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A FULLY NON-STATIONARY STOCHASTIC GROUND MOTION MODEL FOR THE SEISMIC ANALYSIS OF THE ZOSER PYRAMID

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Abstract

Ground motion modelling in the greater Cairo Area and more specifically at Saqqara, in the proximity of the Zoser pyramid is addressed. Although Egypt is not generally considered a seismic prone country, the plethora of archaeological sites require thorough seismic analyses aimed at evaluating the risk of structural collapse and at proposing, therefore, strategies for protecting such important human heritage. The lack of seismic records and detailed information at the site makes the challenge even more difficult. Moreover, the inherent randomness of the seismic action and the epistemic and aleatoric uncertainties involved in the study suggests adopting a suitable stochastic approach to study the seismic response of the Zoser pyramid. To date, the unique universal method of seismic analysis able to accurately assess the probability of failure of such monumental structures is the Monte Carlo Simulation method. Assuming that a reliable numerical model is defined for the structure/site to be investigated, the crucial point for applying the Monte Carlo Simulation technique is the generation of appropriate artificial earthquake accelerograms. After the 1992 Cairo earthquake, that heavily damaged the Zoser pyramid, few works have been proposed to simulate ground motion time histories for specific seismological parameters. Building upon those preliminary studies, in this paper, a ground motion model with random parameters able to mimic the variability of seismic activity at the Zoser pyramid, is proposed. The model combines seismological models to determine the Power Spectral Density function at the bedrock underneath the Zoser pyramid already adopted in previous studies, the unidirectional wave propagation through the soil stratum up to the base of the pyramid and a spatial variability model able to account for incoherency effect of the ground motion at the base of the pyramid. The distributions of the random variables embedded in the model are determined from statistical analyses of the available data. It is noted the derived stochastic ground motion model with random parameters will be able to take simulate ground motion time histories accounting for the natural variability of the seismic activity at the Zoser pyramid. Particular cases of the model include the simulations undertaken in the previous studies. Moreover, the inherent nonstationarity of the frequency content and the lack of coherency of the ground motion at the base of the pyramid are also considered in the model through a frequency-dependent transfer function of the soil stratum and the application of the spectral representation of vector processes method. A Monte Carlo study is finally performed and relevant results are presented in the paper highlighting the impact of the parameters involved in the ground motion model.

Keywords:Ground motion model, fully non-stationary stochastic model, natural variability, Saqqara, Zoser Pyramid.

1. Introduction

Ground motion modelling in the greater Cairo Area and more specifically at Saqqara, in the proximity of the Zoser pyramid, is addressed. Egypt has an extensive cultural heritage reaching back more than five thousand years. The archaeological site of Saqqara, located at about 20 km from Cairo city, is considered the world's most extensive burial ground with monuments of almost every period of ancient Egyptian civilization. In this area is situated the Step Pyramid of Zoser (or Djoser). An important deterioration process that has been recently observed is attributable to seismic events that occurred in the area from the late 1980s, with two main events, the 1992 Cairo earthquake and the 1995 Gulf of Aqaba earthquake that severely damaged the pyramid, which needed an important restoration to avoid the risk of collapse [1]. To assess the seismic response of the pyramid during an earthquake, after a reliable finite element model is determined, it is necessary to perform specific time histories analysis using either recorded or simulated time histories. In this regard, the lack of time histories recorded in the area compelled the seismic study towards the adoption of models consistent with the Saqqara site and the Greater Cairo seismicity. In this regard, there is a substantial body of literature produced after the Cairo 1992 earthquake to determine the source parameters of the earthquakes in Egypt. Specifically, seismic hazard studies in Egypt has been conducted in Reference [2]. Seismic hazard assessment at sphinx has been addressed in Reference [3]. Studies on the seismicity and focal mechanisms of earthquakes in Egypt has been conducted in references [4-7]. In Reference [7] interestingly, seismogenic zones in the Greater Cairo Area have been defined. The authors reported also in their study focal mechanism parameters and basic information to calibrate seismological models for the simulation of ground motion time histories. On the other hand, very few works have been proposed to simulate relevant ground motion time-histories in Egypt. Specifically, in Reference [8], earthquake ground motion simulation at the Zoser pyramid has been addressed through a stochastic approach. The authors, applied a seismological model (Hanks and McGuire [9], Boore [10]) to simulate three specific earthquakes (including the Cairo 1992 earthquake) at the Zoser pyramid. Omar et al [11], adopted the Boore method [10] to simulate the 1992 Cairo Earthquake in the urban area in the north of Cairo. It is noted that the above contributions simulated individual time-histories that are uniformly modulated and generated by an assigned set of seismological parameters. To capture the natural variability of ground motion at a site, few contributions have been published in the past. Just to cite a few, [12] proposed an evolutionary power spectral density function for the K-Net Japanese database, possessing random variables determined through empirical attenuation equations; Rezaeian and Der Kiureghian [13], proposed a fully nonstationary stochastic model with separable temporal and spectral nonstationary characteristics fitted to target ground motions by matching a set of statistical characteristics; Cacciola and Zentner [14] proposed a methodology for simulating artificial earthquake accelerograms matching mean and mean $\pm$ standard deviation response spectra, given either by attenuation relationships or determined by a selected strong-motion database; Vlachos et al. [15] developed a predictive stochastic model based on a bimodal parametric nonstationary Kanai-Tajimi ground motion model and regression relations that inputs a given earthquake scenario description and outputs seismic ground acceleration time histories at a site of interest. A stochastic ground-motion model that allows simulation of nonstationary non-Gaussian time histories that are statistically consistent with data even at sites with very low numbers of records has been proposed by Radu and Grigoriou [16].

