



AMBIENT VIBRATION TESTS AND MODAL PARAMETER IDENTIFICATION OF THE LAXIWA DAM IN CHINA

S. Li⁽¹⁾, J.T. Wang⁽²⁾, G.H. Luo⁽³⁾, Z.Q. Xiang⁽⁴⁾

⁽¹⁾ Ph.D. candidate, Tsinghua University, shuai-li16@mails.tsinghua.edu.cn

⁽²⁾ Professor, Tsinghua University, wangjt@tsinghua.edu.cn

⁽³⁾ Ph.D. candidate, Tsinghua University, luogh18@mails.tsinghua.edu.cn

⁽⁴⁾ Ph.D. candidate, Tsinghua University, xiangzq19@mails.tsinghua.edu.cn

Abstract

Modal parameter identification is essential for understanding the real dynamic characteristics of structures, updating the finite element model, and detecting damage, which is the core of the structural health monitoring. With the construction of super high arch dams in China, it is necessary to identify their modal parameters accurately. In this paper, two ambient vibration tests of the Laxiwa arch dam (250 m high) were conducted in the cases of water discharging and non-discharging cases. The modal parameters are identified using Stochastic Subspace Identification (SSI) algorithm and Frequency Domain Decomposition (FDD). Furthermore, the finite element model is updated using the identified modal parameters. The results show that the identified frequencies using two methods are almost the same for the first five modes. And the identified modal parameters under non-discharging case are consistent with those identified under discharging case. This indicates that the ambient vibration test is the ideal way to identify the modal parameters. The finite element model of the dam is updated according to the identified modal parameters, showing an agreement with the identified modal parameters. The continuous modal tracking and online health evaluation will be conducted in the future.

Keywords: modal parameter identification, Laxiwa dam, ambient vibration tests, finite element modal, SSI and FDD



1. Introduction

With the intensive construction of the infrastructure in the world, such as buildings, bridges, and dams, structural health monitoring (SHM) is a hot subject that has deserved much attention around the international researchers [1]. Especially in China, several super high concrete arch dams have been built or are under construction, such as Jinping I dam (305 m high), Xiaowan dam (294.5 m high), Baihetan dam (289 m high), Xiluodu dam (285.5 m high), in the west region with high seismic intensity and frequent earthquake occurrence. The safety of these super high arch dams is a concern because their failures could result in unacceptable loss of human lives and substantial damage to properties [2]. The modal parameters, including structural frequency, damping ratio, and mode shape, which are commonly considered related to the structural damage and deterioration, may be used for evaluating the health condition of super high dams [3]. Moreover, the identified modal parameters can be used to calibrate and update the seismic analysis model, and thus providing a more realistic response and reliable safety evaluation of super high dams [4]. Meanwhile, the identified modal parameters can be used as the baseline of the damage detection of the dams. Therefore, accurate modal parameter identification of structures using the vibration response is the core of the SHM.

This study focuses on the ambient vibration test and modal parameter identification for high arch dams. The modal parameters of the Laxiwa dam (250 m high) are identified as a case study using Frequency Domain Decomposition (FDD) and Stochastic Subspace Identification (SSI), respectively. And the finite element model is updated using the identified results. This paper is organized as follows: the theories of the two identification algorithms are briefly described in Section 2. The ambient vibration test of the Laxiwa dam is introduced in Section 3. Section 4 shows the identified modal parameters of the Laxiwa dam under water discharging and non-discharging cases, respectively. The finite element model is updated in Section 6, and the conclusions of this work are summarized in Section 7.

2. Theories of two identification methods

2.1 Stochastic subspace identification (SSI)

The discrete stochastic state-space model with white noise excitation can be expressed as follows according to the structural dynamic theory [5]

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \quad (1)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

where \mathbf{x}_k , \mathbf{y}_k , \mathbf{A} , and \mathbf{C} are the discrete-time state vector at time instant k , the sampled outputs vector, the state matrix, and the discrete output matrix, respectively; \mathbf{w}_k and \mathbf{v}_k are the zero-mean white noise.

