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AN INTEGRATED LOCAL DAMPING APPROACH FOR ENHANCED CHARACTERIZATION OF NONLINEAR DYNAMIC RESPONSE

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Abstract

This paper develops a simplified local damping approach for use in the nonlinear dynamic analysis of structures so that different energy dissipation mechanisms can be integrated. It is argued that a local damping approach allows the analyst to control the dissipative mechanisms introduced and check they are meaningful. Furthermore, a local damping approach offers the opportunity for different damping mechanisms to be integrated in the same model and independently specified, signaling a departure from the use of purely viscous damping models. An initial strategy for specifying damping within the integrated local damping framework is proposed, based on providing consistency between the response obtained using local and global viscous damping formulations, for both initial and tangent stiffness proportional damping models. Guidance on the use of such a local damping approach is provided for the characterization of a series of simple case study bridge structures and non-linear dynamic analyses are used to highlight the impact of the new approach.

Keywords: local damping models; tangent-stiffness viscous damping; large displacement analysis; nonlinear analysis

1. Introduction

Nonlinear dynamic analysis is increasing in popularity for the assessment of structures subjected to earthquake excitations due to increased computing power and the potential for more accurate performance evaluations. However, accurate and reliable performance evaluations require, in turn, accurate characterizations of material and geometrical nonlinearities and of the different additional energy dissipation mechanisms.

In the dynamic analysis of elastic systems, damping has been generally introduced by viscous damping forces proportional to the velocity and to a damping constant, c, commonly expressed as a percentage, ξ, of the critical damping coefficient, c_{cr}, in turn, function of the system mass, m, and natural frequency, ω_n, or period, T_n. As explained in classical texts of dynamics of structures [1], the equation of motion for earthquake excitations takes the form reported in Eq. (1).

$$m\ddot{u} + c\dot{u} + ku = m\ddot{u}_g(t) \qquad \text{where} \quad c = \xi\, c_{cr}\, ,\ c_{cr} = 2m\omega_n\, ,\ \omega_n = (k/m)^{0.5} \tag{1}$$

For mathematical simplicity and speed of computation, the same damping is often maintained also for the dynamic analysis of systems with nonlinear hysteretic behavior. As a nonlinear system is characterized not only by an initial elastic stiffness but also by a postyield stiffness, this damping model is referred to as initial stiffness-proportional viscous damping model. For nonlinear systems Eq. (1) can be written as in Eq. (2).

$$m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = m\ddot{u}_g(t) \quad \text{where} \quad c = \xi\, c_{cr}\, ,\ c_{cr} = 2m\omega_n\, ,\ \omega_n = (k/m)^{0.5} \tag{2}$$

However, a number of concerns have been raised in relation to the use of initial stiffness-proportional damping for the analysis of systems responding non-linearly. In his master's thesis [2], Chrisp appears to have been the first to highlight the presence of unrealistic damping forces in nonlinear time history analyses during the phases of yielding of the structure when an initial-stiffness-proportional damping model is used.

Spurious damping forces and artificial damping have also been observed in the elastic to plastic transition phases of the response by Hall [3] and Charney [4].

An alternative viscous damping model that has been recommended by Otani [5], Priestley and Grant [6], and Petrini, et al. [7], is the tangent-stiffness-proportional viscous damping model. In this model, the value of the damping coefficient, c_T, is assumed proportional to the instantaneous value of the stiffness and thus varies during the analysis. The equation of motion for a nonlinear single-degree-of-freedom (SDOF) system in the presence of tangent-stiffness-proportional viscous damping can be expressed as in Eq. (3)

$$m\ddot{u} + c_T\dot{u} + f_s(u,\dot{u}) = m\ddot{u}_g(t)\,) \qquad \text{where} \quad c_T=c_T(u,\dot{u})=c_0(k_T/k_0)= 2m\omega_n(k_T/k_0)=2\omega_n k_T \tag{3}$$