Bearing in mind the necessity to simulate ground motion acceleration in the proximity of the Zoser pyramid and the lack of ground motion recorded in the area, in this paper a versatile ground motion model able to be tuned to mimic the variability of seismic activity at the Zoser pyramid is proposed. The model combines i) the seismological model of Boore [10] to determine the Power Spectral Density function at the bedrock underneath the Zoser pyramid, ii) the one-dimensional vertical wave propagation from the bedrock to the base of the pyramid, and iii) a spatial variability model able to model incoherency effect of the ground motion at the base of the pyramid (see e.g. Deodatis [17], Zerva [18]). The model includes several parameters inherently aleatoric or epistemic, leading to the definition of a Power Spectral Density matrix with random parameters. Model parameters (such as the Magnitude, the epicentral distance, the stress drop) are treated as random variables with probability distribution functions, determined from the seismological data available in the literature. The model is versatile and can be further calibrated using additional data at the site. After the ground motion model

is derived, a Monte Carlo study on a numerical finite element model of the Zoser Pyramid is also performed highlighting the impact of the parameters involved in the proposed model.

2. Stochastic Seismological model at the Zoser Pyramid

In this section, following the stochastic seismological method proposed by Boore [10], the ground motion acceleration at a point on the surface is considered as a realization of a zero-mean Gaussian process, fully defined by the unilateral power spectral density function $G_{\ddot{u}_g}(\boldsymbol{\alpha}, \omega, t)$, herein written in the following form:

$$G_{\ddot{u}_g}(\boldsymbol{\alpha}, \omega, t) = \varphi^2(\boldsymbol{\alpha}, t)(H_P(\boldsymbol{\alpha}, \omega)H_{soil}(\boldsymbol{\alpha}, \omega, t))^2 G_S(\boldsymbol{\alpha}, \omega) \tag{1}$$

where $G_S(\boldsymbol{\alpha}, \omega)$ is the power spectral density function at the source, $H_P(\boldsymbol{\alpha}, \omega)$ is a dimensionless function governing the path of the seismic waves between the source and the bedrock underneath the point at the surface, $H_{soil}(\boldsymbol{\alpha}, \omega, t)$ is the soil amplification function between the bedrock and the surface and $\varphi(\boldsymbol{\alpha}, t)$ is a modulating function governing the time evolution of the ground motion intensity and its duration. Moreover, $\boldsymbol{\alpha}$ is the vector listing the seismological and geotechnical parameters defining the model, such as the seismic moment, the epicentral distance, depth, and the stress-drop. The size of the vector $\boldsymbol{\alpha}$ depends on the number of parameters necessary to calibrate the model. There are various source spectrum models reported in Boore [10] that can be adapted to various scenarios. In this paper, the ω-square model (Aki, [19]), already adopted by various authors (see e.g. [8], [11]) to model the ground motion in Egypt, is adopted. The related unilateral power spectral density function in terms of accelerations is given by:

$$G_S(\boldsymbol{\alpha}, \omega) = 2\,(cM_0)^2 \frac{\omega^4}{\left(1+\left(\frac{\omega}{\omega_c}\right)^2\right)^2} \tag{2}$$

where the constant c is given by

$$c = \frac{r_{\vartheta\varphi} VF}{4\pi\rho V_s^3} \tag{3}$$

in which $r_{\vartheta\varphi}$ is the radiation pattern, V represents the partition of total shear-wave energy into the horizontal components (=0.707), F is the effect of the free surface (=2), ρ is the soil density, Vs is the shear wave velocity in the vicinity of the source and ω_c is the corner frequency given by (Brune [20]):

$$\omega_c(\boldsymbol{\alpha}) = 2\pi\left(4.9\ 10^6\ V_s\ \left(\frac{\Delta\sigma}{M_0}\right)^{\frac{1}{3}}\right) \tag{4}$$

where M_0 is the seismic moment (dyne-cm) and $\Delta\sigma$ is the stress-drop (bars). The path transfer function spectrum $H_P(\boldsymbol{\alpha}, \omega)$ is given by :

$$H_P(\boldsymbol{\alpha}, \omega) = Z(R)exp\left(-\frac{\omega R}{2V_s Q(\omega)}\right)\left(1+\left(\frac{\omega}{\omega_m}\right)^8\right)^{-1/2} \tag{5}$$

where R is the closest distance between the structure and the rupture surface and ω_m is the cut-off frequency (assumed equal to 12Hz [3]). $Z(R)$ is the geometrical spreading function given by a piecewise continuous function (see Boore [10]), and $Q(\omega)$ is the attenuation function. El-Aziz Khairy and El-Aal [7] determined the

values of the function $Q(\omega)$ along with the pertinent parameters in Eq. (5) for three seismogenic zones in and around the Greater Cairo Area. The following quantities have been used in the paper:

$$Q(\omega) = 85.68\left(\frac{\omega}{2\pi}\right)^{0.79} \quad , \tag{6}$$

shear wave velocity, Vs=3.7km/s, and the soil density, ρ =2.7g/cm^3, in vicinity of the source and radiation pattern, $r_{\vartheta\varphi}$ =0.52, while the geometric spreading has been assumed equal $Z(R) = 1/R$. Finally, the distance R, is given by $R = \sqrt{D^2 + h^2}$, where D is the closest distance between to the vertical projection of the rupture surface on the ground surface and h is the pseudo depth (assumed coincident with the average focal depth 23km in the area [3]). The soil amplification function $H_{soil}(\boldsymbol{\alpha}, \omega, t)$ is herein determined from the one-dimensional vertical wave propagation from the bedrock. In the case of a single homogeneous stratum and rigid bedrock can be written in the following form (see e.g. Kramer [21]):