The covariance matrices of the measured structural responses can be expressed as

$$\mathbf{R}_i = E[\mathbf{y}_{k+i}\mathbf{y}_k^T] \approx \frac{1}{j} \sum_{k=0}^{j-1} \mathbf{y}_{k+i}\mathbf{y}_k^T \quad (3)$$

where T is the transpose operator, and j is the number of measured responses involved in the calculation. The matrix Θ is defined as follows

$$\Theta \stackrel{\text{define}}{=} \begin{bmatrix} \mathbf{R}_i & \mathbf{R}_{i-1} & \cdots & \mathbf{R}_1 \\ \mathbf{R}_{i+1} & \mathbf{R}_i & \cdots & \mathbf{R}_2 \\ \cdots & \cdots & \ddots & \vdots \\ \mathbf{R}_{2i-1} & \mathbf{R}_{2i-2} & \cdots & \mathbf{R}_i \end{bmatrix} \quad (4)$$

The matrices \mathbf{A} and \mathbf{C} can be obtained by performing Eq. (4) using singular value decomposition (SVD) and the Moore-Penrose pseudoinverse [1]. The modal parameters can be identified from \mathbf{A} and \mathbf{C} hereafter. For the practical structures, it is not possible to predict the model order that well characterizes the dynamic



behavior of the structure. An overestimated order, which may cause the spurious modes, is usually given for identification. The stabilization diagram [6] and the Density-Based Spatial Clustering of Applications with Noise (DBSCAN) [7] are used for separating the physical and spurious modes.

2.2 Frequency domain decomposition (FDD)

The FDD is a well-known frequency domain technique for operational modal analysis, and the first step of the FDD is to calculate the cross power spectrum matrix $\mathbf{G}(\omega_r)$ of each measured point

$$\mathbf{G}(\omega_r) = \begin{bmatrix} PSD_{11}(\omega_r) & PSD_{12}(\omega_r) & \cdots & PSD_{1N}(\omega_r) \\ PSD_{21}(\omega_r) & PSD_{22}(\omega_r) & \cdots & PSD_{2N}(\omega_r) \\ \vdots & \vdots & \ddots & \vdots \\ PSD_{N1}(\omega_r) & PSD_{N2}(\omega_r) & \cdots & PSD_{NN}(\omega_r) \end{bmatrix} \quad (5)$$

where PSD_{ij} represents the cross-power spectral of the i -th and the j -th measured points at ω_r ; it represents the self-power spectrum of the i -th measured point when $i = j$.

The SVD of the $\mathbf{G}(\omega_r)$ is given as follows

$$\mathbf{G}(\omega_r) = \mathbf{U}_i \Sigma_i \mathbf{U}_i^H \quad (6)$$

where \mathbf{U}_i is an orthonormal matrix that contains the singular vectors of $\mathbf{G}(\omega_r)$, and Σ_i is the diagonal matrix holding the corresponding singular values. According to Ref. [8], the singular vectors are associated with the mode shapes, and the singular values are related to the information of modal frequencies. Therefore, the frequencies of the structure can be obtained from the local maxima of the first singular values plot, whereas the mode shapes are estimated from the associated singular vectors.

3. Ambient vibration test of the Laxiwa dam

The 250-m-high Laxiwa dam is a concrete double curvature arch dam located on the Yellow River in west China, as shown in Fig.1. The crest length is 459.64 m. The base and crest of the dam are at El.2210 m and El.2460 m, respectively. The normal and dead water levels are 2452 m and 2440 m, respectively. There is an underground powerhouse on the right bank with the total installed capacity of 4200 MW.



(a) dam (b) Cross-section of crown cantilever

Fig. 1 Laxiwa double-curvature arch dam

The ambient vibration test was conducted for three weeks in the period of July 7, 2019, to July 27, 2019. During that period, the dam experienced flood discharging and non-discharging conditions. The corresponding water level remains about 2452 m. To reduce the cable length and electrical interferences, three distributed digitizers are used along the crest of the dam, as shown in Fig. 2. In those tests, three digitizers with 24 bit are used to obtain the high-quality vibration data. Each digitizer is composed of 5



uniaxial force balance accelerometers (as shown in Fig. 2 by circle point marks), which are radially installed along with the upper gallery. To perform modal analysis, a good synchronization of the data recorded by all digitizers is assured by the GPS antennas.

