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ASSESSEMENT OF A SIMPLIFIED METHOD TO EVALUATE THE SLIDING OF GRAVITY DAMS UNDER EARTHQUAKE

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Abstract

In 2017, S. Mével proposed a simplified method to evaluate the unrecoverable displacement of a 2D gravity dam under earthquake. Instead of observing the dam as a rigid body, as the Newmark's method does, the method proposed taking into account the flexibility of the structure while considering only the first mode (frequency, damping ratio and fraction of the modal mass of this first mode). This simplified method remains very practical to use from an engineer's point of view and easy to present in a spreadsheet. The aim of this paper is to assess this method by comparing it with already existing results from finite-element analyses taking into account advanced soil structure interaction (foundation with mass and viscous-spring boundaries), fluid-structure interaction (fluid element) and joint element at the dam-foundation interface with a Mohr-Coulomb behaviour. Practical implementation of the method in a spreadsheet will be described, comparatives analyses will be performed on the well-known case of PineFlat dam while proposing recommendations on the simplified model parameters.

Keywords: Seismic analysis, Sliding displacements, Earthquake, Damping, Finite-element

1. Introduction

Under the action of a strong earthquake, a gravity dam may slide over the cracked foundation. The Shear Friction Factor (SFF) may drop under 1 for a short time, so that it does not lead to a global instability or an uncontrolled release of the reservoir, but only to a little unrecoverable displacement of the dam. However, it is necessary to assess the value of this displacement, in order to estimate wisely its consequences, and to decide whether they are acceptable or not.

2. Practical description of the simplified method

2.1 General description of the method

For the presented simplified method, the following hypotheses are made. Only a 2D section is taken into account. The acceleration is only horizontal (an extension of the method taking into account the vertical component is proposed in §4). The fluid-structure interaction is approximated by added-masses. The soil-structure interaction is neglected.

Here are the selected input parameters. α is the fraction of modal mass of the first mode. ω is the pulsation frequency of the first mode. ξ is the damping ratio of the first mode. $a_s(t)$ is the chosen accelerogram. a_{lim} is the limit acceleration, as it has been defined by Newmark [3]. To evaluate this value, a pseudo-static analysis can be performed with an increasing acceleration until the shear friction factor reaches 1.

The proposed method consists in performing a transient nonlinear computation of the system described on Fig 1. This simple system includes: a mass "A": which can oscillate and represents the first mode of the dam, a mass "B": which can slide on the foundation and represents the rigid part of the dam, a spring $k = \alpha M\omega^2$ so that the system has the same frequency than the dam, a dashpot $C = 2\xi\alpha M\omega$, so that the system has the same damping ratio than the dam.

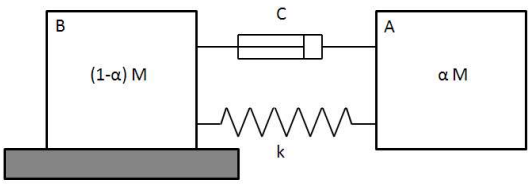

Fig 1 Simplified system

While the mass "B" does not slide, the system has only one degree of freedom and, according to Fig 1, the shear force T between the mass "B" and the foundation is calculated by the following equation:

$$T = M(1-\alpha)a_{sis} - \alpha\omega^2 M q(t) - 2\xi\alpha\omega M\dot{q}(t) \quad (1)$$

Where $q(t)$ is the relative displacement between the mass "A" and the mass "B". The maximum value of T is $T_{max} = Ma_{lim}$. As soon as the calculated value of T is greater than T_{max}, the mass "B" starts to slide, and the system has two degrees of freedom. The first principle of the dynamic leads immediately to the acceleration of each mass, and the displacements can be integrated step by step. In the next chapters the shear force T will be expressed as T/M, which has the same dimension as an acceleration.

2.1 Practical description of the spreadsheet

As there are only two degrees of freedom, an explicit integration scheme is not too expensive and quickly leads to accurate results. The method can thus be easily programmed in a spreadsheet as described in Table 1. For each instant of the chosen accelerogram, equations 2 to 8 describe the values to be computed from the general parameters of the model (frequency, damping, mass ratio, limit acceleration) and from values computed at the previous timestep. Fig 2 shows an example of the simplified analysis for the El-Centro accelerogram, considering a limit acceleration of 1 m/s^2 and the following parameters: f = 2 Hz, α = 0.6, ξ = 5%. When the

shear force (divided by the total mass M) reaches the limit acceleration, the mass B starts to slide. For certain parameters (high frequency for example), the time step of the accelerogram needs to be reduced (from 0.01 to 0.001 s) in order to get reliable results.

