LOAD CONDITION IN RESPONSE OF SDOF SYSTEM

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Abstract

Seismic load considered in structure design is treated as a load/force in a very literal sense and it is treated as same as for dead load by gravity. The load is applied to the structure and the stress by the force is evaluated by comparing with allowable stress. The load is always treated as a force in the structure design. The authors experienced several piping experiments and had doubted whether a seismic load is a force or not. Piping is a kind of flexible structure and the response is likely to be resonated by an earthquake. However, the collapse of piping had never been appeared even by very high-level excitation beyond the design limit for high-level seismic events, although the plastic deformation had appeared enough. The design code for pressure vessel was dividing load type into two categories that were force-controlled and displacement-controlled loads. An inertia force by seismic response is classified in force-controlled load and anchor motions by a seismic input are classified in displacement-controlled load and, generally, they are also called primary and secondary loads. Moreover, force-controlled load/primary load can generate a collapse of structure but displacement-controlled load/ secondary load never does because the structure can deform in a finite displacement. Although seismic load had been being regarded as force-controlled load/primary load for a long time, the design code for piping, ASME Boiler and Pressure Vessel Code Section III, Div.1, NB-3600, incorporated a concept of “Reversing and Non-reversing Dynamic Load” in 1994. The seismic load is classified in reversing dynamic load that is not a pure force-controlled load.

To clarify and understand the behavior of the load called reversing dynamic load, harmonic vibration of single degree of freedom (SDOF) systems are investigated. An equation of motion of the SDOF system consists of inertia force term, damping and spring force terms and external force term, and the total of damping and spring forces can be treated as an internal or elemental force of the system. These force terms are investigated by relating to the deformation of the system. The response can change with the relation of the vibrational properties of the system and an input wave. Three kinds of SDOF systems having the different relations of those are selected. One is a typical resonant condition and the others are rigid and soft conditions that the natural frequencies are respectively higher and lower enough than the frequency of input sinusoidal wave. It is confirmed that the inertia force term in an equation of motion always acts in the reverse direction to the deformation. This means that the inertia force term is generated by the deformation like spring back force and it could be classified as a secondary load. The external force acts in the same direction as the deformation in the rigid condition and it behaves as a primary load. But, in the soft condition, the external force acts in the reverse direction to the deformation and both of the inertia and external forces can be classified as secondary load. The authors try to express the ratio of primary load to the total load contributing to the deformation of the system.

Keywords: Reversing dynamic load, Force-controlled load, Displacement-controlled load, Collapse, Load classification.

1. Introduction

To clarify the failure mode and ultimate strength of piping under seismic excitation, many experiments using a shaking table have been performed by National research Institute Earth science and Disaster resilience (NIED) [1, 2]. The experiments resulted in that the failure mode of piping was the fatigue but not the plastic collapse. Moreover, it was confirmed that there was a large gap between the experimental results and the allowable limit of the design code.

The criterion of the current seismic design code [3, 4] for piping is mainly based on primary stress evaluation and fatigue evaluation. The primary stress is calculated from the force-controlled load called as the primary load. The fatigue is evaluated by the alternating stress calculated from both primary load and the displacement-controlled load called as the secondary load.

From the experimental results, the fatigue is most likely to be as the failure mode of piping under seismic excitation. A load of seismic response categorized as the primary load was generated the fatigue failure but never the plastic collapse. Therefore, it has been considered that the seismic load must not be a pure primary load and a new category called “reversing dynamic load” has been created. The issue of whether the seismic load is the primary load or not and whether the collapse can occur or not in the piping system has not been resolved yet.

The authors calculate the response of a single degree of freedom (SDOF) system excited by several sinusoidal waves. The inertia force term, the element force term including damping and spring forces and the external force term are calculated and the relations with the deformation are analyzed. As a result, the characteristics of the reversing dynamic load can be clarified and interpreted.

2. Plastic collapse and load classification

The basic concept of load classification developed by the design code of Pressure Vessel [3] is rational and practical concept so that design loads are classified by considering the influence on each failure mode and the limitation for the classified design load is provided. The design loads are classified into two kinds of load types that are force-controlled load and displacement-controlled load.

Fig. 1 and 2 show the schematic explanation of the two kinds of load. And the inertia force by the seismic response of structures is classified as force-controlled load. Fig. 1 shows the bending behavior of cantilever by a weight. The deformation will become unstably large and the plastic collapse will occur when the weight is larger than the ultimate strength of the cantilever. Fig. 2 also shows the bending behavior of cantilever but the bending is generated by a jack extension. In this case, the deformation is following the jack motion and stopping at the position of the edge of the jack. The motion of cantilever is stably controlled in a finite deformation. The plastic collapse can be generated just by the force-controlled load. For the displacement-controlled load, just a deformation can be generated and the unstable failure cannot be generated if the structure has enough ductility and the deformation stays within the capacity of it.