$$H_{soil}(\alpha, \omega, t) = \frac{1}{\sqrt{\cos^2\left(\frac{\pi\,\omega}{2\,\omega_g(t)}\right) + \left(\frac{\pi\,\zeta_g(t)\omega}{2\,\omega_g(t)}\right)^2}} \tag{7}$$

where and $\zeta_g(t)$ is the damping ratio of the ground and $\omega_g(t)$ is the soil fundamental frequency that is defined in this paper as a time-dependent function. The use of time-varying soil fundamental frequency and damping in lieu of constant ones has been adopted in the past by several authors (see e.g. Ahmadi and Fan [22]; Deodatis, [17]) to capture the typical frequency non-stationarity of natural earthquakes and embedded in the well-know Kanai-Tajimi and Clough-Penzien power spectral density functions. It is noted that the Boore [10] stochastic method is generally used to model stationary or quasi-stationary (uniformly modulated) ground motion processes. The extension of the Boore's model [10] to account for the soil transfer function along with the time-varying soil fundamental frequency, leads to a fully nonstationary (with both frequency and intensity varying with respect to time) ground motion model, so enhancing the versatility of the method. Furthermore, in the case of large structures, such as the Zoser pyramid, it might be necessary to account for the wave propagation and asynchronous motion at relative short (about 100 meters) distances. The effect of the ground motion spatial variability will be considered in the next section by encompassing the Boore method in the spectral representation method for vector processes proposed in Deodatis [17].

3. Ground motion spatial variability

Let consider now the ground motion time-history at m selected points on the ground surface (i.e. at the base of the Zoser pyramid) as a realization of a zero-mean Gaussian fully non-stationary vector process defined by the evolutionary power spectral density matrix

$$\mathrm{G}_{\ddot{u}_g}(\omega, t; \boldsymbol{\alpha}) = \begin{bmatrix} G_{\ddot{u}_g}^{(1,1)}(\omega, t; \boldsymbol{\alpha}) & \cdots & G_{\ddot{u}_g}^{(1,m)}(\omega, t; \boldsymbol{\alpha}) \\ \vdots & \ddots & \vdots \\ G_{\ddot{u}_g}^{(m,1)}(\omega, t; \boldsymbol{\alpha}) & \cdots & G_{\ddot{u}_g}^{(m,m)}(\omega, t; \boldsymbol{\alpha}) \end{bmatrix} \tag{8}$$

where $G_{\ddot{u}_g}^{(j,j)}(\omega, t; \boldsymbol{\alpha})$, $(j = 1,2,\ldots,m)$ is the unilateral power spectral density function at the j-th point at the surface defined in Eq. (1), while the off-diagonal cross-spectral density functions are given by

$$G_{\ddot{u}_g}^{(j,k)}(\boldsymbol{\alpha}, \omega, t) = \sqrt{G_{\ddot{u}_g}^{(j,j)}(\boldsymbol{\alpha}, \omega, t) G_{\ddot{u}_g}^{(k,k)}(\boldsymbol{\alpha}, \omega, t)}\,\Gamma_{jk}(\boldsymbol{\alpha}, \omega), \qquad j, k = 1,2,\ldots,m; \quad j \neq k \tag{9}$$

$\Gamma_{jk}(\boldsymbol{\alpha}, \omega)$, $(j, k = 1,2,\ldots,m; \quad j \neq k)$ is the complex coherence function between $\ddot{u}_g^{(j)}(t)$ and $\ddot{u}_g^{(k)}(t)$ and can be expressed in the following form

$$\Gamma_{jk}(\boldsymbol{\alpha}, \omega) = \left|\gamma_{jk}(\boldsymbol{\alpha}, \omega)\right| \exp[\, i\theta_{jk}(\boldsymbol{\alpha}, \omega)] \tag{10}$$

in which $|\gamma_{jk}(\boldsymbol{\alpha},\omega)|$ is the incoherence effect and $\exp[\,i\theta_{jk}(\boldsymbol{\alpha},\omega)]$ is the imaginary term describing the site response and the wave passage effect. For a comprehensive description of the ground motion spatial variability, the readers can refer to the monograph by Zerva [18]. The coherency functions are generally determined from dense arrays of closely spaced seismographs. Due to the absence of this information for the Saqqara site, either empirical parametric or analytical models can be adopted. Coherency models at rock sites are limited (Zerva [18]) as the majority of the models were developed for alluvial sites. In the following, the model proposed by Menke et al. [23] will be adopted, specifically:

$$|\gamma_{jk}(\boldsymbol{\alpha},\omega)| = |\gamma_{jk}(\beta,\omega)| = \exp[-\frac{\omega}{2\,\pi}\,\beta\;\,d_{jk}] \tag{11}$$

where β is a parameter ranging between $(0.4-0.7)\times 10^{-3} sec/m$ and d_{jk} is the separation distance between two points j and k projected along the direction of propagation of the seismic wave. Furthermore, under the hypothesis of homogeneous soil, the simplest approximation of the wave passage effect is given by the following equation

$$\theta_{jk}(\boldsymbol{\alpha},\omega) = \theta_{jk}(V_{app},\omega) = -\frac{\omega d_{jk}}{V_{app}} \tag{12}$$

where V_{app} is the apparent propagation velocity of the plane wave [18].

