VIBRATION ATTENUATION THROUGH ONE DIMENSIONAL THREE-LAYER PERIODIC FOUNDATION: ANALYSIS AND EXPERIMENT

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Abstract

Structures subjected to seismic forces are susceptible to heavy damages due to excessive deformation. Various base isolation systems like lead rubber bearing, friction pendulum, elastomeric bearing etc. are used to minimize seismic damages of structures. Recent developments in base isolation techniques have led to development of a new class of isolation systems called periodic foundations. Periodic foundations can be one dimensional (1D), two dimensional (2D) or three dimensional (3D) depending on the periodic arrangement of the component materials. Most of the previous studies on 1D periodic foundations have considered periodic repetitions of two layers of materials. In the present study 1D periodic foundations with repetitions of three layers of materials have been analyzed using plane wave expansion method. Parametric studies have been conducted to understand the effectiveness of the foundation through the change in material and geometric properties of the constituent layers of the three materials. The results showed that wider attenuation zones can be obtained from 1D three-layer periodic foundation which is highly desirable in structural base isolation. An experimental study has also been conducted to show the effectiveness of the three-layer periodic foundation. The results showed that 1D three-layer periodic foundation is highly effective in attenuating the response of structures.

Keywords: Base isolation, Periodic foundation; Band Gap; Attenuation Zone, Wave barrier

1. Introduction

Civil engineering infrastructure tends to undergo high damage under the effect of seismic loads. Thus an earthquake causes an immense loss in life and property in a region. In order to minimize the losses during seismic events, the idea of ductile designing of individual elements and shear resistance design of structures have gained popularity in the last few decades. Base isolation techniques are implemented to provide additional stability to the structures under the seismic forces,. In general, base isolation techniques can be subdivided into two categories namely active base isolation techniques and passive base isolation techniques. In passive base isolation techniques, fundamental frequency of structural systems are modified by addition of either sliding elements or layer of low stiffness between the superstructure and the base of the structural systems. Thus passive base isolation techniques increase the natural time period and thereby decreases the seismic response of the structural systems. In active base isolation techniques, control actuators are employed along with passive base isolators to control the lateral drift of structural systems. Most of the previous studies have shown the effectiveness of both active and passive base isolation systems [1, 2, 3, 4, 5]. However, some of the disadvantages of passive base isolation includes large residual displacement between the base and the superstructures which makes them unfit for structures containing crucial pipelines, sensitive instruments, hospitals etc. Similarly, active base isolation requires a large power source which makes them unsuitable for practical application in civil engineering infrastructure.
From the recent studies, it has been observed that arrangement of crystal of certain materials can regulate the energy of elastic waves passing through them. If materials of different acoustic properties are arranged periodically, elastic waves within certain frequency ranges cannot pass through the given arrangement of materials termed as phononic crystals [6, 7, 8]. The frequency ranges over which waves cannot propagate through the phononic crystals are termed as "Band Gaps" or "Attenuation Zones". Hence all the waves with frequencies falling within the frequency ranges of the band gaps are completely obstructed from passing through the phononic crystal as shown in Fig.1

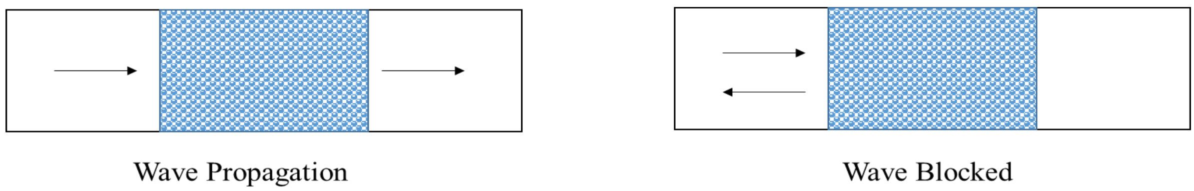


Fig. 1-Wave Propagation Through Periodic Materials

This peculiar property of phononic crystals for wave obstruction are being currently utilized as a new structural base isolation technique for of civil engineering infrastructures by several researchers [9, 10, 11, 12]. The foundations of structural systems with phononic crystal like arrangement being utilized for structural base isolation are termed as "Periodic Foundations". Periodic foundations can totally isolate the buildings from seismic waves as shown in Fig. 2. The periodic foundations can be subdivided into one dimensional (1D), two dimensional (2D) and three dimensional (3D) periodic foundations on the basis of arrangement of the periodic materials as shown in Fig.3.