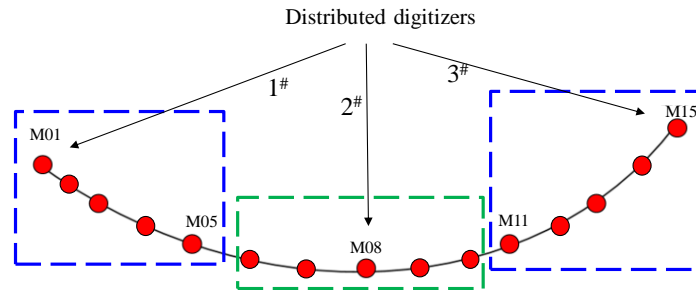
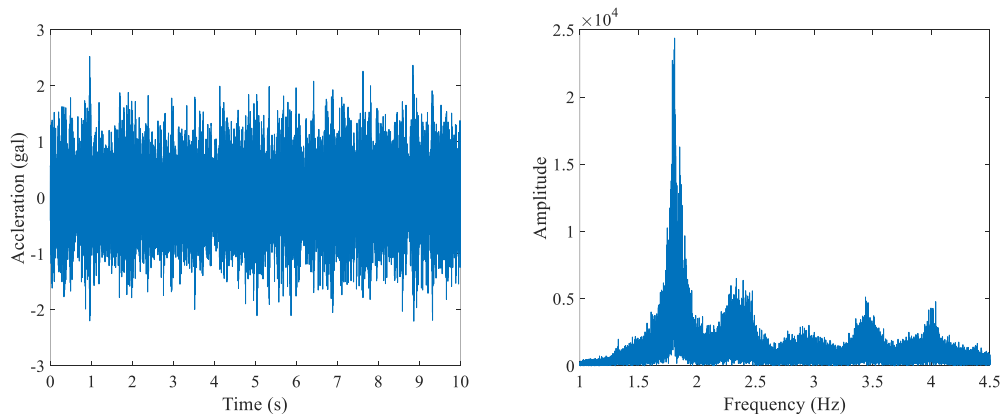


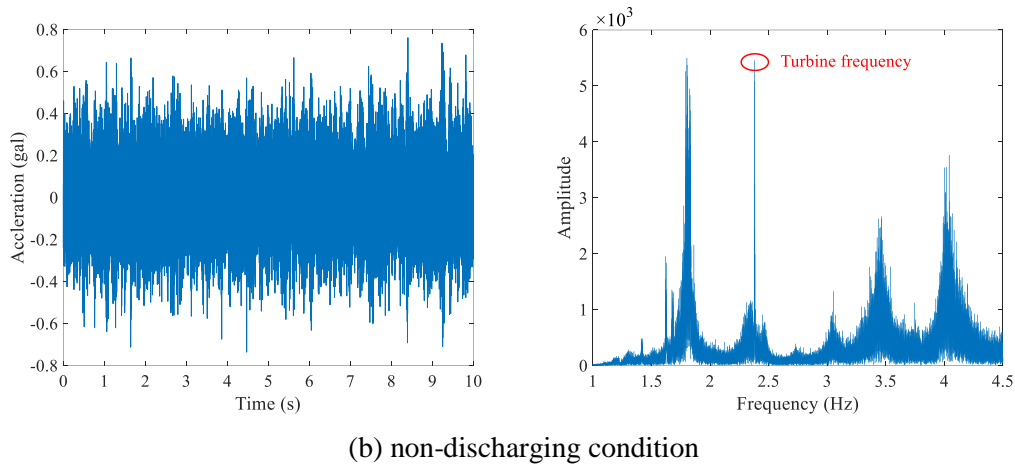
Fig. 2 Layout of the measured points

The force balance accelerometer (941-B from the Institute of Engineering Mechanics, China Earthquake Administration) used in the test has a dynamic range of 140 dB, and the frequency bandwidth goes from DC to 200 Hz. These accelerometers are configured to measure in the range $-0.25\text{ g}/+0.25\text{ g}$ to allow the accurate characterization of low acceleration signals. The monitoring system is configured to continuously record the acceleration time series with a sampling rate of 102.4 Hz. These data are automatically stored in the field digitizers and then downloaded to the computer.

The acceleration time histories of M05 under flood discharging and non-discharging period are selected to show in Fig. 3. Their Fourier spectra are also presented. It can be seen that the acceleration during flood discharging is approximately 1.5 gal, 3 times of that from the non-discharging condition. The peaks near 1.8 Hz, 2.3 Hz, 2.9 Hz, 3.4 Hz, and 4.0 Hz are observed according to the Fourier spectra, and they should be the first five modes of the Laxiwa dam. In addition, a very sharp peak of 2.38 Hz is observed under the non-discharging condition (Fig.3 (b)), which is attributed to the turbine frequency and will be discussed later.



