A STOCHASTIC REAL-TIME HYBRID SIMULATION OF THE SEISMIC RESPONSE OF A MAGNETORHEOLOGICAL DAMPER

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ABSTRACT

Real-time hybrid simulation is an effective method to obtain the response of an emulated system subjected to dynamic excitation by combining loading-rate-sensitive numerical and physical substructures. Albeit, the parameters that characterize the hybrid model are often deterministic. The values of these parameters are regularly determined through deliberate simplifications, ignoring the associated uncertainties. However, the effect of uncertainties may be significant.

Stochastic hybrid simulation is an extension of the state-of-the-art hybrid simulation to address the dynamic response of uncertain structural systems under uncertain operating conditions. Under this concept, the parameters of the emulated system are treated as random variables with known probability distributions. The results of stochastic hybrid simulations are probability distributions of the structural response quantities of interest. The arising question is one of sensitivity, namely, to what extent each of the random variables affects the outcomes of stochastic hybrid simulations.

In this study, the seismic response of a structure with a magnetorheological (MR) damper is examined. The strongly nonlinear behavior of MR dampers amplifies the effects of the uncertainties of the model parameters on the response. A virtual hybrid model is used, where the structure and the MR damper are represented using numerical substructures. In a virtual stochastic hybrid simulation, parameters of the hybrid model and the excitation are treated as random variables in repeated real-time hybrid response simulations to the same random excitation. Based on this simulation data, surrogate models are developed. Multiple additional runs of the surrogate model give a deeper insight into the performance of the examined system under uncertainties. Global Sensitivity Analysis is performed to identify the effect of model and excitation parameters on the response of the system.

Keywords: stochastic real-time hybrid simulation, dynamic response, MR dampers, model predictive control, uncertainty quantification, surrogate modelling, sensitivity analysis

1 Introduction

Magnetorheological (MR) dampers are semi-actively controlled viscous dampers [1–3]. The distinguishing feature of MR dampers is the electromagnetic valve that controls the MR fluid flow between the cylinder chambers. A relatively small electric current is used to change the magnetic field in the valve, thus changing the orientation of the magnetic particles in the MR fluid. Consequently, the apparent viscosity of the MR fluid inside the valve changes, making it possible to control the flow and the magnitude of the viscous damping force delivered by the MR damper. Such control strategy is semi-active, in that a small amount of control power produces a large effect on the response of the controlled system. MR dampers have been used to control vibrations in civil as well as in automotive and aerospace structures.

Dynamic behavior of MR dampers is sensitive to the response rate, non-linear due to changes of MR fluid viscosity, and directly affected by the strategy deployed to control the magnetic field in the MR damper valve. In addition, the dynamic behavior of the structure with MR dampers depends on the parameters of the excitation, the structure, and the MR damper itself and is sensitive to the uncertainties associated with these parameters. Therefore, investigation of the dynamic response of structures with MR dampers requires rate-sensitive stochastic models.

Real-time hybrid simulation (RTHS) is an effective method to obtain the dynamic response of an emulated rate-sensitive system subjected to dynamic excitation by combining loading-rate-sensitive numerical and physical substructures [4, 5]. Conducting a RTHS requires a closed-loop tracking controller to ensure smooth interconnections at substructure interfaces [6, 7] and to compensate for time delays introduced due to inherent dynamics of the actuators while guaranteeing adequate reference signal tracking and robustness under uncertainties and external disturbances [8–10].

A significant aspect in RTHS is the presence of uncertainties. Stochastic real-time hybrid simulation (SRTHS) is an extension of the deterministic RTHS, where parameters of the system are treated as random variables with known probability distributions to capture the effect of uncertainties on the simulation outcomes. Uncertainties originate from the physical substructure (aleatory uncertainties, i.e. natural variability), the numerical substructures (epistemic uncertainties, i.e. lack of knowledge), and from experimental errors (systematic uncertainties, i.e. calibration errors and random uncertainties, i.e. measurement noise) [11, 12].

A probabilistic study of the seismic response of a 3-story building structure with an MR damper, presented in this paper, is conducted using virtual stochastic real-rime hybrid simulations (vSRTHS). The hybrid model consist of a finite element model of a 3-story building (the numerical substructure) and a computer model of the MR damper and its controller (the virtual physical substructure). The tracking controller of the hybrid model is a Model Predictive Controller (MPC) designed in conjunction with polynomial extrapolation and a Kalman filter. Hybrid model uncertainties were modeled by selecting specific parameters of the hybrid model and treating them as random variables with known probability distributions. The outcomes of each vSRTHS are probability distributions of the structural response quantities of interest. Polynomial Chaos Expansion (PCE) [13] and Kriging [14] surrogate models are developed based on repeated vSRTHSs. Global Sensitivity Analysis is performed, using Sobol' indices, to give insight on how uncertainties affect the structural response quantities of interest and in which way. It's worth mentioning that through PCE, Sobol' indices are derived at no extra computational cost, since they are calculated from the already computed PCE coefficients [15].