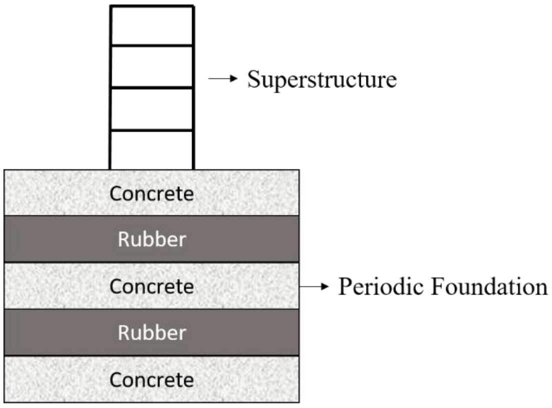


Fig. 2-Schematic Representation of Structure with Periodic Foundation

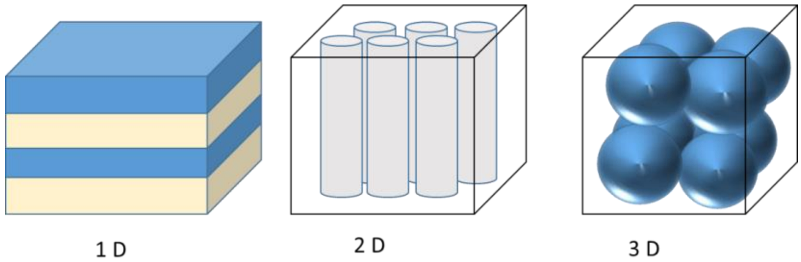


Fig. 3- 1D,2D and 3D Phononic Crystals

The present study is focused on 1D periodic foundations. It has been observed that most of the previous studies on 1D periodic foundations have been conducted on foundations with two layers of material in their unit cells. However, no previous studies have focused on the addition of more than two layers of materials in the unit cells of periodic foundations. Hence, the formulation for 1D three-layer periodic foundation has been derived for the Plane Wave Expansion method (PWE) in the present study to investigate the effect of addition of material layers within the periodic units of the foundations. The derived formulation has been used to perform parametric studies on 1D three-layer periodic foundations. Small scale tests on 1D three-layer periodic foundation specimens have also been conducted to study the attenuation zones of the specimens.

2. Theoretical Computation

The schematic representation of a 1D three-layer periodic foundation is shown in Fig. 4. The wave is assumed to travel along the X direction and the material layers of the periodic foundation are assume to be infinitely extended along the Y and Z directions.

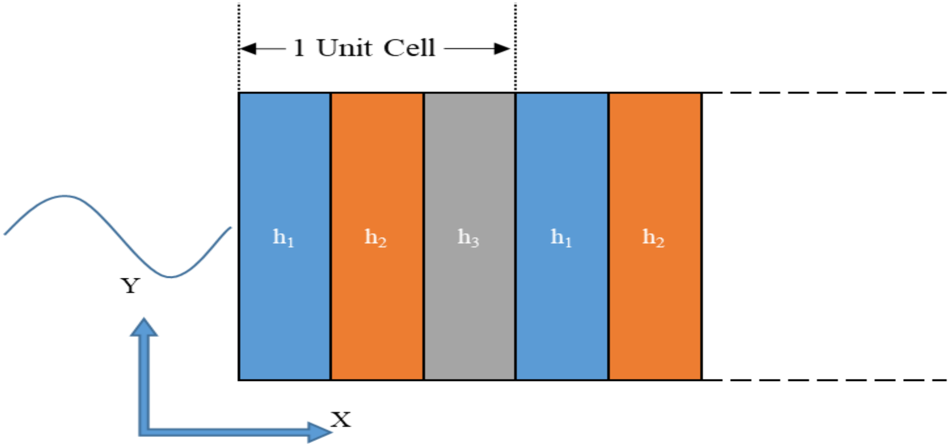


Fig.4- One Dimensional 3 Layer Periodic Foundation

The dispersions curves for a 1D three-layer periodic foundation have been derived through the Plane Wave Expansion method by expressing the material properties in the form of Fourier series expansion in the reciprocal lattice space. Hence the Reciprocal Lattice (RL) vectors $G(G_1, G_2, G_3)$ with respect to the basis $O(e_1, e_2, e_3)$ have been defined in the standard way. The material properties with respect to position vector $\vec{r}$ like density $\rho(\vec{r})$ and elastic constants $C_{11}(\vec{r})$ are also defined as Fourier series expansion in the reciprocal lattice space. The periodicity of the material layers has been captured by application of the Bloch Floquet theorem to provide an Eigen value problem for the wave equation of the 1D three-layer periodic foundation. The dispersion curves from which the attenuation zones (band gaps) of the periodic foundation can be obtained, have been obtained from the Eigen values. The stepwise procedure for obtaining the attenuation zones of the 1D three-layer periodic foundation is presented below.