where k_0 is the system initial stiffness, k_T is the system tangent stiffness, and c_T is the viscous damping coefficient of the tangent damping model, that changes with the system current stiffness as reported in Eq. 3. Priestley and Grant [6] argue that because the damping forces generated using a tangent-stiffness-proportional damping model are significantly reduced in the post-yield range, it better represents the manner with which non-modelled damping sources dissipate energy. Furthermore, Petrini et al. [7] provided some experimental evidence that tangent stiffness damping is more appropriate than initial stiffness proportional damping. Charney [4] showed that artificial viscous damping forces in the presence of a tangent stiffness proportional damping model are drastically reduced but not eliminated. However, concerns have also been highlighted with the tangent-stiffness-proportional damping model. Chopra and McKenna [8, 9] and Hall [10] observed that when a tangent stiffness-proportional viscous damping model is assumed for the characterization of the nonlinear dynamic response to earthquake excitation of an elastoplastic (r=0) SDOF systems, the viscous damping force drops suddenly to zero when structural yielding occurs, and this is judged a nonsensical, [10], physically unacceptable, [9], result even though the authors don't clarify why. An unacceptable feature highlighted in [9] for the tangent damping model is that it implies negative damping coefficient, and forces, when the system stiffness at large deformations becomes negative.

Many other relevant damping models have been proposed in literature. Hall [3] proposed a capped viscous damping model, where a limit is to be imposed on the damping forces, Chopra and McKenna [8, 9] identified as preferred damping models (1) a modified Rayleigh damping model in which the stiffness proportional term is based on a k that omits contributions from all rotational springs used to model plastic hinges; and (2) a damping matrix defined by superposition of modal damping matrices. Luco and Lanzi [11] proposed a damping model where damping forces are assumed proportional to the first derivative of the restoring forces with respect to time. Carr, Puthanpurayil, Lavan, & Dhakal [12] proposed a new approach where inherent damping is modelled at the element level, as detailed in Puthanpurayil et al. [13]. Charney [4] observed that an ideal solution would be to eliminate the use of viscous damping altogether, and suggests the use of nonlinear frictional or hysteretic devices to represent inherent damping, since frictional and hysteretic damping is much more in tune with the actual behavior of a structure. A similar recommendation was made by Wilson [14], who highlighted that most physical energy dissipation mechanisms in real structures tends to be a nonlinear function of the magnitude of the displacements rather than proportional to the velocity.

With the aim of accurately identifying the seismic collapse capacity of SDOF systems with tangent stiffness-proportional viscous damping in the presence of large material and geometrical nonlinearities, De Francesco and Sullivan [15] proposed the introduction of damping at a local level through the use of viscous damper elements, rather than at a global level by a viscous damping matrix. The local formulation introduced was shown to overcome the main shortcomings with traditional applications of the tangent damping model associated with (1) negative damping forces that develop in the post-yield range when the system stiffness becomes negative, and (2) unrealistic damping forces arising at degrees of freedom undergoing second-order movements only. The local approach was developed for spring-mass-damper and inverted pendulum SDOF systems with bilinear plastic hysteretic behavior and it is further developed in this paper for a series of simple bridge systems.

2. Integrated local approach for the representation of non-modelled energy dissipation mechanisms

There are several benefits of adopting a local damping approach for modelling the various sources of additional energy dissipation involved in nonlinear dynamic analysis of structures. A local approach, where damping mechanisms are specified by a discrete number of elements, allows the structural analyst to easily identify and check whether the dissipative mechanisms specified are physically acceptable or not. This is in agreement with the approach suggested by Hall in [10] who states that the key to an appropriate damping formulation is a realistic mechanism that allow all the damping forces and moments to be meaningfully assessed. The same author, in [16], for an accurate seismic performance evaluation of a series of multi-story steel-frame buildings, specified the main component of damping by inter-story shear dampers [10], and only a small percentage (0.5 per cent) of viscous damping was specified as stiffness proportional damping at a global level [16].

Several, and different, are the energy dissipation mechanisms involved in nonlinear dynamic response of structures and it seems improbable that all of them can be accurately described by a single damping mechanism. It is the opinion of the authors that reliable prediction of seismic performance may require the main sources of energy dissipation to be quantified (numerically and experimentally) and modelled separately. The three major sources of energy dissipation commonly represented by viscous damping, as suggested by Priestley in [6], are associated with effects of inherent material characteristics of the structural system, foundation damping, and non-structural components. The damping effects associated with these three classes of damping should ideally be modelled separately, and not necessarily with a viscous mechanism. The effects of energy dissipation from damage to non-structural components are likely to be more appropriately modelled by a frictional or hysteretic mechanism, [4, 14] while those associated with inherent material properties of the structural system and foundation damping could arguably be efficiently described by a viscous model. However, damping associated with foundations is likely to be influenced not only from the response of the super-structure but also from the characteristics of the soil and of the foundation system. In [18], Hall suggests that when soil-structure interaction is relevant and the structural response undergoes inelastic behavior, foundations and soils should be modelled explicitly. A local damping approach which offers the possibility of integrating different damping mechanisms, such as viscous, frictional and material, or more viscous damping mechanisms with independent properties, seems appealing.

