EFFECT OF SHEAR—EXTENSION COUPLING IN A MEMBRANE ELEMENT WITH ASYMMETRY OF REINFORCEMENT

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Abstract

The non-linear behaviour of a reinforced concrete (RC) membrane element under increasing in-plane shear stress, can be predicted by the Modified Compression Field Theory and Softened Membrane Model (SMM). These models can be used in a suitable finite element analysis (FEA) programme, to predict the behaviour of a wall-type RC structure under lateral loads generated during an earthquake. The current formulation of the Poisson's effect in SMM assumes symmetry of reinforcement with respect to the principal axes of tensile and compressive stresses, generated due to shear applied on the element. This condition may be violated in an element of the FEA model, which leads to the under-estimation of the generated shear strain.

This paper proposes a modified formulation to extend the applicability of SMM to an element without symmetry of reinforcement. A two-dimensional anisotropic formulation is developed to replace the current orthotropic formulation. This considers the additional shear strain (γ_{12}) generated due to the principal stresses (σ_2 and σ_1), which is referred to as the effect of shear–extension coupling.

An experimental program was carried out to quantify γ_{12} by testing RC panels under tension-compression using a biaxial panel tester. The test set-up, reinforcement details, instrumentation and results of 16 panels are presented. It is observed that substantial shear strain is generated in the asymmetric panels, after the cracking of concrete under tension, and subsequent damage under compression. Based on these results, a model for the additional shear strain will be developed to incorporate the effect of shear–extension coupling in the SMM.

Keywords: Biaxial stresses; Membrane element; Reinforced concrete; Shear–extension coupling; Shear strain.

1. Introduction

Performance-based approach for analysis of structures requires understanding of the non-linear behaviour of the members, under extreme loads that may act during the service life of the structure. A non-linear finite element analysis package is required to predict the in-plane shear versus deformation behaviour of two-dimensional (2D) wall-type structures made with reinforced concrete (RC). A common application is the analysis of a shear wall under in-plane lateral load during an earthquake. A computational model of a wall can be developed using membrane elements. If the in-plane shear stress versus strain behaviour of an element can be computed, then the behaviour of the assemblage of elements can be numerically predicted. The Modified Compression Field Theory [1] and Softened Membrane Model [2] are capable of accurately predicting the behaviour of an RC membrane element under increasing in-plane shear stress.

In a membrane element, applied shear stress generates an in-plane biaxial tensile–compressive stress field. To accurately evaluate the behaviour, the additional tensile strain generated due to compression in the orthogonal direction has to be considered, especially after the cracking of concrete. This is analogous to the Poisson's effect in a linear elastic material. An orthotropic model is used in the formulation of SMM to capture the Poisson's effect.

One of the primary assumptions of the orthotropic model is that the rectangular reinforcement grid is symmetric with respect to the principal axes of applied normal stresses. This assumption holds true when the reinforcement grid is along the axes or inclined at 45° with equal amount of reinforcement in the two directions. However, if the reinforcement is placed asymmetric with respect to the principal axes of stresses, the axes do not remain as principal axes for strains. Shear strain generates in addition to normal strains in the principal axes of stresses. The generation of additional shear strain along with normal strains is termed as shear–extension coupling.

The current orthotropic model of the SMM does not consider the additional shear strain in a membrane element with asymmetric reinforcement. In the present paper, an anisotropic formulation for an RC membrane element is derived, similar to the concept of shear–extension coupling coefficients used in linear elastic composite materials [3]. In the ongoing research, an experimental program was undertaken to quantify the generated shear strain by testing RC panels under tension–compression using a biaxial panel tester.

First, a brief introduction and the limitation of the SMM are presented. Next, the proposed analytical formulation is explained. Then, the objectives of the experimental programme, the test setup and test programme are described. The test results of selected panels are discussed at the end.

2. Research Significance

The SMM, in its current form is truly applicable only when the reinforcement grid in a membrane element is symmetric with respect to the principal axes of applied normal stresses. Though, SMM predicts the shear stress versus deformation behaviour accurately for such elements, it needs to be generalised to include a membrane element with reinforcement which is asymmetric about the principal axes of loading. The present research proposes a formulation to incorporate the effect of shear–extension coupling in the SMM.

3. Softened Membrane Model

Hsu [4] proposed the softened truss model (STM) to analyse a membrane element subjected to in-plane shear. The anomaly in the STM was that, beyond the peak shear strength, to maintain the equilibrium condition the shear strain was found to decrease with decrease in shear stress contrary to the experimentally observed behaviour. This anomaly occurred because the tensile strain generated due to the compression in the orthogonal direction, is not deducted from the total tensile strain, while calculating the tensile stress in the reinforcement. This was rectified by introducing the concept of apparent Poisson's ratios to incorporate the Poisson's effect in the STM [5]. Later Hsu and Zhu [2] introduced the Poisson's effect in the STM and termed the model as Softened Membrane Model (SMM). The apparent Poisson's ratio for the generation of tensile strain due to compression can be high. For a certain load stage, it was expressed in terms of the instantaneous stresses in concrete and reinforcement [6].