4. Simulation of fully non-stationary ground motion vector processes at the Zoser pyramid

The simulation of ground motion time histories at the Zoser pyramid consistent with the seismicity of the site is addressed in this section. Specifically, the elements of the vector $\boldsymbol{\alpha}$ are considered random variables fully defined by the joint probability density function $p_{\boldsymbol{A}}(\boldsymbol{\alpha})$, $\boldsymbol{A}$ being the domain of the vector of random variables $\boldsymbol{\alpha}$. In this paper the distance R, the angle of incidence δ, the seismic moment M_0, the stress drop $\Delta\sigma$ and the parameter β governing the coherency of the spatial variable model are assumed independent random variables whose distribution is determined from the database available in the literature (see e.g. [7]). The joint probability density function $p_{\boldsymbol{A}}(\boldsymbol{\alpha})$ can be, therefore, written in the following form

$$p_{\boldsymbol{A}}(\boldsymbol{\alpha}) = p_M(M_0)p_R(R)p_S(\Delta\sigma)p_\Delta(\delta) \tag{13}$$

It is noted that additional parameters can be also those parameters can be also added to the model enhancing its capability to better represent local seismicity. Once established the probabilistic nature of the parameters involved in the model, following the procedure proposed by Deodatis [7] for the generation of the random vector $\boldsymbol{\alpha}$, the simulation of non-stationary ground motion realization $\ddot{u}_g^{(j)}(t)$ at selected points $(j = 1,2,\ldots,m)$ is pursued through the application of the spectral representation method. The evolutionary cross-spectral density matrix $\mathbf{G}_{\ddot{u}_g}(\omega,t;\boldsymbol{\alpha})$, defining the ground motion vector process is first decomposed at every time instant t through the Cholesky's method into the following product:

$$\mathbf{G}_{\ddot{u}_g}(\omega,t;\boldsymbol{\alpha}) = \mathbf{H}(\omega,t;\boldsymbol{\alpha})\mathbf{H}^T(\omega,t;\boldsymbol{\alpha}) \tag{14}$$

where $\mathbf{H}(\omega,t;\boldsymbol{\alpha})$ is a lower triangular matrix given by

$$\mathbf{H}(\omega,t) = \begin{bmatrix} H_{11}(\omega,t) & 0 & \cdots & 0 \\ H_{21}(\omega,t) & H_{22}(\omega,t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{m1}(\omega,t) & H_{m2}(\omega,t) & \cdots & H_{mm}(\omega,t) \end{bmatrix}. \tag{15}$$

and the superscript T denotes the transpose conjugate of a matrix. Once the cross-spectral density matrix $\mathbf{G}_{\ddot{u}_g}(\omega, t; \boldsymbol{\alpha})$ is decomposed according to Eqs. (14) and (15), a realization $(r) of$ non-stationary ground motion vector process $\ddot{u}_g^{(j,r)}(t)$ at the point, $j = 1,2,\ldots,m$ on the surface (or at the base of the Zoser pyramid) can be simulated by the following series as $N \to \infty$

$$\ddot{u}_g^{(j,r)}(t) = \sum_{k=1}^{m}\sum_{s=1}^{N}\left|H_{jk}(\omega_s, t; \boldsymbol{\alpha})\right|\sqrt{2\,\Delta\omega}\,cos\left[\omega_s t - \vartheta_{jk}(\omega_s, t; \boldsymbol{\alpha}) + \varphi_{ks}^{(r)}\right], j = 1,2,\ldots,m \tag{16}$$

where:

$$\vartheta_{jk}(\omega, t; \boldsymbol{\alpha}) = tan^{-1}\left(\frac{Im[H_{jk}(\omega,t;\boldsymbol{\alpha})]}{Re[H_{jk}(\omega,t;\boldsymbol{\alpha})]}\right) \tag{17}$$

with $Im[\quad]$ and $Re[\quad]$ denoting the imaginary and real part of a complex number, respectively and $\varphi_{ks}^{(r)}$ $(k = 1,2,\ldots,m;\quad s = 1,2,\ldots,N)$ are m sequences of N independent random phase angles distributed uniformly over the interval $[0,2\pi]$. It is noted that the generation of ground motion time histories at a single point (i.e. ignoring the ground motion spatial variability) reduces to

$$\begin{aligned}\ddot{u}_g^{(r)}(t) &= \sum_{s=1}^{N}\left|H_{11}(\omega_s, t; \boldsymbol{\alpha})\right|\sqrt{2\,\Delta\omega}\,cos\left[\omega_s t + \varphi_{1s}^{(r)}\right] \\ &= \sum_{s=1}^{N}\sqrt{2\,\mathrm{G}_{\ddot{u}_g}(\omega_s, t; \boldsymbol{\alpha})\Delta\omega}\,cos\left[\omega_s t + \varphi_{1s}^{(r)}\right]\end{aligned} \tag{18}$$

Eq. (18) can be used under the hypothesis of uniform ground motion at the pyramid base.

5. Simplified FE model of Zoser pyramid

The Zoser pyramid consists of six steps of limestone and clay with a height of 62 meters and a base with an area of about 109 meters x 125 meters, that was built about 4600 years ago (see e.g. ref [24] for additional information). Surveying with a 3D scanner has been carried on to determine the geometry of the Zoser pyramid. Soil investigations have been also conducted at the site. Specifically, samples from the stones inside the site were tested in the laboratory for determining the properties of the materials used in building the pyramid. Moreover, a borehole with a depth of 10 meters was dug to determine the soil properties in Saqqara. The total borehole was limestone or sandy limestone interspersed with layers of silt at the depth of the borehole. No groundwater has been observed during the borehole. Relevant mechanical parameters are reported in Table 1. A simplified model has been developed in SAP2000.