Step 1- Assume wave solution

The equation of wave propagation for displacement u_1 in 1D is given by Eq. (1).

$$\rho(\vec{r})\frac{\partial^2 u_1(\vec{r},t)}{\partial t^2} = \frac{\partial}{\partial x_1}\left[C_{11}(\vec{r})\frac{\partial u_1}{\partial x_1}\right] \tag{1}$$

The solution of Eq. 1 satisfying the Bloch Floquet theorem is assumed as shown in Eq. (2).

$$\vec{u}(\vec{r},t) = e^{i(\vec{K}.\vec{r}-wt)}\vec{U}_{\vec{K}}(\vec{r}) \tag{2}$$

where $\vec{K}(K_1, K_2, K_3)$ is the Bloch wave vector and $U_k(\vec{r})$ has the periodicity of the direct lattice space.

Step 2- Express parameters in Reciprocal Lattice space

The parameters like displacement, density and elastic constants are expressed in Fourier series expansion in reciprocal lattice space as shown in Eqs. (3) - Eq. (5).

$$\vec{U}_{\vec{K}}(\vec{r}) = \sum_{G'}\vec{U}_{\vec{K}}(G')e^{i\vec{G}'.\vec{r}} \qquad \text{Where} \quad \vec{G}' \in RL \tag{3}$$

$$C_{ijkl}(\vec{r}) = \sum_{G''} C_{ijkl}(G'') e^{i\vec{G}''.\vec{r}} \quad \text{Where} \quad \vec{G}' \in RL \tag{4}$$

$$\rho(\vec{r}) = \sum_{G''} \rho(G'') e^{i\vec{G}''.\vec{r}} \quad \text{Where} \quad \vec{G}' \in RL \tag{5}$$

Step 3- Evaluate Kronecker delta function over unit cell

The Kronecker delta function defined over the complete unit cell is evaluated using Eq. (6).

$$\frac{1}{V_{UC}} \int e^{i(\vec{G}'+\vec{G}''-\vec{G}).\vec{r}} d\vec{r} = \delta_{\vec{G}'+\vec{G}''-\vec{G},0} = \begin{cases} 1 & \text{If } (\vec{G}'+\vec{G}''-\vec{G}) = \vec{0} \\ 0 & \text{If } \vec{G}'+\vec{G}''-\vec{G} \neq \vec{0} \end{cases} \quad \vec{G}'' = \vec{G} - \vec{G}' \tag{6}$$

Where V_{UC} is the volume of the complete unit cell. For one dimensional periodic foundation V_{UC} is the total thickness of a single unit cell of the periodic foundation as defined in Eq. (7).

$$H = h_1 + h_2 + h_3 \tag{7}$$

Step 4- Simplify LHS and RHS of wave equation

Simplify LHS and RHS of Eqs. (1) as shown in Eqs. (8) and (9) respectively.

$$\rho(\vec{r}) \frac{\partial^2 u_1(\vec{r},t)}{\partial t^2} = -w^2 e^{i(\vec{K}.\vec{r}-wt)} \sum_{G',G''} \rho(G'') U_{1,\vec{K}}(G') e^{i(G'+G'').\vec{r}} \tag{8}$$

$$\frac{\partial}{\partial x_1}\left[C_{11}(\vec{r}) \frac{\partial u_1}{\partial x_1}\right] = -e^{i((\vec{K}+\vec{G}).\vec{r}-wt)} \sum_{G'} \left[(K_1 + G'_1)(K_1 + G_1) C_{11}(\vec{G}-\vec{G}')\right] U_{1,\vec{K}}(\vec{G}') \tag{9}$$

Step 5- Formulate Eigen value problem

The depth of the unit cell up to the n[th] layer is defined as H_n as shown in Eq. (10).