3.1 Limitations of SMM

Although SMM can predict the post-peak behaviour, the shear stress for a certain shear strain is overestimated when the requirement of orthotropy in the formulation of the Poisson's effect is violated. For an element with unequal amounts of reinforcements in the two orthogonal directions (one case of asymmetry), the capacity is reduced empirically by introducing a reduction factor in the softening coefficient for concrete (Equation 1).

$$\zeta = F_1(\varepsilon_1)F_2(f_c^{/})F_3(\beta) \tag{1a}$$

$$= \left(\frac{1}{\sqrt{1+400\varepsilon_1}}\right)\left(\frac{5.8}{\sqrt{f_c^{/}}}\right)\left(1-\frac{|\beta|}{24^{\circ}}\right) \tag{1b}$$

The function $F_3(\beta)$ is the empirical factor which was determined by regression analysis of deviation angle, β [7]. To demonstrate the limitation, a numerical simulation of the behaviour of an asymmetric panel (Panel B3 with unequal amounts of reinforcement along the longitudinal and transverse axes, tested under increasing equal biaxial tension and compression [8]) was carried out using the algorithm of SMM, with and without the reduction factor. The algorithm was implemented using a MATLAB program. Fig. 1 shows the comparison of the predicted behaviours with the experimental data. It can be noted that the SMM without the factor over predicts the shear strength and underestimates the shear strain at a certain level of stress.

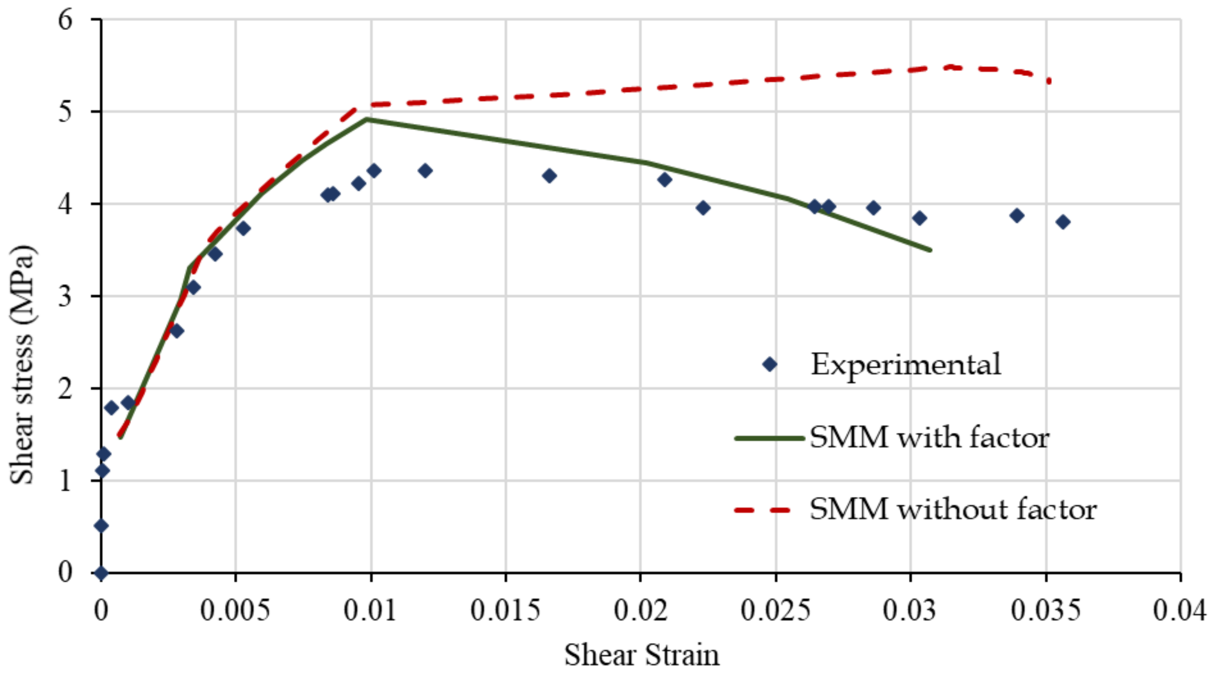


Fig. 1 – Shear stress versus shear strain behaviour for Panel B3 [8]

4. Cases of Asymmetry in Membrane Elements

In an orthotropic material, the directions of applied principal stresses coincide with the directions of generated principal strains. This is referred to as the principle of coaxiality. In SMM, an orthotropic model for the Poisson's effect can truly be used only when the reinforcement is placed symmetric about the loading axes. But, when the reinforcement is not symmetric about the loading axes, the principle of coaxiality is violated. Asymmetry of reinforcement can occur in two cases as demonstrated in Fig. 2. Here, tension and compression are applied along 1- and 2- axes, respectively. The axes of reinforcement are denoted as longitudinal (*l*-) and transverse (*t*-). The reinforcement ratios in the two directions are denoted as ρ_l and ρ_t, respectively. The inclination of the principal stresses with respect to the reinforcement grid is measured as the angle from the *l*- to the 2- axis. This is denoted as α_2.

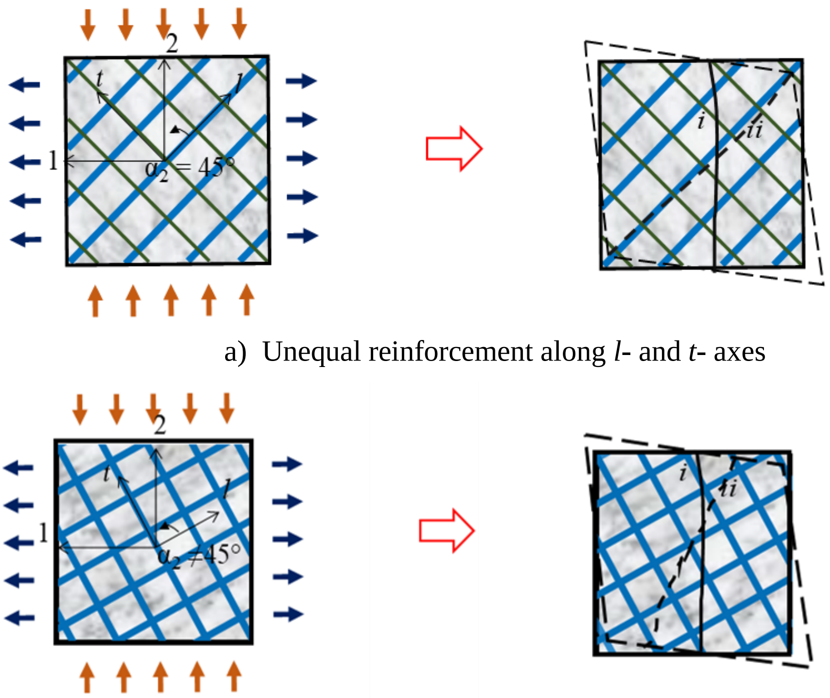
Case a) $\rho_l > \rho_t$ with $\alpha_2 = 45°$