Table 1 Exemple of spreadsheet

		A : oscillating mass			B : sliding mass		
accelerogram $a_{sis}(t)$	shear force T(t)	acceleration $a_A(t)$	velocity $v_A(t)$	displacement $d_A(t)$	acceleration $a_B(t)$	velocity $v_B(t)$	displacement $d_B(t)$

In the following equations, all these variables are *soil-relative*, that-is-to say that $v_B = 0$ means that the mass B is sticking to the foundation. At time-step t+1, the shear-force T^{t+1}, between the foundation and the mass B can be approximated as:

$$T^{t+1} = \min\begin{cases} a_{lim} \\ (1-\alpha)a_{sis}^t - \alpha\omega^2(d_A^t - d_B^t) - 2\xi\alpha\omega v_A^t \end{cases} \tag{2}$$

Where the soil-relative displacements d_A and d_B can be integrated with the explicit scheme. For the mass A, it gives immediately:

$$a_A^{t+1} = -\omega^2(d_A^t - d_B^t) - 2\xi\omega(v_A^t - v_B^t) - a_{sis}^t \tag{3}$$

$$v_A^{t+1} = v_A^t + \frac{1}{2}(a_A^t + a_A^{t+1})\Delta t \tag{4}$$

$$d_A^{t+1} = d_A^{t-1} + v_A^t\Delta t + \frac{1}{2}a_A^t\Delta t^2 \tag{5}$$

For the mass B, the equations depend on the contact-state: *sliding* or *sticking*. If $T^t < a_{lim}$ and $v_B^{t-1} = 0$ then the mass B is *sticking*, which leads immediately to equation (6a):

$$a_B^{t+1} = 0 \tag{6a}$$

If the previous condition is not verified, then the mass B is *sliding*, and the equation of the motion is (6b):

$$a_B^{t+1} = \frac{1}{1-\alpha}\left(a_{lim} + \alpha\omega^2(d_A^{t-1} - d_B^{t-1}) + 2\xi\alpha\omega(v_A^{t-1} - v_B^{t-1})\right) - a_{sis}^t \tag{6b}$$

The displacement of the mass B is then computed with the explicit scheme:

$$v_A^{t+1} = v_A^t + \frac{1}{2}(a_A^t + a_A^{t+1})\Delta t \tag{7}$$

$$d_A^{t+1} = d_A^{t-1} + v_A^t\Delta t + \frac{1}{2}a_A^t\Delta t^2 \tag{8}$$

A little correction can be added to this behaviour. As the dam is supposed to slide only in the downstream direction, the limit-acceleration in the upstream direction is supposed unreachable, and then:

