17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 13th to 18th 2020

A CLOSED-FORM PROBABILISTIC SEISMIC DEMAND MODEL FOR PREDICTING THE FLOOR SPECTRAL PSEUDO-ACCELERATION

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Abstract

A closed-form probabilistic seismic demand model is proposed for predicting the conditional probability distribution of the elastic floor spectral response of linear structures in terms of pseudo-acceleration. The considered conditioning intensity measure is the ground spectral pseudo-acceleration calculated at the fundamental period of vibration of the structure. The model, which assumes the floor spectral response as a lognormal variable, consists of analytical expressions for the direct calculation of the log mean and standard deviation. The required inputs are the modal properties of the structure, as well as the period and the damping ratio of the floor spectral ordinate of interest. Knowledge of the conditional probability distribution of the spectral pseudo-acceleration of the ground motion is also needed. The model is a computationally more efficient alternative to commonly adopted seismic demand analysis procedures, such as stripe or cloud analysis, which require the selection of ground motion records and the response-history analysis of the structure. The proposal is validated through comparisons with exact predictions obtained numerically for a six-story reinforced concrete frame.

Keywords: nonstructural components; acceleration-sensitive; linear response; multi-degree-of-freedom structure

1. Introduction

Probabilistic methods employed in the seismic performance assessment of buildings usually make use of so-called probabilistic seismic demand models for expressing the conditional probability of demand exceedance given the seismic intensity. The latter, typically, is measured in terms of the ground spectral pseudo-acceleration calculated at the fundamental period of vibration of the building, while the parameter that describes the seismic demand is selected depending on the specific component under consideration. If the prediction of nonstructural damage is the target, in the case of acceleration-sensitive components the commonly used demand parameter is the floor spectral pseudo-acceleration. Usually, derivation of the demand model is time-consuming, because it involves a multiple-stripe or a cloud analysis that need the selection of ground motion records, consistent with the seismic hazard at the site, and the response-history analysis of the building structure. This paper presents a closed-form model that allows direct calculation of the elastic floor spectral pseudo-acceleration of a generic multi degree of freedom structure, using as conditional intensity measure the ground spectral pseudo-acceleration. The probability distribution of the floor pseudo-acceleration is assumed lognormal and it is described by the mean and standard deviation of the variable's natural logarithm. These two parameters are calculated through a set of analytic expressions from the modal properties of the structure, as a function of the period and the damping ratio of the floor spectral ordinate of interest. The conditional probability distribution of the ground spectral pseudo-accelerations obtained from the seismic hazard analysis is also required. The analytic expressions, which derive from an extension of a closed-form model proposed by the authors for predicting the floor spectral pseudo-acceleration of a single degree of freedom structure, assume that the response of the structure is linear and that the dynamic interaction between the structure and the component is negligible. In order to illustrate the implementation of the model, a reinforced concrete frame is analyzed, and the results are then used for the evaluation of the proposal. Because of its characteristics, the use of the model is especially suitable for functionality loss analyses in which the seismic behavior of numerous nonstructural components is of primary interest and where the structure is required to respond almost elastically to the earthquake excitation. With negligible computational effort the model can in fact be applied to predict the seismic demand of components with different periods of vibration and damping ratios that are attached to several floor levels of the structure.

2. Proposed model

Consider a building with a fundamental period of vibration T_1, subjected to a seismic excitation with an intensity level of $S_a(T_1) = s_a^1$, where S_a is the pseudo-acceleration response spectrum of the ground shaking. Let's denote with $S(T^*)$ the floor spectral pseudo-acceleration of the building calculated at period T^*. Assume that $S(T^*)$ is lognormally distributed and that is described by the following conditional probabilistic seismic demand model (PSDM) [1]:

$$\ln S^*|s_a^1 = \mu_{\ln S^*|s_a^1} + \sigma_{\ln S^*|s_a^1}\varepsilon_{\ln S^*|s_a^1} \quad (1)$$

in which the shorthand notation S^* is used for $S(T^*)$, and where $\mu_{\ln\ ^*|s_a^1}$ and $\sigma_{\ln S^*|s_a^1}$ denote the mean and standard deviation of the natural logarithm of S^* conditioned on s_a^1, being $\varepsilon_{\ln S^*|s_a^1}$ a standard normal random variable that describes the record-to-record variability of $\ln S^*$ given s_a^1.

For each mode of vibration of the building, a PSDM similar to (1) can be also defined but for the modal floor spectral response:

$$\ln S_i^*|s_a^1 = \mu_{\ln S_i^*|s_a^1} + \sigma_{\ln S_i^*|s_a^1}\varepsilon_{\ln S_i^*|s_a^1} \quad (2)$$

where S_i^* is the floor spectral pseudo-acceleration of a single degree of freedom system, representing the ith mode of vibration of the building (see Fig. 1), $\mu_{\ln S_i^*|s_a^1}$ and $\sigma_{\ln S_i^*|s_a^1}$ denote the conditional mean and standard deviation of the natural logarithm of S_i^*, respectively, and $\varepsilon_{\ln S_i^*|s_a^1}$, again, is a standard normal random variable.