Fig. 1 shows a local damping approach that can be easily implemented in practice to permit definition of different damping mechanisms for a bridge system. Note that the local viscous damping mechanism shown in Fig. 1 dissipates energy due to the relative rotation of nodes at either end of the member, recognizing that this is sufficient for the lateral response of the bridge in question. When, however, the lateral response of elements depends on varying amounts of curvature and shear deformations (for example, when the point of contra-flexure in a member changes during response) then an additional damping element would be required so as to account for both the relative translation and rotation of the element ends.

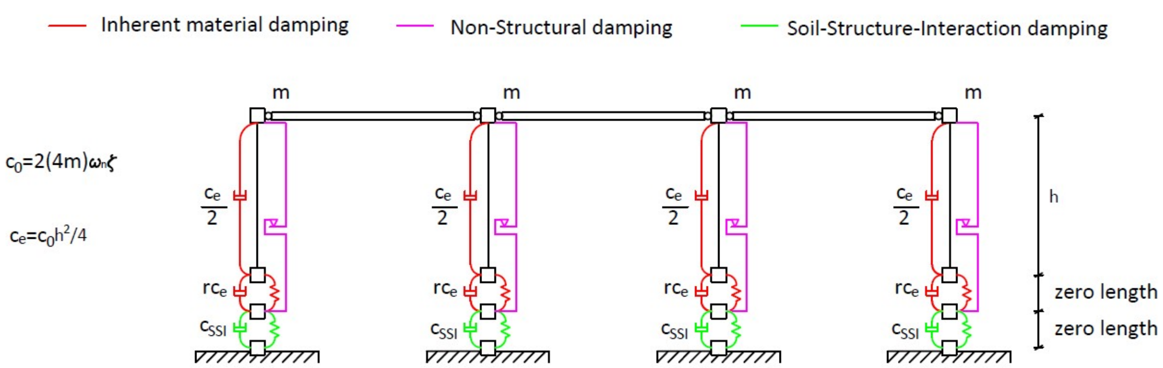

Fig. 1 – Integrated local approach for modelling additional energy dissipation mechanisms

An integrated local damping approach could help reduce the large uncertainty observed in a fundamental area of research where it still appears unclear if damping effects should be modelled by initial or tangent stiffness-proportional damping models [21]. Presently, the authors of this paper advocate the use of the tangent-stiffness proportional damping model for modeling damping of the main structural system but also recognize that there are no specific reasons for tying all the sources and mechanisms of energy dissipation to the yielding and stiffness characteristics of the structural system, as stated by Hall in [19], and that some energy dissipation could also be present when the main structural system is in the yielding phase [9].

3. Consistency between local and global damping formulations for bridge systems

While in general the local damping approach allows one to assign any combination of viscous damping coefficients to the elastic and plastic components of the motion, the viscous damping coefficients of the local damping model can also be assigned to result in the same small displacement response of a global damping model, as shown in [15]. Although the properties of the elements of the local damping model should be tied to the characteristics of the structural system analyzed, by distributing damping elements in manner that is consistent with a global initial or tangent damping model, one can set realistic local damping values that could be later refined. This approach recognizes that traditional global damping models have been largely implemented in the past, with some experimental support, while the same cannot be said for a local damping models.

To show how consistency of results between local and global damping formulations can be obtained, the local damping approach will be next applied to two simple schemes of bridge. The first scheme consists of the single pier bridge system shown in Fig.1. The mass properties of the system are assumed to be concentrated at the top of the pier while the material nonlinearities are concentrated at the bottom, in the plastic hinge region. As in Chopra and McKenna [9], analyses are conducted for the North-South component of the El Centro ground motion and the properties of the system are specified to result in a small amplitude vibration period of T_n = 0.5 s and in a normalized yield strength of f_y = 0.125. The viscous damping ratio was assumed equal ξ= 5%. A seismic weight of W=10000 kN and height of h=4 m have also been specified. All the analyses have been conducted with the inelastic time history analysis program Ruaumoko [17].