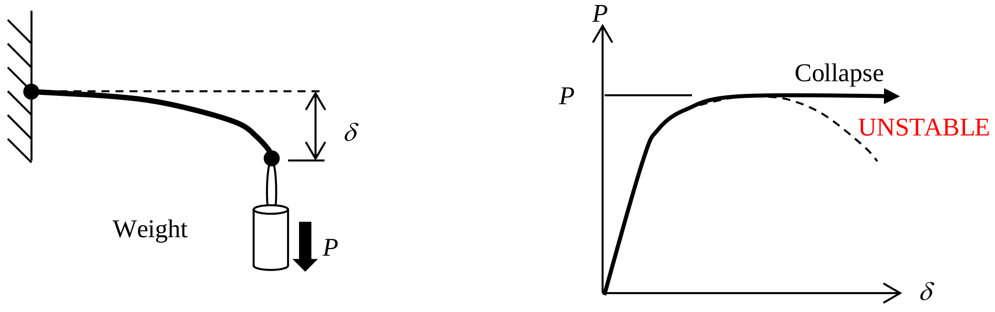


Fig. 1 – Force-controlled load can cause a collapse

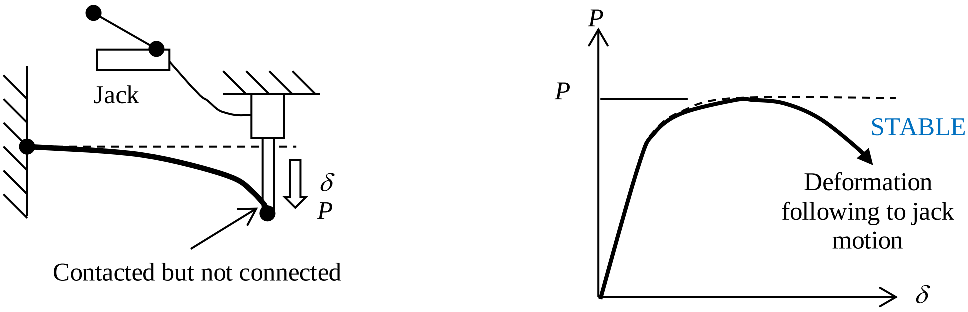


Fig. 2 – Displacement-controlled load can cause stable plastic deformation

The authors consider that the inertia force by seismic response is not a pure force-controlled load but may have the characteristic like the displacement-controlled load because the excitation experiments of piping of the past researches could not generate the plastic collapse and the failure mode in the experiments was fatigue. The authors consider that the collapse could be generated only by the unstable conditions shown in Fig. 3 when the directions of the inertia force could coincide with the deformation.

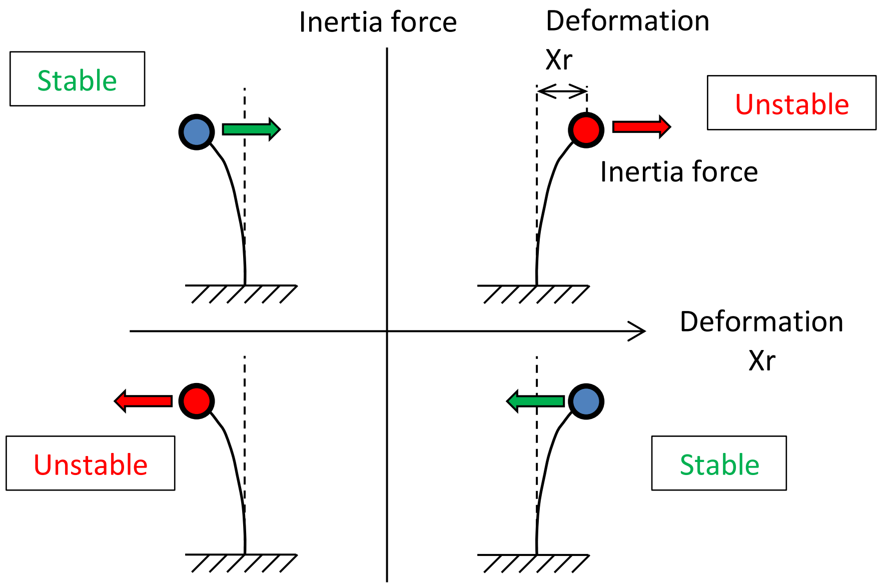


Fig. 3 – Correlation of load and deformation directions and failure behavior

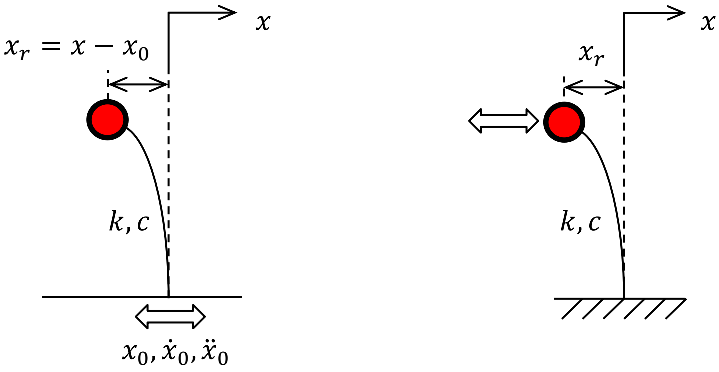
3. Equation of motion for SDOF system

Earthquake means that quakes of the ground literally and vibration of the SDOF system excited by the earthquake is expressed in the schematic model on which the earthquake motions are applied on the anchor point as shown in Fig. 4(a). The equation of motion for the schematic model is given in Eq. (1).