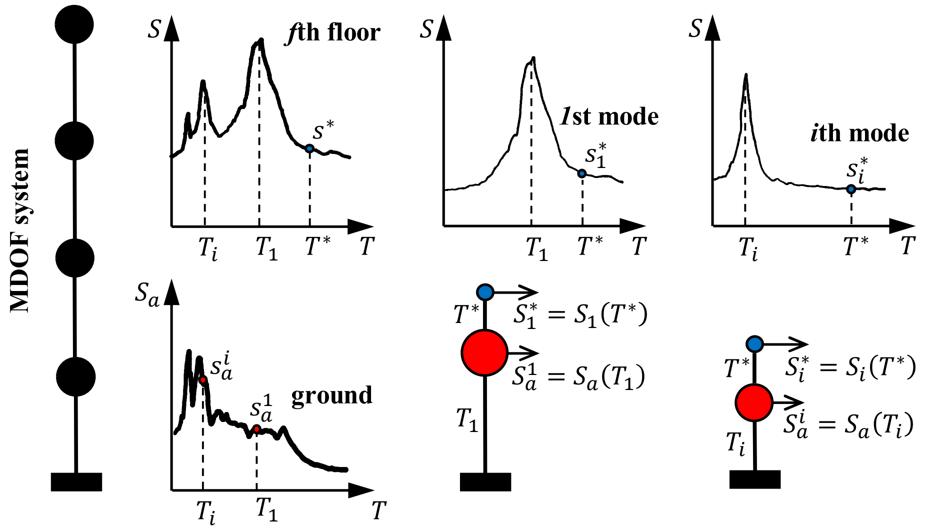

Fig. 1 – Modal floor response spectra of a multi degree of freedom system

The idea of the present work is to determine the parameters of the PSDM of S^* analytically, from the combination of those describing the conditional probability distribution of each modal contribution. The expressions to determine the latter will be derived first, and then the rule for combining such contributions will be illustrated.

2.1 Modal floor spectral response

Analytic expressions for $\mu_{\ln S_i^*|s_a^1}$ and $\sigma_{\ln\ {}_i^*|s_a^1}$ can be easily derived if the modal floor spectral response S_i^* is expressed as follows:

$$\ln S_i^* = \ln S_a^i + \ln S_i^*/S_a^i \quad (3)$$

as a linear combination, in the log-space, of the ground pseudo-acceleration of the ith mode of vibration of the structure S_a^i, and of the amplification factor S_i^*/S_a^i of the modal floor spectral response. Solution for the prediction of such two variables, in fact, has already been found.

The conditional distribution of S_a^i can be obtained from existing ground motion prediction equations (GMPE) for the spectral pseudo-acceleration, by means of the following expressions proposed by Baker and Cornell [2]:

$$\mu_{\ln S_a^i|s_a^1} = \mu_{\ln S_a^i}(M,R) + \rho_{\ \ln S_a^1,\ln S_a^i}\ \sigma_{\ln S_a^i}\left(\frac{\ln s_a^1 - \mu_{\ln S_a^1}}{\sigma_{\ln S_a^i}}\right) \quad (4)$$

$$\sigma_{\ln S_a^i|s_a^1} = \sigma_{\ln S_a^i} \cdot \sqrt{1-\rho^2_{\ln S_a^1,\ln S_a^i}} \quad (5)$$

$$\rho_{\ln S_a^1,\ln S_a^i} = 1 - 0.33 \cdot \ln(T_1/T_i) \quad 0.1s \le T_i, T_1 \le 4s \quad (6)$$

where M is the magnitude and R the distance of the earthquake scenario. Eq. (6), developed by Inoue and Cornell [3], can be also replaced by alternative empirical relationship proposed for the correlation $\rho_{\ln S_a^1,\ln S_a^i}$ [4]. In the case of multiple causal earthquakes, the cumulative effect of all possible earthquakes shall be accounted for by substituting Eq. (4) and (5) with:

$$\mu_{\ln S_a^i|s_a^1} = \sum_j p_j \mu_{\ln S_{a,j}^i|s_a^1} \quad (7)$$

$$\sigma_{\ln S_a^i|s_a^1} = \sqrt{\sum_j p_j \left[\sigma^2_{\ln S_a^i|s_a^1} + \left(\mu_{\ln S_{a,j}^i|s_a^1} - \mu_{\ln S_a^i|s_a^1}\right)^2\right]} \tag{8}$$

in which $\mu_{\ln S_{a,j}^i|s_a^1}$ and $\sigma_{\ln S_a^i|s_a^1}$ denote the conditional mean and standard deviation of $\ln S_a^i$ produced by the jth earthquake (i.e., by the jth (M, R) combination), and the weighting factor p_j is the probability of the single event as obtained from seismic hazard deaggregation [5].