$$H_n = h_1 + h_2 + ...h_n \quad \text{and} \quad H_0 = 0 \tag{10}$$

The formula for $\rho(G)$ is derived as shown in Eqs. (11) and (12).

$$\rho(G) = \frac{1}{H}\left(\int_0^{H_1} \rho_1 e^{-iGx} + \int_{H_1}^{H_2} \rho_2 e^{-iGx} + \int_{H_2}^{H_3} \rho_3 e^{-iGx} \right) \tag{11}$$

$$\rho(G) = \sum_{n=1}^{3} \rho_n \left[\frac{e^{-iGH_{n-1}} - e^{-iGH_n}}{iGH} \right] \tag{12}$$

Similarly, the formula for elastic constant as shown in Eq. (13) can be derived.

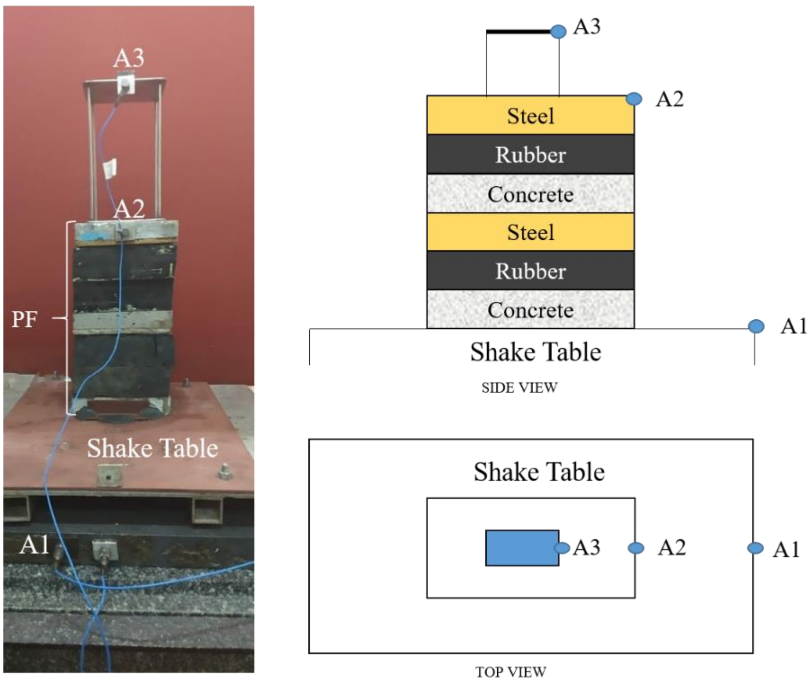
$$C_{11}(G) = \sum_{n=1}^{3} C_{11n} \left[\frac{e^{-iGH_{n-1}} - e^{-iGH_n}}{iGH} \right] \tag{13}$$

The final Eigen value problem is obtained as shown in Eq. (14) by substituting of formulae of material parameters shown in Eqs (12) and (13) into Eqs. (8) and (9) to solve the wave equation shown in Eq. (1). The attenuation zones (or band gaps) of the three-layer periodic unit cell considered in the present study is obtained from the solution of the Eigen value problem represented in Eq. (14).

$$w^2 \sum_{G'} \rho(\vec{G}-\vec{G}')U_{1,\vec{K}}(\vec{G}') = \sum_{G'} \left[(K_1+G'_1)(K_1+G_1)C_{11}(\vec{G}-\vec{G}')\right]U_{1,\vec{K}}(\vec{G}') \tag{14}$$

3. Experimental Study

A 1D three-layer periodic foundation having dimensions of 150 mm x 150 mm and consisting of 30 mm thick concrete layers, 100 mm thick rubber layers and a 10 mm thick steel plates has been tested on a 1.5 ton shake table of size 490 mm x 490 mm as shown in Fig. 5. The geometric and material properties of the tested specimen of the periodic foundation are presented in Table 1. A single story steel frame consisting of two 120 mm x 120 mm x 2.5 mm steel plates connected by four cylindrical columns each of diameter 2 mm and length 200 mm, has been used as the superstructure in the experiment. Accelerometers have been mounted at three locations of the test setup, namely at the base of the shake table, at the top of the periodic foundation and at the upper level of the steel frame superstructure. A sweep sinusoidal signal has been provided with an amplitude of 2 m/s^2 and frequency variation from 5 Hz to 1200 Hz. The three accelerometer readings have been collected through HBM DAQ (Data Acquisition System).