Soil			Pyramid		
$\rho_s\ (kg/m^3)$	$E_s\ (kPa)$	υ_s	$\rho_p\ (kg/m^3)$	$E_p\ (kPa)$	υ_p
2200	45x10^6	0.25	2900	45x10^6	0.25

Table 1. Material properties for Soil and Pyramid

Figure 1 shows a sketch of the model with the relevant geometrical measures adopted. Numerical Finite Element models of the Zoser Pyramid has been created through the software SAP2000 by using solid elements. The model is considered fully fixed at the base.

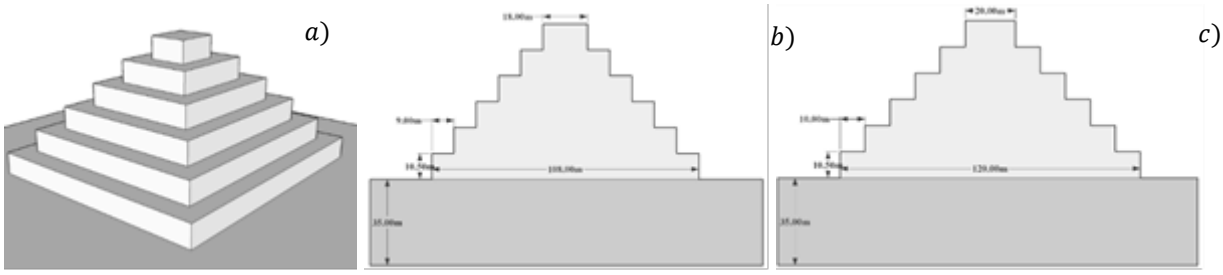

Figure 1. Simplified model Zoser Pyramid, (a) 3D view, (b) North and c)West sections of the stepped Pyramid

A modal analysis is first carried out for the fixed-base Pyramid model: the first mode, associated with the smallest width of the pyramid (Figure2a) has a natural frequency of 14.6 Hz, while the second mode, associated with the largest width (Figure2b) is characterized by the natural frequency of 14.95 Hz while the third mode is torsional (Figure2c) with frequency 20.11Hz. It is noted that those frequencies are derived from the simplified model and not validated (or calibrated) with dynamic tests on the pyramid.

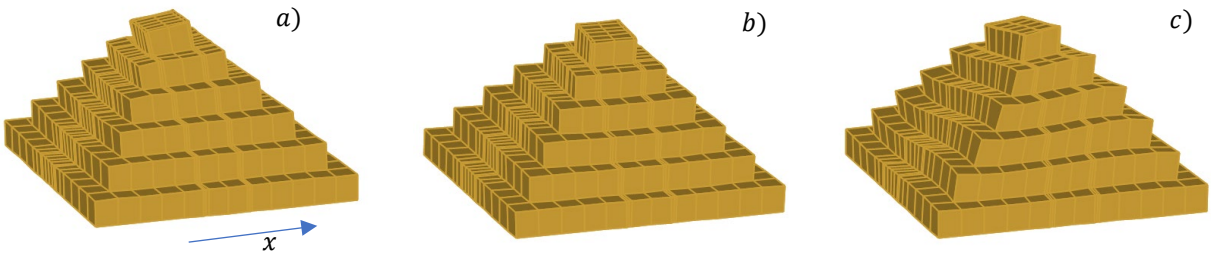

Figure 2. First three mode shapes of the Zoser pyramid FE model

It has to be emphasized that soil structure interaction effects are not considered in the present study. A reduction of the frequencies is however expected due to the similar characteristics of the soil and structure materials. Once established a reliable model of the pyramid the pertinent ground motion model is defined to undertake the seismic analysis of the Zoser pyramid.

6. Numerical Results

A preliminary study on the seismic activity in Saqqara relevant to the Zoser pyramid has been undertaken. Using the data reported in Reference [7] the following map (Figure 3) of the earthquakes that occurred in the effective seismogenic zone in and around the Cairo Area has been developed. From the distribution of the epicentres (and the distance from the Zoser pyramid), it will be clearly expected a large variability of ground motion excitation at the base of the pyramid.

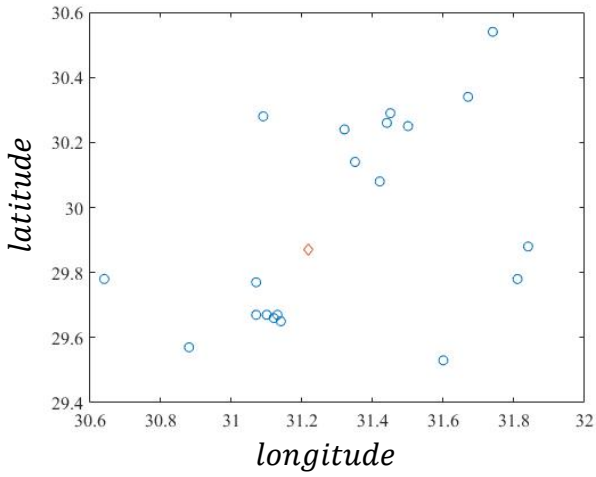

Figure 3. Location of the earthquake epicentres (circles) with magnitude $M \geq 3$ after [7] which occurred in the effective zones in and around the Greater Cairo Area and location of the Zoser pyramid (diamond).