The variable $\ln S_i^*/S_a^i$, which represents the log amplification factor of the response of an oscillator with period T^* with respect to that of a supporting single degree of freedom structure with period T_i, was already investigated by the authors in a study on uniform hazard floor response spectra [6], in which the following predictive equations are proposed for the parameters of the unconditional probability distribution:

$$\mu_{\ln S_i^*/S_a^i} = a = \begin{cases} a^t\, {r_i^*}^{n_1} & r_i^* \le 1 \\ a^t + n_2\left({r_i^*}^{n_3} - 1\right) & r_i^* > 1 \end{cases} \tag{9}$$

$$\sigma_{\ln S_i^*/S_a^i} = \sigma = \begin{cases} \sigma^t\,[1 - (1 - r_i^*)^{n_4}] & r_i^* \le 1 \\ \sigma^t + n_5(r_i^* - 1) & r_i^* > 1 \end{cases} \tag{10}$$

where r_i^* (simply denoted as r in [6]) is the period ratio T^*/T_i, and the coefficients a^t, σ^t, n_1, n_2, n_3, n_4 and n_5 depend on the oscillator's damping ratio ξ^* (denoted ξ in [6]) through third-order polynomials of $\ln(100\xi^*)$. It is important to note that when the considered modal contribution of interest is the one given by the fundamental mode of vibration of the structure, Eq. (9) and (10) can be also used to estimate $\mu_{\ln S_i^*/S_a^i|s_a^1}$ and $\sigma_{\ln S_i^*/S_a^i|s_a^1}$. Because of the homoscedasticity of the $\ln S_i^*/S_a^i$ model, which was demonstrated in [6], $\ln S_1^*/S_a^1$ can indeed be assumed as independent on S_a^1, and its conditional distribution considered the same as the unconditional one. In the case of the higher modes, on the contrary, the applicability of Eq. (9) and (10) for $\mu_{\ln S_i^*/S_a^i|s_a^1}$ and $\sigma_{\ln S_i^*/S_a^i|s_a^1}$ is not straightforward. The effect on S_a^i (and therefore on the amplification factor as well) of conditioning on s_a^1 cannot in fact be neglected in this case, especially when $\ln S_i^*/S_a^i$ is estimated at T^* values around T_1. For oscillators with periods close to or longer than T_1, besides, the value of r_i^* is usually quite high and the predictive accuracy of Eq. (9) and (10) reduces. This is because calibration of the equations were carried out on r_i^* values ranging only between 0 and 2, since in [6] an approximate solution was proposed for the uniform hazard floor spectral response at longer periods. Based on all such considerations, the following modified version of the closed-form model of [6] is proposed for the conditional distribution of $\ln S_i^*/S_a^i$, both for the fundamental and the higher modes of vibration of the structure:

$$\mu_{\ln S_i^*/S_a^i|s_a^1} = \begin{cases} a & r_i^* \le \bar{r}_i^* \\ \mu_{\ln S_a^*/S_a^i|s_a^1} & r_i^* > \bar{r}_i^* \end{cases} \tag{11}$$

$$\sigma_{\ln S_i^*/S_a^i|s_a^1} = \begin{cases} \sigma & r_i^* \le \bar{r}_i^* \\ \min\left(\sigma, \sigma_{\ln S_a^*/S_a^i|s_a^1}\right) & r_i^* > \bar{r}_i^* \end{cases} \tag{12}$$

in which $\bar{r}_i^* = \bar{T}^*/T_i$ denotes the period ratio of the oscillator where the value of a is equal to that of $\mu_{\ln S_a^*/S_a^i|s_a^1}$, namely, $\mu_{\ln S_i^*/S_a^i} = \mu_{\ln S_a^*/S_a^i|s_a^1}$, being the log-ratio of the ground spectral responses $\ln S_a^*/S_a^i\,|s_a^1$ given by [7]:

$$\mu_{\ln S_a^*/S_a^i|s_a^1} = \mu_{\ln S_a^*|s_a^1} - \mu_{\ln S_a^i|s_a^1} \tag{13}$$

$$\sigma_{\ln S_a^*/S_a^i|s_a^1} = \sqrt{\sigma^2_{\ln S_a^*|s_a^1} + \sigma^2_{\ln S_a^i|s_a^1} - 2\sigma_{\ln S_a^*|s_a^1}\sigma_{\ln S_a^i|s_a^1}\rho_{\ln S_a^*,\ln S_a^i|s_a^1}} \tag{14}$$

$$\rho_{\ln S_a^*,\ln S_a^i|s_a^1} = \frac{\rho_{\ln S_a^*,\ln S_a^i} - \rho_{\ln S_a^*,\ln S_a^1}\rho_{\ln S_a^i,\ln S_a^1}}{\sqrt{1 - \rho^2_{\ln S_a^*,\ln S_a^1}}\sqrt{1 - \rho^2_{\ln S_a^i,\ln S_a^1}}} \tag{15}$$