Here, the longitudinal (*l*-) and transverse (*t*-) bars are inclined at 45° to the directions of loading. However, when the reinforcement along *l*- axis is more than the reinforcement along *t*- axis ($\rho_l > \rho_t$), the crack which initially forms perpendicular to 1- axis (marked as *i* in Fig. 2.1a) tends to rotate clockwise and become perpendicular to the *t*- axis (marked as *ii* in Fig. 2.1a), especially after the yielding of the transverse bars. This generates shear strain (γ_{12}) along the principal stress axes 2-1. Similarly, if $\rho_l < \rho_t$ then the cracks will rotate anti-clockwise, generating γ_{12} of opposite sign. After the yielding of the bars, the capacities of the bars in the two directions expressed as $\rho_l f_{yl}$ and $\rho_t f_{yt}$ are the relevant quantities for comparison.

Case b) $\rho_l = \rho_t$ with $\alpha_2 \neq 45°$

Here, the reinforcements along the *l*- and *t*- axes are equal. However, when the reinforcement is asymmetrically inclined to the loading axes (with an angle other than 45°, within the range of 0° and 90°), the crack which initially forms along *i* tends to rotate and bisect the angle between the bars (marked as *ii* in Fig. 2.1b).

The above two cases can occur either separately or simultaneously.



a) Unequal reinforcement along *l*- and *t*- axes

b) Asymmetric orientation of reinforcement with respect to 2-1 axes

Fig. 2 – Cases of asymmetry in membrane elements

5. Model for shear–extension coupling

The modelling of shear–extension coupling for a membrane element using a 2D anisotropic model is first explained in terms of linear elastic material.

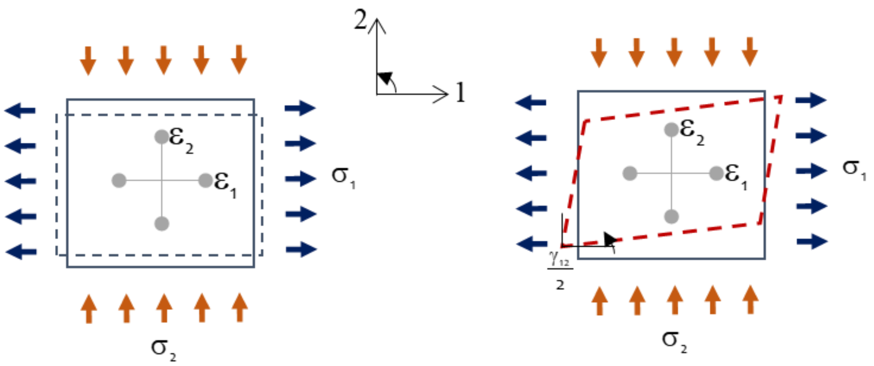
5.1 Linear elastic material

For a 2D anisotropic membrane element, the required number of independent constants to characterise an explicit stress versus strain behaviour is 9, as shown in Equation 2.

It is to be noted that, even in the absence of shear stress τ_{12}, shear strain γ_{12} will be generated (Fig. 3).

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{pmatrix} 1 & -\nu_{12} & \eta_{1,12} \\ -\nu_{21} & 1 & \eta_{2,12} \\ \eta_{12,1} & \eta_{12,2} & 1 \end{pmatrix} \begin{Bmatrix} \dfrac{\sigma_1}{E_1} \\ \dfrac{\sigma_2}{E_2} \\ \dfrac{\tau_{12}}{G_{12}} \end{Bmatrix} \tag{2}$$

Here, ε_1, ε_2, and γ_{12} are the normal and shear strains, and σ_1, σ_2, and τ_{12} are the applied normal and shear stresses, E_1, E_2, and G_{12} are the elastic moduli, ν_{12} and ν_{21} are the Poisson's ratios. $\eta_{1,12}$ and $\eta_{2,12}$ are the extension-due-to-shear coupling coefficients, $\eta_{12,1}$ and $\eta_{12,2}$ are the shear-due-to-extension coupling coefficients [3].



a) Orthotropic material b) Anisotropic material

Fig. 3 – Linear elastic materials under biaxial stresses

5.2 Modelling of shear-extension coupling for RC elements

To give importance to compression carried by concrete after cracking, the SMM considers 2- and 1- as the leading and trailing principal axes of applied stresses, respectively (here, $\tau_{12} = 0$). Maintaining the same convention, Equation 2 can be rewritten as shown in Equation 3.

$$\begin{Bmatrix} \varepsilon_2 \\ \varepsilon_1 \\ \gamma_{12} \end{Bmatrix} = \begin{pmatrix} 1 & -\nu_{21} \\ -\nu_{12} & 1 \\ \eta_{12,2} & \eta_{12,1} \end{pmatrix} \begin{Bmatrix} \varepsilon_{2u} \\ \varepsilon_{1u} \end{Bmatrix} \tag{3}$$