$$v_B(t) < 0 \longrightarrow v_B(t) = 0$$

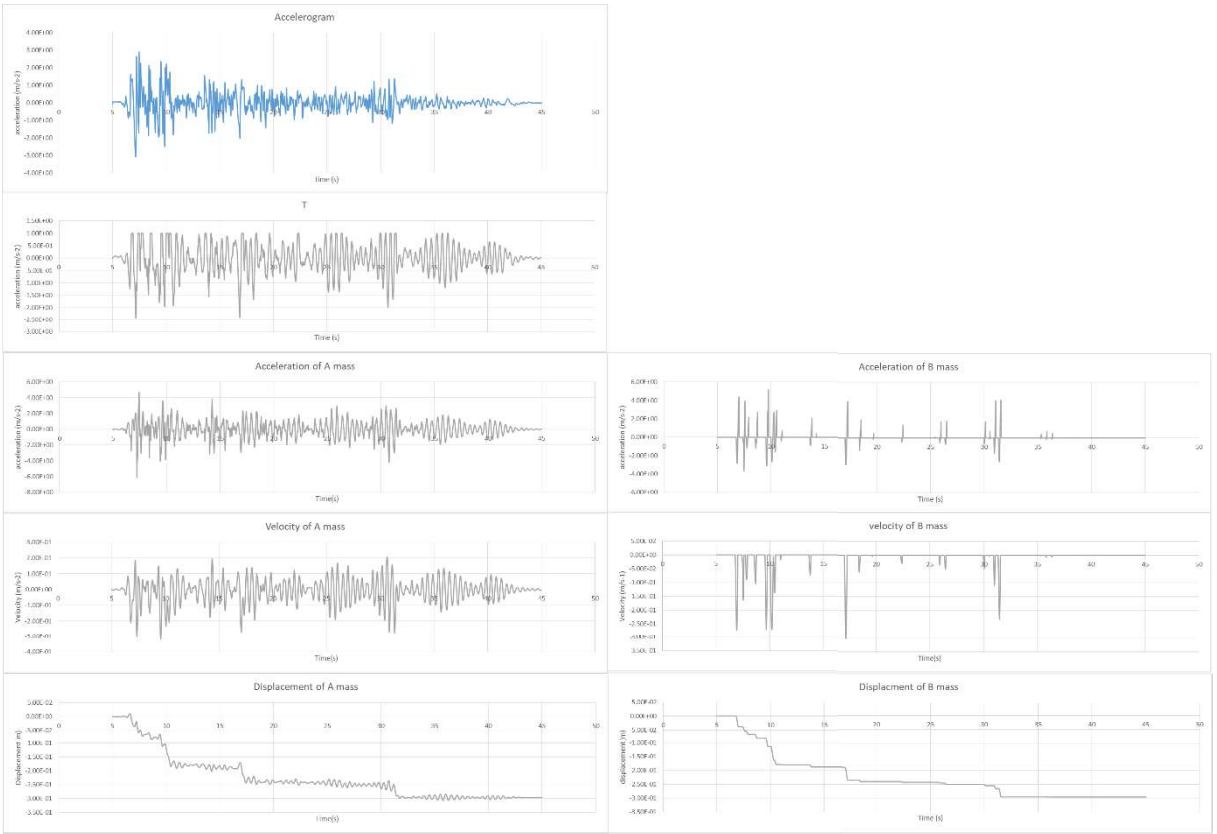

Fig 2 Example of analysis with El-Centro accelerogram, f=2Hz, α=0.6, ξ=5%, alim=1 m/s2

3. Comparison with FE analyses

In order to qualify and to select adequate input parameters for the simplified model, comparison with non-linear finite-element analyses have been performed. Assumptions and results are shown here.

3.1 Pine Flat case study

In 2019, the international benchmark workshop organized by ICOLD in Milan [4] proposed the seismic analysis of Pine Flat concrete dam as case study with the following assumptions. The geometry of the dam is presented in Fig 3.The material properties for concrete and foundation are : Module of elasticity of 22,410 MPa, Density of 2,483 kg/m^3, Poisson's ratio of 0.2 normal reservoir level (El 290) is considered, no uplift is considered, an intensifying acceleration time-history (ETAF signal) as an upstream/downstream acceleration input (Fig 4). This artificial record proves to be particularly adequate for the comparison.

3.2 Finite-element methods

Finite element analyses of this study case have been performed by the author [5] for the ICOLD benchmark workshop using the open-source software Code_Aster [6], developed by EDF. The mesh of the system is presented on Fig 6.

Linear behaviour is considered for the dam and the foundation but joint-element are introduced at the contact between the dam and the foundation. A non-linear Mohr-Coulomb mechanical behaviour is used in order to allow the sliding of the dam on its foundation. The following parameters are considered for the joint-elements: friction angle is 45°, no cohesion, normal and tangential stiffness are 1E11 Pa.