2 Problem formulation - The hybrid model

The reference structure under consideration is the 3-story building shown in Fig.1 [16], equipped with an MR damper between the ground and the first floor. The equations of motion (EOM) that describe its dynamics are:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{\Lambda}\ddot{x}_g + \mathbf{\Gamma} f \tag{1}$$

where $\mathbf{x} = [x_1, x_2.x_3]^T$, $\dot{\mathbf{x}} = [\dot{x}_1, \dot{x}_2.\dot{x}_3]^T$ and $\ddot{\mathbf{x}} = [\ddot{x}_1, \ddot{x}_2.\ddot{x}_3]^T$ correspond to the displacement, velocity and acceleration relative to the ground, $\ddot{x}_g$ is the ground motion and f corresponds to the force generated from the MR damper. The $\mathbf{M}, \mathbf{C}, \mathbf{K}$ matrices represent the mass, damping and stiffness of the structure, respectively, as follows:

$$\begin{aligned} \mathbf{M} = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix} Kg, \mathbf{C} = 1e4 * \begin{bmatrix} 1.408 & -0.787 & 0.044 \\ -0.787 & 1.494 & -0.635 \\ 0.044 & -0.635 & 0.722 \end{bmatrix} \frac{Ns}{m}, \\ \mathbf{K} = 1e7 * \begin{bmatrix} 2.605 & -2.313 & 0.594 \\ -2.313 & 3.256 & -1.442 \\ 0.594 & -1.442 & 0.927 \end{bmatrix} \frac{N}{m} \end{aligned} \tag{2}$$

Vector $\mathbf{\Lambda} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ is the ground motion influence vector, while vector $\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ represents the effect of the MR damper on the structure.

A state-space representation of Eq. 1 is used in vSRTHSs:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \tag{3}$$

where $\boldsymbol{x} = [x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3]^T$, $\boldsymbol{u} = [\ddot{x}_g, f]^T$, $\boldsymbol{y} = [x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3, \ddot{x}_1, \ddot{x}_2, \ddot{x}_3]^T$, and

$$\begin{aligned} \boldsymbol{A} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} & \boldsymbol{B} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & -\boldsymbol{M}^{-1}\boldsymbol{\Gamma} \end{bmatrix} \\ \boldsymbol{C} &= \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} & \boldsymbol{D} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & -\boldsymbol{M}^{-1}\boldsymbol{\Gamma} \end{bmatrix} \end{aligned} \tag{4}$$

For the virtual MR damper, the viscous + dahl model [17–19] was used, described by:

$$\begin{aligned} f(t) &= \big[k_{x_a} + k_{x_b} v(t)\big]\dot{x}_d(t) + \big[k_{w_a} + k_{w_b} v(t)\big]w(t) \\ \dot{w}(t) &= \rho\big(\dot{x}_d(t) - |\dot{x}_d| w(t)\big) \\ w(0) &= \frac{f(0) - \big[k_{x_a} + k_{x_b} v(0)\big]\dot{x}_d(0)}{k_{w_a} + k_{w_b} v(0)} \end{aligned} \tag{5}$$

where $\dot{x}_d(t)$ denotes the MR damper piston velocity, $v(t)$ the voltage input command, $f(t)$ the damping force, w the damper's nonlinear behavior, k_{x_a} and k_{x_b} the viscous friction coefficient, k_{w_a} and k_{w_b} the dry friction coefficient [18] and t refers to the simulation time. The parameter ρ is calculated as in [20] and selected to be $\rho = 47.95\ (cm^{-1})$. The friction parameters are calculated from linear regression as $k_{x_a} = 9.78\ (Nscm^{-1})$, $k_{x_b} = 40.75\ (Nscm^{-1}V^{-1})$, $k_{w_a} = 60.11\ (N)$ and $k_{w_b} = 344.78\ (NV^{-1})$. The inputs of the MR damper model are the displacement $x_d(t)$ and the voltage $v(t)$ and the output the force f.