Here, ε_{2u} and ε_{1u} represent the uniaxial strains due to applied compressive and tensile stresses, respectively. The apparent Poisson's ratios are denoted as ν_{21} and ν_{12}. The *apparent shear–extension coupling coefficients* are denoted as $\eta_{12,2}$ and $\eta_{12,1}$. These four quantities are not intrinsic material properties, but they are analogous smeared properties for an RC membrane element after cracking of concrete or yielding of the bars. This type of formulation is suitable for implementation in a nonlinear finite element program. The generated additional shear strain due to lack of symmetry of the reinforcement is expressed in Equation 4.

$$\gamma_{12} = \eta_{12,2}\varepsilon_{2u} + \eta_{12,1}\varepsilon_{1u} \tag{4}$$

The coefficients are defined as ratios of average strains as given in Equation 5.

$$\eta_{12,2} = \frac{\gamma_{12}|_{\sigma_2}}{\varepsilon_{2u}} \tag{5a}$$

$$\eta_{12,1} = \frac{\gamma_{12}|_{\sigma_1}}{\varepsilon_{1u}} \tag{5b}$$

To model the behaviour of an asymmetric membrane element precisely, $\eta_{12,2}$ and $\eta_{12,1}$ should be quantified. This requires modelling of $\gamma_{12}|\sigma_2$ and $\gamma_{12}|\sigma_1$ only, as ε_{2u} and ε_{1u} can be estimated from the applied stresses σ_2 and σ_2, respectively (using the uniaxial material models).

6. Experimental investigation

An experimental programme was undertaken to evaluate the shear strain generated, due to lack of symmetry in reinforcement with respect to principal loading axes, by testing panels under sequential biaxial tension–compression.

The following two parameters were identified as the variables to quantify the asymmetry of reinforcement.

1) $H = \frac{\rho_l f_{yl}}{\rho_t f_{yt}}$: Measure of difference in reinforcement in the two directions, (for Case a)

2) α_2 : Measure of inclination of the reinforcement grid, (for Case b)

Considering the amount of reinforcement in the *l*- axis to be always more than the *t*- axis, H is assumed to be greater than 1.0. The details of test parameters along with material properties for the specimens tested, are shown in Table 1. In a panel designation, say P45-2-1A, P45 refers to a panel with α_2 = 45°. The number 2 implies the rounded off value of H. The identification 1A corresponds to one panel of the group based on the maximum amount of tension applied. Symmetric panels (Series P45-1) were tested as reference cases. These are reported in Kosuru and Sengupta [9]. Panels P45-2-2B, P45-2-3B, P45-4-2B and P45-4-3B are yet to be tested.

6.1 Test setup

The available panel tester is a self-equilibrating system for applying biaxial loading of capacity 2000 kN in each horizontal direction. A schematic sketch of the setup is shown in Fig. 4a. The components are:

a) Two stiff beams and high strength tie rods of 32 mm diameter, for transferring tension.

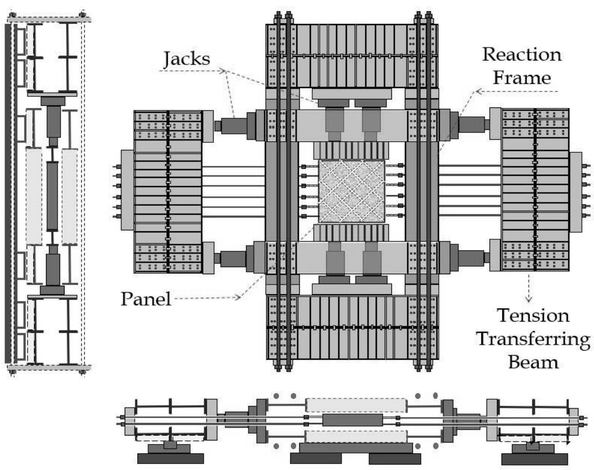
b) Two stiff reaction beams and high strength tie rods for self-equilibration along the compression direction.

c) Eight load controlled hydraulic jacks, a set of four jacks in each direction, for applying compression and tension.

The two sets of jacks are operated separately by two pumps, and two pairs of distribution blocks. The oil pressure from each pump is controlled by a hand operated lever. Each distribution block maintains approximately equal pressure in the four jacks connected to it. A view of the test setup is shown in Fig. 4b.