A local damping model, consistent at a global level with an initial stiffness-proportional viscous damping model, can be specified for the bridge piers shown by introducing two rotational dampers. The damping coefficients are to be assumed equal to c_0h^2 and $0.5c_0h^2$, where $c_0=2m\omega_n\xi$, for the damper acting in parallel with the rigid-plastic rotational spring at the base and the elastic pier, respectively, as shown in Fig.2. Similarly, the local damping formulation for tangent-stiffness-proportional viscous damping can be specified by introducing two rotational dampers, with damping coefficient rc_0h^2 and $0.5c_0h^2$, acting in parallel with the rigid-plastic rotational spring at the base hinge and over the portion of the elastic pier, as shown in Fig. 1. In both cases no damping is specified at a global level. Details of the modeling approach can be found in [15].

A comparison of the displacement response obtained from non-linear time-history analyses unsing a small displacement analysis regime for initial and tangent stiffness-proportional viscous damping models is shown in Fig. 3. The results confirm the consistency between the local and global damping formulations.

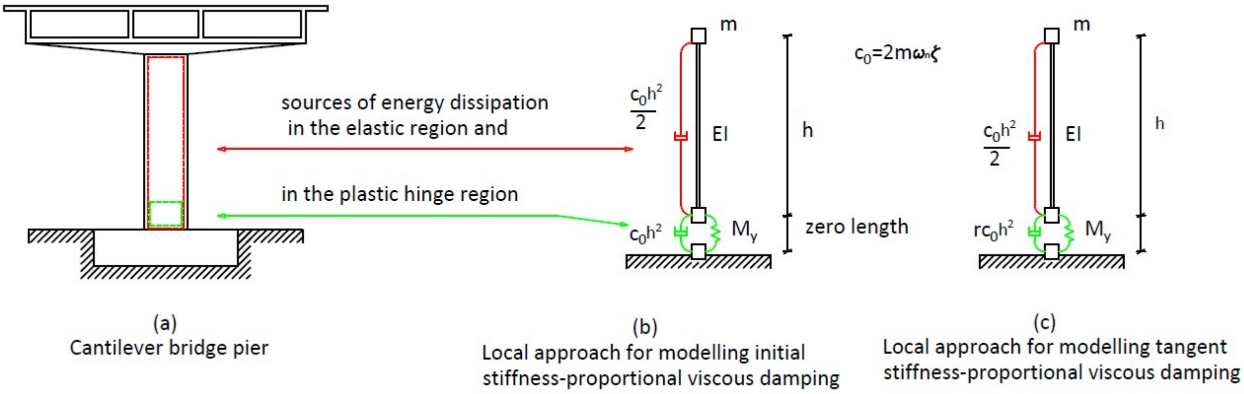

Fig. 2 – Local approaches for initial and tangent stiffness-proportional damping models

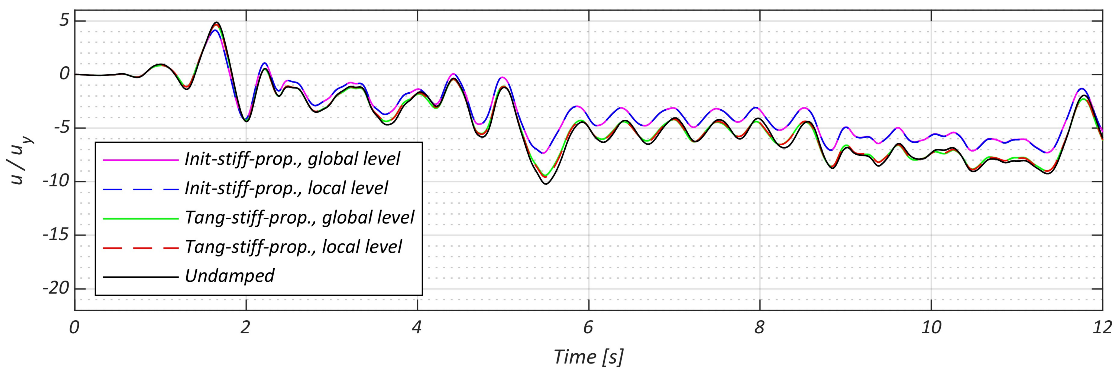

Fig. 3 – Displacement response for local and global, initial and tangent, stiffness-proportional damping