The proposed ground motion model aims to capture such variability using available data to calibrate the probability distributions of the relevant parameters. From the analysis of Figure 3 it is evident that the distance and the angle of incidence with respect to the Zoser pyramid need to be considered as a random quantity along with the Magnitude and the stress drops. A preliminary study considering the data available [7] led to the distributions of the seismological data given in Figure 4.

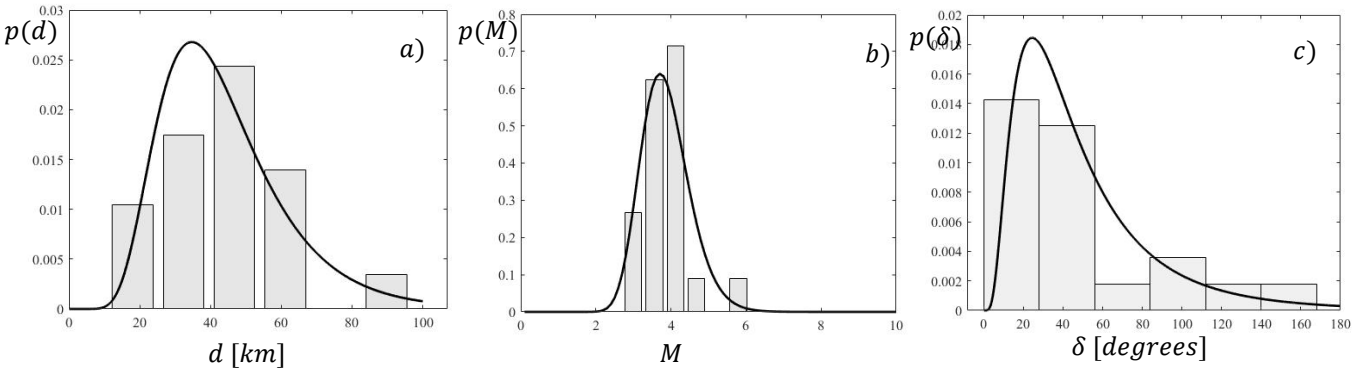

Figure 4. Comparison between the distribution (bars) and fitted probability density functions (solid line) for the seismological parameters distance, magnitude ($M \geq 3$) and angle of incidence in and around the Greater Cairo: a) distance; b) Magnitude and c) angle of incidence (+90 degrees).

The variables are assumed lognormal distributed. The distances are determined between the coordinates of the epicentre of the past earthquakes and the centre of the Zoser pyramid using an equirectangular approximation. Furthermore, the incidence angle is determined with respect to the determined E-W side of the pyramid. The stress drop Δσ, due to the lack of data, is taken as uniformly distributed between 15-50bars. Clearly more refined distributions can be used in the model for all those parameters including statistical correlations. Once established the probability distribution of the seismological model, the vertical propagation is addressed. The position of the bedrock has been determined from previous studies available in the literature [25], [26] and it is considered to be located at 35m depth. The soil stratum between the bedrock and the base of the pyramid is assumed homogeneous considering the site investigation undertaken in the first 10m. In absence of specific records to calibrate frequency nonstationary the soil fundamental frequency governing in Eq. (7) is considered varying linearly with respect to time.

$$\omega_g(t,\boldsymbol{\alpha}) = \omega_g(0) - 0.1\omega_g(0)\frac{t}{t_f(\boldsymbol{\alpha})} \tag{19}$$

where

$$\omega_g(0) = \frac{\pi Vs}{2h_s} \tag{20}$$

is the constant fundamental frequency determined from the linear theory, h_s=35m the depth of the soil stratum and $Vs = 2860\ m/sec$ the velocity of the shear waves and damping has been assumed constant with a value $\zeta_g = 0.6$. In equation (19) $t_f(\boldsymbol{\alpha})$ is the duration of the seismic event that is also a random variable. In this regard the distance-dependent duration proposed by Atkinson [28] and adopted in Reference [11] for Egypt is considered in the following analyses, that is:

$$t_f(\boldsymbol{\alpha}) = t_f(\Delta\sigma, M_0, R) = \frac{1}{2\pi\omega_c(\Delta\sigma, M_0)} + 0.05R \tag{21}$$

with $\omega_c(\Delta\sigma, M_0)$ the corner frequency defined in Eq. (4) and M_0 the seismic moment related to the moment magnitude M through the following equation (see Boore [10]]

$$M = \frac{2}{3}\log_{10}(M_0) - 10.7 \tag{22}$$

The single ground motion component simulated from Eq. (18) is determined first and used for the preliminary seismic analysis of the pyramid. The adopted modulating function $\varphi^2(\boldsymbol{\alpha}, t)$ is defined in [10]. Randomly selected trajectories simulated through the proposed approach are reported in Figure 5

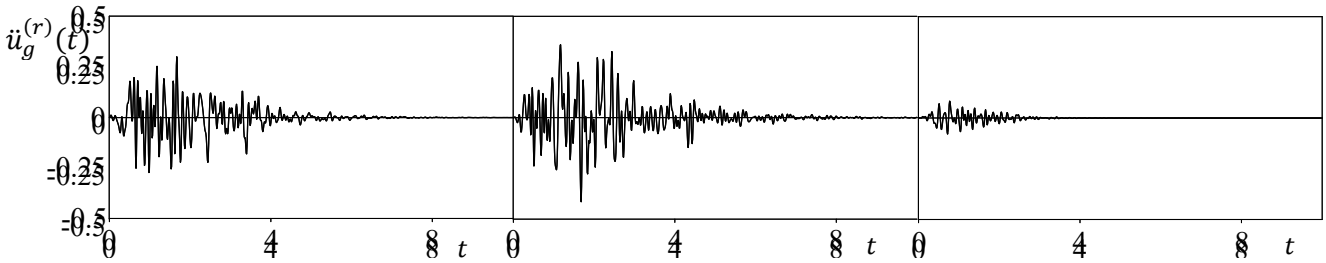

Figure 5. Randomly selected realizations of ground motion simulated at the base of the Zoser pyramid.