As mentioned above, a relatively small electric current can change the behavior of the MR damper. In order to ensure optimal response, a command voltage rule is implemented. It consists of a bang-bang controller, designed for optimal voltage input computation (Fig.3). More specific when $sign(x_1(t)) = sign(\dot{x}_1(t))$ then the bang-bang controller provides the MR damper with the maximum input voltage, and as a result with the maximum force, otherwise with the minimum. This bang-bang controller is part of the MR damper and it's exclusively responsible for the internal behavior of the MR damper.

In order to interconnect the two substructures of the hybrid model, an actuator is essential. For this purpose a virtual linear actuator is used, modeled by a second-order transfer function. Such order of transfer function can represent quite accurately the dynamics of the actuator [21, 22]. The actuator model follows:

$$G_{act} = \frac{3060}{s^2 + 267s + 3060} \tag{6}$$

In Fig.2 the overall vSRTHS block diagram is displayed. The reference system of Fig.1 is separated in two parts; the numerical and the virtual physical substructures. The numerical substructure corresponds to the 3-story structure defined by the matrices of Eq.2, while the virtual physical to the actuator in series with the MR damper described by Eq.6 and 5 respectively. In between the substructures, a tracking controller is implemented to ensure smooth interaction of the two parts. This will be addressed in detail later on in section 3. At every time step of the simulation process the numerical substructure provides the virtual physical with a desirable displacement to be tracked, x_1. This is the displacement of the first floor. Then the generated force, f, from the MR damper is being calculated and feed back in the numerical substructure, completing the coupling of the two parts. Additionally, in order to capture more realistic results, white noise is added in the calculated force, f, representing measurement noise from the load cell. The force time series generated from the MR damper is shown in Fig.5. The El Centro 1940 ground acceleration record is used as the reference ground motion $\ddot{x}_g$ (Fig.4). The sampling frequency was set to 4096 Hz. For the numerical integration scheme, the RK4 (Runge–Kutta) method is used with a fixed time step of $1/4096$ sec.

In this study, the selected Quantities of Interest (QoI) of the overall hybrid model are the: 1) absolute mean displacement of the first story, x_1 (m) and 2) absolute mean force generated from the MR damper, f (N). Six parameters of the hybrid model (four from the numerical substructure and two from the physical) are selected to be random with known probability distributions. The distributions characteristics of these random variables are shown in Table 1. The values of Eq.2 are calculated based on the mean values of the selected random parameters. In section 4 the random parameters and the overall SRTHS framework are explained in detail.