A1 = Accelerometer at Base,

A2 = Accelerometer at Top Level of Periodic Foundation,

A3 = Accelerometer at Top Level of Steel Frame

Fig. 5- Test Setup

Table 1- Properties of Periodic Foundation

Material	Thickness (mm)	Young's Modulus (Pa)	Poison Ratio	Density (kg/m^3)
Concrete	30	4.14 x 10^{10}	0.2	2300
Rubber	100	3.6 x 10^6	0.4633	1277
Steel	10	2.05 x 10^{11}	0.28	7850

4. Results and Discussion

4.1 Dispersion Curves

The dispersion curves for a 1D three-layer periodic foundation having similar configuration as the tested periodic foundation specimen is shown in Fig. 6. It can be observed from Fig. 6 that the starting frequency (SF) and the band gap the band gap (BG) of the first attenuation zone of the periodic foundation are 316 Hz and 1556 Hz respectively. The analysis for 1D two layer periodic foundation consisting of 30 mm concrete layer and 100 mm rubber layers gave SF and BG of periodic foundation as 104 Hz and 51 Hz respectively. Hence a wide band gap is obtained for the 1D three-layer periodic foundation.

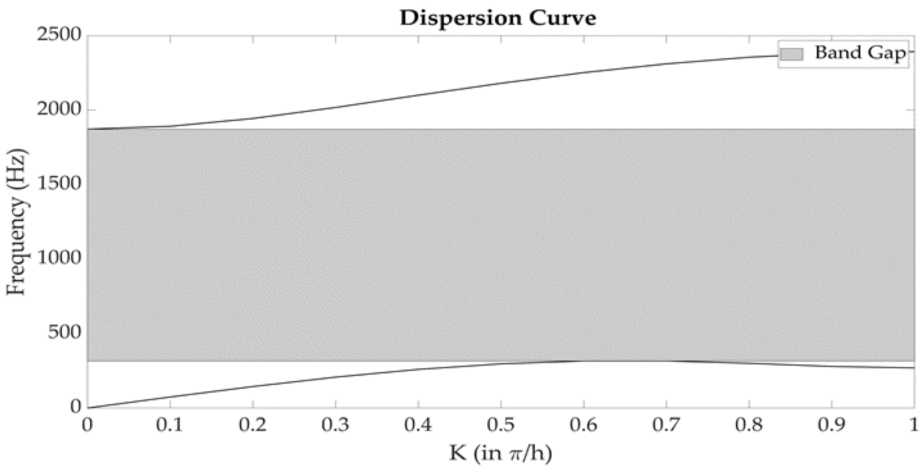


Fig. 6- Dispersion Curve

4.2 Parametric Study

The parametric studies with the material properties of the 1D three-layer periodic foundation has been conducted by changing any one of the material properties and keeping all other material properties identical to the basic material properties shown in Table 1.

4.2.1 Variation of Attenuation Zone with Material Density

A parametric study has been conducted to obtain the variation in the attenuation zones of the basic periodic foundation (described in Section 3) due to the variation of the material densities of the constituent layers. All other properties of the constituent materials have been kept identical to the basic material properties shown in Table 1. The density of concrete has been varied from 1000 kg/m^3 to 2500 kg/m^3 in the present study. The variation in SF and BG of the first attenuation zone of the three-layer periodic foundation with the variation of the concrete density is shown in Fig. 7. It can be observed that SF decreased from 497 Hz to 302 Hz and BG increased from 50 Hz to 1702 Hz with the increase of concrete density from 1000 kg/m^3 to 2500 kg/m^3. The density of rubber has also been varied from 1000 kg/m^3 to 2500 kg/m^3 in the present study and the corresponding variation in SF and BG of the first attenuation zone of the three-layer periodic foundation is shown in Fig. 8. It was observed that SF of the first attenuation zone of the three-layer periodic foundation increased slightly from 316 Hz to 317.2 Hz and BG decreases drastically from 2147 Hz to 323 Hz with the increase of rubber density from 1000 kg/m^3 to 2500 kg/m^3. Similarly, the density of steel has been varied from 3000 kg/m^3 to 9000 kg/m^3 in the parametric study. It can be observed from Fig. 9 that SF changes from 306 Hz to 321 Hz and BG changes from 1715 Hz to 1492 Hz with an increase in the steel density from 3000 kg/m^3 to 9000 kg/m^3.