$$m \cdot (\ddot{x} - \ddot{x}_0) + c \cdot (\dot{x} - \dot{x}_0) + k \cdot (x - x_0) = 0 \quad (1)$$

Herein, x_r is defined as a relative displacement between the anchor point and the mass point, with the relationship of $x_r = x - x_0$, Eq. (1) can be transformed into Eq. (2) rendering the equation of motion of the model shown as Fig. 4(b).

$$m \cdot \ddot{x}_r + c \cdot \dot{x}_r + k \cdot x_r = -m \cdot \ddot{x}_0 \Rightarrow F \quad (2)$$



(a) Dynamic displacement loading from the ground (Actual phenomenon)

(b) Dynamic loading on mass point (Equivalent model)

Fig. 4 – Vibration model of SDOF system

Then, since typically a natural frequency and a critical damping ratio are written by generic expressions ω_n and ζ respectively, Eq. (2) can also be expressed as the generic expressions.

$$\ddot{x}_r + 2 \cdot \zeta \cdot \omega_n \cdot \dot{x}_r + {\omega_n}^2 \cdot x_r = -\ddot{x}_0 \quad (3)$$

$$\left[\because \zeta = \frac{c}{2\sqrt{m \cdot k}}, \qquad \omega_n = \sqrt{\frac{k}{m}} \right]$$

Eq. (3) means the dynamic response of the SDOF system can also be obtained by loading an equivalent acceleration on the mass point instead of applying a motion at the anchor point.

This concept is generally used to the basis of seismic response analysis for structures. Eq. (3) superficially presents the same responses of the structure as those from Eq. (1). But this conversion of the equation of motion sometimes confuses while determining an external force which is loaded on the structure as the force-controlled load, because it does not render the actual movement of the ground properly from the aspect of the actual balance of forces. In other words, the left side of the equation of motion originally should be described as internal forces of a structure due to both deformation and kinetic energy, and the right side should be described as external forces but the conversion sometimes replaces those definitions. This kind of transformation tends to make engineers misunderstand to consider what kind of load type is dominant during seismic excitations and the plastic collapse might be generated.

Here, to confirm the characteristics of the reversing dynamic load, several investigations regarding the relations between the forces in the equation of motion and the deformation have been conducted.

4. The response of free vibration

Regarding the free vibration of the SDOF system, the equation of motion can be expressed as Eq. 4, with eliminating the external force term on the right side of Eq. (3).

$$\ddot{x}_r + 2 \cdot \zeta \cdot \omega_n \cdot \dot{x}_r + {\omega_n}^2 \cdot x_r = 0 \quad (4)$$

The general solution of Eq. (4) considering initial conditions: $x_r = a, \dot{x}_r = 0 \; at \; t = 0$ is given as follows:

$$x_r = a \cdot e^{-\zeta \cdot \omega_n \cdot t} \cdot \left(\cos\sqrt{1-\zeta^2} \cdot \omega_n \cdot t + \frac{\zeta}{\sqrt{1-\zeta^2}} \cdot \sin\sqrt{1-\zeta^2} \cdot \omega_n \cdot t \right)$$

$$= a \cdot \frac{e^{-\zeta \cdot \omega_n \cdot t}}{\sqrt{1-\zeta^2}} \cdot \cos\left(\sqrt{1-\zeta^2} \cdot \omega_n \cdot t - \phi\right) \quad (5)$$

$$\left[\because \phi = \tan^{-1} \frac{\zeta}{\sqrt{1-\zeta^2}} \right]$$

Dividing both sides of Eq. (5) by the initial deformation a, a non-dimensional displacement mentioned as "reduced displacement" in this study from here, which stands for a kind of amplification ratios, is obtained as Eq. (6). Then, by differentiating the reduced displacement by time, the reduced velocity and the reduced acceleration can be obtained as well, and shown as Eqs. (7) and (8).

Reduced displacement: $$\frac{x_r}{a} = \frac{e^{-\zeta \cdot \omega_n \cdot t}}{\sqrt{1-\zeta^2}} \cdot \cos\left(\sqrt{1-\zeta^2} \cdot \omega_n \cdot t - \phi\right) \quad (6)$$

Reduced velocity: $$\frac{\dot{x}_r}{a \cdot \omega_n} = -\frac{e^{-\zeta \cdot \omega_n \cdot t}}{\sqrt{1-\zeta^2}} \cdot \sin\sqrt{1-\zeta^2} \cdot \omega_n \cdot t \quad (7)$$