The second bridge system considered, shown in Fig. 4, consists of four piers with equal height, h, and elastic stiffness, EI. The piers are assumed pinned at the top to an axially rigid bridge deck, and are fixed at the base. The central piers are assumed elastic while the external piers elastoplastic. The mass of the system and the elastic stiffness properties of the piers are assigned to give a small amplitude vibration period of the bridge equal to T_n=0.5 seconds. The yielding moment at the base of the short piers is assigned to provide the system with a normalized yield strength of f_y = 0.125. As a result of the geometrical and material properties specified, the bridge system at a global level presents a bilinear plastic hysteretic behavior with postyield stiffness ratio r=0.5.

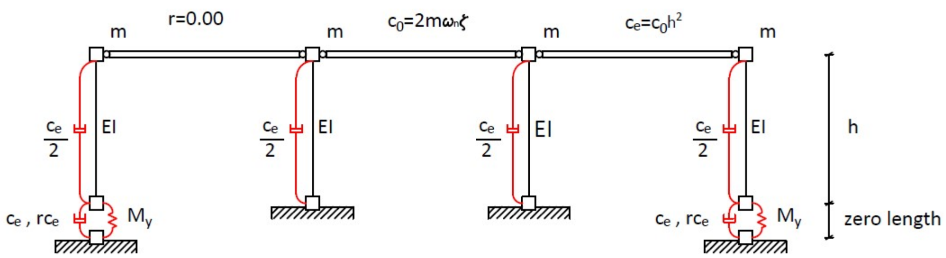

Fig. 4 – Local approaches for initial and tangent stiffness-proportional damping models for a bridge system

A local damping model that is consistent at a global level with the initial-stiffness-proportional viscous damping model can be specified by introducing six rotational dampers for the bridge system. Four, with damping coefficient $c_e=(c_0h^2/2)/4$, acting in parallel with the four bridge piers and two, with damping coefficient $c_e=c_0h^2/4$, acting in parallel with the rigid-plastic base springs.

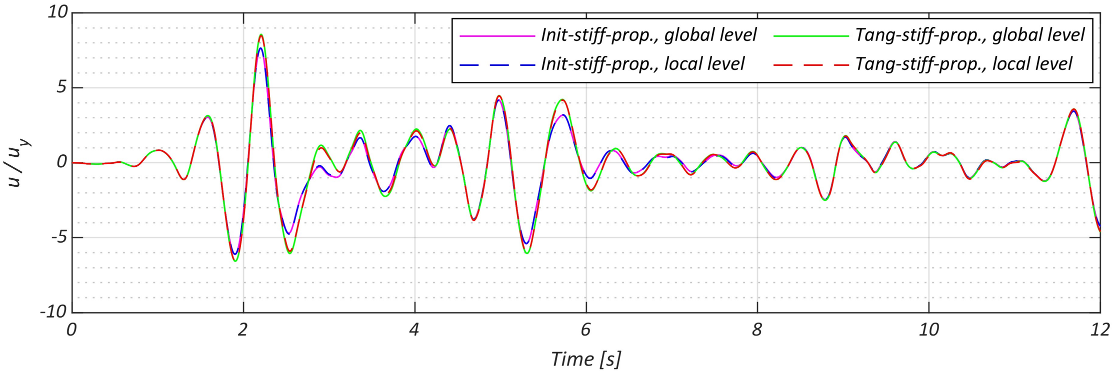

Fig. 5 – Ductility demand for local and global, initial and tangent, stiffness-proportional damping models

A local damping model consistent at a global level with the tangent stiffness-proportional damping model can specified for the bridge system as per the previous case but reducing the damping coefficient of the two dampers acting in parallel with the rigid plastic spring at the base by a factor equal to the local bilinear factor of the piers, $c_p=rc_e$. For elastoplastic ($r=0$) piers, this is equivalent to removing the rotational dampers coupled with the rigid-plastic springs. The displacement response for local and global, initial and tangent, stiffness-proportional viscous damping models is presented in Fig. 5 and show the consistency between local and global damping formulation results.

4 Distribution of damping forces for harmonic and earthquake excitations

To have a first insight into the local distribution of the damping forces for the bridge structure with both initial and tangent stiffness-proportional viscous damping models, the steady-state response of the second configuration shown in Fig. 4 is subject to a generic harmonic excitation.