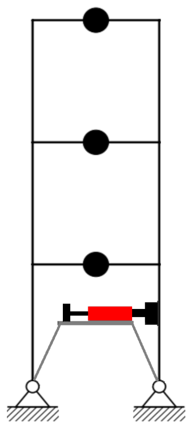


Fig. 1: Reference structure

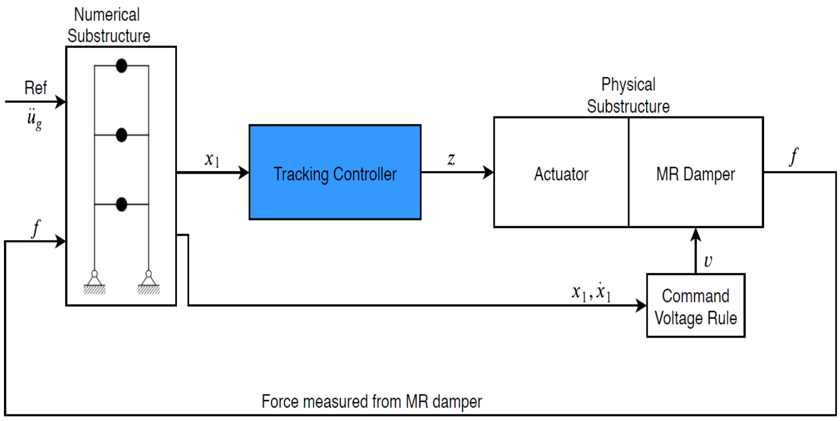


Fig. 2: Block diagram of the hybrid model in vSRTHS

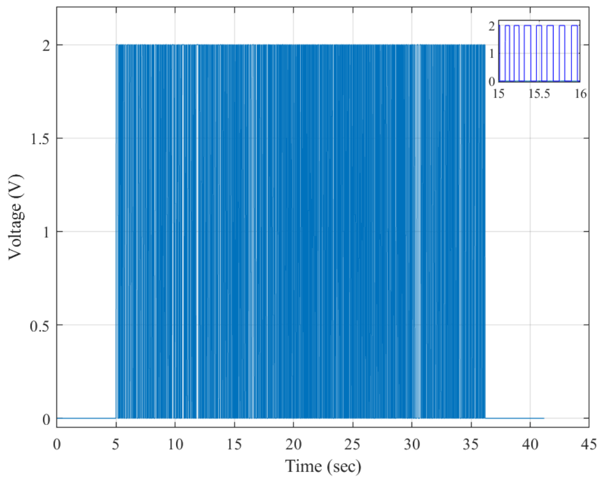


Fig. 3: Voltage command rule

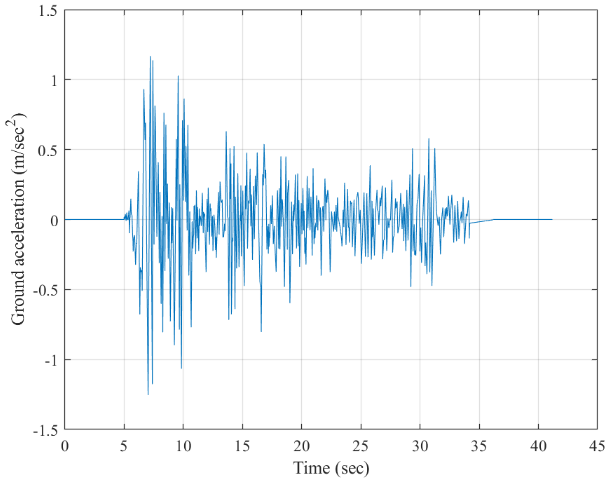


Fig. 4: Reference ground motion, El Centro 1940

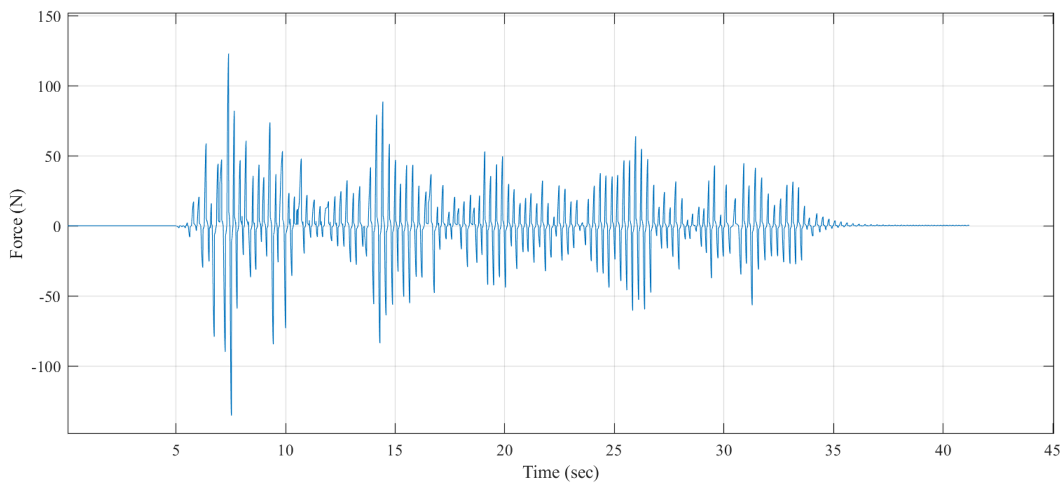


Fig. 5: Force generated from MR damper, single deterministic case

Table 1: Stochastic input variables and their characteristics

Input Variable	Probability Distribution	Nominal Value (μ)	Standard Dev. (σ)	Coef. of Variat. (CV) (%)	Parameter Description	Units
L_b	Lognormal	0.762	0.1524	20	Beam length	m
L_c	Lognormal	0.635	0.127	20	Column length	m
M	Lognormal	1000	200	20	Floor mass	kg
Z	Lognormal	0.05	0.01	20	Damping ratio	-
K_{x_a}	Lognormal	9.78	1.956	20	Viscous friction coef. of MR	Ns/cm
K_{w_a}	Lognormal	60.11	12.022	20	Dry friction coef. of MR	N

3 The tracking controller

The goal of the tracking controller is for the control plant (in this study, the control plant corresponds to the actuator, which is part of the virtual physical substructure) to follow the reference signal generated from the numerical substructure at every time step of the numerical integration scheme. This translates in adequate reference tracking, time delay compensation and robustness under disturbances and uncertainties. In this paper, the tracking controller consists of an MPC [23, 24] along with a polynomial extrapolation algorithm [25]. The architecture of the tracking controller can be found in Fig.6.