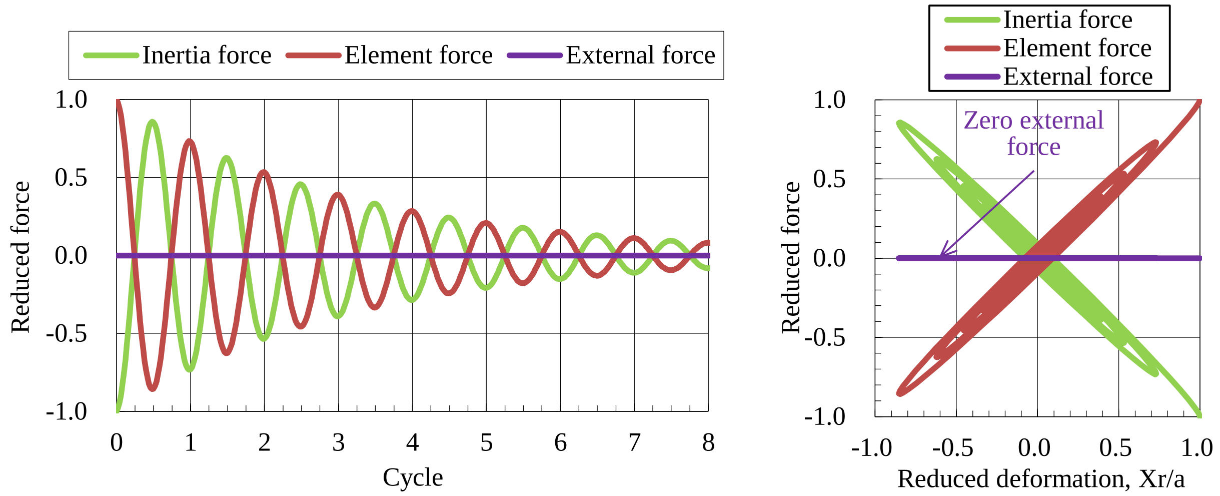
For reduced acceleration: $$\frac{\ddot{x}_r}{a \cdot {\omega_n}^2} = -\frac{e^{-\zeta \cdot \omega_n \cdot t}}{\sqrt{1-\zeta^2}} \cdot \cos\left(\sqrt{1-\zeta^2} \cdot \omega_n \cdot t + \phi\right) \quad (8)$$

Besides, the equation of motion under free vibration can be transformed as Eq. (9) with dividing both sides of Eq. (4) by $a \cdot {\omega_n}^2$. Considering the free vibration condition, since there are no terms on the right side of the equation, it indicates no external loads are being loaded on the structure.

$$\underbrace{\frac{\ddot{x}_r}{a \cdot {\omega_n}^2}}_{\text{Inertia force term}} + \underbrace{2 \cdot \zeta \cdot \frac{\dot{x}_r}{a \cdot \omega_n} + \frac{x_r}{a}}_{\text{Element force term}} = 0 \quad (9)$$

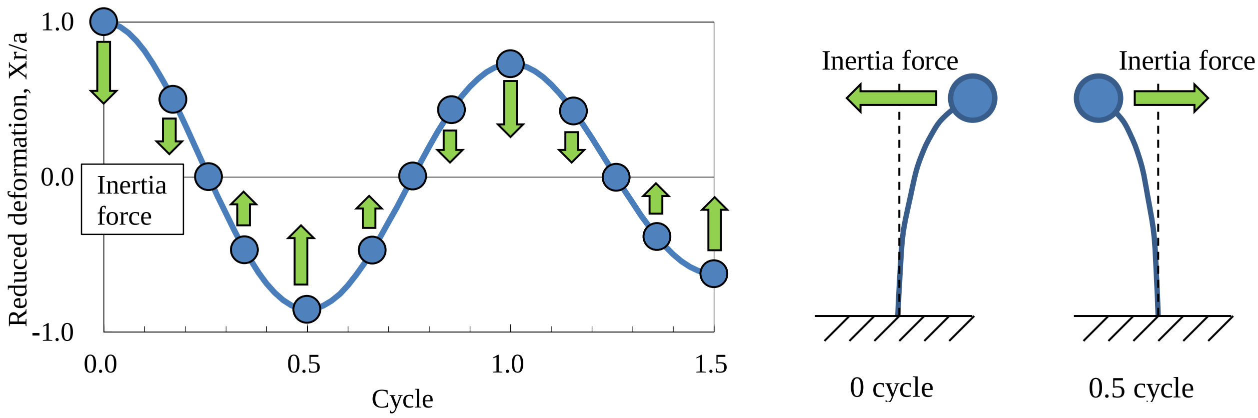
As for the left side of Eq. (9), the first term corresponds to the inertia force term on the mass point. The 2nd term and 3rd term correspond to the generalized damping and the spring force of the SDOF system respectively and are regarded as the element forces term. Then, Fig. 5 shows the relationship between those conceptual force terms and the deformation. Fig. 6 shows the relationship between the deformation and the inertia force.

It can be found that the inertia force term and the element force term are always balanced each other and the sum of those two terms indicates zero, shown as the lines indicated as "External force" in Fig. 5. It is assumed that the entire inertia force in the system is generated from the internal and potential energy accumulated in the element force term, because of Eq. (9) coming from the equation of motion declares no external forces. This means that the inertia forces during the free vibration can be regarded as the displacement-controlled load because the inertia force term originates from the element force term controlled by the deformation. During the free vibration, the system keeps trading potential and kinetic energy.