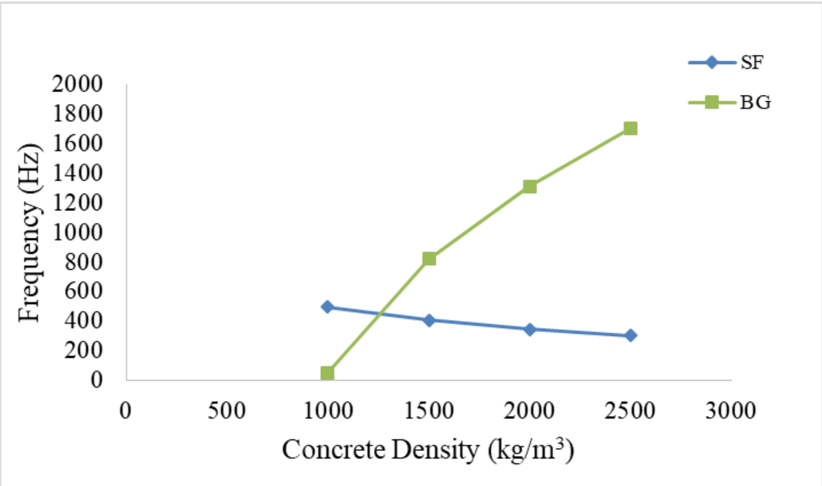


Fig 7- Variation of First Attenuation Zone with Density of Concrete

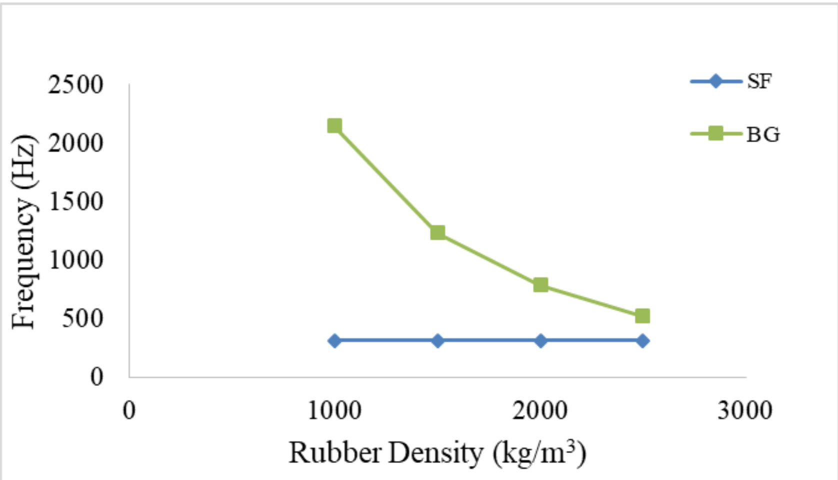


Fig. 8- Variation of First Attenuation Zone with Density of Rubber

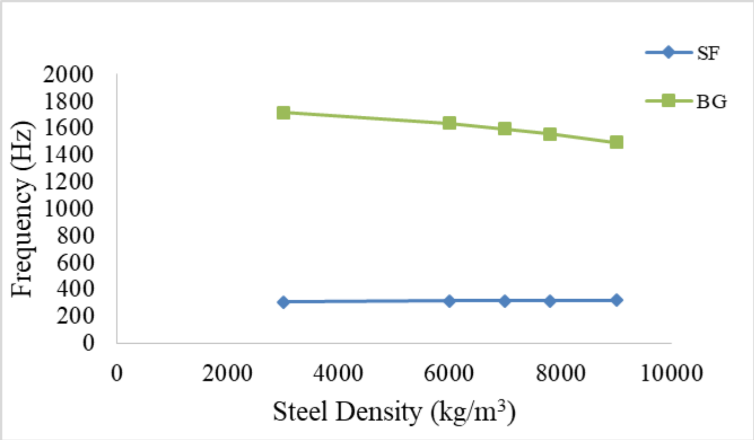


Fig. 9- Variation of First Attenuation Zone with Density of Steel

4.2.2 Variation of Attenuation Zone with Modulus of Elasticity of Materials

The variation of the attenuation zone of the 1D three-layer periodic foundation due to the individual variation in the modulus of elasticity of the three constituent materials has also been investigated in the present study. The modulus of elasticity of concrete has been varied from 20 GPa to 50 GPa. The variation in SF and BG of the first attenuation zone due to the variation in the concrete modulus of elasticity is shown in Fig. 10. It can be observed that SF of the first attenuation zone increased from 128 Hz to 410 Hz with an increase in the concrete modulus of elasticity from 20 GPa to 50 GPa. On the other