in terms of the conditional mean, standard deviation and correlation of the spectral responses $S_a^* = S_a(T^*)$ and S_a^i calculated according to Eq. (4)-(8). Eq. (11) and (12) derive from the observation that, with the increase of the oscillator's period T^* with respect to that of the structural mode T_i, because the filtering effect of the latter becomes negligible, the floor spectral response S_i^* converge to the ground spectral response S_a^*. As shown in the left and center panel of Fig. 2, reporting the results of the case study which will be discussed in the following section, $\mu_{\ln S_i^*/S_a^i|s_a^1}$ and $\sigma_{\ln S_i^*/S_a^i|s_a^1}$ converge as a consequence to $\mu_{\ln S_a^*/S_a^i|s_a^1}$ and $\sigma_{\ln S_a^*/S_a^i|s_a^1}$, respectively. The equations are also based on the approximation that, by assuming T_i to be sufficiently lower than T_1, the effect on the amplification factor of conditioning on s_a^1 can be considered as negligible for short-period oscillators.

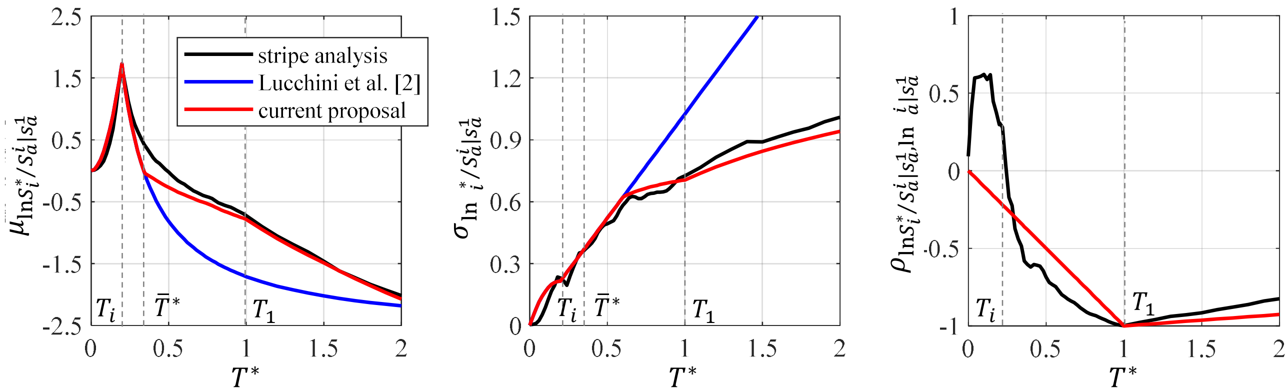

Fig. 2 – Prediction of the log amplification factor of the modal floor spectral response

In order to describe the joint probability distribution of $\ln S_a^i|s_a^1$ and $\ln S_i^*/S_a^i\,|s_a^1$, so as to be able to derive the parameters of the conditional PSDM of S_i^* from Eq. (3), the correlation between such two variables is also needed. This can be calculated by means of the following theoretical equation (shown in the right panel of Fig. 2):

$$\rho_{\ln S_i^*/S_a^i|s_a^1,\ln S_a^i|s_a^1} = \begin{cases} -T^*/T_1 & T^* \leq T_1 \\ -1 + 0.3\dfrac{T_i(T^* - T_1)}{T_1(T_1 - T_i)} & T_1 < T^* < T_1^2/T_i \\ -0.7 & T^* \geq T_1^2/T_i \end{cases} \quad (16)$$

that is a multilinear approximation of the correlation as a function of T^*, given T_1 and T_i, which is built on the consideration that $\rho_{\ln S_i^*/S_a^i|s_a^1,\ln S_a^i|s_a^1} = 0$ at $T^* = 0$ (being $\ln S_i^*/S_a^i$ constant and equal to 0, because $S_i^* = S_a^i$), and on the assumption that $\rho_{\ln S_i^*/S_a^i|s_a^1,\ln S_a^i|s_a^1}$ is equal to -1 and -0.7 at $T^* = T_1$and $T^* = T_1^2/T_i$, respectively. These last two assumptions on the values of the correlation come from the approximation that for $T^* \gg T_i$ the filtering effect of the structure can once again be considered as negligible and therefore $S_i^* \cong S_a^*$. Then, considering that:

$$\rho_{\ln S_i^*/S_a^i|s_a^1,\ln S_a^i|s_a^1} = \frac{\rho_{\ln S_i^*|s_a^1,\ln S_a^i|s_a^1}\sigma_{\ln S_i^*|s_a^1} - \sigma_{\ln S_a^i|s_a^1}}{\sqrt{\sigma^2_{\ln S_i^*|s_a^1} + \sigma^2_{\ln S_a^i|s_a^1} - 2\rho_{\ln S_i^*|s_a^1,\ln S_a^i|s_a^1}\sigma_{\ln S_i^*|s_a^1}\sigma_{\ln S_a^i|s_a^1}}} \quad (17)$$

because for $T^* \gg T_i$ $\rho_{\ln S_i^*|s_a^1,\ln S_a^i|s_a^1} \cong \rho_{\ln S_a^*|s_a^1,\ln S_a^i|s_a^1}$ and approximately equal to 0 it follows:

$$\rho_{\ln S_i^*/S_a^i|s_a^1,\ln S_a^i|s_a^1}(T^* \gg T_i) \cong -\frac{1}{\sqrt{\dfrac{\sigma^2_{\ln S_a^*|s_a^1}}{\sigma^2_{\ln S_a^i|s_a^1}} + 1}} \quad (18)$$