When an initial stiffness-proportional viscous damping model is adopted, the damping forces generated at each of the bridge top nodes are harmonic, because the common variation of the translational velocity of the top nodes is harmonic and the damping coefficient, c_0, constant. The common hysteretic behavior of these damping forces generated is shown on top of Fig. 6, first row. Due to the symmetric properties of the bridge system, these damping forces are equally distributed between the four piers, for levels of displacement lower than the yielding displacement, and are equally distributed between the two elastic central piers, for levels of displacement larger than yield displacement, when the two external elastoplastic piers yield. The hysteretic response of the damping and restoring forces distributed between the external and central piers is shown at the bottom of Fig. 6. Note that for the initial-stiffness-proportional viscous damping model, the damping forces generated in the plastic hinge of the short elastoplastic piers when the system yields migrate to the elastic central piers. This is due to the damping forces being distributed in relation to the elements stiffness. Also, note the hysteretic response of the damping forces for the elastic central piers after the external short piers have yielded is not elliptical as is typical for a system responding linear elastically.

When a tangent stiffness-proportional viscous damping model is adopted, the damping forces generated at the top four nodes are proportional to the common harmonic velocity of the top four nodes and to the instantaneous value of the damping coefficient, which changes with changes in the tangent stiffness. As a result, the damping forces generated possess the hysteretic response shown on top of Fig. 6, second row. For symmetry, these damping forces are equally distributed between the four piers for levels of displacement lower than the yielding displacement, and are equally distributed between the two elastic central piers for larger levels of displacement when the two external elastoplastic piers are in their yielding phases.