(a) discharging condition



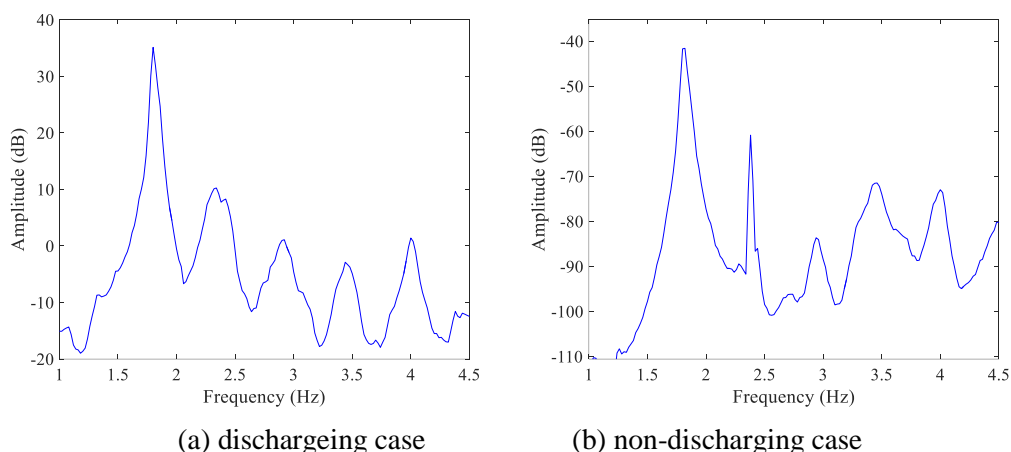
(b) non-discharging condition
Fig.3 Acceleration responses at M05 and their Fourier spectra

4. Modal identification based on ambient vibration data

In this section, the modal parameters of the Laxiwa dam are identified using FDD and SSI methods based on the ambient vibration test, respectively. The pre-processing of the measured data is accomplished before analysis, which includes the elimination of offsets and filtering to 10 Hz with an eighth-order low-pass Butterworth filter (the first five vibration frequencies are below 5 Hz).

4.1 Modal parameters identified from FDD

Fig. 4 shows the first singular value plots of the FDD under discharging and non-discharging cases. It can be seen that the peaks about 1.8 Hz, 2.3 Hz, 3.0 Hz, 3.4 Hz, and 4.0 Hz should be selected as the modal parameters. However, the peak of the second mode, about 2.3 Hz shown in Fig.4(b), is very sharp. The reason is that it was affected by the turbine frequency of about 2.38 Hz, which is close to the second mode. In contrast, the turbine frequency has less effect on the discharging case. This may be due to the amplitude of the vibration signal of the discharging case is larger than the turbine vibration. The turbine-induced vibration problem needs further investigation.



(a) discharging case (b) non-discharging case
Fig. 4 Singular value plots of FDD

The identified frequencies under discharging and non-discharging cases using FDD are summarized in Table 1, and it shows that the identified frequencies under the two cases are almost the same. In addition, it is worth noting that the first two modal frequencies of the Laxiwa dam are not close. This is different from the prior knowledge about the high arch dam.



Table 1 Identified modal parameters based on ambient vibration tests using FDD

Cases	Identified frequency (Hz)				
	1st	2nd	3rd	4th	5th
Non-discharging	1.80	2.35	2.93	3.44	4.00
Discharging	1.80	2.34	2.92	3.44	4.00
MAC	0.98	0.99	0.97	0.96	0.96

For the sake of simplicity and clarity, only the mode shapes under the discharging case are presented in Fig. 5. It can be seen that the first five mode shapes are antisymmetric, symmetric, symmetric, antisymmetric, and antisymmetric, respectively. This is consistent with the general law of arch dams. The MAC values between the identified mode shapes under non-discharging and discharging cases are summarized in Table 1. It can be seen that they are all larger than 0.95 for the five modes, indicating a well matching of the mode shapes.