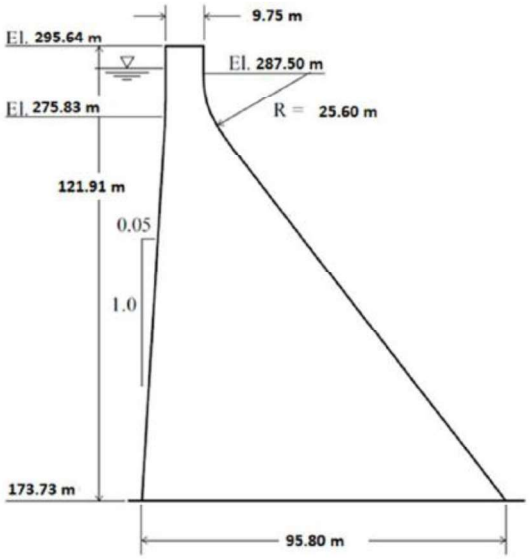

Fig 3 Geometry of the Pine Flat case study

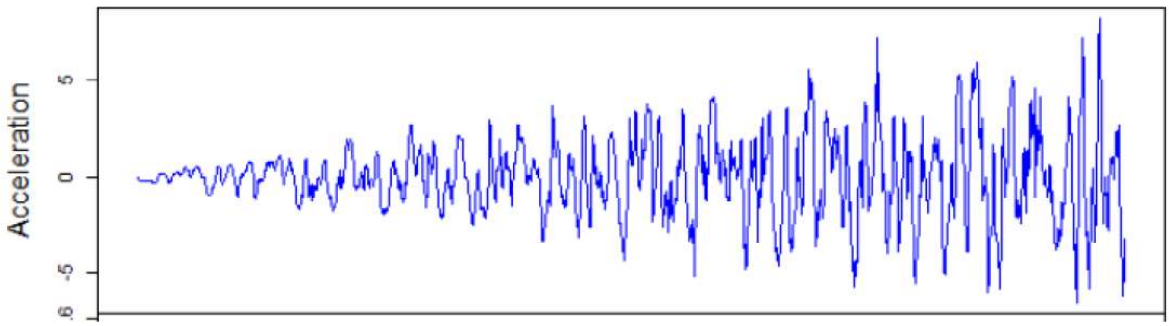

Fig 4 ETAF signal: intensifying acceleration time-history

For the seismic analyses of the system, 2 methods are considered. A simplified soil-fluid-structure interaction method considering a massless foundation and Westergaard added masses (as they were the standard hypotheses for decades in dam engineering computations). An advanced soil-fluid-structure interaction method taking into account the mass of the foundation and absorbing boundaries (viscous-spring boundaries) around the foundation, and fluid elements which behave as compressible water. In Code_Aster, the viscous spring boundary model is implemented as proposed and well described in [7] and [8], and briefly summarized in Fig 5. It is employed to absorb the wave energy radiating away from the dam and the foundation. In this method, earthquake input is introduced as compression and shear waves, vertically propagating from the bottom to the top of the foundation. Side boundaries should not be neglected using free-field column providing the propagation of the wave in an unbounded foundation.

A Rayleigh damping is considered for concrete, in order to get around 3% of structural damping. A modal analysis of the structure (considering the foundation massless and the fluid elements), showed that the first natural frequency is around 2.1 Hz. This modal analysis, performed with a linear model, does not take into account the non-linear joint. In consequence, the value of the first frequency might be slightly lower, around 2% probably. In order to get roughly 3% damping between 2 and 15 Hz, the characteristic values for the Rayleigh damping are : α = 0.00056 and β = 0.63575, for $\underline{\underline{C}} = \alpha\underline{\underline{K}} + \beta\underline{\underline{M}}$.

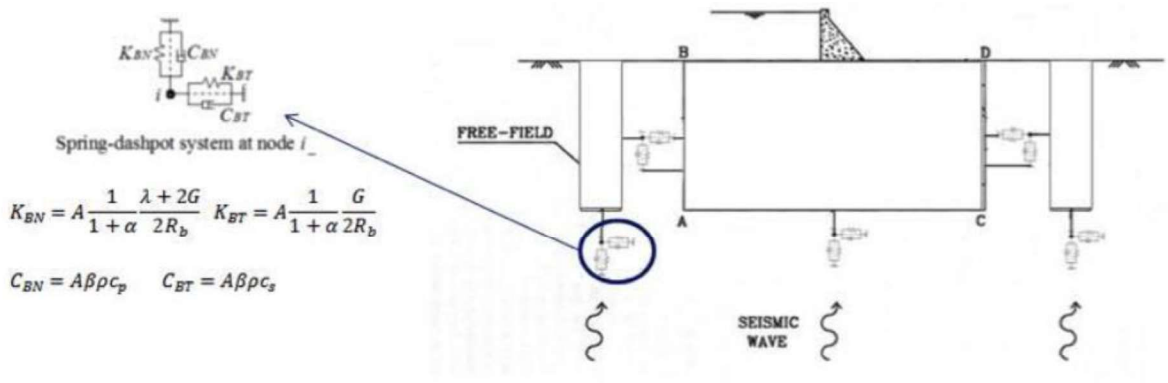

Fig 5 Viscous spring boundaries model

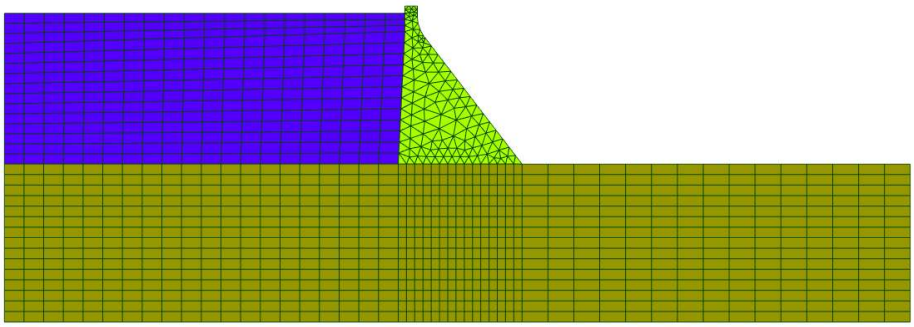

Fig 6 mesh of the dam, fluid and foundation

3.3 Simplified method

For the simplified method, a conventional stability assessment analysis, based on forces evaluation at the dam-foundation interface, is required to evaluate the limit acceleration. The weight of the dam and the hydrostatic forces are computed considering the values chosen in §**Erreur ! Source du renvoi introuvable.** (no uplift are considered in this case study). A pseudo-static analysis with an increasing horizontal acceleration is performed, leading to additional inertia forces due to the dam and Westergaard added masses. The limit acceleration is reached once the shear friction factor equals 1.0. (Equation 9 can also be used to evaluate the limit acceleration). In this case study, the limit acceleration computed is 0.372 g. In order to be consistent with the finite-element analyses, the following parameters are considered for the simplified model: f = 2.1 Hz, α = 60%, and ξ = 3% (f and α are coming from a modal analysis of the system).

