PARAMETRIC OPTIMIZATION OF TUNED MASS DAMPERS FOR HIGH-RISE STRUCTURES CONSIDERING STRUCTURAL BENDING DEFORMATION

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Abstract
This paper puts forward a simple practical method for designing a tuned mass damper (TMD) for a high-rise structure considering structural bending deformation. In order to investigate how the bending deformation of high-rise structures works on the performance of TMD, the equivalent bending-shear model was developed to simulate the bending deformation of a high-rise structure, which may induce the "uphill" and "downhill" effects of TMD. On the basis of that, H_2 criterion is employed to design TMD. The closed-form exact solution of TMD considering structural bending was derived in this paper. Numerical parametric studies of TMD for high-rise structure were conducted to verify the validity of the closed-form exact solution of TMD. Research results show that the proposed closed-form solution of TMD can be employed for high-rise structures and achieve a better performance level than the classic TMD design.

Keywords: TMD; high-rise structure, bending deformation, parametric optimization, H_2 criterion

1. Introduction

The living comfort of a high-rise structure caused by undesired vibrations has recently received more and more attention. The application of TMD is an effective way to reduce undesired vibrations of high-rise structures[1,2]. TMD is a passive control devices, which composed of a mass, a spring and viscous damper attached to primary system[3]. Many optimization criteria were proposed for optimum design of TMD, and the three most popular criterion are H_∞ optimization, H_2 optimization and the stability maximization criterion[4,5]. To solve the closed-form exact solution of optimal TMD parameters, the above three optimization criteria are based on undamped single degree-of-freedom primary system[6], which structural bending deformation may be in consequence ignored. For a high-rise structure, the storey deformation can be separated into bending components and shear components[7-9]. Moreover, with the increment of structural height, the storey deformation may be dominated by bending components[10]. Therefore, the investigation of how the bending deformation of high-rise structures works on the effectiveness of supplemental damping devices seems to be of great importance.

Previous studies mostly concentrate on inventing novel damping devices[11,12], with scarce attention given to the design methods of a TMD for a high-rise structure considering structural bending deformation, which may result in the "uphill" and "downhill" effects of TMD. In this study, the closed-form exact solution of TMD for a high-rise structure with considering structural bending deformation is derived. The "uphill" and "downhill" effects of TMD caused by the structural bending deformation is fully considered. The performances of TMD designed by the proposed H_2 closed-form solution is verified by a realistic 32-storey concrete tube structure and compared with that of a classical TMD design.

2. The equivalent bending-shear model

Referring to Fig. 1, A TMD is installed on the roof of a high-rise structure considering structural bending deformation, in which x_i and θ_i denote lateral displacement and bending angle of the *i*-th floor, respectively. m_i and J_i denote the mass and mass moment of inertia of the *i*-th floor. m_t, c_t and k_t are the mass, damping coefficient and stiffness property of the TMD system, respectively. To illustrate the process of solving the optimal parameters of TMD for a high-rise structure, an equivalent bending-shear model are formed as shown in Fig. 2, in which x and θ represent the modal displacement and bending angle responses, respectively. m and J denote the modal mass and mass moment of inertia, respectively. k_s and k_b denote the modal shear and bending stiffness, respectively. As is seen from Fig. 2, the equivalent bending-shear model can take both structual modal displacement and bending angle into accout, thus the closed-form exact solution of TMD deduced based on the equivalent bending-shear model is cosidered to be more reasonable for high-rise structures.

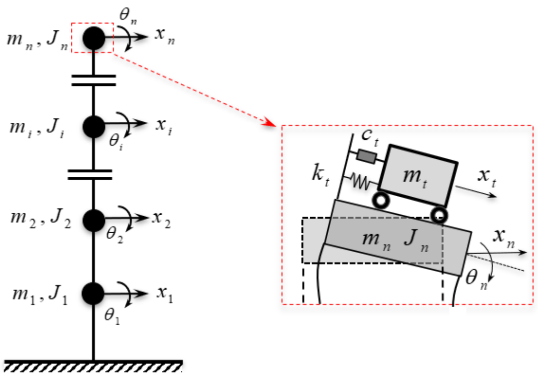


Fig. 1- modelling of a high-rise structure with a TMD.