Eq. (18) shows that at $T^* = T_1$ the correlation is equal to -1, being $S_a^* = s_a^1$ and therefore $\sigma^2_{\ln S_a^*|s_a^1} = 0$. When $T^* = T_1^2/T_i$, because the log-distance of T^* and T_i from T_1 is the same, it is reasonable to assume that $\sigma^2_{\ln S_a^*|s_a^1}/\sigma^2_{\ln S_a^i|s_a^1} \cong 1$, and that the correlation is equal to $-1/\sqrt{2} = = -0.7$. Note that an empirical equation (whose development is beyond the scope of the present work) as those proposed in [4] but for the ground spectral responses, could be used for $\rho_{\ln S_i^*/S_a^i|s_a^1, \ln S_a^i|s_a^1}$ as an alternative to Eq. (18).

Once the joint probability distribution of $\ln S_a^i|s_a^1$ and $\ln S_i^*/S_a^i\,|s_a^1$ is fully characterized, $\mu_{\ln S_i^*|s_a^1}$ and $\sigma_{\ln\ {}_i^*|s_a^1}$ can be finally obtained as follows:

$$\mu_{\ln S_i^*|s_a^1} = \mu_{\ln S_a^i|s_a^1} + \mu_{\ln S_i^*/S_a^i|s_a^1} \tag{19}$$

$$\sigma_{\ln S_i^*|s_a^1} = \sqrt{\sigma^2_{\ln S_a^i|s_a^1} + \sigma^2_{\ln S_i^*/S_a^i|s_a^1} + 2\sigma_{\ln S_a^i|s_a^1}\sigma_{\ln S_i^*/S_a^i|s_a^1}\rho_{\ln S_i^*/S_a^i|s_a^1, \ln S_a^i|s_a^1}} \tag{20}$$

as direct functions, in conclusion, of the periods T_i of the modes of vibration of the structure, and of the period T^* and the damping ratio ξ^* of the oscillator representing the floor spectral response of interest.

2.3 Combination rule

In order to determine the parameters of the PSDM of $S^*|s_a^1$, from those of the modal floor spectral responses $S_i^*|s_a^1$, the following complete quadratic combination (CQC) rule applied to the non-logarithmic form of the floor spectral pseudo-acceleration is proposed:

$$\mu_{S^*|s_a^1} = \sqrt{\sum_i \sum_j \Gamma_i \phi_{i,f} \cdot \Gamma_j \phi_{j,f} \cdot \rho_{u_i^*, u_j^*} \cdot \mu_{S_i^*|s_a^1} \cdot \mu_{S_j^*|s_a^1}} \tag{21}$$

$$\sigma_{S^*|s_a^1} = \sqrt{\sum_i \sum_j \Gamma_i \phi_{i,f} \cdot \Gamma_j \phi_{j,f} \cdot \rho_{u_i^*, u_j^*} \cdot \sigma_{S_i^*|s_a^1} \cdot \sigma_{S_j^*|s_a^1}} \tag{22}$$

in which Γ_i is the participation factor and $\phi_{i,f}$ the component at floor f of the ith mode of vibration of the structure, and $\rho_{u_i^*, u_j^*}$ is the correlation between relative displacements u_i^* and u_j^* measured between the oscillator and the supporting floor respectively produced by the seismic excitation filtered only by the ith and the jth mode of the structure. Because of the assumed lognormal distribution for both $S^*|s_a^1$ and $S_i^*|s_a^1$, it follows that:

$$\mu_{S_i^*|s_a^1} = \exp\left(\mu_{\ln S_i^*|s_a^1} + \sigma^2_{\ln S_i^*|s_a^1}/2\right) \tag{23}$$

$$\sigma_{S_i^*|s_a^1} = \mu_{S_i^*|s_a^1}\sqrt{\exp\left(\sigma^2_{\ln S_i^*|s_a^1}\right) - 1} \tag{24}$$

and:

$$\mu_{\ln S^*|s_a^1} = \ln\left(\mu^2_{S^*|s_a^1}/\sqrt{\mu^2_{S^*|s_a^1} + \sigma^2_{S^*|s_a^1}}\right) \tag{25}$$

$$\sigma_{\ln S^*|s_a^1} = \sqrt{\ln\left(\sigma^2_{S^*|s_a^1}/\mu^2_{S^*|s_a^1} + 1\right)} \tag{26}$$

The combination rule for the mean given by Eq. (21) was demonstrated by Jiang et al. [8], but for the floor spectral response expressed in terms of the absolute acceleration $\ddot{u}^{t*}$. In [8] an analytic expression for the correlation $\rho_{\ddot{u}_i^{t*}, \ddot{u}_j^{t*}}$ is also proposed, which is a function of angular frequency and damping ratio of both the oscillator and the modes of vibration of the structure. Considering that for oscillators with low damping levels the absolute acceleration is well approximated with the pseudo-acceleration, it is reasonable to assume that Eq. (21) is also valid for the latter.