3.4 Results

The Fig 8 compares the unrecoverable displacements computed with simplified and finite-element analyses. The displacement computed with the Newmark method [3] is also represented. Results show that the simplified method proposed is quite close with the best estimate FE method taking into account advanced soil-fluid-structure method.

On the contrary, Newmark analysis that does not take into account the flexibility of the dam strongly underestimates the sliding by a factor 10. The simplified FE method with massless foundation and Westergaard added masses clearly overestimates the displacement: Fig 9 compares the time-history of normal and shear forces at the dam-foundation interface and shows that this is mainly due to the overestimation of the normal forces by the massless foundation analysis.

In the simplified method, the acceleration is only horizontal, while both directions are taken into account in the FE methods. That is why the vertical force between dam and foundation is always equal to zero in the first one, while it is oscillating in the others.

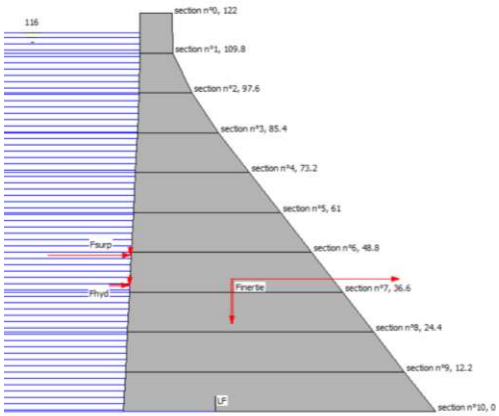

Fig 7 Simplified beam-structure like analysis

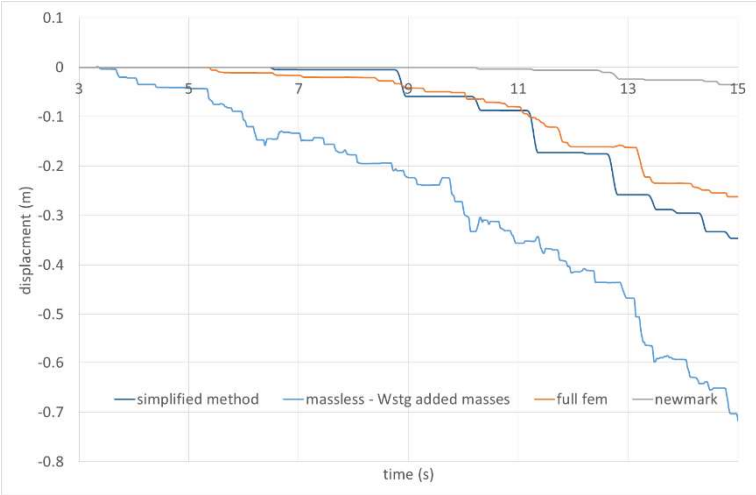

Fig 8 comparison of the unrecoverable displacement computed from simplified and finite-element analyses

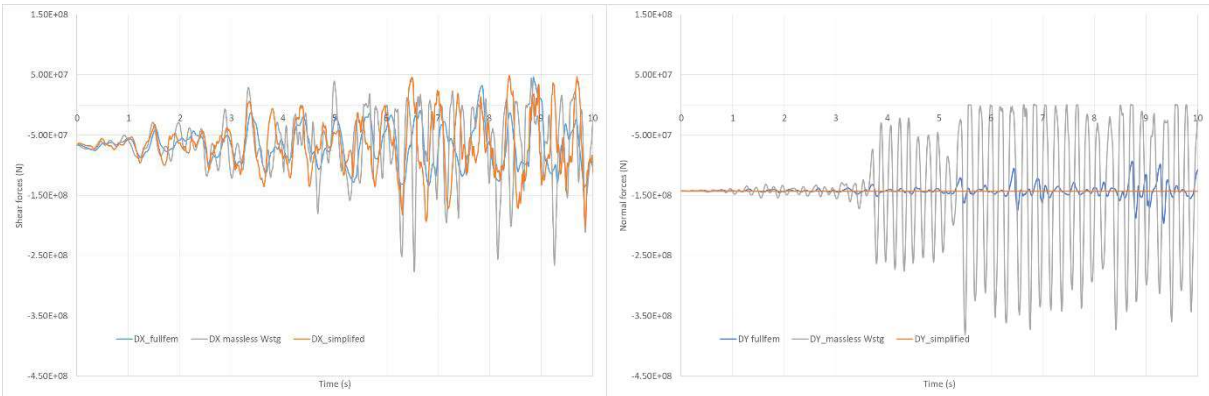

Fig 9 comparison of the shear (left) and normal (right) forces at the dam foundation interface

3.4 Sensibility analyses

In order to evaluate the sensibility of the results obtained by the simplified model to its parameters, 5 sensibility analyses are performed.