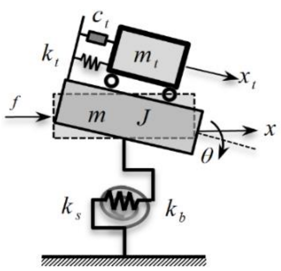


Fig. 2- Schematic diagrams of a undamped equivalent bending-shear model model with a TMD.

When TMD is employed to control the structural response of the first mode, the modal mass and mass moment of inertia of the high-rise structure can be expressed as

$$m=\{X\}_1^T[M]\{X\}_1,\quad J=\{\theta\}_1^T[J]\{\theta\}_1 \tag{1}$$

where

$$\{X\}_1=\{\varphi_1,\varphi_2,\cdots,\varphi_n\}_1^T,\{\theta\}_1=\{\varphi_{n+1},\varphi_{n+2},\cdots,\varphi_{2n}\}_1^T \tag{2}$$

in which $\{\varphi\}$ is the mode shape. Assume that the stiffness matrix of the equivalent bending-shear model can be described as[13]

$$[K']=\begin{bmatrix} k_s & -\frac{h}{2}k_s \\ -\frac{h}{2}k_s & k_b+\frac{k_s h^2}{4}\end{bmatrix} \tag{3}$$

Let ω_1 represent the angular frequency at the first mode, the modal shear and bending stiffness parameters can be written as

$$k_s=\frac{2\bar{\varphi}}{2\bar{\varphi}-h}m\omega_1^2,\quad k_b=\left(J+\frac{\bar{\varphi}mh}{2}\right)\omega_1^2 \tag{4}$$

In which, $\bar{\varphi}=\max\left(\{X\}_1\right)/\max\left(\{\theta\}_1\right)$. In the case of $\sin\theta\approx\theta$, the equations of motion of the undamped equivalent bending-shear model with a TMD can be expressed as

$$\begin{bmatrix} m+m_t & 0 & m_t \\ 0 & J & 0 \\ m_t & 0 & m_t\end{bmatrix}\begin{Bmatrix}\ddot{x}\\ \ddot{\theta}\\ \ddot{x}_t\end{Bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&c_t\end{bmatrix}\begin{Bmatrix}\dot{x}\\ \dot{\theta}\\ \dot{x}_t\end{Bmatrix}+\begin{bmatrix} k_s & -\frac{k_s h}{2} & 0\\ -\frac{k_s h}{2} & k_b+\frac{k_s h^2}{4} & -m_t g\\ 0 & -m_t g & k_t\end{bmatrix}\begin{Bmatrix}x\\ \theta\\ x_t\end{Bmatrix}=\begin{Bmatrix}f\\0\\0\end{Bmatrix} \tag{5}$$

If the external excitation is a ground motion excitation $\ddot{x}_g(t)$, Eq. (5) can then be rewritten as

$$\begin{bmatrix} m+m_t & 0 & m_t \\ 0 & J & 0 \\ m_t & 0 & m_t \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{x}_t \end{Bmatrix}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c_t \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{x}_t \end{Bmatrix}+\begin{bmatrix} k_s & -\frac{k_s h}{2} & 0 \\ -\frac{k_s h}{2} & k_b+\frac{k_s h^2}{4} & -m_t g \\ 0 & -m_t g & k_t \end{bmatrix}\begin{Bmatrix} x \\ \theta \\ x_t \end{Bmatrix}=\begin{Bmatrix} -(m+m_t)\ddot{x}_g \\ 0 \\ -m_t\ddot{x}_g \end{Bmatrix} \tag{6}$$

It can be observed from Eq.(5) and Eq.(6), the gravity component of TMD, $m_t g\theta$, may result in the "uphill" and "downhill" effects of TMD.

3. H_2 optimization of TMD parameters

3.1 Definition of H_2 performance index

In this study, a H_2 criterion is employed to obtain the closed-form exact solution of TMD for a undamped high-rise structure modeled as a bending-shear model. Firstly, the frequency response function of primary system can be written as:

$$H(i\omega)=\frac{A(i\omega)}{B(i\omega)} \tag{7}$$

where

$$\begin{cases} A(i\omega)=\sum_{k=0}^{4} a_k\times(i\omega)^k \\ B(i\omega)=\sum_{k=0}^{6} b_k\times(i\omega)^k \end{cases} \tag{8}$$

To suppress the vibration of the primary system subjected to random excitation, the H_2 performance index can be defined as[5]

$$PI=\frac{1}{2\pi}\int_{\infty}^{\infty}\left|H(i\omega)\right|^2 d\omega \tag{9}$$