The use of the CQC rule for the standard deviation is justified on the other hand by the simplifying assumption that the same coefficient of variation CV characterizes $S^*|s_a^1$ and $S_i^*|s_a^1$. Note, in fact, that Eq. (21) can be rewritten as follows:

$$\frac{\sigma_{S^*|s_a^1}}{\mathrm{CV}_{S^*|s_a^1}} = \sqrt{\sum_i \sum_j \Gamma_i \phi_{i,f} \cdot \Gamma_j \phi_{j,f} \cdot \rho_{u_i^*,u_j^*} \cdot \frac{\sigma_{S_i^*|s_a^1}}{\mathrm{CV}_{S_i^*|s_a^1}} \cdot \frac{\sigma_{S_j^*|s_a^1}}{\mathrm{CV}_{S_j^*|s_a^1}}} \tag{27}$$

which gives:

$$\sigma_{S^*|s_a^1} = \sqrt{\sum_i \sum_j \Gamma_i \phi_{i,f} \cdot \Gamma_j \phi_{j,f} \cdot \rho_{u_i^*,u_j^*} \cdot \frac{\sigma_{S_i^*|s_a^1}}{\mathrm{CV}_{S_i^*|s_a^1}} \cdot \frac{\sigma_{S_j^*|s_a^1}}{\mathrm{CV}_{S_j^*|s_a^1}} \cdot \mathrm{CV}^2_{S^*|s_a^1}} \tag{28}$$

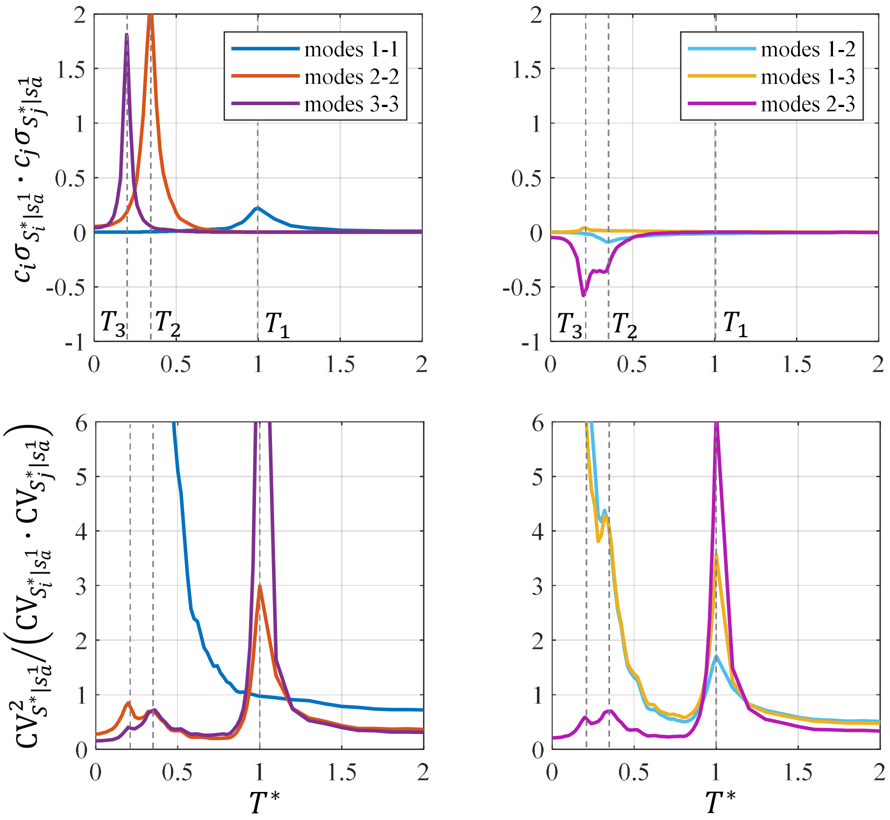

Fig. 3 – Evaluation of the combination rule for the standard deviation of $S^*|s_a^1$ as given by Eq. (28): modal contributions $\sigma_{S_i^*|s_a^1}$ weighted by the $c_i = \Gamma_i \phi_{i,f}$ factors, and ratios of the coefficients of variations CV

Proof that this assumption can be considered as acceptable is given by the case study results reported in Fig. 3, which show the $\mathrm{CV}^2_{S^*|s_a^1}/\left(\mathrm{CV}_{S_i^*|s_a^1} \cdot \mathrm{CV}_{S_j^*|s_a^1}\right)$ ratios of Eq. (28) for the different modal contributions $\Gamma_i \phi_{i,f} \sigma_{S_i^*|s_a^1}$. By observing the first row plots of this figure, it can be noted, as expected, that the most important contributions of the combination are: near T_1 and for longer periods, that given by the fundamental mode of vibration of the structure, denoted as "modes 1-1" in the plot; at lower periods, instead, those from the higher modes of vibration and from the corresponding cross-term "2-3". The values of the CV ratios of all these terms is always close to 1, with a significant difference observed only for the higher modes in the very low period range.