Table 1 – Details of test programme

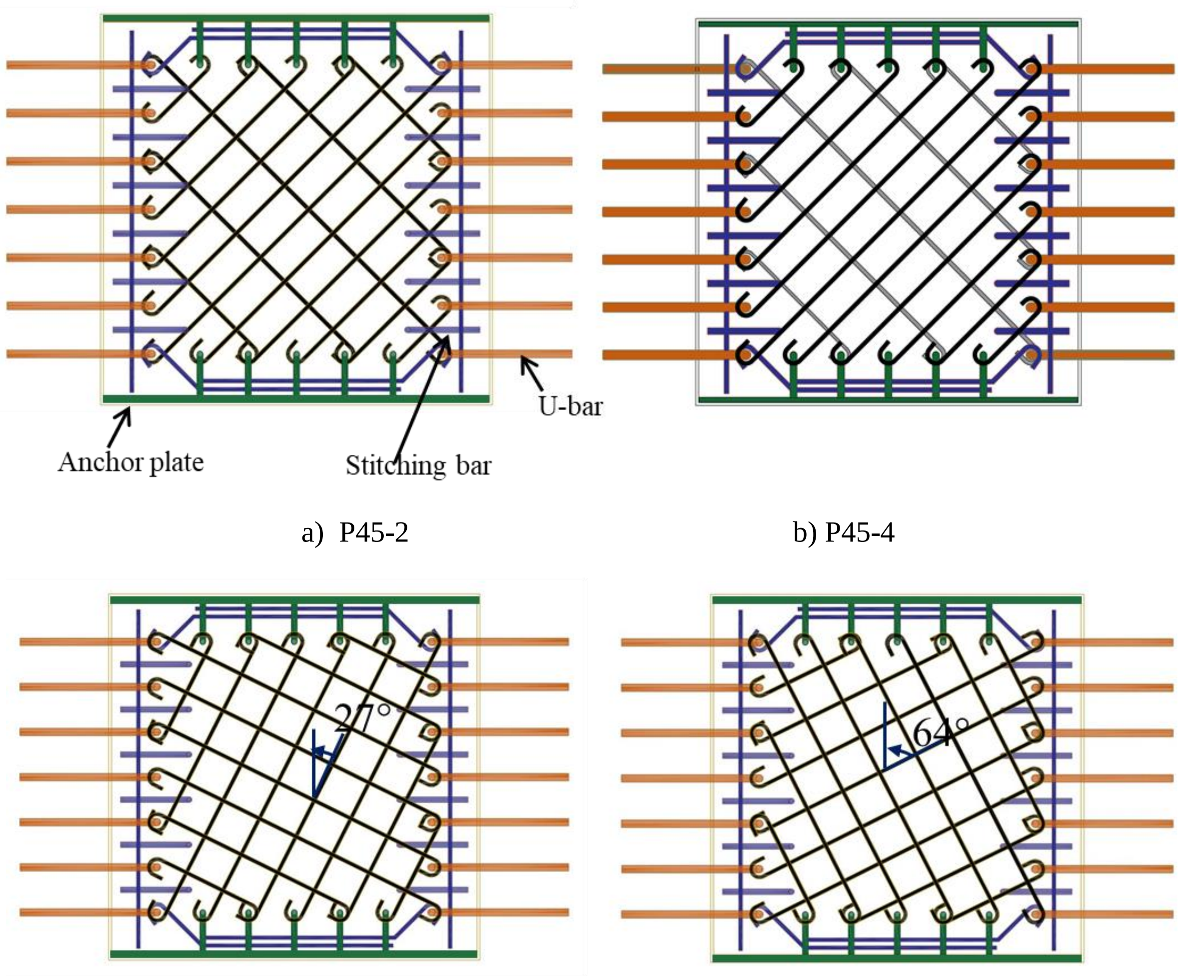
Series	Panel	Reinforcement in each layer in *l*- direction	Reinforcement in each layer in *t*- direction	f_c' (MPa)	$f_{yl} = f_{yt}$ (MPa)	α_2	H	$\frac{\sigma_1}{\sigma_y}$
Case a: Panels with different amounts of reinforcement in the *l*- and t- directions								
P45-2	P45-2-1A	8mm dia @ 71 mm on centre	8mm dia @ 142 mm on centre	29.2	530.9	45°	1.8 ≈ 2	-
	P45-2-1B			27.7				0.72
	P45-2-2A			30.7				0.98
	P45-2-3A			28.9				1.03
P45-4	P45-4-1A	8mm dia @ 71 mm on centre	6mm dia @ 142 mm on centre	31.6			3.91 ≈ 4	0.73
	P45-4-1B			33.0				0.72
	P45-4-2A			28.9				0.99
	P45-4-3A			29.7				1.04
Case b: Panels with reinforcement grid inclined to the directions of loading								
P27-1	P27-1-1A	8mm dia @ 90 mm on centre	8mm dia @ 90 mm on centre	36.6	530.9	26.5° ≈ 27°	1	0.71
	P27-1-1B			33.9				0.70
	P27-1-2A			34.1				0.96
	P27-1-2B			31.2				0.90
P64-1	P64-1-1A	8mm dia @ 90 mm on centre	8mm dia @ 90 mm on centre	31.5		63.5° ≈ 64°	1	0.74
	P64-1-1B			33.8				0.70
	P64-1-2A			32.1				0.99
	P64-1-2B			32.3				0.94



a) Schematic sketch

b) Photograph

Fig. 4 – Test setup



a) P45-2 b) P45-4

c)P27-1 d) P64-1

Fig. 5 – Reinforcement details of panel series

6.2 Loading protocol

To investigate the effect of the chosen parameters, panel specimens were tested under sequential tension–compression. A sequential tension–compression load path was selected to segregate the components of γ_{12}, $\gamma_{12}|\sigma_1$ and $\gamma_{12}|\sigma_2$. Initially, tension (σ_1) was applied along the 1-direction up to a certain target level. This is expressed as the terminal value of the ratio of σ_1/σ_y, where σ_y is the estimated yield stress (Table 1). It was maintained constant during the subsequent compression phase, to have constant estimated stress in the bars. Next, compression (σ_2) was applied along the 2-direction up to the crushing of concrete.

6.3 Test specimens

All the panel specimens were of dimensions 800 mm × 800 mm × 100 mm. The dimensions in plan were fixed based on the requirement that a minimum of three to four cracks form along the direction of tension within the test region. The thickness of 100 mm was selected such that the capacity of a panel with normal strength concrete, when tested in uniaxial compression, was less than the capacity of the tester i.e., 2000 kN. The reinforcement was provided in two layers and details are shown in Figure 5.

The following features were added to avoid premature failures.

1) Stitching reinforcement was provided along the tension edges of the panel to avoid premature cracking of the edges.