Variation of the material damping of concrete, from 1 to 5%: even with complex finite-element analysis, this parameter needs to be carefully chosen. Considering that the results of the simplified analysis are similar

to the results with absorbing boundaries and fluid elements, a realistic value of damping should be selected for concrete, between 1 and 3%.

Variation of the natural frequency (0.4 Hz on each side of the chosen 1st frequency): evaluation of the natural frequency of the dam is not always precise with an uncertainty around 0.5 Hz. For engineering practice, the following methods can be used for the evaluation of the natural frequency: ambient vibration measures, Tardieu simplified formula [9] (N = 0,23 S / H for full reservoir, or N = 0,17 S / H for empty reservoir with S the velocity of the shear waves in concrete and H the height of the dam), JCOLD Formula from the analyses of earthquake records on Japanese dams [11]: N = 526/H, modal analyses with finite-element software.

Variation of the modal mass fraction: for the 'usual' shape of a gravity dam, the mass fraction of the 1st mode is generally around 60%,

Variation of the limit acceleration: strongly dependent on several assumptions chosen for the stability assessment of the dam (friction angle, uplift assumptions etc.),

Direction of the accelerogram: by changing the sign of the time-history accelerogram, results might vary.

Fig 10 shows the sensibility of the results for each of these parameters: results remain in the same range. That means that even if the parameters are not perfectly chosen for a given dam, the simplified method provides an adequate approximation. It is then possible to evaluate the sensibilities to the parameters, by estimating the ratio between the relative variation of the result (the unrecoverable displacement δ) to the relative variation of each input parameter.

Table 2 Sensibilities to the input parameters

$\left(\frac{d\delta}{\delta}\right)/\left(\frac{d\xi}{\xi}\right)$	From **-7** (for $\xi \approx 1\%$) to **-1** (for $\xi \approx 5\%$)
$\left(\frac{d\delta}{\delta}\right)/\left(\frac{d\alpha}{\alpha}\right)$	**+ 0,5**
$\left(\frac{d\delta}{\delta}\right)/\left(\frac{da_{lim}}{a_{lim}}\right)$	**- 0,8**

These results are numerically different from those obtained in 0, but the signs of these ratios are always known: the damping and the limit-acceleration are negatively correlated to the displacement, and the mass fraction is positively correlated.

The impact of a frequency variation cannot be easily quantified, as the sign of the variation of the unrecoverable displacement cannot be predicted: sometimes positive, sometimes negative. This fact may be highlighted: it is not possible to select a natural frequency which necessarily leads to pessimistic results. Several computations have to be performed in order to determine the optimistic and pessimistic natural frequencies.

3.5 Comparison for other dam's height

In order to evaluate the performance of the simplified analysis for dams with lower height, comparisons are performed for 3 additional case studies dams of the following heights: 10, 35 and 90 m, with a similar profile and mechanical characteristics as Pine Flat dam. Fig 11 shows the geometry of the dams and Table 3 summarizes the main characteristics and parameters needed for the analyses. The ETAF increasing horizontal acceleration is used for the analyses. No vertical acceleration is used. Finally, Fig 12 compares the time-history displacement at the dam-foundation interface for simplified and finite-element analyses taking into account the mass of the foundation and fluid elements. Despite having similar limit accelerations, the case studies lead to different displacements and the comparison shows that the simplified method allows to catch these discrepancies.