(a) Time histories of reduced forces (b) Relation against reduced deformation

Fig. 5 – Response of free vibration



(a) Time histories of reduced forces (b) The direction of inertia force

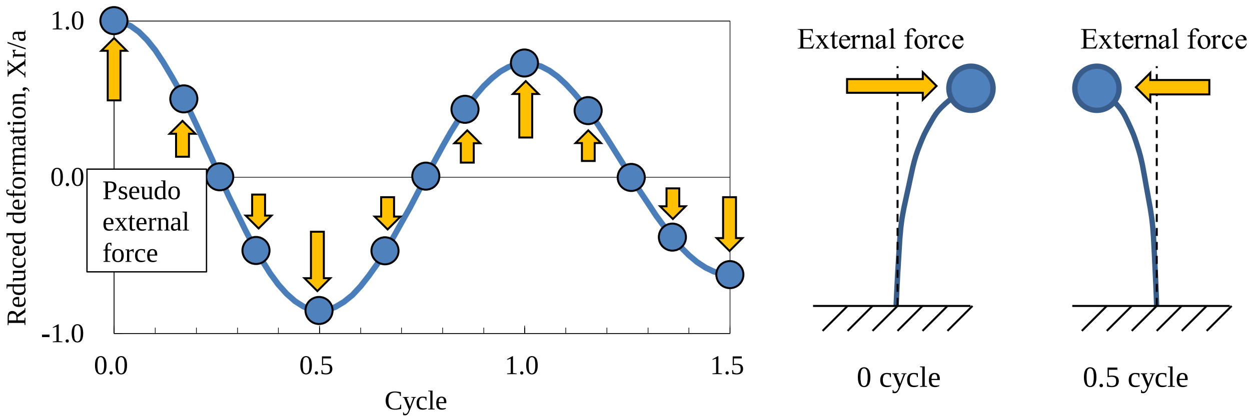
Fig. 6 – The relationship between deformation and inertia force

The schematic conditions at the initial condition and the deformation peaked condition are shown in Fig. 6. The figure shows that the inertia force originates in the opposite direction to the deformation. The relation means that the inertia force is generated by the deformation such as an arrow that can be shot from a deformed bow. However, in seismic engineering, this phenomenon must be misunderstood as shown in Fig. 7. The misunderstanding is considered to be coming from an assumption of pseudo external force that is expressed as Eq. (10) and Fig.7.

$$\boxed{2 \cdot \zeta \cdot \frac{\dot{x}_r}{a \cdot \omega_n} + \frac{x_r}{a}} = \boxed{-\frac{\ddot{x}_r}{a \cdot {\omega_n}^2}} \tag{10}$$

Element force term Pseudo external force term

The force terms are balanced between the left and right sides but the inertia force term on the left side must be switched to a pseudo external force on the right side that would be applied in the same direction as the deformation. This invalid switching must have made engineers misunderstand the inertia force would be a primary load.



(a) Time histories of reduced forces (b) The direction of inertia force

Fig. 7 – The misunderstood relationship between deformation and inertia force

5. Response by sinusoidal wave excitation

The equation of motion by sinusoidal wave excitation can be obtained by substituting the displacement x_0, velocity $\dot{x}_0$, and acceleration $\ddot{x}_0$ as Eqs. (11) to (13) into Eq. (2), where a and ω are the amplitude of displacement and the angular frequency of the excitation. The equation of motion can be expressed as Eq. (14) and the solution of the equation can be obtained as Eq. (15).

$$x_0 = a \cdot \cos \omega t \tag{11}$$

$$\dot{x}_0 = -a \cdot \omega \cdot \sin \omega t \tag{12}$$

$$\ddot{x}_0 = -a \cdot \omega^2 \cdot \cos \omega t \tag{13}$$

$$\ddot{x}_r + 2 \cdot \zeta \cdot \omega_n \cdot \dot{x}_r + {\omega_n}^2 \cdot x_r = a \cdot \omega^2 \cdot \cos \omega t \tag{14}$$

$$x_r = \frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\zeta\eta)^2}} \cdot a \cdot \sin(\omega t + \gamma) \tag{15}$$

$$\left[\because \gamma = \tan^{-1}\frac{1-\eta^2}{2\zeta\eta},\ \eta = \frac{\omega}{\omega_n}\right]$$

Dividing Eq. (15) by $a \cdot \eta^2$ where η is the ratio of the angular frequency of the sinusoidal wave and the natural frequency of the system, the reduced displacement can be written as Eq. (16). Then, by differentiating the reduced displacement by time, the reduced velocity and the reduced acceleration can be obtained as well, and shown as Eqs. (17) and (18).

Reduced displacement:
$$\frac{x_r}{a \cdot \eta^2} = \frac{1}{\sqrt{(1-\eta^2)^2+(2\zeta\eta)^2}} \cdot \sin(\omega t + \gamma) \tag{16}$$

Reduced velocity:
$$\frac{\dot{x}_r}{a \cdot \eta^2 \cdot \omega_n} = \frac{\eta}{\sqrt{(1-\eta^2)^2+(2\zeta\eta)^2}} \cdot \cos(\omega t + \gamma) \tag{17}$$

Reduced acceleration:
$$\frac{\ddot{x}_r}{a \cdot \eta^2 \cdot {\omega_n}^2} = \frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\zeta\eta)^2}} \cdot \sin(\omega t + \gamma) \tag{18}$$

By dividing by $a \cdot \eta^2 \cdot \omega_n{}^2$, the equation of motion can be transformed to the non-dimensional equation as Eq. (19) and expressed with the reduced force terms.

$$\frac{\ddot{x}_r}{a \cdot \eta^2 \cdot \omega_n{}^2} + 2 \cdot \zeta \cdot \frac{\dot{x}_r}{a \cdot \eta^2 \cdot \omega_n} + \frac{x_r}{a \cdot \eta^2} = \cos \omega t \quad (19)$$

As mentioned for the response of free vibration, the first term on the left side can correspond to the inertia force, the pair of the second and the third terms on the left side corresponds to the element force and the right side term corresponds to the external force. The relations of the terms against the reduced displacement are shown in Fig. 8 to 12 when the damping ratio ζ is 0.05 and the frequency ratios of η are 0.1, 0.5, 1.0, 2.0 and 10. From these figures, the following results can be found.