The power spectral densities of input excitation are constant over all frequencies if the excitation is the ideal white noise, and the performance index can then be evaluated in analytical form[14,15]

$$PI=\frac{1}{2\Delta_6}\left[a_4^2\Delta_1+\left(a_3^2-2a_2a_4\right)\Delta_2+\left(a_2^2-2a_1a_3+2a_0a_4\right)\Delta_3+\left(a_1^2-2a_0a_2\right)\Delta_4+a_0^2\Delta_5\right] \tag{10}$$

where

$$\begin{aligned} &\Delta_1=b_0b_3^2+b_1^2b_4-b_0b_1b_5-b_1b_2b_3,\quad \Delta_2=b_1^2b_6+b_0b_3b_5-b_1b_2b_5,\quad \Delta_3=b_0b_5^2+b_1b_3b_6-b_1b_4b_5 \\ &\Delta_4=\frac{1}{b_0}\left(b_2\Delta_3-b_4\Delta_2+b_6\Delta_1\right),\quad \Delta_5=\frac{1}{b_0}\left(b_2\Delta_4-b_4\Delta_3+b_6\Delta_2\right),\quad \Delta_6=b_0\left(b_1\Delta_5-b_3\Delta_4+b_5\Delta_3\right) \end{aligned} \tag{11}$$

According to H_2 criterion, the objective function is set as follows

$$PI_{\text{opt}}=\min\left[PI(\gamma,\zeta_t)\right] \tag{12}$$

3.2 Closed-form exact solution of TMD parameters

For simplicity, some symbols are introduced in Table1, note that shear frequency ω_s may not equal to natural frequency of the primary system. Generally, the mass ratio of TMD is determined by the practical situation in advance, thus the optimal design of TMD can be conducted to derive the optimal frequency and damping ratio of TMD. Therefore, to obtain PI_{opt}, the design equations for the primary system with a TMD can be expressed as

$$\frac{\partial PI}{\partial\gamma}=0 \quad \frac{\partial PI}{\partial\zeta_t}=0 \tag{13}$$

Table 1- Description of symbols

Symbol	Description
$\omega_s = \sqrt{k_s/m}$	Shear frequency of the primary system
$\alpha = J/m$	Ratio of moment of inertia to mass of the primary system
$\beta = k_b/k_s$	Ratio of bending stiffness to shear stiffness
$\mu = m_t/m$	Mass ratio of TMD to the primary system
$\omega_t = \sqrt{k_t/m_t}$	Natural frequency of TMD
$\gamma = \omega_t/\omega_s$	Frequency ratio of TMD to the primary system
$\zeta_t = c_t/(2m_t\omega_t)$	Damping ratio of TMD

For a undamped primary system, a closed-form exact solution of TMD optimal frequency ratio γ_{opt} can be derived as

$$\gamma_{opt} = \sqrt{\frac{\lambda_1\mu + \lambda_2}{\lambda_3\mu^2 + \lambda_4\mu + \lambda_5}} \tag{14}$$

where

$$\begin{aligned} &\lambda_0 = \left(4\beta + h^2\right),\ \ \lambda_1 = 2\beta\lambda_0^2, \lambda_2 = 4\beta\left(\lambda_0^2 + 4\alpha h^2\right), \lambda_3 = \lambda_0^3 \\ &\lambda_4 = 2\lambda_0\left(\lambda_0^2 + 4\alpha h^2\right),\ \ \lambda_5 = h^6 + \left(8\alpha + 12\beta\right)h^4 + 16\left(\alpha^2 + 2\alpha\beta + 3\beta^2\right)h^2 + 64\beta^3 \end{aligned} \tag{15}$$

and optimal damping ratio ζ_{topt} can then be obtained as

$$\zeta_{topt} = \sqrt{\frac{\delta_1\mu^2 + \delta_2\mu + \delta_3}{\delta_4\mu^2 + \delta_5\mu + \delta_6}} \tag{16}$$