2) A panel consisted of an anchorage plate along each compression edge for adequate anchorage of bars during the application of tension load.
3) The compression edges were also strengthened by placing confining steel plates along the edges, to avoid premature crushing of the edges during the application of compression.

6.3.1 Instrumentation

Load cells of capacity 500 kN were used to measure the tension load applied by the hydraulic jacks. As there was no gap to place load cells on the compression side, a hydraulic jack connected in series to the compression jacks was placed in a separate reaction frame outside the panel tester to measure the compression load.

Deformations were measured using linear variable differential transducers (LVDTs). LVDTs were fixed only on the top face of the panel. As the bottom face was inaccessible, no LVDT was placed below the panel. The average strains were calculated from the measured deformations. Arrangement of the LVDTs is shown in Fig. 6. LVDTs 1 and 2 were used to record deformation along the compression direction (ε_2). LVDTs 3 and 4 recorded the deformation along tension direction (ε_1). LVDTs 5 to 8 recorded the deformations along the diagonals to quantify the average shear strain (γ_{12}).

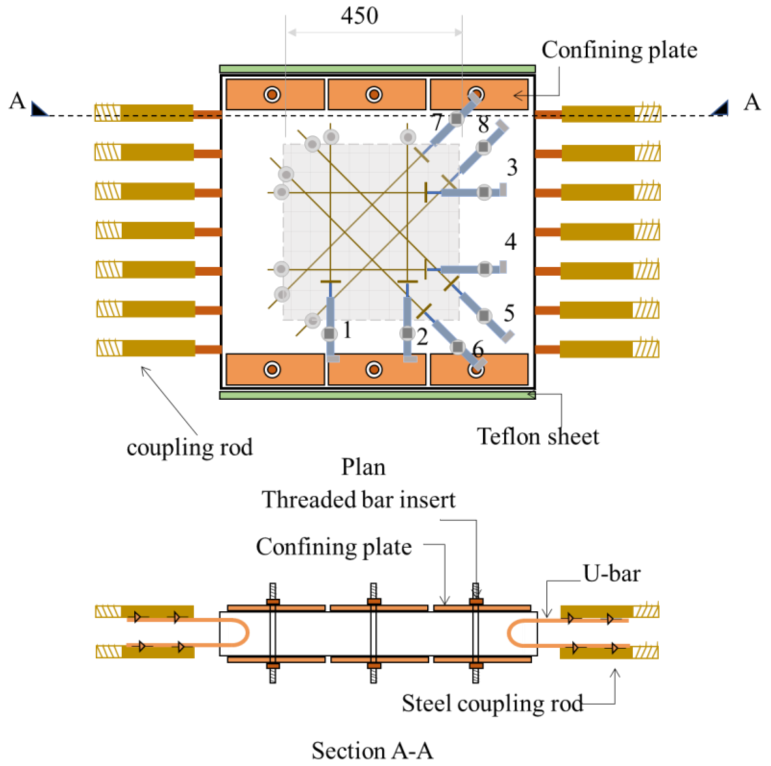
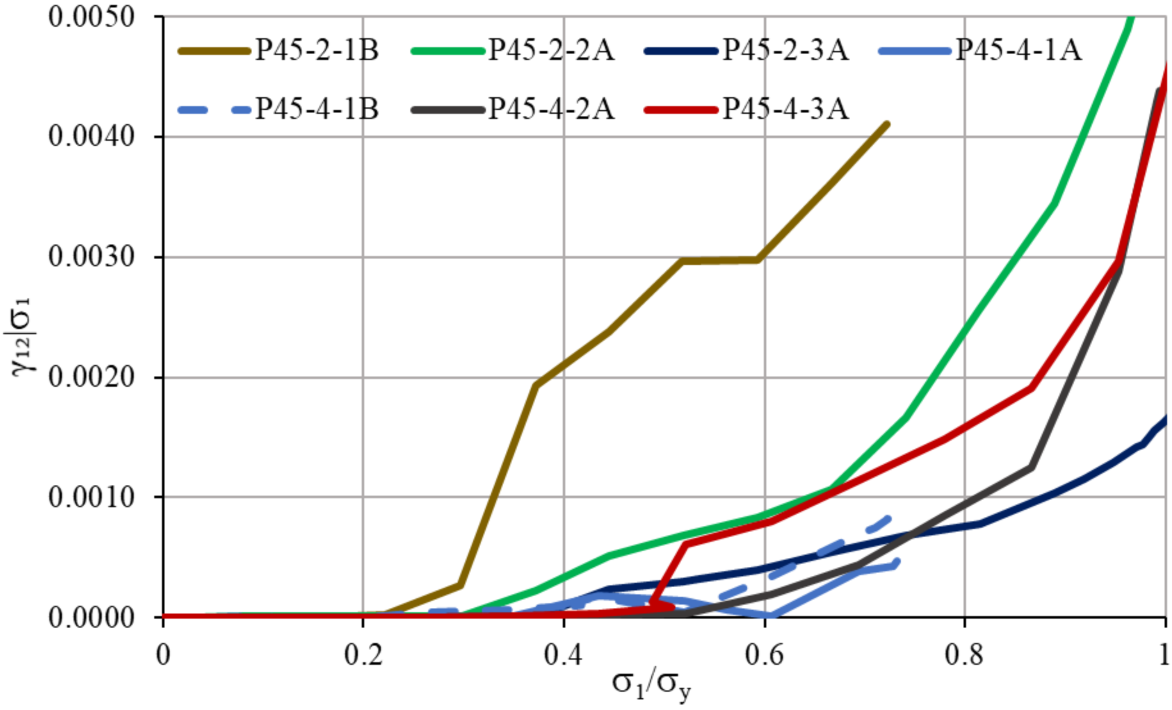