In MPC, an optimization problem is being solved at every control interval. A control interval is a set of continuously time steps of the simulation. It serves as an internal time step for the MPC in order to gather enough measurement feedback and compute the following new control laws for the control plant. This result in a control sequence that is applied to the hybrid model at every interval, providing each time the control plant with new commands while satisfying hard constraints on the inputs, states and/or outputs of the system under consideration. This corresponds also to the main difference between MPC and classical control. In the latter, only one control law is being used during the whole simulation process and it's being computed offline prior to the start of the simulation, remaining the same for the overall duration. MPC is desirable for SRTHS applications, since uncertainties and physical limitations in the experimental equipment are always present. Since a new control law is computed at every control interval, the controller can compensate for the uncertainties that are being introduced in the system at that specific interval.

The cost function that was used in the MPC formulation is:

$$\mathbf{J}^*(\hat{x}_1, \hat{x}_m, z_k) = \sum_{i=1}^{P} \left\{ w^{x_m} \left[\hat{x}_1(k+i|k) - \hat{x}_m(k+i|k) \right] \right\}^2 \tag{7}$$

where k represents the current control interval, P the prediction horizon, w^{x_m} the tuning weight of the measured displacement x_m, $\hat{x}_1(k+i|k)$ the reference value to be tracked at the i-th prediction horizon step, $\hat{x}_m(k+i|k)$ the predicted values of the measured displacement at the i-th prediction horizon step and $z_k^T = [u(k|k)^T u(k+1|k)^T \dots u(k+1-P|k)^T]$ consists of the control laws for every control interval k. The control interval is set to a sampling frequency of 1024 Hz, one fourth of the overall simulation frequency, the prediction horizon is set to $P = 10$, the tuning weight $w^{x_m} = 64.072$, the next reference value $\hat{x}_1(k+i|k)$ is calculated from the numerical substructure only, and the $\hat{x}_m(k+i|k)$ is predicted using a Kalman filter, designed for optimal state estimation. The selection of the control interval, the prediction horizon and the weight is made through trail and error as there exists a trade-off between optimal controller performance and computational effort. The latter can effect significantly the simulation speed, crucial characteristic of RTHS.

The control law is derived in every control interval k, from the solution of the following optimization problem.

$$\min_{z_k} \mathbf{J}^*(\hat{x}_1, \hat{x}_m, z_k) \tag{8}$$

subjected to the following constraints:

$$\begin{aligned} -250 \leq \hat{x}_1(k+i-1|k) \leq 250, \quad (mm) \quad i = 1, \dots, P \\ -100 \leq \dot{\hat{x}}_1(k+i-1|k) \leq 100, \quad (\frac{mm}{sec}) \quad i = 1, \dots, P \end{aligned} \tag{9}$$

The above constraints correspond to the physical limitations of the actuator, which has a maximum stroke of $\pm 250\ mm$ and maximum velocity of 100 $\frac{mm}{sec}$.

MPC can guarantee very good tracking performance and robustness under uncertainties and disturbances. However, since in such kind of applications even small tracking errors can alter significantly the simulation output resulting in not representative results, a fourth-order polynomial extrapolation method is used [25], in order to further compensate for the time delays and additionally improve MPC's performance. Is expressed as follows:

$$\hat{x}1_k = 5x1_{(0,k)} - 10x1_{(1,k)} + 10x1_{(2,k)} - 5x1_{(3,k)} + x1_{(4,k)} \tag{10}$$

,where $x1_{(i,k)} = x_1(t_k - iT_d)$ is the discrete reference signal by adding shifts of a pure time delay (T_d) by integer values of i. The polynomial coefficients were obtained using the Lagrange basis functions.