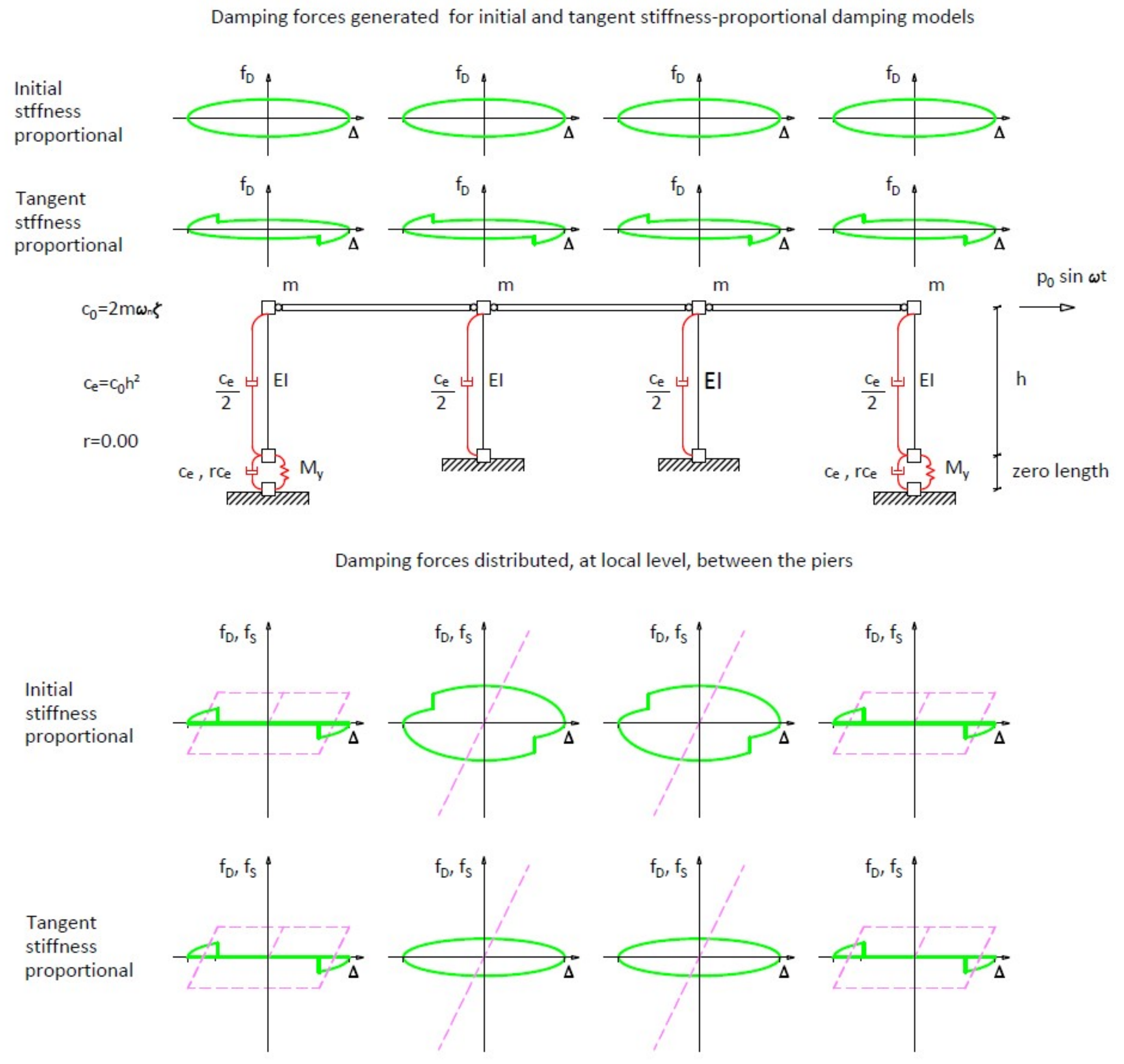

Fig. 6 – Damping and restoring forces in the steady-state response to harmonic excitations

The local hysteretic response of the damping forces for the central piers do not present the same discontinuity in the damping forces observed for initial-stiffness-proportional viscous damping. This is due to the reduction of the damping forces generated at a global level in the tangent model when the stiffness reduces. In terms of local viscous damping formulation, this can be associated with the absence of viscous dampers coupled with the plastic hinges of the short piers. The damping forces when the system responds elastically are now compatible with the hysteretic response of a linear elastic system with elastic stiffness equal to the tangent stiffness of the bilinear system.

The damping force distribution between the different components of the bridge system is further investigated for the North-South component of the El Centro earthquake record and the results, in terms of damping forces acting on the central elastic piers and to the external elastoplastic piers, are shown in Fig.7 and Fig.8, respectively.

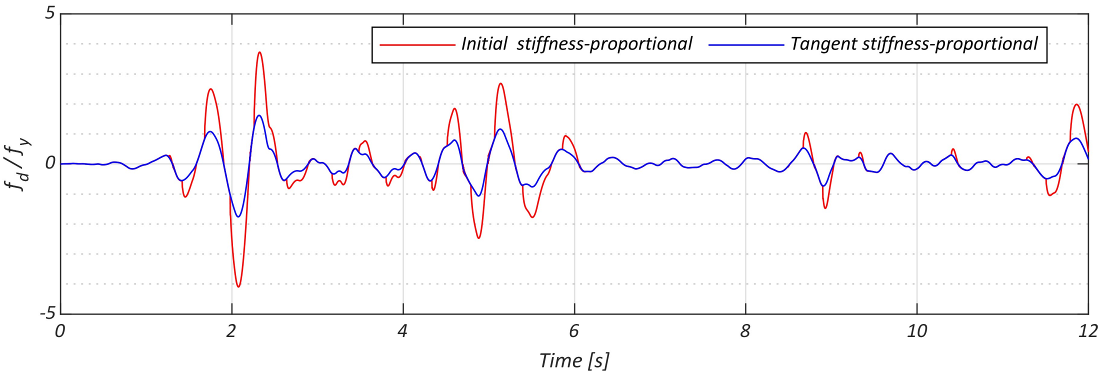

Fig. 7 – Adimensional damping forces in the central elastic piers for initial and tangent stiffness-proportional viscous damping models