hand, BG of the first attenuation zone increased from 231 Hz to 1922 Hz. A variation in the modulus of elasticity of rubber from 1 MPa to 5 MPa has led to the minor increment of both SF and BG of the first attenuation zone of the three-layer periodic foundation from 310 Hz to 320 Hz and from 1549 Hz to 1559 Hz respectively, as shown in Fig. 11. A reduction in both SF and BG of the first attenuation zone of the periodic foundation from 489 Hz to 102 Hz and 2090 Hz to 466 Hz respectively has been observed due to a variation in the modulus of elasticity of steel from 100 GPa to 400 GPa as shown in Fig. 12. However, both SF and BG increased from 102 Hz to 686 Hz and 466 Hz to 1906 Hz respectively as the modulus of elasticity of steel increased from 400 GPa to 500 GPa.

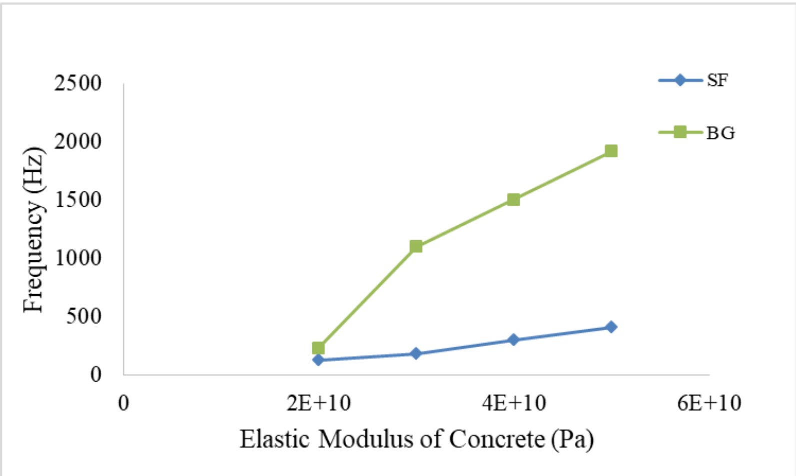


Fig. 10 Variation of First Attenuation Zone with Modulus of Elasticity of Concrete

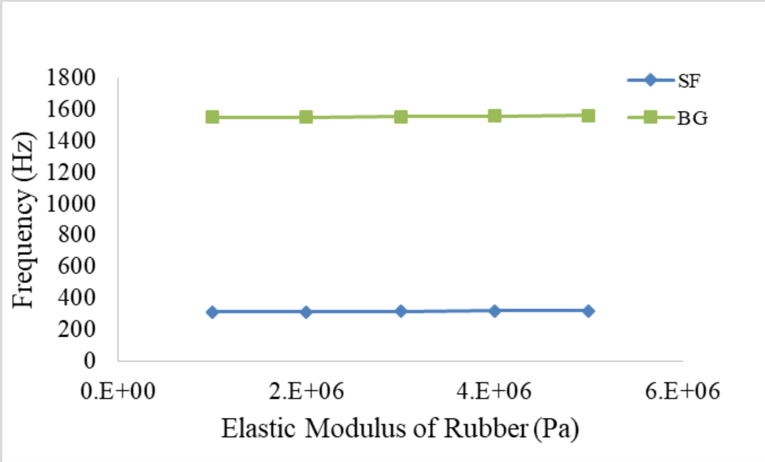


Fig. 11: Variation of First Attenuation Zone with Modulus of Elasticity of Rubber