where

$$\begin{aligned} &\delta_0 = 16\beta^2 + 8\beta h^2 + 4\alpha h^2 + h^4,\ \ \delta_1 = \lambda_0^3\gamma_{opt}^6 h^2,\ \ \delta_2 = 2\lambda_0\gamma_{opt}^4 h^2\left[\gamma_{opt}^2\delta_0 - 2\beta\lambda_0\right] \\ &\delta_3 = 16\lambda_0\beta^2 h^2\gamma_{opt}^2 - 8\delta_0\beta h^2\gamma_{opt}^4 + h^2\gamma_{opt}^6\left[h^6 + \left(8\alpha + 12\beta\right)h^4 + \left(16\alpha^2 + 32\alpha\beta + 48\beta^2\right)h^2 + 64\beta^3\right] \\ &\delta_4 = -64\lambda_0^2\beta g h\gamma_{opt}^4,\ \ \delta_5 = 16\lambda_0^2\beta h^2\gamma_{opt}^4,\ \ \delta_6 = 16\beta\gamma_{opt}^4\left(16\beta^2h^2 + 8\beta h^4 + h^6 + 4\alpha h^4\right) \end{aligned} \tag{17}$$

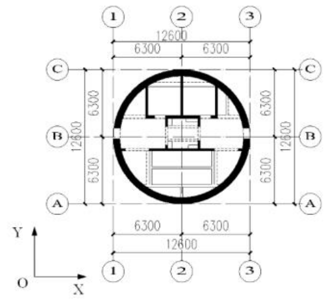
It is worth to mention that, if high-rise structure is assumed to be undamped, the optimal parameters of TMD are the same when the structure is subjected to the external force or ground motion acceleration.

4. Numerical example

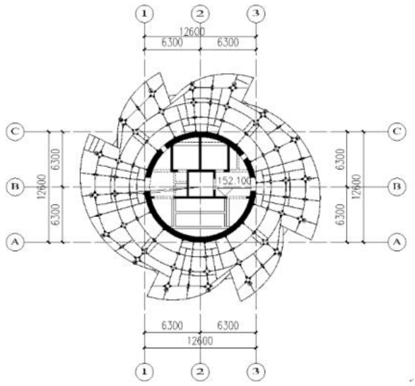
To verify the validity and accuracy of the proposed H_2 closed-form exact solution of TMD optimal parameters, a realistic 32-storey concrete tube structure was selected as numerical example as illustrated in Fig. 3. The total height of structure is 167.4 meters and the diameter of the tube is 12 meters. The aspect ratio of the tower is about 14 so the bending deformation of the structure is obvious. Considering the space limit of the tower, a 258-ton TMD is installed on top of the structure. The modal analysis is conducted and the first five mode of structure are shown in Fig. 4. It can be observed that the structure has a typical character of bending-shear deformation at the first mode. To depict the bending deformation components of the first mode, the equivalent bending-shear model of the first mode was obtained from the modal analysis results, and the details of the equivalent bending-shear model are presented in Table 2. It is noted that, the shear frequency of the tower 1.73rad/s is quite different from the natural frequency of the tower 0.98rad/s. The design damping level of the tower is assumed to be small enough, therefore, the optimal stiffness and damping coefficients of TMD for the tower can be designed by the proposed closed-form exact solution and the classical solution in

Reference[15] respectively, as tabulated in Table 2. For convenience, TMD1 and TMD2 represent TMD designed by the proposed solution and classical solution, respectively. In particular, TMD1a herein represents TMD designed by the proposed solution in the case of ignoring the "uphill" and "downhill" effects.

It can be observed from Table 2 that both the stiffness and damping coefficients of TMD1a and TMD2 are the same because the "uphill" and "downhill" effects of TMD are ignored. That means the proposed H_2 closed-form solution of TMD optimal parameters has an acceptable accuracy. It can also be observed that although the optimal stiffness coefficients of TMD1 and TMD2 are the same, the optimal damping coefficient of TMD1 is quite different from that of TMD2, suggesting the bending deformation of high-rise structure has a considerable influence in the optimal damping coefficient of TMD. Frequency domain analysis is then carried out based on the equivalent bending-shear model. It can be seen from Fig. 5 that TMD1 designed by the proposed H_2 closed-form exact solution can achieve a better performance level than the classical TMD2 .