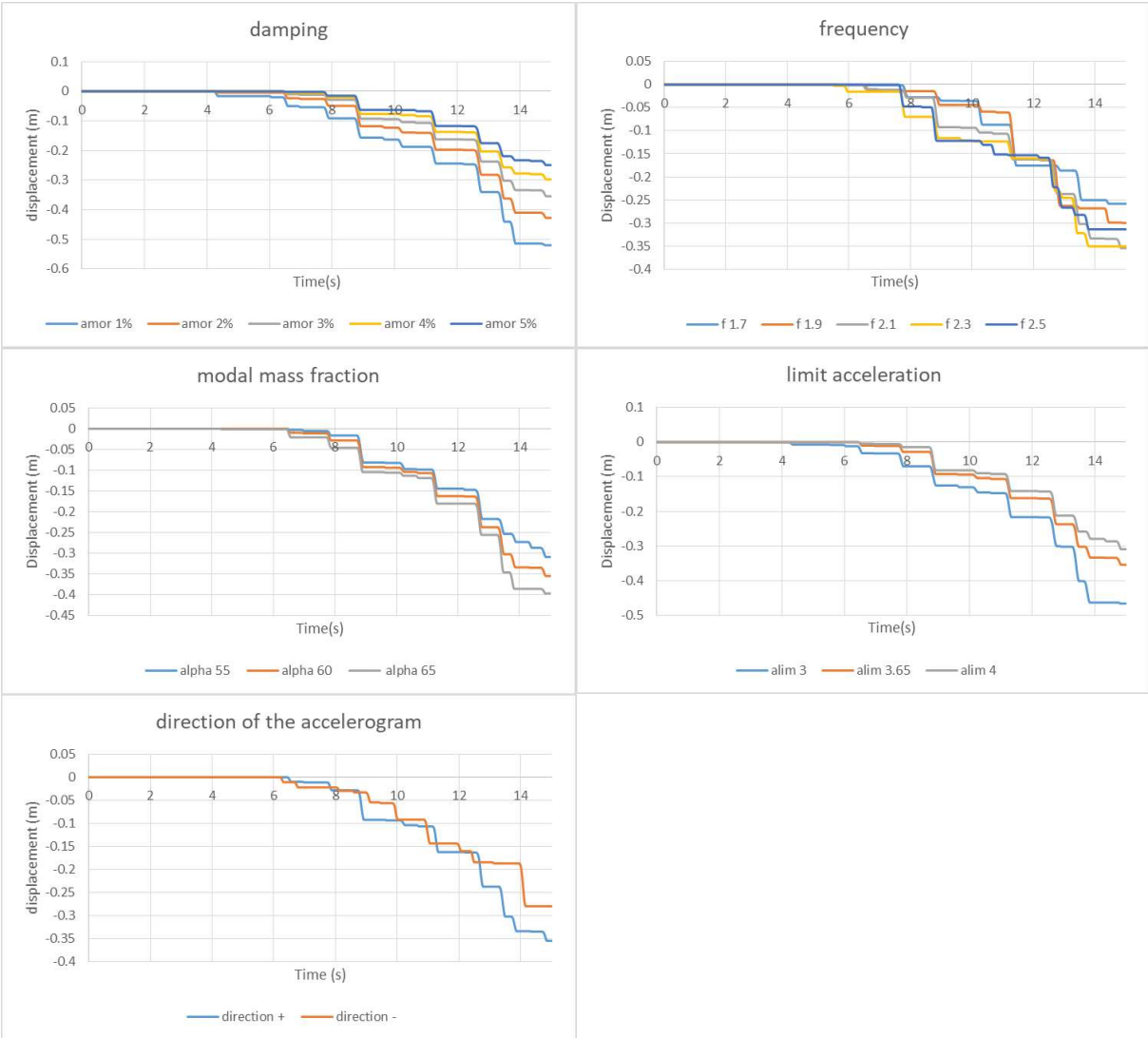

Fig 10 sensibility analysis with the simplified model parameters

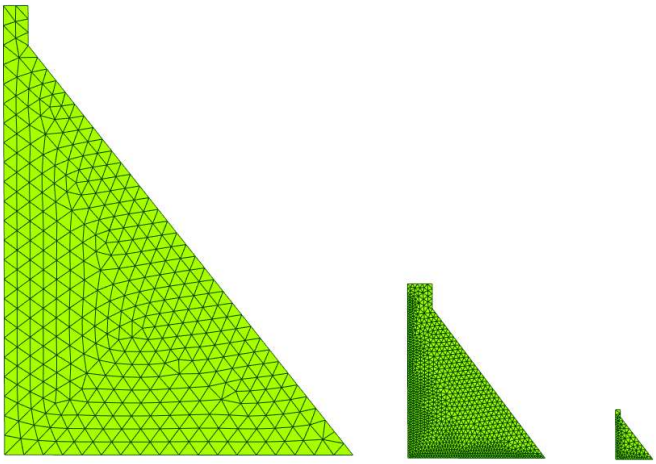

Fig 11 geometry of the case studies dams B90, B35 and B10

Table 3 characteristics of the 3 case studies

	Height (m)	Water level (m)	Limit acceleration (g)	1st frequency (Hz)	Modal mass fraction of the 1st mode	Rayleigh damping coefficient for 3% equivalent damping
B10	10	9	0.377	24.5	0.61	α=0.000148 β=5.7279
B35	35	33	0.382	6.7	0.62	α=0.00044 β=1.7459
B90	90	86	0.336	2.7	0.62	α=0.0005395 β=0.8626