It is worth noting that in the case of structures with non-closely spaced modes, correlation $\rho_{u_i^*,u_j^*}$ tends to 0 and therefore the square root of sum of squares (SRSS) rule can be applied as a simpler alternative to the CQC rule.

3. Validation

The proposed closed-form PSDM is evaluated in this section by analyzing a six-story three-bay reinforced concrete frame already used in [6] as a case study. The structure is characterized by three significant modes of vibration, with periods approximately equal to 1s, 0.35s and 0.2s, respectively. A 5% nominal damping ratio is assigned to each mode. The structure is assumed to be located in Milan on a soil characterized by a shear wave velocity V_{S30} equal to 800 m/s. Validation is carried out with reference to two values of the return period T_R, equal to 250 and 500 years, respectively, for which two suites of hazard-consistent ground motions of 100 records each are obtained. The conditioning intensity measure used for the selection is the spectral pseudo-acceleration of the arbitrary horizontal component of the ground motion, calculated at the fundamental period of the structure T_1 = 1s. Records are selected by means of the conditional spectrum method [5], from the Italian Accelerometric Archive ITACA [9] integrated by the NGA-West2 database [10]. The equation of Boore and Atkinson [11] is used to predict the geometric mean horizontal component of the spectral pseudo-acceleration, and the arbitrary component then obtained in accordance with Beyer and Bommer [12]. The correlation between the arbitrary component spectral ordinates is estimated using the empirical equation of Baker and Jayaram [4].

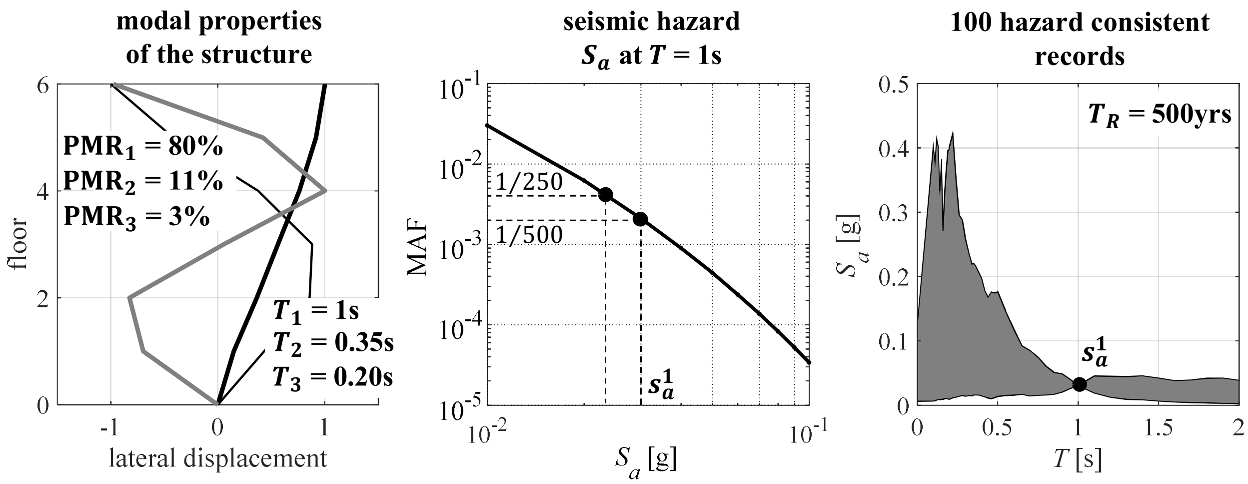

Fig. 4 – Case study structure: modal properties (left), seismic hazard at the site (center), and response spectra of hazard consistent ground motion records (right)

Fig. 5 shows the parameters of the conditional PSDM of the floor spectral response at different floor levels of the structure under the seismic excitation corresponding to a mean return period T_R = 500 years. The results obtained in the case of T_R = 250 years, which are not reported here for the sake of brevity, present similar trends in the match between the numerical and the analytical prediction. These two are determined as follows. The numerical prediction is derived from the S_k^* values of the floor spectral responses obtained by exciting the structure with the set of ground motion records, by simply estimating the (sample) mean and standard deviation of $\ln S^*|s_a^1$ as follows:

$$\hat{\mu}_{\ln S^*|s_a^1} = \sum_{k=1}^{N} \ln S_k^*/N \tag{29}$$

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$$\hat{\sigma}_{\ln S^*|s_a^1} = \sqrt{\sum_{k=1}^{N} \left(\ln S_k^* - \hat{\mu}_{\ln S^*|s_a^1}\right)/(N-1)} \tag{30}$$

where the number of values (i.e., of records) N is equal to 100. The analytical prediction is derived from the parameters of the PSDM of the modal floor spectral responses S_i^* calculated through Eq. (19)-(20) and (9)-(16), in which the ground spectral response is obtained from Eq. (4)-(5) and (7)-(8) using the models [11]-[12] and [4] in order to be consistent with the seismic hazard analysis. Modal contributions are then combined by means of the SRSS rule.