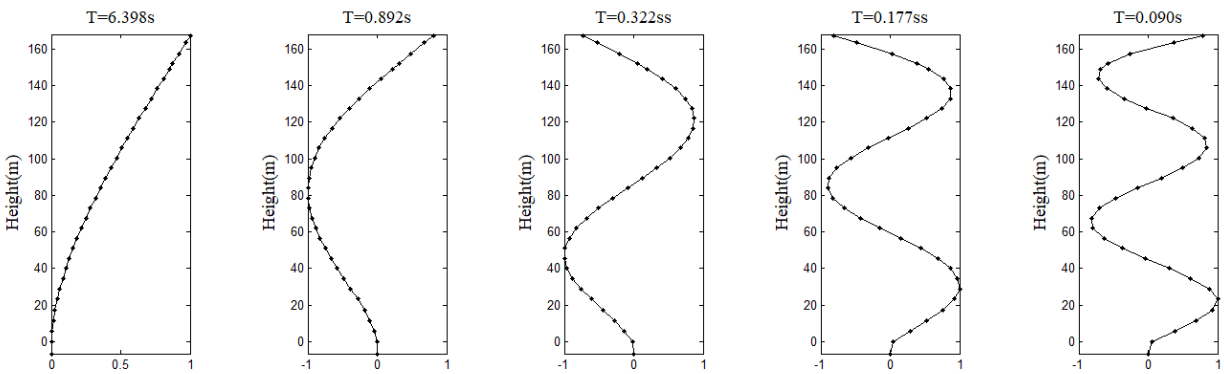
(b) The plane layout of standard floor



(c) The plane layout of 28th floor

(a) Finite element model

Fig. 3 - Details of 32-storey concrete tube structure



(a) first (b) second (c) third (d) fourth (e) fifth

Fig. 4 - The first five mode of structure.

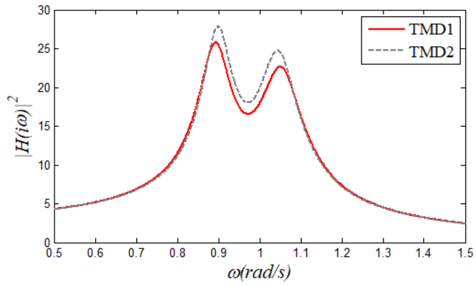
Table 2 - Details of the equivalent bending-shear model of the first mode

h (m)	**Natural frequency ω_1 (rad/s)**	**Mode ratio $\bar{\varphi}$**	**M (kg)**	**J (kg/m^2)**	**k_b (Nm/rad)**	**k_s (N/m)**	**ω_s (rad/s)**	**β**
167.4	0.98	123.24	8.40×10^6	2.45×10^3	8.36×10^{10}	2.53×10^7	1.73	3.31×10^3

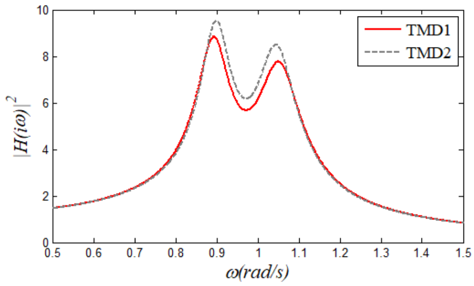
Note: shear frequency is not equal to natural frequency($\omega_s \neq \omega_1$)

Table 3 - optimal design results of TMD parameters

Type		**Mass (kg)**	**Mass ratio (%)**	**Stiffness k_t(N/m)**	**Damping coefficient c_t (Ns/m)**
TMD1	Considering “uphill”and “downhill” effects	2.58×10^5	3.07	2.38×10^5	4.66×10^4
TMD1a	Ignoring “uphill”and “downhill” effects	2.58×10^5	3.07	2.38×10^5	4.29×10^4
TMD2	—	2.58×10^5	3.07	2.38×10^5	4.29×10^4



(a) Force excitation



(b) Ground motion excitation

Fig. 5 - Frequency domain analysis based on the equivalent bending-shear model.

In this paper, β is defined to represent the “uphill” and “downhill” effects of TMD. With the decrease of β value, structural bending deformation becomes more remarkable, which means the “uphill” and “downhill” effects of TMD would be more vital. To make a further understanding of how the “uphill” and “downhill” effects works on the optimal parameters and performance of TMD, the variations of the optimal parameters of TMD and objective function PI_{opt} with β are shown in Fig. 6 and 7.

One can see from Fig. 6 that β has a considerable influence on the optimal damping coefficient of TMD. In fact, as the decrease in β, the difference between the damping coefficients of TMD1 and TMD2 becomes obvious. The damping coefficient of TMD plays a key role in adjusting the movement phase of TMD, which is important for a high-rise structure considering bending deformation. The performance of TMD will get worse if TMD is always in the “uphill” state. Furthermore, the proposed closed-form exact solution is very close to classic solution when the “uphill” and “downhill” effects of TMD is ignored, indicating the validity and accuracy of the proposed H_2 closed-form exact solution of TMD optimal parameters.