From the figure, it can be observed the large variability in terms of peak ground acceleration and duration. The distribution of those quantity determined by a generation of 500 samples is depicted in Figure 6.

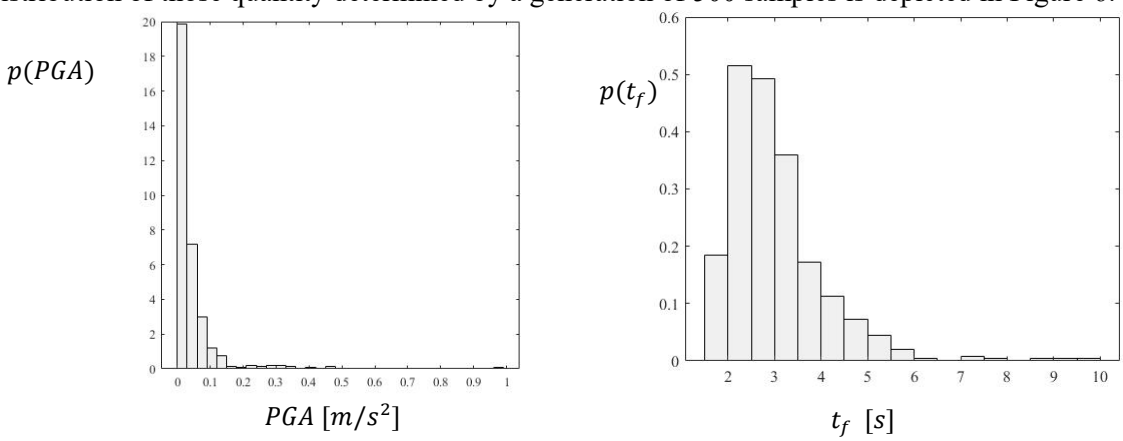

Figure 6. PGA (a) and duration (b) distributions at the base of the Zoser pyramid

The PGAs are in general relatively small according to the parameters adopted and in line with the results presented in [8] and [11]. Interestingly, from the exams of Figure 6a and the distributions in Figure 4, the parameters relevant to the Cairo Earthquake 1992 (M=5.8 and distance from the Zoser pyramid 14km [8]) fall in the tails of the distributions. That combination of data leads to the higher PGAs as simulated in [8]. The Monte Carlo study of the Zoser pyramid to 500 samples of ground motion generated with the proposed model has been then undertaken in SAP2000. It has to be emphasized that the random incidence angles are generated using the distribution determined in Figure 4c. Peak Von Mises stress distribution pertinent to the randomly selected time histories is depicted in Figure 7

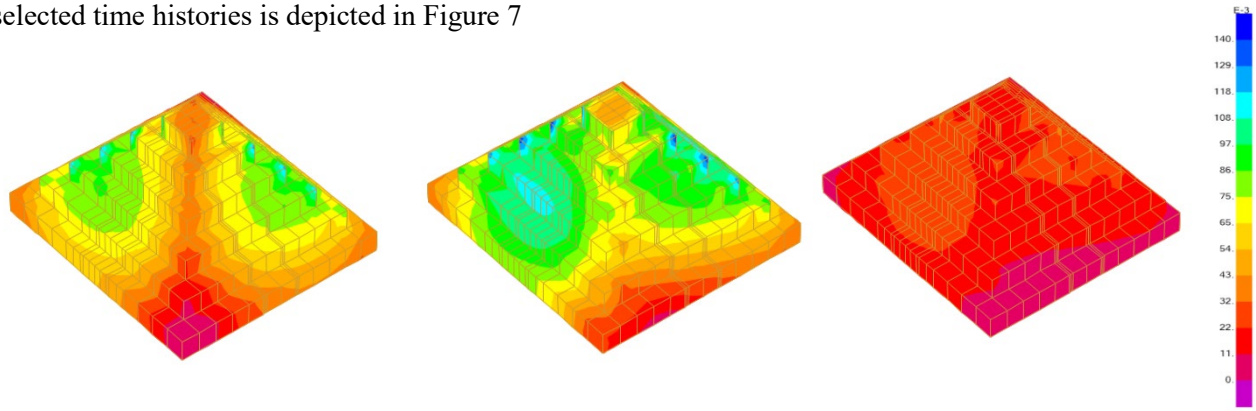

Figure 7. Peak Von Mises stress (MPa) for three selected randomly generated time histories.

Due to the different directionality of the input, the portion of the pyramid manifesting the highest stress are in general different, however (neglecting stress concentrations in the corners) the second and third steps appear to be the zone with higher stresses. Interesting those areas are among the ones that have been damaged by the Cairo earthquake. The distribution of the accelerations at the top of the pyramid in two orthogonal directions is also reported in Figure 8. The distributions are similar due to the distribution of the angles of incidence Figure 4c (angle to be rotated -90 degrees) and also evident from the epicentre distributions with respect to the Zoser pyramid depicted in Figure 3.