- ✓ $\eta = 0.1$:
  When the excitation frequency is one-tenth of the natural frequency of the structure, the structure is rigid enough against the excitation. The element force is almost the same as the external force and the inertia force is negligibly small on this condition. Therefore, the dynamic force by the excitation can be said as a quasi-static force. Moreover, the deformation of the structure is comparatively small because of the rigid condition. The element force is dominated by the external force and categorized into the force-controlled load.
- ✓ $\eta = 0.5$:
  The inertia force appears and acts on the structure in the opposite direction to the deformation. The element force is a mixed condition of the force-controlled and the displacement-controlled loads on this condition. The external force can be categorized into the force-controlled load and the inertia force can be categorized into the displacement-controlled load.
- ✓ $\eta = 1.0$:
  This case means a resonant condition and the phase gap between external force and the response of the structure. The response acceleration and displacement become large enough compared with the external force and the element force mostly balance with the inertia force. So, this case can be categorized into the displacement-controlled load. Moreover, this tendency becomes more significant when the damping is smaller.
- ✓ $\eta = 2.0$:
  The excitation frequency is two times the natural frequency. The higher frequency excitation than the natural frequency reverses the phase between the excitation force and the deformation. This means that the external force acts in the opposite direction of the deformation like the external force pulls back the deformation. This can be categorized into the displacement-controlled load.
- ✓ $\eta = 10$:
  The excitation frequency is higher enough than the natural frequency and the condition is like a vibrationally isolated condition. The element force becomes very small because the structure is soft enough against the excitation and the inertia force mostly balances with the external force. The inertia force acts in the opposite direction of the deformation. This can be categorized into the displacement-controlled load.

On the response under sinusoidal wave excitation, the load categorization of the response depends on the parameter η that is the ratio of the excitation frequency and the natural frequency of the structure. When the parameter η becomes smaller than 1.0, the directions of the external force coincides with the direction of the deformation. And the collapse of the structure can be generated if the external force is larger than the ultimate strength of the structure. Only this condition can be said as the force-controlled load or the primary load.

When the parameter η becomes larger than 1.0, the external force acts in the opposite direction to the deformation. This condition can be stable and the deformation is dominated by the displacement of the excitation, so the collapse cannot be generated if the deformation capacity of the structure is larger than the displacement of the excitation wave and the structure can follow the motion of the excitation wave. This condition can be categorized into the displacement-controlled load or the secondary load. As a result, the excitation force having a higher frequency than the natural frequency like high speed or impact force cannot generate the plastic collapse because the condition is vibrationally isolated.

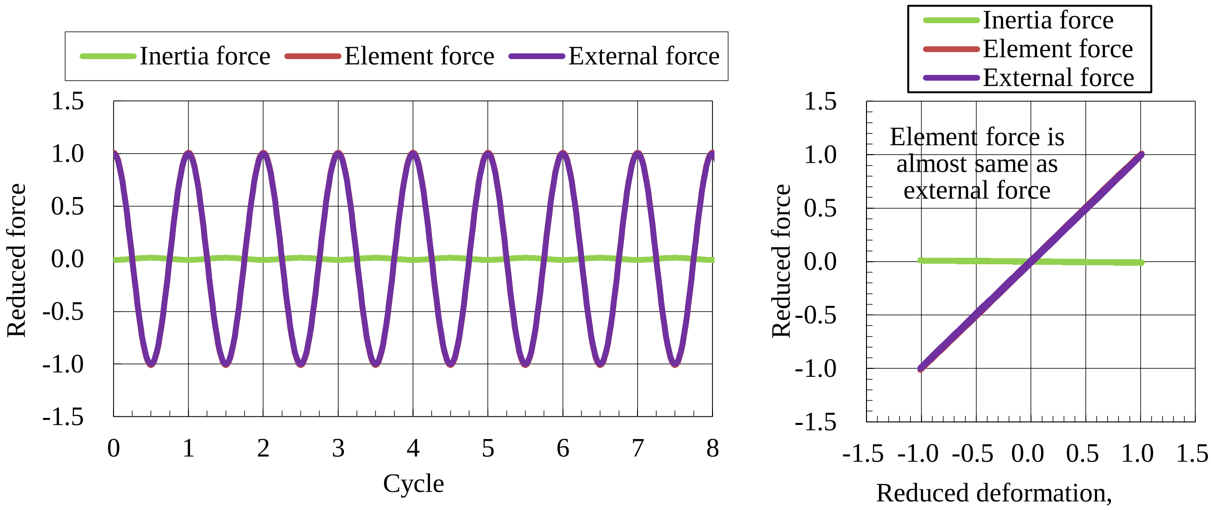


Fig. 8 – The response by sinusoidal wave excitation ($\eta = 0.1, \zeta = 0.05$)