As shown in Fig. 7, with the reduction of β, TMD1 designed by the proposed H_2 closed-form exact solution can achieve a much better performance level than the classicial TMD2. For instance, when structure is subjected to a force excitation, PI_{opt} of TMD1 at β=10^3 is 173, taking up 78% of PI_{opt} of TMD2. However, “uphill” and “downhill” effects of TMD can be ignored when β is larger than 10^4 . This is because the “uphill” and "downhill" effects of TMD mainly controlled by structural bending deformation, which is not remarkable anymore when β is larger than 10^4.

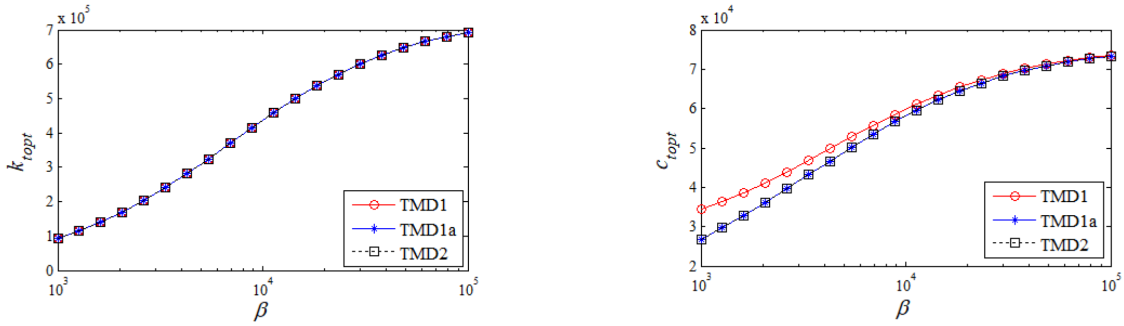
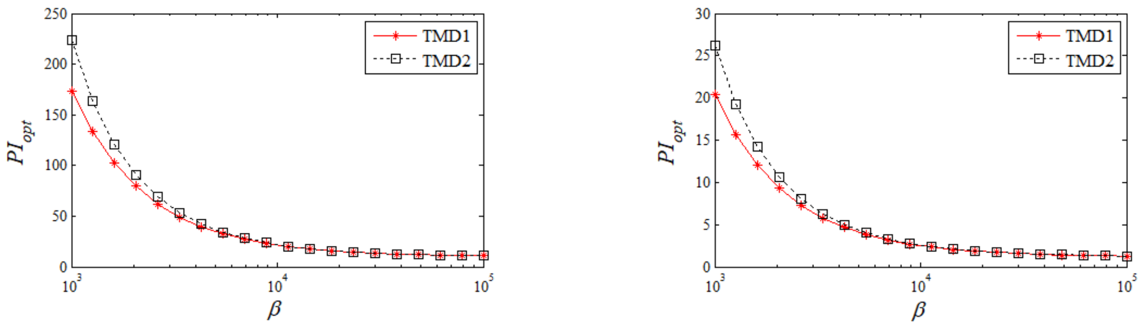


Fig. 6 - Comparison results of solutions for $\beta \in [10^3, 10^5]$.



(a) Force excitation (b) Ground motion excitation

Fig. 7 - PI_{opt} of a damped structure with a TMD for $\beta \in [10^3, 10^5]$.

5. Conclusion

1. A closed-form exact solution to H_2 optimization of the tuned mass dampers for high-rise structures was proposed, and the validity and accuracy of the proposed closed-form solution were verified by a realistic 32-storey concrete tube structure.

2. The equivalent bending-shear model was presented to depict the bending deformation components of structural modal responses. When the structural bending deformation becomes more remarkable, the "uphill" and "downhill" effects of TMD have a considerable influence in the optimal damping coefficient of TMD.

3. TMD designed by the proposed H_2 closed-form solution can achieve a better performance level than the classical TMD for a high-rise structure in which structural bending deformation is remarkable.

6. Acknowledgements

The authors gratefully acknowledge the partial support of this research by the National Key Research and Development Program of China under Grant No. 2017YFC0703600, the National Natural Science Foundation of China under Grant No. 51978185, and the Program for Innovative Research Team of Department of Education of Guangdong Province of China No. 2016KCXTD016.

7. Copyrights

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