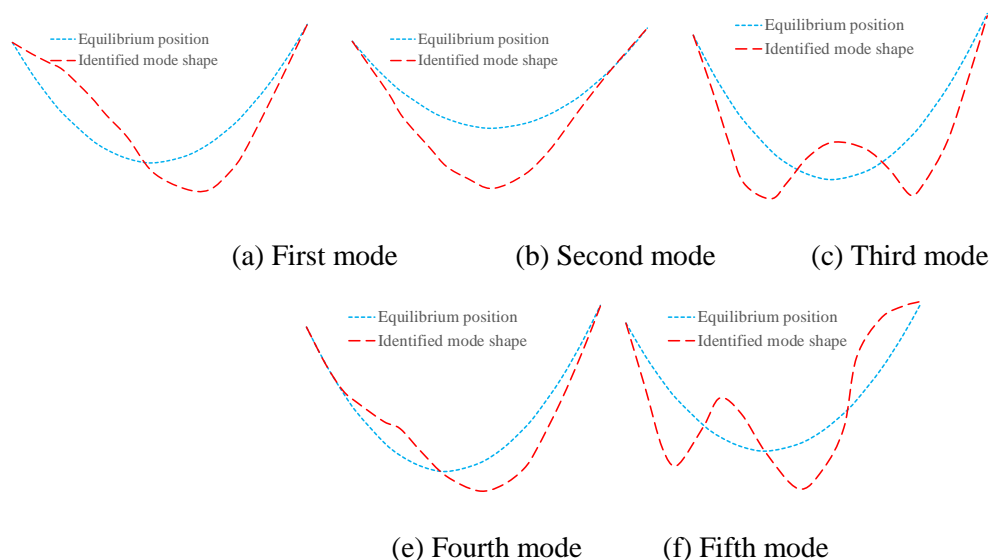


Fig. 5 Identified mode shapes of the Laxiwa dam using FDD under discharging case

Therefore, it can be concluded that the ambient vibration test is an ideal way to identify the modal parameters of the high arch dam, and the non-discharging case and the discharging case give similar results. In the following analysis, the non-discharging case is not presented.

It is worth noting that FDD algorithm relies on the selection of peaks. In this analysis, the peaks are easily selected because of the long high-quality data. FDD has certain limitations on the identification of damping ratios.

4.2 Modal parameters identified from SSI

In general, the user-defined parameters of the SSI should be adjusted iteratively to obtain a good-quality stabilization diagram. According to the selection suggestions proposed in Ref. [2], the user-defined parameters of the SSI in this case are set as follows: (1) $N_{max} = 60$; (2) $i = 60$; and (3) $j = 20000$, where N_{max} is the maximum order of the stabilization diagram. Fig. 6(a) shows the stabilization diagram of the Laxiwa dam, in which five stable alignments can be clearly observed. It indicates that the first five modes are identified. The clustering diagram of the stabilization diagram using DBSCAN clustering analysis is shown in Fig. 6(b). It can be seen that the spurious modes and the physical modes can be clearly distinguished. The five physical modes are clustered tightly in five clusters shown in different colors, while the spurious modes do not belong to any cluster, marked by a circle. It means that the modal identification without user interaction can be realized using the SSI algorithm coupled with the clustering analysis, making the



identified result more objective. This will be very beneficial to the online automatic modal identification and health monitoring. The identified frequencies and damping ratios using SSI in this analysis are summarized in Table 2, as well as the MAC values between discharging case and non-discharging case. It can be seen that the differences of the frequencies are very small, and the MAC values are larger than 0.96. It shows a good match of the identified modal parameters between discharging and non-discharging cases. In addition, the MAC values of the identified mode shapes between FDD and SSI are larger than 0.94, indicating the two identification methods could obtain the similar results.