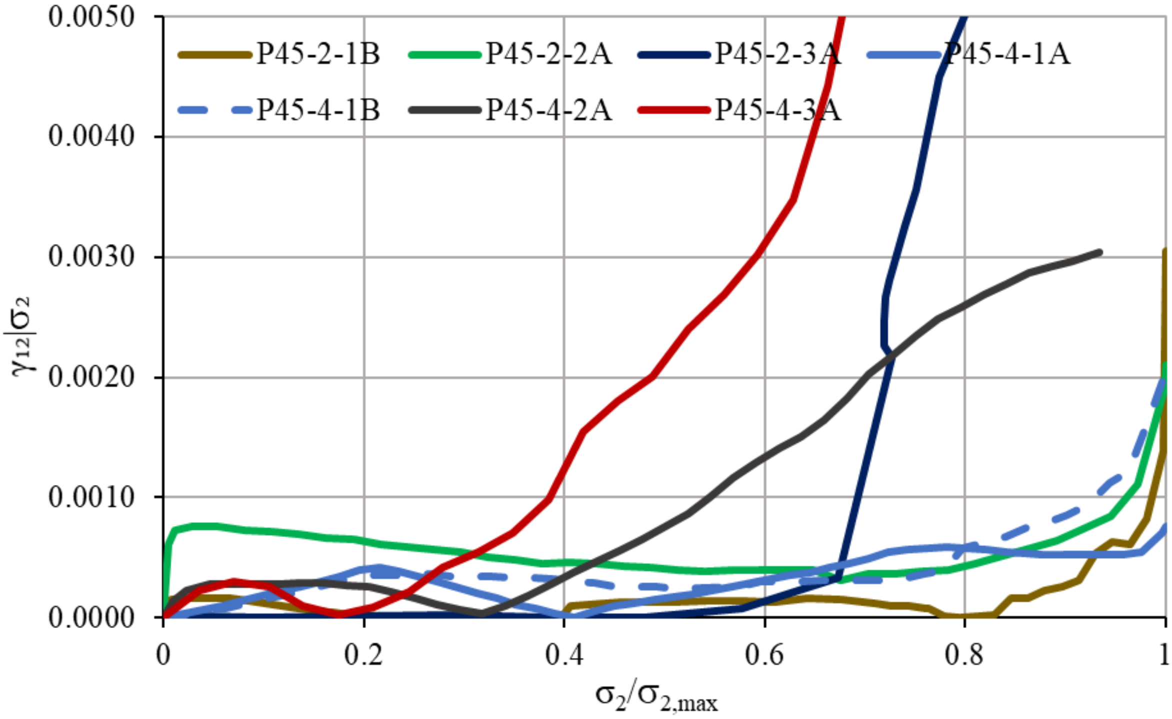
Fig. 6 – Instrumentation and additional features

7. Test Results

The data from the test results is presented as plots of numerical values of shear strains ($\gamma_{12}|\sigma_1$ and $\gamma_{12}|\sigma_2$ for the two phases of loading) versus applied normalised loads in Fig. 7 and Fig. 8 for Case a and Case b, respectively. The instantaneous tension, σ_1 is normalised with the calculated yield stress, σ_y. The instantaneous compression σ_2 is normalised with the recorded crushing stress, $\sigma_{2,max}$. In Fig. 7, the results for panels in Series P45-2 and P45-4 are presented. The plots do not show the results of Panel P45-2-1A as it failed prematurely. In Fig. 8, the results for panels in Series P27-1 and P64-1 are presented.