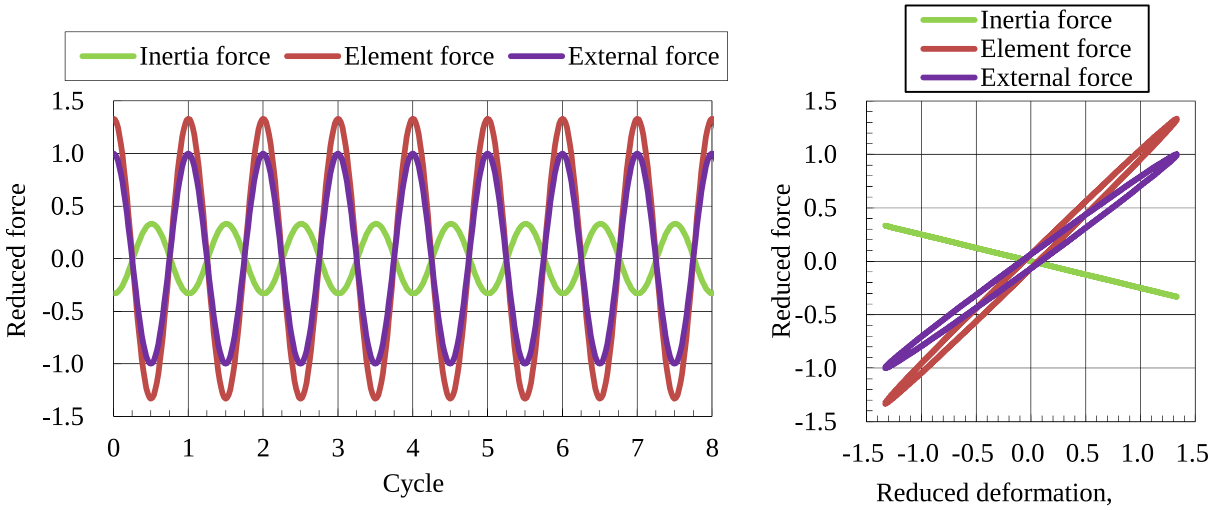


Fig. 9 – The response by sinusoidal wave excitation ($\eta = 0.5, \zeta = 0.05$)

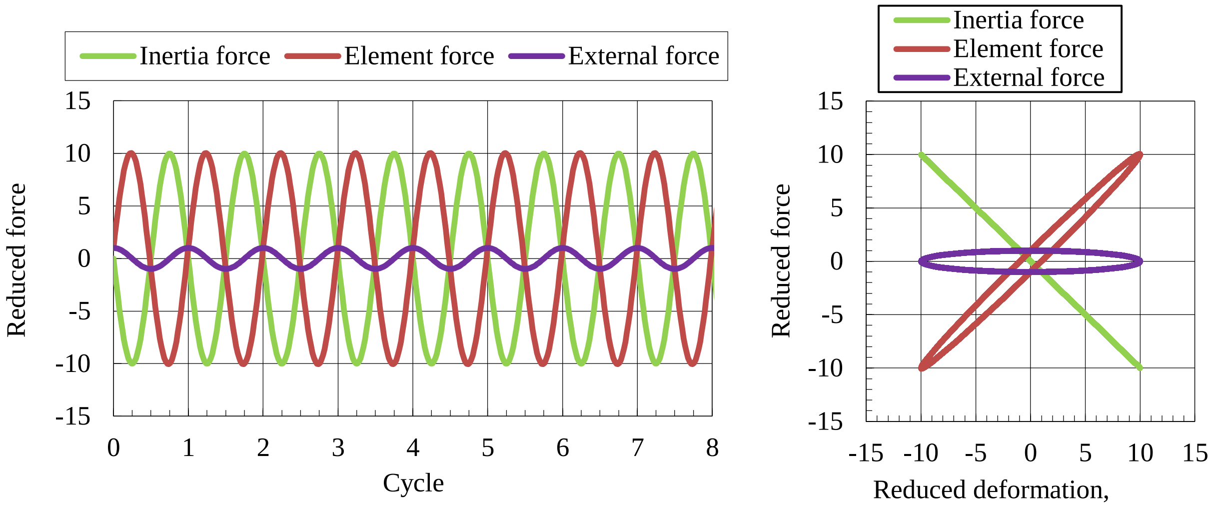


Fig. 10 – The response by sinusoidal wave excitation ($\eta = 1.0, \zeta = 0.05$)