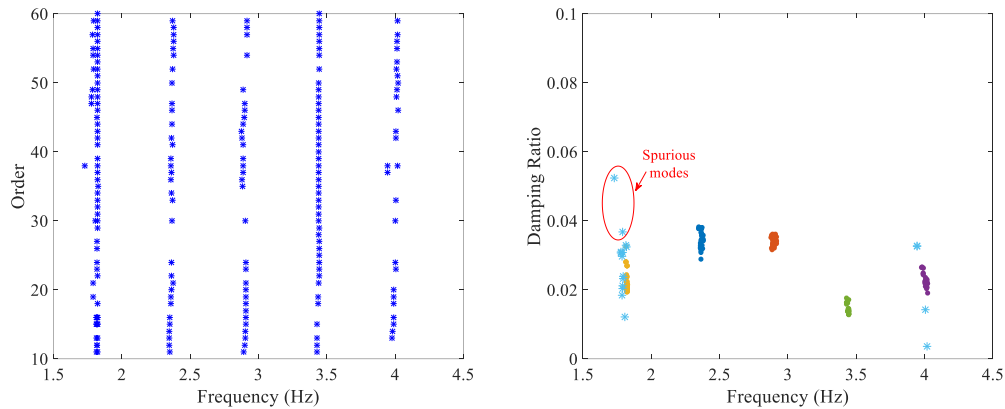


Fig.6 Stabilization diagram and clustering diagram using SSI algorithm

Table 2 Identified modal parameters based on ambient vibration tests using SSI

Cases	Modes	1st	2nd	3rd	4th	5th
Discharging	Freq. (Hz)	1.81	2.33	2.93	3.43	3.97
	Damp. (%)	2.23	3.48	3.40	1.42	2.30
Non-discharging	Freq. (Hz)	1.81	2.33	2.97	3.44	3.98
	Damp. (%)	1.90	3.01	2.57	2.31	1.58
MAC of Dis. And Non-dis.		0.97	0.96	0.98	0.96	0.99
MAC of FDD and SSI		0.95	0.97	0.96	0.94	0.96

5. Updating the finite element model

The frequencies obtained from the ambient vibration tests were used to update the finite element model of the Laxiwa dam developed by Tsinghua University. In this model shown in Fig.7 with the dam–foundation system, a flexible foundation rock and a reservoir corresponding with the real type are taken into account. Considering that the dam is in a linear elastic state because of the small amplitudes of the ambient vibration test, the contraction joints are ignored to simplify the model. The dynamic elastic modulus of the concrete and rock were adjusted in the calibration to reproduce the frequencies identified from the ambient vibration test.

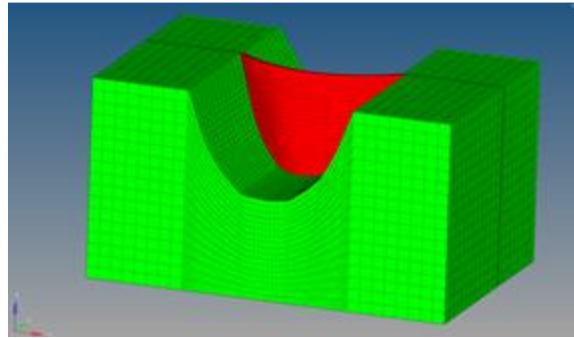


Fig. 7 Finite element model of the Laxiwa dam

The good case is selected as follows: The elastic modulus of the dam concrete is 45 Gpa, and the elastic modulus of the foundation rock is 40 Gpa. And the frequencies of the updated numerical model are summarized in Table 3. In comparison, the numerical model, in spite of its simplifications, also produces results very close to those obtained from the earthquake data and ambient vibration test. The first, third, fourth, and fifth modes are identical, and the differences are 1.10%, 0.68%, 3.20%, 1.50%, respectively. Though the second mode is a little less accurate, with the difference of 10.30%, the updated numerical model still could describe the dynamic characteristics of the dam.

Table 3 The frequency obtained from the FEM and identification

Mode	First	Second	Third	Fourth	Fifth
FEM	1.83	2.09	2.92	3.55	3.93
Identification	1.81	2.33	2.94	3.44	3.99
Error (%)	1.10	10.30	0.68	3.20	1.50

Fig. 8 shows the numerical modal shapes of the Laxiwa dam. It can be seen that they also approximate fairly well the experimental results shown in Fig. 5. It indicates that the updated finite element model can well reflect the actual dynamic characteristics of the dam. It will be beneficial for health monitoring and damage detection in the future.