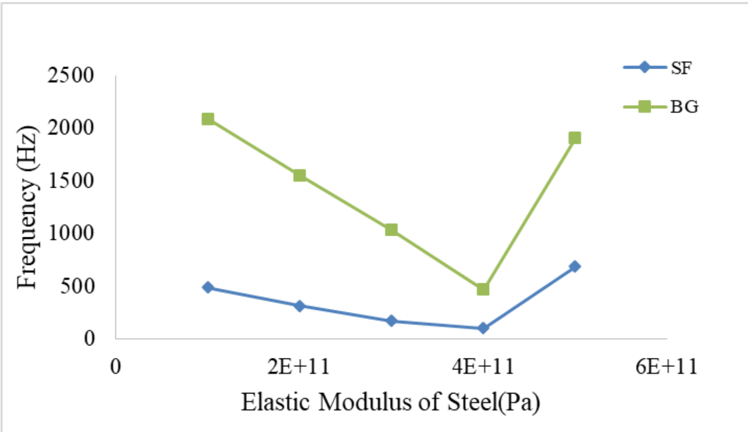


Fig. 12- Variation of First Attenuation Zone with Modulus of Elasticity of Steel

4.3 Experimental Results

The acceleration responses recorded using the accelerometers at the top of the steel frame and at the top periodic foundation have been normalized with respect to the shake table input acceleration values recorded at the base of the shake table. The normalized amplitudes of the acceleration responses are plotted in Fig. 13. The response of the steel frame without any base isolation has been compared with the response of the frame with the periodic foundation. It can be observed that the response of the steel frame is reduced up to 4.4% of the input acceleration when the excitation frequencies are within the attenuation zones of the periodic foundation. It can also be observed that the response of the steel frame without the periodic foundation is significantly amplified at excitation frequencies of higher order natural frequencies (986 Hz and 1163 Hz) within the attenuation zones of the periodic foundation as compared to steel frame resting on periodic foundation as shown in Fig.13.

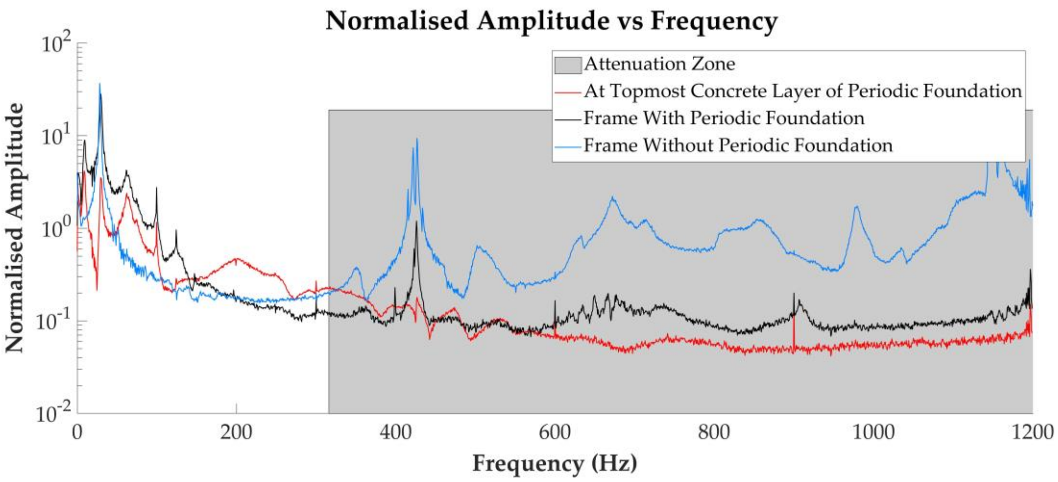


Fig. 13- Response Under Sinusoidal Excitation

5. Conclusions

The following major conclusions can be drawn from the present study:

1. 1D three-layer periodic foundation can have wider attenuation zones than 1D two-layer periodic foundations.
2. The starting frequency of the attenuation zone is decreased and the width of the attenuation zone is increased by increasing the density of concrete in a 1D three-layer periodic foundation.
3. The starting frequency of the attenuation zone is slightly increased with a significant decrease in the width of the attenuation zone due to an increase in the density of rubber in a 1D three-layer periodic foundation.
4. The starting frequency of the attenuation zone is slightly increased with a decrease in the width of the attenuation zone due to an increase in the density of steel in a 1D three-layer periodic foundation.
5. The starting frequency and width of the attenuation zone are both increases on increasing the modulus of elasticity of concrete in a 1D three-layer periodic foundation..
6. The starting frequency and width of the attenuation zone are both negligibly increased on increasing the modulus of elasticity of rubber in a 1D three-layer periodic foundation.
7. The starting frequency and width of the attenuation zone are both decreased on increasing the modulus of elasticity of steel up to a value of 400 GPa in a 1D three-layer periodic foundation. Both the starting frequency and the width of the attenuation zone increase if the modulus of elasticity of steel is increased above 400 GPa.
8. Test results show that 1D three-layer periodic foundation is effective in significant response reduction of the superstructure under excitation frequencies within its attenuation zone.

6. References

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