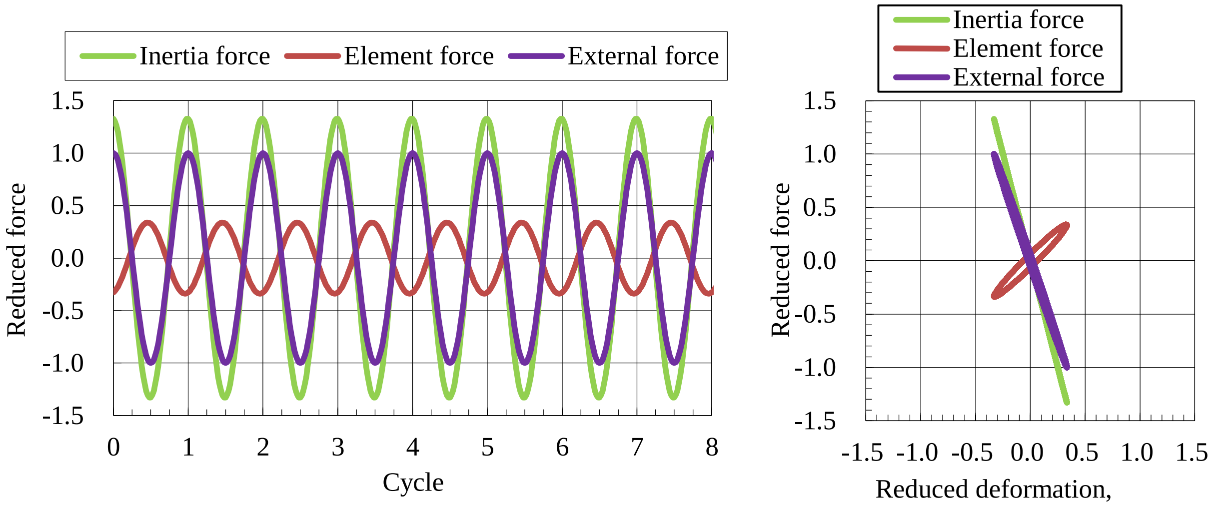


Fig. 11 – The response by sinusoidal wave excitation ($\eta = 1.0, \zeta = 0.05$)

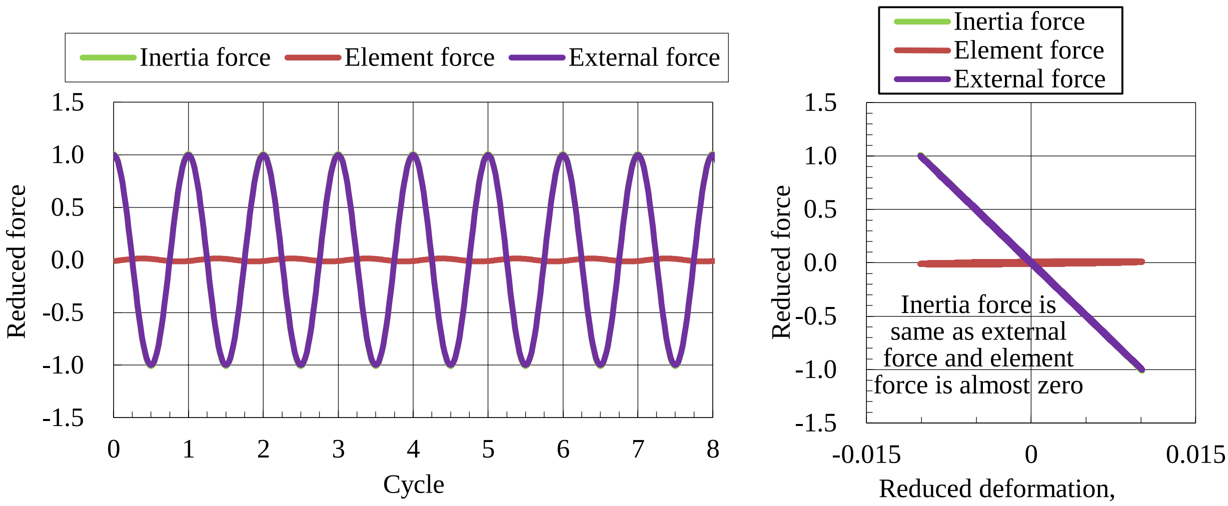


Fig. 12 – The response by sinusoidal wave excitation ($\eta = 10, \zeta = 0.05$)

6. The primary ratio of response by sinusoidal wave excitation

It is found that the load of the response by sinusoidal wave excitation has both characteristics of the force-controlled/primary and the displacement controlled/secondary. Here, the authors tried to calculate a ratio regarding the primary characteristic.

For simplification, the damping ratio ζ is set to be zero.

$$\text{External force} = \cos \omega t \tag{20}$$

$$\text{Element force} = \frac{1}{1-\eta^2} \cdot \sin\left(\omega t + \frac{\pi}{2}\right) \tag{21}$$

For $\eta \leq 1$, the ratio of the external load and the element force that means the primary load ratio can be expressed as the following formula.

$$\text{Primary load ratio} = \frac{\text{External force}}{Element\ force} = 1 - \eta^2 \tag{22}$$

And for $\eta > 1$, the primary load ratio is zero because the external load acts in the opposite direction of the deformation and also acts as the displacement-controlled load.

7. Conclusion

The present seismic design code provides the primary stress evaluation for preventing the plastic collapse of the structure because the load by the seismic response is treated as the force-controlled load called as the primary load. However, It is found that the load by the seismic response behaves as a mixture of the force-controlled load and the displacement-controlled load. And the force-controlled load is dominated when the natural frequency of the structure is higher than the frequency of the excitation wave. This is a rigid structure condition, so it can be said that the collapse can be generated just for the rigid structure. But, generally, the rigid structure cannot be resonated by the excitation and the response cannot become so large. Therefore, the plastic collapse hardly occurs when the structure has sufficient ductility.

Finally, the primary load ratio of the response by sinusoidal wave excitation was proposed. The ratio must be useful for a rational evaluation to prevent the plastic collapse of the structure.

8. Acknowledgments

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9. References

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