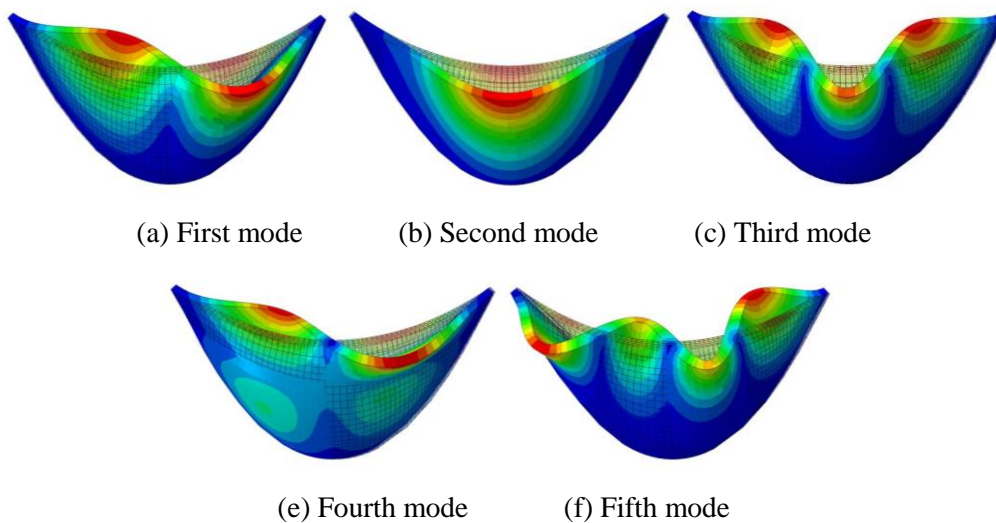


Fig. 8 Mode shapes of the Laxiwa dam obtained from the numerical model



6. Conclusions

This paper implements the modal identification of the Laxiwa dam using two identification methods based on the ambient vibration data under water discharging and non-discharging. The dynamic characteristics identified with different methods (SSI and FDD) show good consistency, confirming the reliability of the two identification methods and of the identified frequencies. The following conclusions can be drawn.

(1) The first two frequencies of the Laxiwa dam are 1.81 Hz and 2.33 Hz. The corresponding damping ratios are 1.83% and 3.86%, respectively. The first two mode shapes are anti-symmetric and symmetric. It is worth noting that the first two modes are not closely-spaced, which is different from the prior knowledge about the high arch dams. In addition, it should be noted that the turbine frequency is close to the second mode of the dam system.

(2) For the ambient vibration test, the differences of the identified frequencies using FDD and SSI are small, and the identified modal shapes match well.

(3) The identified modal parameters from ambient vibration tests and the finite element model agree well with each other. The ambient vibration test is an ideal alternative way to identify the modal parameters of the super high arch dams.

7. Acknowledgements

This research is financially supported by the National Natural Science Foundation of China (Nos. 51725901, 51639006, and 51579133). The authors express their sincerest gratitude for the support. Many thanks to Prof. Álvaro Cunha, Dr. Filipe Magalhães, and Dr. Sérgio Pereira for their guidance and help during the first author's visit to the University of Porto.

8. Copyrights

17WCEE-IAEE 2020 reserves the copyright for the published proceedings. Authors will have the right to use content of the published paper in part or in full for their own work. Authors who use previously published data and illustrations must acknowledge the source in the figure captions.

9. References

- [1] Magalhães F, Cunha A, Caetano E (2009): Online automatic identification of the modal parameters of a long span arch bridge. *Mechanical Systems & Signal Processing*, 23 (2), 316-329.
- [2] Li S, Jin AY, Wang JT, Luo GH (2020): Parametric analysis of SSI algorithm in modal identification of high arch dams. *Soil Dynamics & Earthquake Engineering*, 129, 105929.
- [3] Cheng L, Zheng D (2014): The identification of a dam's modal parameters under random support excitation based on the Hankel matrix joint approximate diagonalization technique. *Mechanical Systems & Signal Processing*, 42(1–2), 42-57.
- [4] Li S, Pan JW, Luo GH, Wang JT (2020): Improvements in the HHT for modal parameter identification of structures with closely spaced modes. *Journal of Earthquake Engineering*, accepted.
- [5] Chopra AK (2000): Dynamics of structures: theory and applications to earthquake engineering (2nd ed.), Prentice Hall, New Jersey.
- [6] Reynders E, Roeck GD (2008): Reference-based combined deterministic–stochastic subspace identification for experimental and operational modal analysis. *Mechanical Systems and Signal Processing*, 22 (3), 617-637.
- [7] Demarie GV, Sabia D (2019): A machine learning approach for the automatic long-term structural health monitoring. *Structural Health Monitoring*, 18 (3), 819-837.



[8] Magalhães F, Cunha Á (2011): Explaining operational modal analysis with data from an arch bridge. *Mechanical Systems & Signal Processing*, 25 (5), 1431-1450.