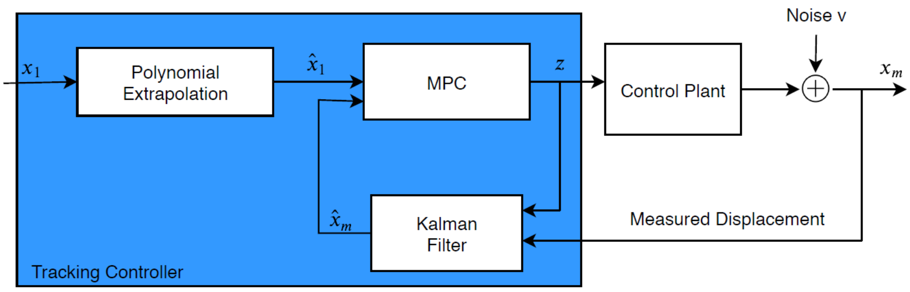


Fig. 6: Block diagram of the vSRTHS tracking controller

However, the control plant corresponds only to the actuator and doesn't include the remaining virtual physical substructure, MR damper, the output of the overall virtual physical substructure follows quite well its input, the reference signal. In this way the complexity of the control system is reduced providing at the same time adequate performance and robustness. The reference tracking performance of the controller can be shown in Fig.7 for a single deterministic case. The time delay is almost zero and the Root-Mean-Square (RMS) of the tracking error is 1.9%. RMS represents the difference between the measured and desired displacement of the actuator.

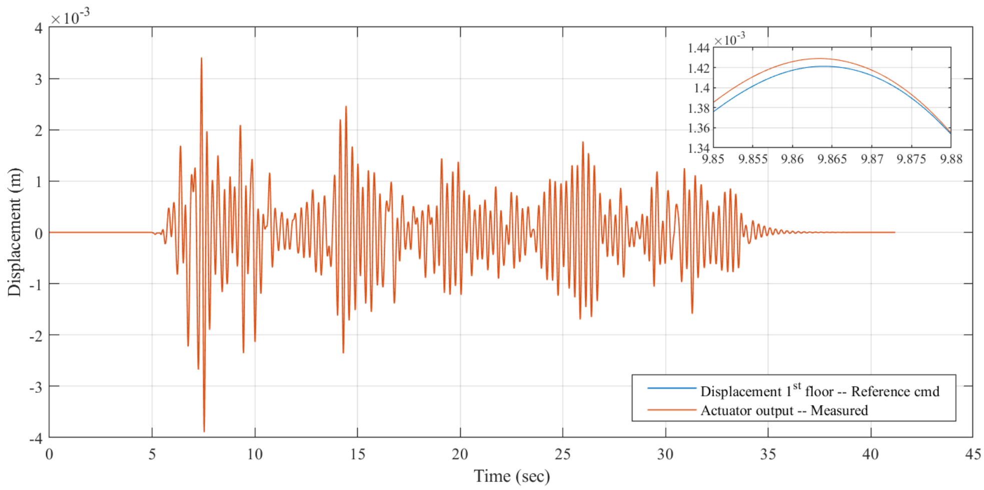


Fig. 7: Performance of tracking controller, single deterministic case

4 The vSRTHS framework

In order to investigate and understand how uncertainties propagate throughout the hybrid model and moreover in which way they affect the output QoI, the proposed vSRTHS framework is used. The six parameters shown in Table 1 were selected in order to vary the dominant parameters of the hybrid model. For the numerical substructure these are the mass, damping and stiffness of the 3-story structure, while for the physical substructure these are the parameters which correspond to the nonlinear behavior of the MR damper. The latter is of particular importance since MR dampers have a strongly nonlinear dynamic response and modelling these devices results in deliberate simplifications, ending up with introducing even more uncertainties.

Out of the above six probability distributions, 200 samples were generated using the Latin Hypercube Sampling (LHS) methodology. The hybrid model (Fig.2) was simulated 200 times, one for every sample, combining all six stochastic input variables at every iteration. The output of the 200 simulations are probability distributions of the two QoI. The stochastic inputs along with the output distributions are shown in Fig.8. In this study the QoI are not focusing on the control performance and in the time delay compensation of the actuation system, since the interest is how uncertainties affect the structure performance. However, because the goal is to conduct real-time hybrid simulation, is crucial to adequately compensate for time delays in order to obtain representative results. The results are quite promising that the tracking controller can compensate well enough for the inherent dynamics of the actuation system and therefore encourage us to move forward and conduct SRTHS.

For each QoI, two surrogate models, based on PCE and Kriging are created and trained on data captured from the 200 simulation runs. 80% of the 200 data was used to train the meta-models and the remaining 20% to validate them. The validation error of the surrogates was for both QoI below 4%. The adaptive sparse Least Angle Regression (LAR) methodology is used to construct the PCE, while for the Kriging a quadratic trend type is used along with the Matérn correlation family. Maximum-likelihood and the hybrid generic algorithm has been employed as of the estimation and optimization methods respectively [13, 14]. In order to obtain a better insight on how uncertainties propagate throughout the hybrid model, one millions samples are generated from the surrogates of the QoI. The outputs are probability distributions of the QoI. These are illustrated in the Fig.9a and 9b along with the originally obtained data from the 200 runs of the hybrid model. For QoI.1: x_1, the PCE surrogate is chosen, while for QoI.2: f the Kriging, since those had the best fit with the initial data. Surrogates are used because they can reproduce quite accurate the initial model with a significantly lower computational cost.