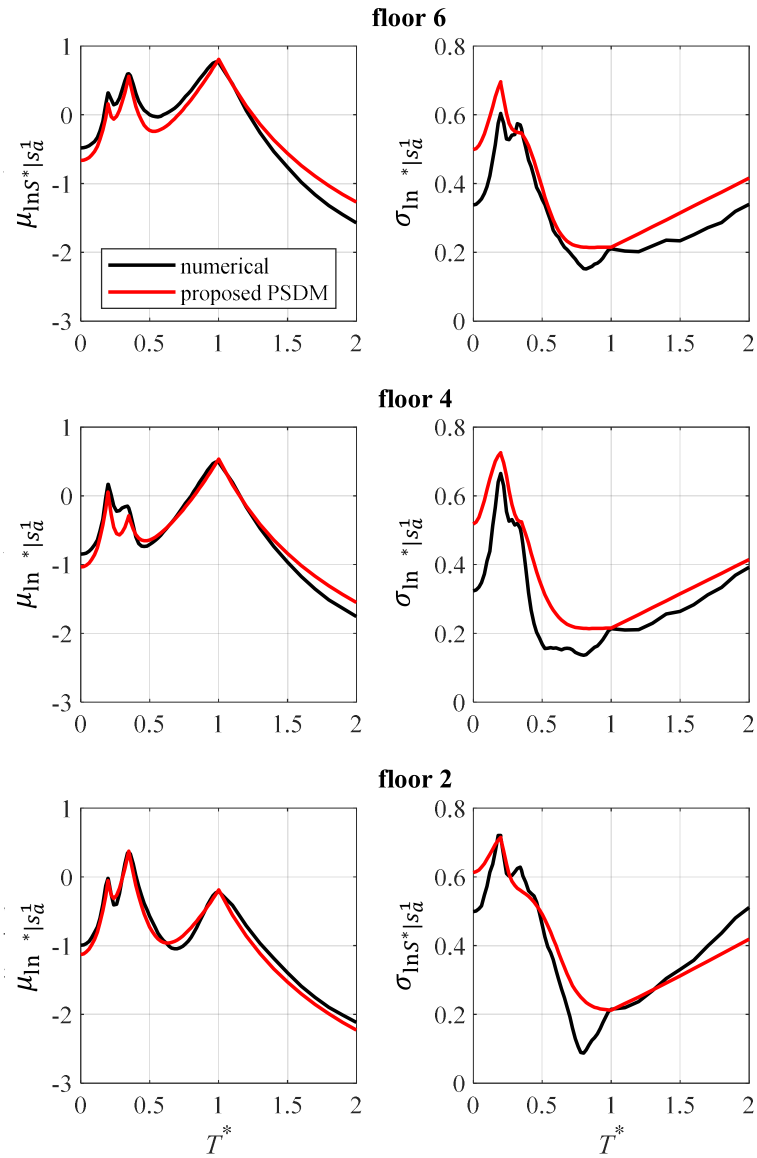

Fig. 5 – Parameters of the conditional PSDM of the floor spectral response obtained for the case study structure under a seismic excitation corresponding to a mean return period T_R = 500 years

As shown by the results reported in the figure, the proposed analytical PSDM matches well the exact parameters from the numerical simulation. The larger errors can be observed in the prediction of the standard deviation within the very low period range, produced by the approximation on the CV ratios assumed in Eq. (28) which has already been discussed in the previous section.

4. Conclusions

A closed-form probabilistic seismic demand model for the floor spectral response in terms of pseudo-acceleration has been proposed and then validated through comparisons with exact predictions obtained numerically. The main advantage of using the proposed model, with respect to commonly adopted stripe or cloud analyses, is that the selection of ground motion records and the response-history analysis of the structure is not required. Because of its analytical form, the model can be considered as a useful tool for design purposes, allowing, for example, to quickly evaluate the effect on floor spectra, and therefore on nonstructural response, of the variation of the structural properties. Besides, the epistemic uncertainty in these latter, which has already been found to strongly affect floor spectra [13] and which may be significant in the assessment of existing structures, can be easily evaluated.

Future work will be focused on extending to the case of multi degree of freedom structures the model proposed by the authors for the correlation of floor spectral responses of single degree of freedom systems [14]. The final goal will be to develop a model that fully describes the joint probability distribution of the floor spectral response, which can be used to directly determine the seismic demand of nonstructural components, within seismic risk analyses, or to generate target spectra for the simulation, or selection, of floor acceleration histories to use as input in time-history analysis of (possibly nonlinear) components.

5. Acknowledgements

The financial support of the Sapienza University of Rome (Research Grant Program 2017, project RG11715C782BD382 "Assessing the seismic risk of functionality loss in hospitals caused by non-structural damage") is gratefully acknowledged.

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