi-chi record, γ_N = 66.3%

Fig.3 Top displacement responses of the uncontrolled chimney and chimney with TMIS

As is shown in Fig. 3, the top displacement of the chimney can be effectively reduced under different excitations by installing the TMIS with the optimal design proposed in this study. And also, γ_N for different excitations are close to the target performance of $\gamma_{N,\lim}$=70%, which indicates the proposed optimal design is effective.

5 Conclusion

In this study, an inerter-based lightweight device, called tuned mass inerter system (TMIS), is proposed for the seismic response mitigation of the RC chimney. The analysis results indicate that the TMIS is efficient for seismic mitigation of RC chimney and behaves a better vibration control effect with a lighter tuned mass compared to conventional TMD. And also, the proposed optimal design of TMIS is considered an effective and accurate method for the seismic control of the RC chimney. To sum up, the TMIS can be a suitable device for improving the performance of chimney and the proposed design method is potential to extend to other similar type of inerter-based lightweight devices.

Acknowledgement

This study was supported by the National Natural Science Foundation of China (Grant 51978525, 51778490), and the Key Program for International S&T Cooperation Projects of China (No.2016YFE0127600).

References

[1] Kilic SA, Sozen MA (2003): Evaluation of effect of August 17, 1999, Marmara earthquake on two tall reinforced concrete chimneys. *ACI Structural Journal*, 100, 357-364.
[2] Zhou C, Zeng X, Pan Q, Liu B (2015): Seismic fragility assessment of a tall reinforced concrete chimney. *The Structural Design of Tall and Special Buildings*, 24, 440-460.
[3] Longarini N, Zucca M (2014): A chimney's seismic assessment by a tuned mass damper. *Engeering Structures*, 79, 290-296.
[4] Brownjohn JMW, Carden EP, Goddard CR, Oudin G (2010): Real-time performance monitoring of tuned mass damper system for a 183m reinforced concrete chimney. *Journal of Wind Engineering and Industrial Aerodynamics*, 98, 169-179.
[5] Ikago K, Saito K, Inoue N (2012): Seismic control of single-degree-of-freedom structure using tuned viscous mass damper. *Earthquake Engineering & Structural Dynamics*, 41, 453-474.
[6] Pan C, Zhang RF, Luo H, Li C, Shen H (2018): Demand-based optimal design of oscillator with parallel-layout viscous inerter damper. *Structural Control & Health Monitoring*, 25, e2051.
[7] Zhao ZP, Zhang RF, Jiang YY, Pan C (2019): Seismic response mitigation of structures with a friction pendulum inerter system. *Engeering Structures*, 193, 110-120.
[8] Zhao ZP, Zhang RF, Lu Z (2019): A particle inerter system for structural seismic response mitigation. *Journal of the Franklin Institute-Engineering and Applied Mathematics*, 356, 7669-7688.
[9] Zhang RF, Zhao ZP, Pan C, Ikago K, Xue ST (2020): Damping enhancement principle of inerter system. *Structural Control and Health Monitoring*. DOI:10.1002/stc.2523.
[10] Marian L, Giaralis A (2014): Optimal design of a novel tuned mass-damper–inerter (TMDI) passive vibration control configuration for stochastically support-excited structural systems. *Probabilistic Engineering Mechanics*, 38, 156-164.
[11] Lazar I, Neild S, Wagg D (2014): Using an inerter-based device for structural vibration suppression. *Earthquake Engineering & Structural Dynamics*, 43, 1129-1147.
[12] Chen QJ, Zhao ZP, Xia Y, Pan C, Luo H, Zhang RF (2019): Comfort based floor design employing tuned inerter mass system. *Journal of Sound and Vibration*, 458, 143-157.
[13] Zhang RF, Zhao ZP, Dai KS (2019): Seismic response mitigation of a wind turbine tower using a tuned parallel inerter mass system. *Engeering Structures*, 180, 29-39.
[14] Zhao ZP, Zhang RF, Jiang Y, Pan C (2019): A tuned liquid inerter system for vibration control. *International Journal of Mechanical Sciences*, 164, 105171.
[15] Pan C, Zhang RF (2018): Design of structure with inerter system based on stochastic response mitigation ratio. *Structural Control & Health Monitoring*, 25, e2169.
[16] Datta T, Jain A (1987): An analytical study of the across-wind response of cylinders due to vortex shedding. *Engeering Structures*, 9, 27-31.
[17] Elias S, Matsagar V, Datta TK (2016): Effectiveness of distributed tuned mass dampers for multi-mode control of chimney under earthquakes. *Engeering Structures*, 124, 1-16.
[18] Hartog JPD (1956): *Mechanical vibrations, 4 ed*. New York: McGraw-Hill, Dover.