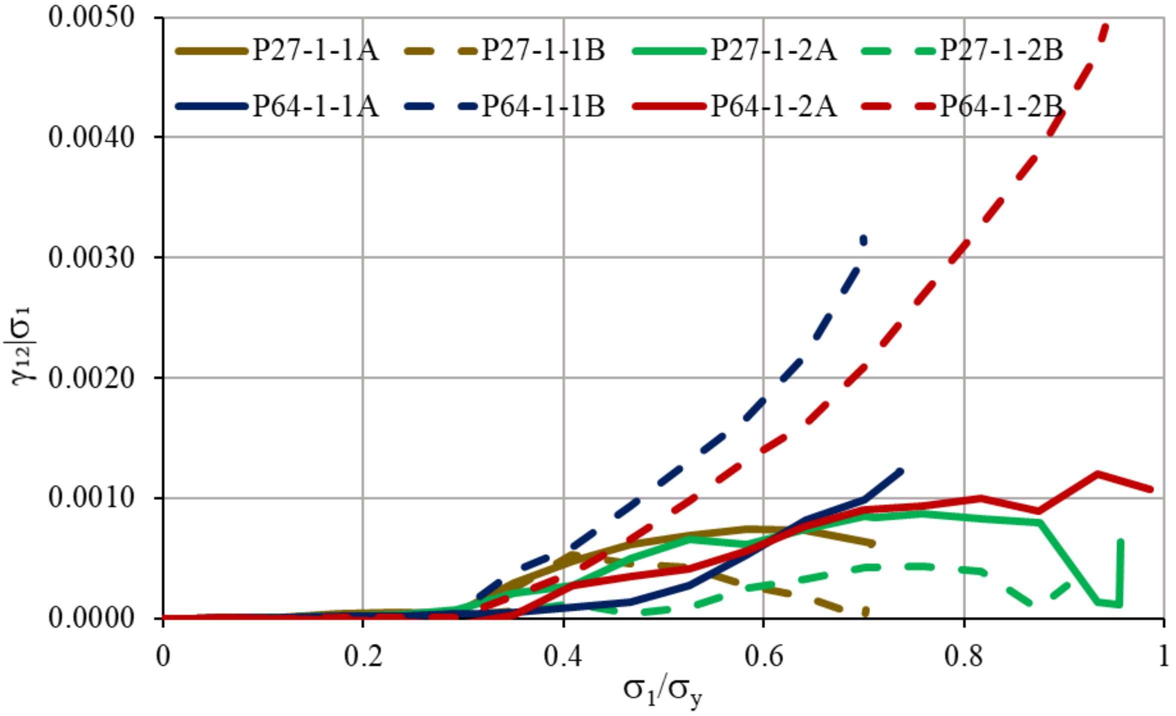


a) Shear strain versus normalized tensile stress

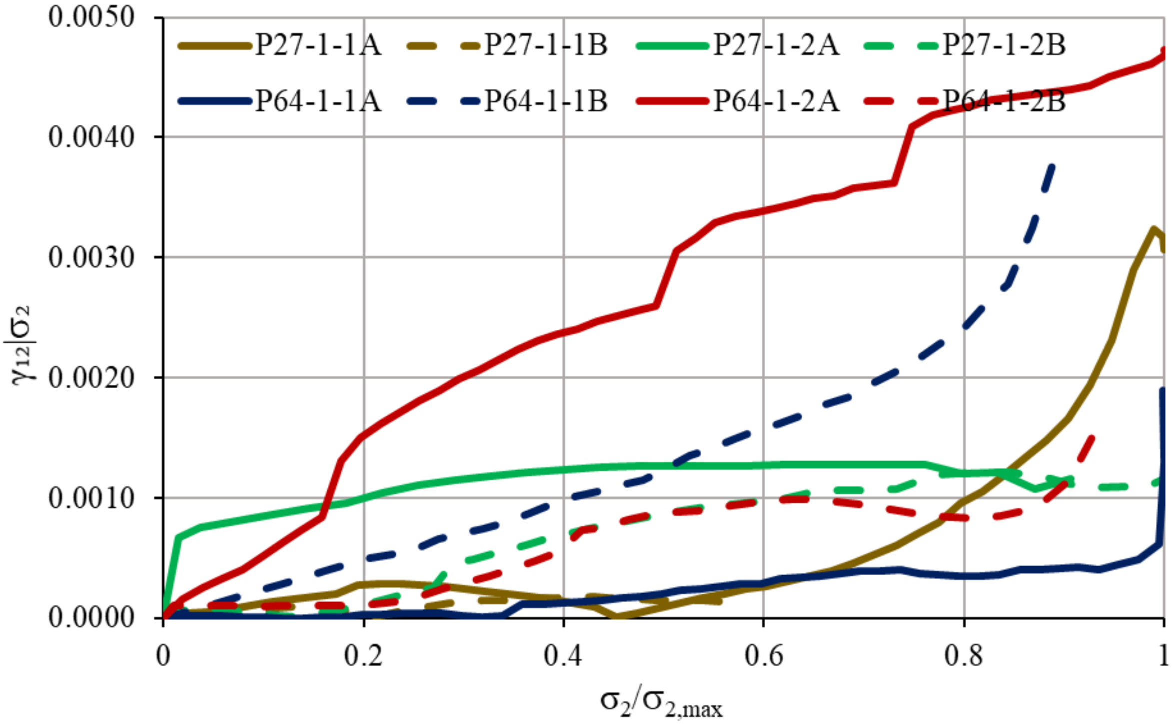


b) Shear strain versus normalized compressive stress

Fig. 7 – Panels with different amounts of reinforcement in *l*- and *t*- directions (Case a)



a) Shear strain versus normalized tensile stress



b) Shear strain versus normalized compressive stress

Fig. 8 – Panels with reinforcement grid inclined to the directions of loading (Case b)

8. Summary and Conclusions

The summary and conclusions of the work presented in the paper are as follows.

a. The formulation of the Softened Membrane Model (SMM) considers an empirical coefficient to reduce the shear capacity of a membrane element, with reinforcement asymmetric about the principal axes of applied stresses. This is demonstrated in the predicted shear stress versus strain behaviour of a panel tested under increasing biaxial tension–compression.

b. An approach to incorporate the effect of shear–extension coupling for asymmetric membrane elements, is presented based on the formulation of SMM. Two cases of asymmetry were discussed. First, for reinforcement inclined at 45° to the axes of applied principal stresses, if the amounts or capacities of reinforcement in the longitudinal and transverse directions vary, then shear strain develops in the principal axes of stresses. Second, when the orthogonal reinforcement grid is inclined to the principal axes with an angle other than 45° (within the range of 0° and 90°), then also shear strain develops in the principal axes of stresses.

c. An experimental program was carried out to quantify the generated shear strain by testing panels with asymmetric reinforcement, under sequential biaxial tension–compression. The test set-up, reinforcement details, instrumentation and results for 16 panels are presented. The shear strain in a panel for each phase of loading is plotted with respect to the corresponding normalized applied stress.

d. It is observed that shear strain is generated in the asymmetric panels, in the tension phase after the cracking of concrete. Similarly, in the compression phase, shear strain is generated with further cracking and damage of concrete.

The analysis to quantify the generated shear strain in terms of the geometric variables and identified stress variables, is in abeyance. The model developed for the shear strain will be used to incorporate the effect of shear–extension coupling in the algorithm of SMM. This will enhance the capability of SMM to predict the in-plane shear behaviour of a membrane element without symmetry of reinforcement.

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