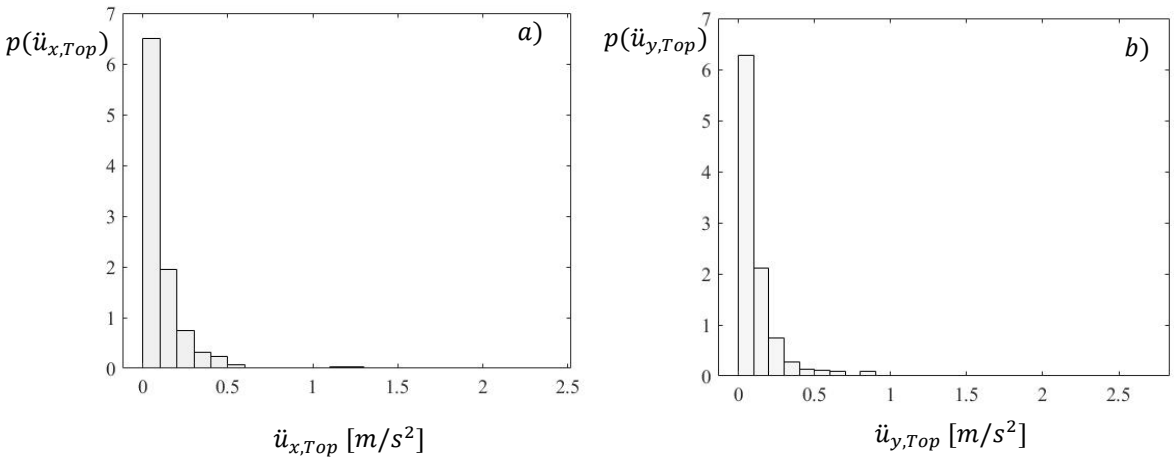

Figure 8. Distribution of the absolute response acceleration at the top of the Zoser pyramid, a) N-S direction; b) E-W, direction

Finally, the samples of the ground motion vector process have been generated through Eq. (16). The direction of wave propagation has been assumed the x-direction (See Figure 2a), i.e the direction of the first mode shape. The simulation of ground motion time histories can be quite computational demanding if pursued in several points due to the double summation in Eq. (16). To reduce the computational effort the ground motion is considered identical for a length of about 30meters, so to subdivide the total length (108m) of the base of the

pyramid in the direction of the seismic wave in three segments. The ground motion has been then generated at 3 different points distant respectively $d_{21} = d_{32} = 54m$; $d_{31} = 108m$. Figure 9a shows a realization of the vector process at the three different points.

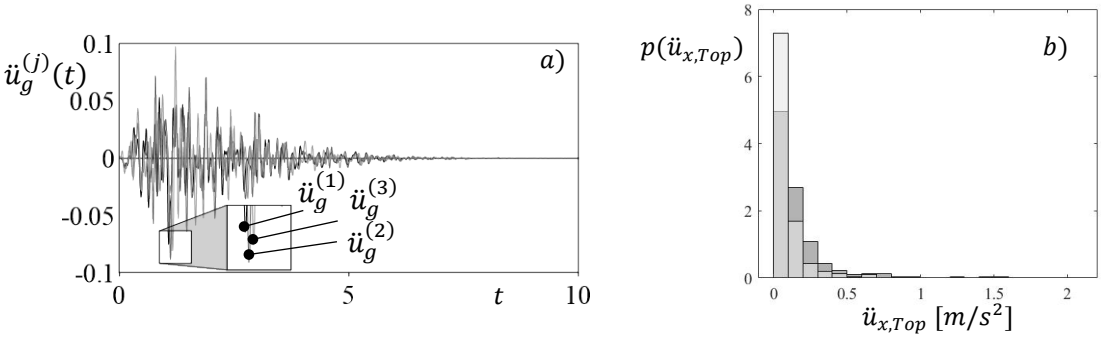

Figure 9. a) Sample of the ground motion vector process including spatial variability [m/s2]; b) Distribution of the absolute response acceleration at the top of the Zoser pyramid for uniform ground motion (dark grey) and considering the spatial variability (light grey).

Figure 9b shows the comparison of the distribution of peak absolute accelerations at the top of the Zoser pyramid in the direction of the input considering 500 samples for the case of uniform base acceleration (i.e. only the first component) and considering the three different components. From the figure it can be observed a reduced mean acceleration in the case of ground motion including the spatial variability in comparison with the uniform case. Differences in terms of peak stress distributions changes from element to element, but not significant discrepancies have been observed in this present study.

7. Concluding Remarks

In this paper ground motion model able to capture the natural variability of the seismic activity at the Zoser pyramid has been proposed. The proposed model combines a seismological model to characterize the ground motion at the bedrock underneath the Zoser pyramid, the vertical wave propagation and the ground motion spatial variability to account for frequency nonstationarity and the ground motion spatial variability at the base pyramid. The model considers relevant seismological parameters as random variables whose distribution is determined from relevant databases. Specifically, the epicentral distance, the magnitude, the stress-drop, the incidence angle are considered random variables as well as the parameter governing the coherency. Pertinent distributions have been determined using available data reported in the literature. The model, therefore, is able to capture both natural and spatial variability of ground motion accelerations in the Saqqara area and encompass as a particular case the 1992 Cairo earthquake that heavily damaged the Zoser pyramid. Despite the limitation of the model related to a so complex phenomenon the approach proposed led to results in agreement with published literature and has the potentiality to be continuously updated/validated with local registrations and/or experimental data.

5. Acknowledgements

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