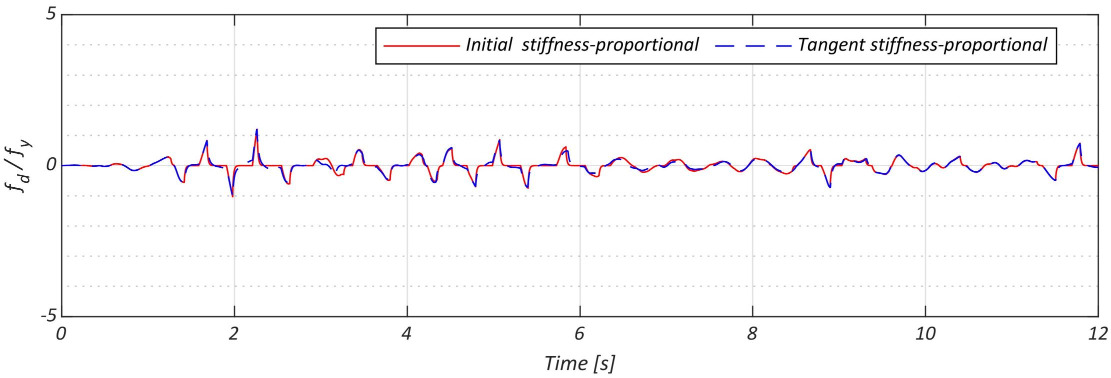

Fig. 8 – Adimensional damping forces in the external elastoplastic piers for initial and tangent stiffness-proportional viscous damping model

A comparison between the displacement responses for initial and tangent stiffness-proportional viscous damping shown in Fig. 4, and the damping forces in the external elastoplastic and central elastic piers, shown in Fig. 7 and Fig. 8, highlights that the difference between the observed displacement responses for initial and tangent stiffness proportional damping models is due to the difference in the damping forces in the elastic central piers. The concentration of the damping forces observed in the central elastic piers, after yielding, for the initial-stiffness-proportional viscous damping model is questionable. It is noted that the localized variation of the damping force in the central piers could be still larger, being a function of the elastic properties of the external and central piers, and increasing for stiffer properties of the external columns and softer properties of the central columns. For example, assuming the two central piers of the bridge in Fig. 6 have negligible translational stiffness, the overall bridge behavior would be elastoplastic (r=0) and the damping forces in the central piers after yielding would lead to even more physically unacceptable variations in the local damping forces. Because the unrealistically high damping forces (approximately 4 times the yielding strength) in the central elastic piers are the sole reasons for changes in the global response in the post-yield phase, this concentrated variation of the damping forces highly influences the bridge response. This suggests that a tangent stiffness-proportional damping model is a more rational damping model to be implemented in nonlinear dynamic analysis.

5. Summary and Recommendations

Reliable predictions of seismic performance requires accurate characterization of the energy dissipation mechanisms involved in the nonlinear dynamic response of structures. This, in turn, requires the main alternative sources of energy dissipation to be independently quantified and specified. Three main, alternative and independent sources of energy dissipation have been identified in this paper which are associated with inherent material characteristics of the structural system, nonstructural components, and foundation damping.

There is still no convergence of opinions on the most suitable damping model and, as observed by Jehel and Leger in [21], some researchers advocate the use of initial stiffness-based damping models while others recommend tangent stiffness-based models. As in [1, 4, 5, 6, 7, 8, 11, 20, 22], this paper argues that the energy dissipation mechanism associated with the inherent material characteristics of a structural system (which is herein assumed alternative to the damping mechanisms associated with nonstructural components and foundation damping) is more accurately modelled by a tangent stiffness-proportional viscous damping model. This approach avoids unacceptable local concentrations of damping forces in the elastic-to-plastic transition regions of the motion (see Fig. 7-8), which influence the quality of the response at a local and at a global level. However, a single purely viscous damping mechanism is not an accurate way of modelling the inherent energy dissipation mechanisms of a complete structural system, and other damping mechanisms, such as material and/or frictional, should be integrated for more accurate characterization of the inelastic dynamic response.

An integrated local damping approach where alternative damping mechanisms can be independently specified is proposed in this paper to permit more accurate representation of the nonlinear dynamic response of structures. In [4], Charney claimed that the best strategy for modeling damping is for viscous damping to be eliminated altogether, and more real damping mechanisms specified by local frictional and hysteretic [14] devices. While the authors support this notion, it is also observed that a viscous damping mechanism, integrated with other material and/or frictional mechanisms, may be appropriate for more accurate characterizations of the non-modelled energy dissipation mechanisms. When a viscous dissipative mechanism is judged desirable it should be specified at a local level via viscous damping elements, rather than at a global level by a damping matrix, to allow analysts to identify and control the relevance and effects of damping.

The integrated local damping approach proposed herein provides a general framework to move from the characterization of additional energy dissipation mechanisms as purely viscous towards an approach that permits a combination of different dissipative mechanisms.

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