Furthermore, Global Sensitivity Analysis (GSA) is conducted using second order Sobol' indices in order to understand how the selected input variables, the uncertainties, affect the QoI and in which way [26]. In Fig.10a and 10b the total Sobol' indices of the two QoI are presented and is observed that the selected uncertainties affect both QoI almost in the same way. L_c has the most significant effect with 85.13% while M follows with 13.76%. The development and implementation of the surrogate modelling as well as the GSA was performed with the UQLab software framework developed by the Chair of Risk, Safety and Uncertainty Quantification in ETH Zurich [27].

5 Conclusions

In this study, a vSRTHS framework has been developed in order to exceed traditional deterministic simulations and investigate how actual uncertainties alter the dynamic response of the structure under consideration. For this reason, a hybrid model was developed consisting of a 3-story building as of the numerical substructure and an actuator in series with a MR damper as of the virtual physical substructure. The output QoI of the hybrid model are the absolute mean displacement of the first story and the absolute mean force generated from the MR damper. A tracking controller based on MPC and polynomial extrapolation was designed in order to ensure smooth interconnection between the substructures and to provide adequate reference signal tracking, time delay compensation and robustness under uncertainties. Selected parameters of the hybrid model were selected to be random with known probability distributions. These random parameters represent potential uncertainties of the overall hybrid model as if it was physical. The random parameters were propagated through the hybrid model 200 times generating probability distributions of the QoI. Based on these data, surrogate models were trained in order to investigate the QoI under larger sample data. One millions runs of the surrogates are conducted providing us with more confident probability distributions of the QoI, understanding better how the random parameters propagate throughout the hybrid model. Finally, GSA is conducted, obtaining metrics on how each one of the random parameters affect the QoI.