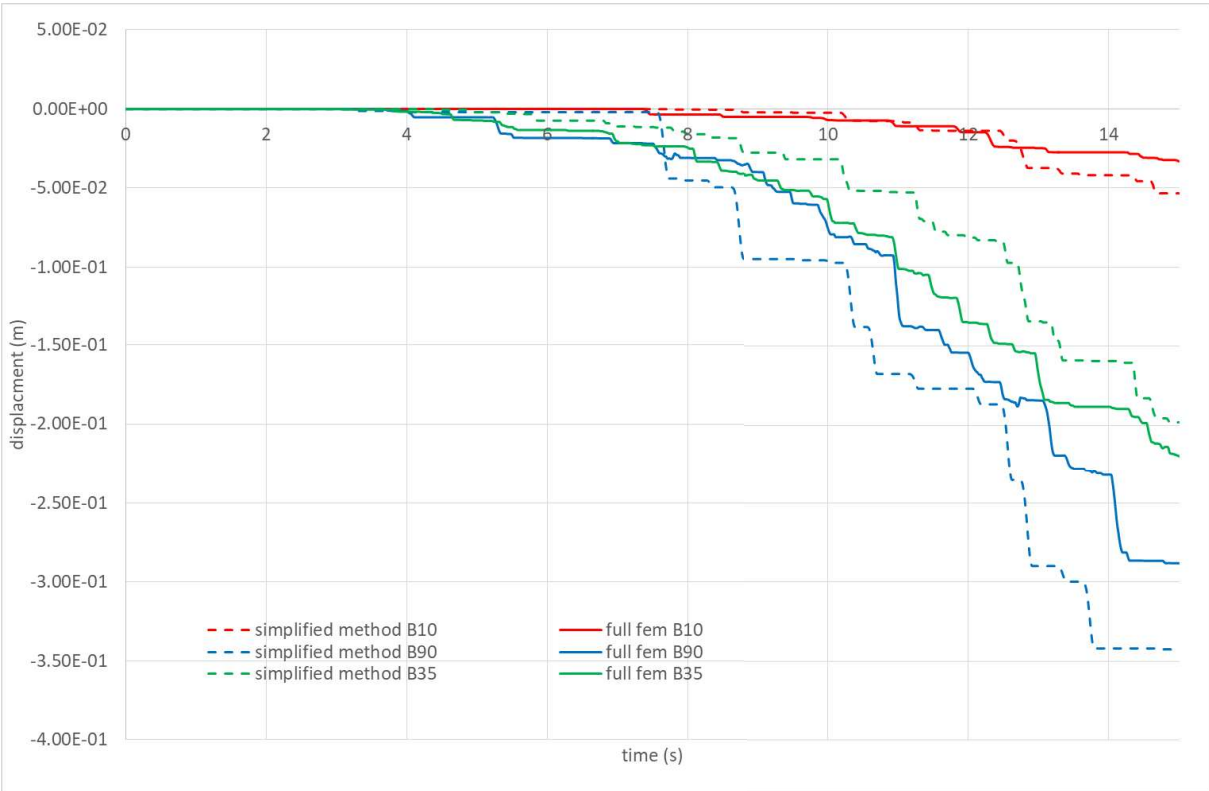

Fig 12 Comparison of the displacement computed by simplified method and FE analyses for the 3 case-study dams

In these cases, the simplified method overestimates the displacement once, and underestimates it twice. The difference between the two methods is about 10-15%, which is quite acceptable for that kind of computations, in comparison with the other uncertainties in the input parameters.

4. Conclusion

This paper describes the practical implementation of a simplified method to evaluate the unrecoverable displacement of a 2D gravity dam under earthquakes. Comparatives analyses show that the results of this model are of the same magnitude than results computed with advanced finite-element analyses taking into account soil-structure and fluid-structure interaction with absorbing boundaries, mass of the foundation, fluid elements and non-linear behaviour at the dam-foundation interface.

Finally, with the feed-back of the analyses performed, the following guidelines are proposed concerning the 'practical' use of such type of analysis with the usual geometry of gravity dam:

- Frequency of the first mode can be evaluated by finite-element analysis or with the formula proposed by Tardieu [9] or JCOLD [11],
- The ratio of the modal mass of the first mode can be taken at 60%,
- A realistic value of damping should be considered for the concrete (1 to 3%) considering that the results are similar with advanced finite-element analysis taking into account radiative boundaries,
- Considering that results are strongly dependent of the chosen accelerogram, multiple analyses with several time-history accelerations are recommended in order to evaluate an average value of the unrecoverable displacement,
- As some input parameters have a strong influence on the results, and are hard to measure or estimate for projected dams, it is recommended to always perform some sensibility analyses, and to select the design parameters very carefully.

An example of the spreadsheet with the simplified method can be sent on request (by email to the author). The authors would like to add that other improvements of this simplified method can be imagined. For instance, the following:

- The effect of the vertical component of the earthquake can be considered with a slight modification of the simplified method. Analyses not presented here show that even if the displacement computed remains in the same magnitude, the displacement increases by a factor 1.4 to 1.5.
- Analytical models of soil-structure interaction could be added under the dam (for instance, as proposed by [10]);
- The importance of the effect of the vertical acceleration of the reservoir has been highlighted by many authors, and an analytical estimation of this effect can be found in [2].

But one has to keep in mind that the main advantage of this simplified method is the drastically low number of input parameters. Improving the method may help to catch and understand more phenomena, but leads to a less easy-to-use tool.

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