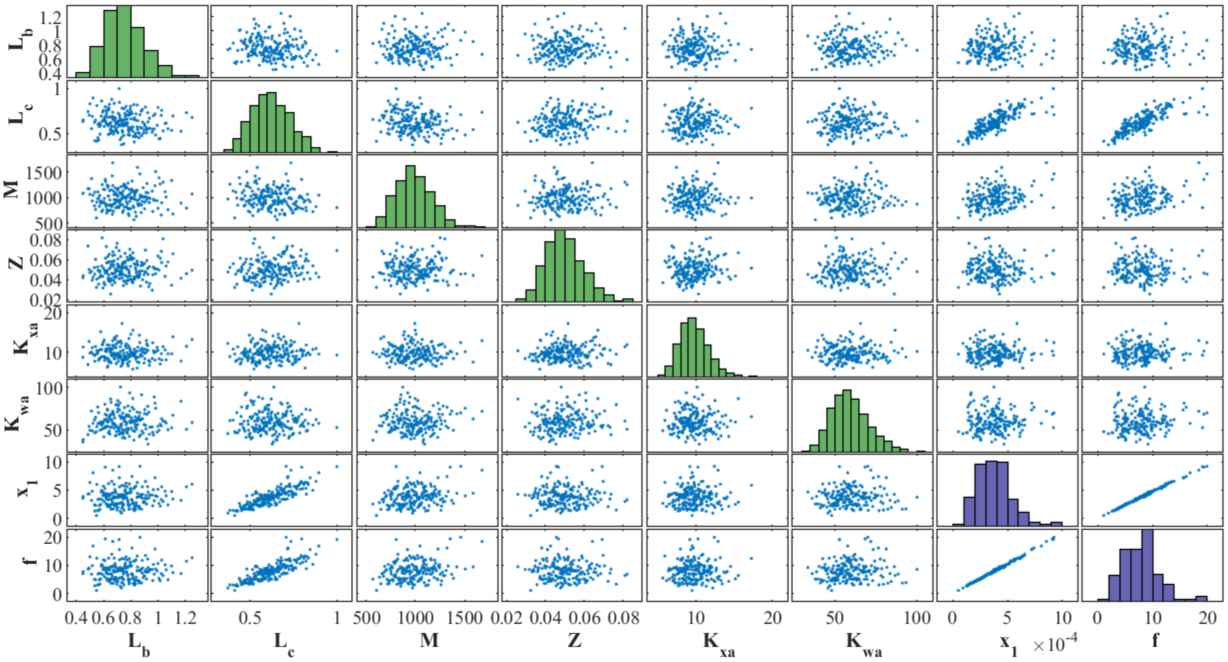
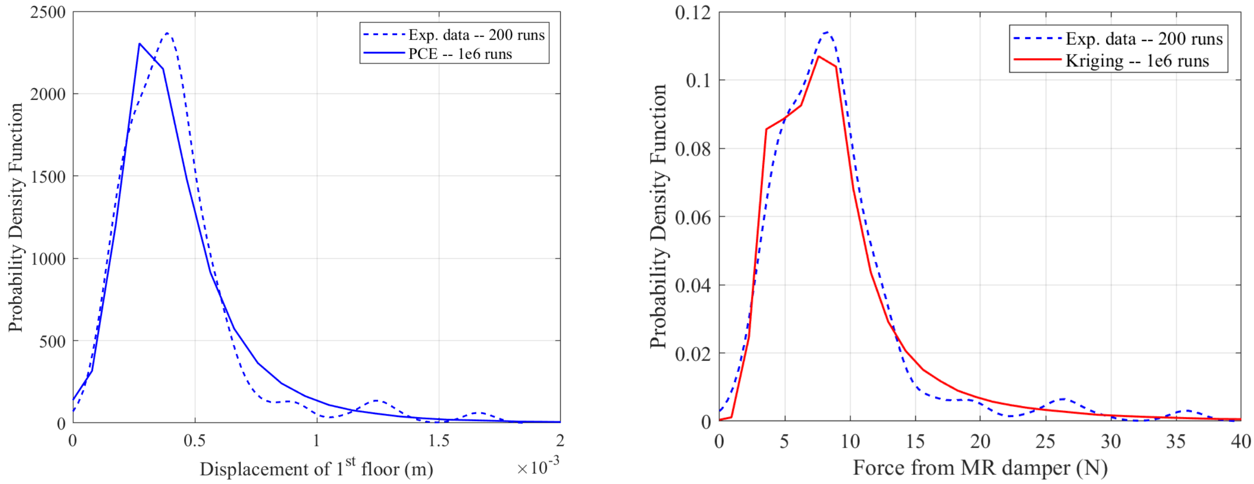
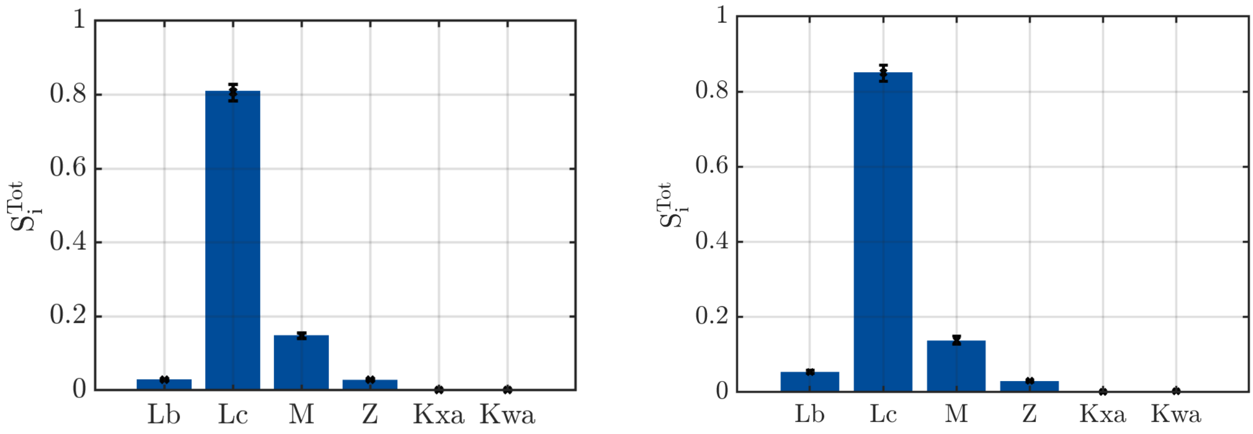


Fig. 8: Probability distributions of stochastic input variables and output QoI



(a) QoI.1: Absolute mean value of 1st floor displacement, x_1 **(b)** QoI.2: Absolute mean value of MR damper's force, f

Fig. 9: Kernel densities of QoI



(a) QoI.1: x_1 **(b)** QoI.2: f

Fig. 10: Total Sobol's indices of QoI

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